#419887

Topic: Matrices

In the matrix, write :

$$A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$$

(i) The order of the matrix

(ii) The number of elements

(iii) Write the elements $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$

Solution

(i) In the given matrix, the number of rows is 3 and the number of columns is 4.

Therefore, the order of the matrix is 3×4.

(ii) Since the order of the matrix is 3×4, there are $3 \times 4 = 12$ elements in it.

(iii) $a_{13} = 19, a_{21} = 35, a_{33} = -5, a_{24} = 12, a_{23} = \dfrac{5}{2}$

#419888

Topic: Matrices

If a matrix has 24 elements, what are the possible order it can have? What, if it has 13 elements?

Solution

We know that if a matrix is of the order $m \times n$, it has mn elements.

Thus to find all the possible orders of a matrix having 24 elements, we have to find all the ordered pairs of natural numbers whose product is 24.

The ordered pairs are$(1, 24), (24, 1), (2, 12), (12, 2), (3, 8), (8, 3), (4, 6)$ and $(6, 4)$

Hence, the possible orders of a matrix having 24 elements are:

$1 \times 24, 24 \times 1, 2 \times 12, 12 \times 2, 3 \times 8, 8 \times 3, 4 \times 6, 6 \times 4$

$(1, 13)$ and $(13, 1)$ are the ordered pairs of natural numbers whose product is 13.

Hence, the possible orders of a matrix having 13 elements are 1×13 and 13×1

#419889

Topic: Matrices

If a matrix has 18 elements , what are the possible orders it can have? what, if it has 5 elements?

Solution

We know that if a matrix is of the order $m \times n$, it has mn elements.

Thus, to find all the possible orders of a matrix having 18 elements, we have to find all the ordered pairs of natural numbers whose product is 18.

The ordered pairs are : $(1, 18), (18, 1), (2, 9), (9, 2), (3, 6)and(6, 3)$

Hence the possible orders of a matrix having 18 elements are:

$1 \times 18, 18 \times 1, 2 \times 9, 9 \times 2, 3 \times 6 \quad$ and $\quad 6 \times 3$

$(1, 5)$ and $(5, 1)$ are the only ordered pairs of natural numbers whose product is 5.

#419890

Topic: Matrices

Construct a 3×4 matrix, whose elements are given by

(i) $a_{ij} = \dfrac{1}{2}|-3i + j|$

(ii) $a_{ij} = 2i - j$

Solution

In general a 3×4 matrix is given by,

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

(i)

$a_{ij} = \frac{1}{2}|-3i+j|, i = 1, 2, 3 \quad and \quad j = 1, 2, 3, 4$

$\therefore a_{11} = \frac{1}{2}|-3 \times 1 + 1| = \frac{1}{2}|-3+1| = \frac{1}{2}|-2| = \frac{2}{2} = 1$

$a_{21} = \frac{1}{2}|-3 \times 2 + 1| = \frac{1}{2}|-6+1| = \frac{1}{2}|-5| = \frac{5}{2}$

$a_{31} = \frac{1}{2}|-3 \times 3 + 1| = \frac{1}{2}|-9+1| = \frac{1}{2}|-8| = \frac{8}{2} = 4$

$a_{12} = \frac{1}{2}|-3 \times 1 + 2| = \frac{1}{2}|-3+2| = \frac{1}{2}|-1| = \frac{1}{2}$

$a_{22} = \frac{1}{2}|-3 \times 2 + 2| = \frac{1}{2}|-6+2| = \frac{1}{2}|-4| = \frac{4}{2} = 2$

$a_{32} = \frac{1}{2}|-3 \times 3 + 2| = \frac{1}{2}|-9+2| = \frac{7}{2}$

$a_{23} = \frac{1}{2}|-3 \times 2 + 3| = \frac{1}{2}|-6+3| = \frac{1}{2}|-3| = \frac{3}{2}$

$a_{33} = \frac{1}{2}|-3 \times 3 + 3| = \frac{1}{2}|-9+3| = \frac{1}{2}|-6| = \frac{6}{2} = 3$

$a_{14} = \frac{1}{2}|-3 \times 1 + 4| = \frac{1}{2}|-3+4| = \frac{1}{2}|1| = \frac{1}{2}$

$a_{24} = \frac{1}{2}|-3 \times 2 + 4| = \frac{1}{2}|-6+4| = \frac{1}{2}|-2| = \frac{2}{2} = 1$

$a_{34} = \frac{1}{2}|-3 \times 3 + 4| = \frac{1}{2}|-9+4| = \frac{1}{2}|-5| = \frac{5}{2}$

Therefore, the required matrix is

$$A = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{3}{2} & 3 & \frac{5}{2} \end{bmatrix}$$

(ii)

$a_{ij} = 2i - j, i = 1, 2, 3 \quad and \quad j = 1, 2, 3, 4$

$\therefore a_{11} = 2 \times 1 - 1 = 2 - 1 = 1$

$a_{21} = 2 \times 2 - 1 = 4 - 1 = 3$

$a_{31} = 2 \times 3 - 1 = 6 - 1 = 5$

$a_{12} = 2 \times 1 - 2 = 2 - 2 = 0$

$a_{22} = 2 \times 2 - 2 = 4 - 2 = 2$

$a_{32} = 2 \times 3 - 2 = 6 - 2 = 4$

$a_{13} = 2 \times 1 - 3 = 2 - 3 = -1$

$a_{23} = 2 \times 2 - 3 = 4 - 3 = 1$

$a_{33} = 2 \times 3 - 3 = 6 - 3 = 3$

$a_{14} = 2 \times 1 - 4 = 2 - 4 = -2$

$a_{24} = 2 \times 2 - 4 = 4 - 4 = 0$

$a_{34} = 2 \times 3 - 4 = 6 - 4 = 2$

Therefore, the required matrix is

$$A = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$$

#419905

Topic: Matrices

The number of all possible matrices of order 3×3 with each entry 0 or 1 is:

A 27

B 18

C 81

$\boxed{\text{D}}$ 512

Solution

The given matrix of the order 3×3 has 9 elements and each of these elements can be either 0 or 1.

Now, each of the 9 elements can be filled in two possible ways.

Therefore, by the multiplication principle, the required number of possible matrices is $2^9 = 512$

#419906

Topic: Operations on Matrices

Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

Find each of the following

(i) $A + B$

(ii) $A - B$

(iii) $3A - C$

(iv) AB

(v) BA

Solution

(i) $A + B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2+1 & 4+3 \\ 3-2 & 2+5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$

(ii) $A - B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2-1 & 4-3 \\ 3-(-2) & 2-5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 5 & -3 \end{bmatrix}$

(iii) $3A - C = 3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

$= \begin{bmatrix} 3\times 2 & 3\times 4 \\ 3\times 3 & 3\times 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

$= \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

$= \begin{bmatrix} 6+2 & 12-5 \\ 9-3 & 6-4 \end{bmatrix}$

$= \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$

(iv) Matrix A has 2 columns. This number is equal to the number of rows in matrix B. Therefore, AB is defined as:

$AB = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2(1)+4(-2) & 2(3)+4(5) \\ 3(1)+2(-2) & 3(3)+2(5) \end{bmatrix}$

$= \begin{bmatrix} 2-8 & 6+20 \\ 3-4 & 9+10 \end{bmatrix}\begin{bmatrix} -6 & 26 \\ -1 & 19 \end{bmatrix}$

(v) Matrix B has 2 columns, This number is equal to the number of rows in matrix A. Therefore, BA is defined as :

$BA = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1(2)+3(3) & 1(4)3(2) \\ -2(2)+5(3) & -2(4)+5(2) \end{bmatrix}$

$= \begin{bmatrix} 2+9 & 4+6 \\ -4+15 & -8+10 \end{bmatrix} = \begin{bmatrix} 11 & 10 \\ 11 & 2 \end{bmatrix}$

#419907

Topic: Operations on Matrices

Compute the following:

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

(ii) $\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$

(iii) $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$

(iv) $\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$

Solution

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a+a & b+b \\ -b+b & a+a \end{bmatrix} = \begin{bmatrix} 2a & 2b \\ 0 & 2a \end{bmatrix}$

(ii) $\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$

$= \begin{bmatrix} a^2+b^2+2ab & b^2+c^2+2bc \\ a^2+c^2-2ac & a^2+b^2-2ab \end{bmatrix}$

$= \begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$

(iii) $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$

$= \begin{bmatrix} -1+12 & 4+7 & -6+6 \\ 8+8 & 5+0 & 16+5 \\ 2+3 & 8+2 & 5+4 \end{bmatrix}$

$= \begin{bmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}$

(iv) $\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad \left(\because \sin^2 x + \cos^2 x = 1 \right)$

#419909

Topic: Operations on Matrices

If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$, then compute $(A+B)$ and $(B-C)$. Also, verify that $A+(B-C)=(A+B)-C$

Solution

$$A + B = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3 & 2-1 & -3+2 \\ 5+4 & 0+2 & 2+5 \\ 1+2 & -1+0 & 1+3 \end{bmatrix} = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix}$$

$$B - C = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3-4 & -1-1 & 2-2 \\ 4-0 & 2-3 & 5-2 \\ 2-1 & 0-(-2) & 3-3 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$$

$$A + (B - C) = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+(-1) & 2+(-2) & -3+0 \\ 5+4 & 0+(-1) & 2+3 \\ 1+1 & -1+2 & 1+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

$$(A + B) - C = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4-4 & 1-1 & -1-2 \\ 9-0 & 2-3 & 7-2 \\ 3-1 & -1-(-2) & 4-3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

Hence, we have verified that $A + (B - C) = (A + B) - C$.

#419910

Topic: Properties of Matrices

If $A = \begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix}$ and $B = \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$, then compute $3A - 5B$.

Solution

$3A - 5B = 3\begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix} - 5\begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$

$= \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

#419911

Topic: Operations on Matrices

Simplify: $\cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

Solution

$\cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

$= \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{bmatrix} + \begin{bmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$

$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \sin\theta\cos\theta \\ -\sin\theta\cos\theta + \sin\theta\cos\theta & \cos^2\theta + \sin^2\theta \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

#419912

Topic: Operations on Matrices

Find X and Y, if

(i) $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

(ii) $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

Solution

(i) $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \ldots\ldots (1)$

$X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \ldots\ldots (2)$

Adding equations (1) and (2), we get:

$2X = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 7+3 & 0+0 \\ 2+0 & 5+3 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$

$\therefore X = \frac{1}{2}\begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$

Now, $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$

$\Rightarrow Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$

$\Rightarrow Y = \begin{bmatrix} 7-5 & 0-0 \\ 2-1 & 5-4 \end{bmatrix}$

$\therefore Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

(ii) $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \ldots . (3)$

$3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} \ldots . (4)$

Multiplying equation (3) with (4), we get :

$2(2X + 3Y) = 2\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$

$\Rightarrow 4X + 6Y = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix} \ldots ..(5)$

Multiplying equation (4) with (3), we get :

$3(3X + 2Y) = 3\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

$\Rightarrow 9X + 6Y = \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix} \ldots ..(6)$

From (5) and (6), we have:

$(4X + 6Y) - (9X + 6Y) = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix} - \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix}$

$\Rightarrow -5X = \begin{bmatrix} 4-6 & 6-(-6) \\ 8-(-3) & 0-15 \end{bmatrix} = \begin{bmatrix} -2 & 12 \\ 11 & -15 \end{bmatrix}$

$\therefore X = -\frac{1}{5}\begin{bmatrix} -2 & 12 \\ 11 & -15 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}$

$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$

Now,

$\Rightarrow 2\begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix} + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \frac{4}{5} & -\frac{24}{5} \\ -\frac{22}{5} & 6 \end{bmatrix} + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \frac{4}{5} & -\frac{24}{5} \\ -\frac{22}{5} & 6 \end{bmatrix} + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$

$\Rightarrow 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} \frac{4}{5} & -\frac{24}{5} \\ -\frac{22}{5} & 6 \end{bmatrix}$

$$\Rightarrow 3Y = \begin{bmatrix} 2-\frac{4}{5} & 3+\frac{24}{5} \\ 4+\frac{22}{5} & 0-6 \end{bmatrix} = \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -2 \end{bmatrix}$$

$$\therefore Y = \frac{1}{3}\begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -2 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

#419913

Topic: Operations on Matrices

Find X, if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

Solution

$$2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow 2X + \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1-3 & 0-2 \\ -3-1 & 2-4 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$\therefore X = \frac{1}{2}\begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$$

#419914

Topic: Operations on Matrices

Find X and Y, if $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

Solution

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we have:

$2 + y = 5 \ \Rightarrow y = 3$

and $2x + 2 = 8 \ \Rightarrow x = 3$

$\therefore x = 3$ and $y = 3$

#419915

Topic: Operations on Matrices

Solve the equation for x, y, z and t if $2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

Solution

$2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x+3 & 2z-3 \\ 2y & 2t+6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$

Comparing the corresponding elements of these two matrices, we get:

$2x + 3 = 9 \Rightarrow 2x = 6 \Rightarrow x = 3$

$2y = 12 \Rightarrow y = 6$

$2z - 3 = 15 \Rightarrow 2z = 18 \Rightarrow z = 9$

$2t + 6 = 18 \Rightarrow 2t = 12t = 6$

$\therefore x = 3, y = 6, z = 9$ and $t = 6$

#419916

Topic: Operations on Matrices

If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

Find values of x and y.

Solution

$x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x - y \\ 3x + y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

Comparing the corresponding elements of these two matrices,

we get $2x - y = 10$ and $3x + y = 5$

Solving these we get,

$x = 3$ and $y = -4$

#419917

Topic: Operations on Matrices

Given : $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$

Find the values of x, y, z and w.

Solution

$$3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ -1+z+w & 2w+3 \end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we get:

$3x = x + 4$

$\Rightarrow 2x = 4$

$\Rightarrow x = 2$

$3y = 6 + x + y$

$\Rightarrow 2y = 6 + x = 6 + 2 = 8$

$\Rightarrow y = 4$

$3w = 2w + 3$

$\Rightarrow w = 3$

$3z = -1 + z + w$

$\Rightarrow 2z = -1 + w = -1 + 3 = 2$

$\Rightarrow z = 1$

$\therefore x = 2, y = 4, z = 1$ and $w = 3$

#419918

Topic: Operations on Matrices

If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $F(x)F(y) = F(x+y)$

Solution

$F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}, F(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$F(x+y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$F(x)F(y)$

$= \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} \cos x\cos y - \sin x\sin y + 0 & -\cos x\sin y - \sin x\cos y + 0 & 0 \\ \sin x\cos y + \cos x\sin y + 0 & -\sin x\sin y + \cos x\cos y + 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= F(x+y)$

$\therefore F(x)F(y) = F(x+y)$

#419919

Topic: Operations on Matrices

Show that

(*i*) $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$

(*ii*) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

Solution

(i) $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

$= \begin{bmatrix} 5(2) - 1(3) & 5(1) - 1(4) \\ 6(2) + 7(3) & 6(1) + 7(4) \end{bmatrix}$

$= \begin{bmatrix} 10 - 3 & 5 - 4 \\ 12 + 21 & 6 + 28 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$

$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$

$= \begin{bmatrix} 2(5) + 1(6) & 2(- 1) + 1(7) \\ 3(5) + 4(6) & 3(- 1) + 4(7) \end{bmatrix}$

$= \begin{bmatrix} 10 + 6 & -2 + 7 \\ 15 + 24 & -3 + 28 \end{bmatrix} = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$

$\therefore \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$

$= \begin{bmatrix} 1(- 1) + 2(0) + 3(2) & 1(1) + 2(- 1) + 3(3) & 1(0) + 2(1) + 3(4) \\ 0(- 1) + 1(0) + 0(2) & 0(1) + 1(- 1) + 0(3) & 0(0) + 1(1) + 0(4) \\ 1(- 1) + 1(0) + 0(2) & 1(1) + 1(- 1) + 0(3) & 1(0) + 1(1) + 0(4) \end{bmatrix}$

$= \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1(1) + 1(0) + 0(1) & -1(2) + 1(1) + 0(1) & -1(3) + 1(0) + 0(0) \\ 0(1) + (- 1)(0) + 1(1) & 0(2) + (- 1)(1) + 1(1) & 0(3) + (- 1)(0) + 1(0) \\ 2(1) + 3(0) + 4(1) & 2(2) + 3(1) + 4(1) & 2(3) + 3(0) + 4(0) \end{bmatrix}$

$= \begin{bmatrix} -1 & -1 & -3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$

$\therefore \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

#420560

Topic: Properties of Matrices

Find $A^2-5A+6I$, if $A=\begin{bmatrix}2&0&1\\2&1&3\\1&-1&0\end{bmatrix}$

Solution

We have $A^2=A\times A$

$$=\begin{bmatrix}2&0&1\\2&1&3\\1&-1&0\end{bmatrix}\begin{bmatrix}2&0&1\\2&1&3\\1&-1&0\end{bmatrix}$$

$$=\begin{bmatrix}2(2)+0(2)+1(1)&2(0)+0(1)+1(-1)&2(1)+0(3)+1(0)\\2(2)+1(2)+3(1)&2(0)+1(1)+3(-1)&2(1)+1(3)+3(0)\\1(2)+(-1)(1)+0(1)&1(0)+(-1)(1)+0(-1)&1(1)+(-1)(3)+0(0)\end{bmatrix}$$

$$=\begin{bmatrix}4+0+1&0+0+1&2+0+0\\4+2+3&0+1-3&2+3+0\\2-2+0&0-1+0&1-3+0\end{bmatrix}$$

$$=\begin{bmatrix}5&-1&2\\9&-2&5\\0&-1&-2\end{bmatrix}$$

$\therefore A^2-5A+6I$

$$=\begin{bmatrix}5&-1&2\\9&-2&5\\0&-1&-2\end{bmatrix}-5\begin{bmatrix}2&0&1\\2&1&3\\1&-1&0\end{bmatrix}+6\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}5&-1&2\\9&-2&5\\0&-1&-2\end{bmatrix}-\begin{bmatrix}10&0&5\\10&5&15\\5&-5&0\end{bmatrix}+\begin{bmatrix}6&0&0\\0&6&0\\0&0&6\end{bmatrix}$$

$$=\begin{bmatrix}5-10&-1-0&2-5\\9-10&-2-5&5-15\\0-5&-1+5&-2-0\end{bmatrix}+\begin{bmatrix}6&0&0\\0&6&0\\0&0&6\end{bmatrix}$$

$$=\begin{bmatrix}-5&-1&-3\\-1&-7&-10\\-5&4&-2\end{bmatrix}+\begin{bmatrix}6&0&0\\0&6&0\\0&0&6\end{bmatrix}$$

$$=\begin{bmatrix}-5+6&-1+0&-3+0\\-1+0&-7+6&-10+0\\-5+0&4+0&-2+6\end{bmatrix}$$

$$=\begin{bmatrix}1&-1&-3\\-1&-1&-10\\-5&4&4\end{bmatrix}$$

#420561

Topic: Properties of Matrices

If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$

Solution

$$A^2 = AA = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+4 & 0+0+0 & 2+0+6 \\ 0+0+2 & 0+4+0 & 0+2+3 \\ 2+0+6 & 0+0+0 & 4+0+9 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

Now, $A^3 = A^2 . A$

$$= \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5+0+16 & 0+0+8 & 10+0+24 \\ 2+0+10 & 0+8+0 & 4+4+15 \\ 8+0+26 & 0+0+0 & 16+0+39 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

$\therefore A^3 - 6A^2 + 7A + 2I$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 21+7+2 & 0+0+0 & 34+14+0 \\ 12+0+0 & 8+14+2 & 23+7+0 \\ 34+14+0 & 0+0+0 & 55+21+2 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix}$$

$$= \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

$\therefore A^3 - 6A^2 + 7A + 2I = 0$

#420562

Topic: Properties of Matrices

If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA - 2I$

Solution

$A^2 = A.A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

$= \begin{bmatrix} 3(3) + (-2)(4) & 3(-2) + (-2)(-2) \\ 4(3) + (-2)(4) & 4(-2) + (-2)(-2) \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$

Now, $A^2 = kA - 2I$

$A^2 = kA - 2I$

$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & 2k \\ 4k & -2k-2 \end{bmatrix}$

Comparing the corresponding element, we have:

$3k - 2 = 1$

$\Rightarrow 3k = 3$

$\Rightarrow k = 1$

#420563

Topic: Properties of Matrices

If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that $I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Solution

L.H.S.

$= I + A$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix} \ldots (1)$

and R.H.S.

$= (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}\right)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} \cos\alpha + \sin\alpha\tan\frac{\alpha}{2} & -\sin\alpha + \cos\alpha\tan\frac{\alpha}{2} \\ -\cos\alpha\tan\frac{\alpha}{2} + \sin\alpha & \sin\alpha\tan\frac{\alpha}{2} + \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} 1 - 2sin^2\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\tan\frac{\alpha}{2} & -2\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2} + \left(2cos^2\frac{\alpha}{2} - 1\right)\tan\frac{\alpha}{2} \\ -\left(2cos^2\frac{\alpha}{2} - 1\right)\tan\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\tan\frac{\alpha}{2} + 1 - 2sin^2\frac{\alpha}{2} \end{bmatrix}$

$= \begin{bmatrix} 1 - 2sin^2\frac{\alpha}{2} + 2sin^2\frac{\alpha}{2} & -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} - \tan\frac{\alpha}{2} \\ -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\tan\frac{\alpha}{2} + 2\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2} & 2sin^2\frac{\alpha}{2} + 1 - 2sin^2\frac{\alpha}{2} \end{bmatrix}$

$= \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$

Clearly L.H.S. $=$ R.H.S. Hence proved

#420565

Topic: Properties of Matrices

The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are Rs 80, Rs 60 and Rs 40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.

Solution

The bookshop has 10 dozen chemistry books, 8 dozen physics books, and 10 dozen economics books.

The selling prices of a chemistry book, a physics book and an economics book are respectively given as Rs 80, Rs 60 and Rs 40.

The total amount of money that will be received from the sale of all these books can be represented in the form of a matrix as:

$$12\begin{bmatrix} 10 & 8 & 10 \end{bmatrix}\begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}$$

$= 12[10 \times 80 + 8 \times 60 + 10 \times 40]$

$= 12(800 + 480 + 400)$

$= 12(1680)$

$= 20160$

Thus, the bookshop will receive Rs 20160 from the sale of all these books.

#420566

Topic: Matrices

Passage

Assume X, Y, Z, W and P are matrices of order $2 \times n, 3 \times k, 2 \times p, n \times 3$ and $p \times k$, respectively.

The restriction on n, k and p so that $PY + WY$ will be defined are:

A (boxed) $k = 3, p = n$

B k is arbitary, $p = 2$

C p is arbitary, $k = 3$

D $k = 2, p = 3$

Solution

In this, order of $P = p \times k$

Order of $W = n \times 3$

Order of $Y = 3 \times k$

Thus, order of $PY = p \times k$, when $k = 3$.

And the order of $WY = p \times k$, where $p = n$

Thus option (A) is correct.

#420567

Topic: Operations on Matrices

Passage

Assume X, Y, Z, W and P are matrices of order $2 \times n, 3 \times k, 2 \times p, n \times 3$ and $p \times k$, respectively.

If $n = p$, then the order of the matrix $7X - 5Z$ is:

A $p \times 2$

B (boxed) $2 \times n$

C $n \times 3$

D $p \times n$

Solution

In this, order of $X = 2 \times n$

and order of $Z = 2 \times p$

Therefore, $n = p$

Hence order of $7X - 5Z = 2 \times n$.

Thus option (B) is correct.

#420568

Topic: Transpose of a Matrix

Find the transpose of each of the following matrices:

$(i) \begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$

$(ii) \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$(iii) \begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

Solution

(i) Let $A = \begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$, then $A^T = \begin{bmatrix} 5 & \frac{1}{2} & -1 \end{bmatrix}$

(ii) Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$, then $A^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

(iii) Let $A = \begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$, then $A^T = \begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}$

#420569

Topic: Transpose of a Matrix

If $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, then verify that

$(i) (A + B)^T = A^T + B^T$

$(ii) (A - B)^T = A^T - B^T$

Solution

We have:

$$A^T = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} B^T = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$$

(i) $A + B = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$

$$\therefore (A + B) = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}$$

$$A^T + B^T = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$$

Hence, we have verified that $(A + B)^T = A^T + B^T$

(ii) $A - B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$

$$\therefore (A - B)^T = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$$

$$A^T - B^T = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$$

Hence, we have proved that $(A - B)^T = A^T - B^T$.

#420570

Topic: Transpose of a Matrix

If $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then verify that:

$(i) (A + B)' = A' + B'$

$(ii) (A - B)' = A' - B'$

Solution

(i) It is known that $A = \left(A'\right)'$

Therefore, we have:

$A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$

$B' = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

$A + B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

$\therefore (A + B)' = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$

$A' + B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$

Thus, we have verified that $(A + B)' = A' + B'$.

(ii) $A - B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix}$

$\therefore (A - B)' = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

$A' - B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

Thus, we have verified that $(A - B)' = A' - B'$

#420571

Topic: Transpose of a Matrix

If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find $(A + 2B)'$.

Solution

We know that $A = \left(A'\right)'$

$\therefore A + 2B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + 2\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$

$\therefore (A + 2B)' = \begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$

#420572

Topic: Transpose of a Matrix

For the matrices A and B, verify that $(AB)' = B'A'$ where

(i) $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$

Solution

(i) $AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}\begin{bmatrix} -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$

$\therefore \quad (AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$

Now, $A' = \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$, $B' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$

$\therefore B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & -4 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$

Hence, we have verified that $(AB)' = B'A'$

(ii) $AB = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}\begin{bmatrix} 1 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}$

$\therefore (AB)' = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$

Now, $A' = \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}$, $B' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$

$\therefore B'A' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$

Hence, we have proved that $(AB)' = B'A'$

#420573

Topic: Transpose of a Matrix

If (i) $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then verify that $A'A = I$

(ii) $A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$, then verify that $A'A = I$

Solution

(i) $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$\therefore A' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$A'A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} (\cos\alpha)(\cos\alpha) + (-\sin\alpha)(-\sin\alpha) & (\cos\alpha)(\sin\alpha) + (-\sin\alpha)(\cos\alpha) \\ (\sin\alpha)(\cos\alpha) + (\cos\alpha)(-\sin\alpha) & (\sin\alpha)(\sin\alpha) + (\cos\alpha)(\cos\alpha) \end{bmatrix}$

$= \begin{bmatrix} cos^2\alpha + sin^2\alpha & \sin\alpha\cos\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \sin\alpha\cos\alpha & sin^2\alpha + cos^2\alpha \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Hence, we have verified that $A'A = I$

(ii) $A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$

$\therefore A' = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix}$

$A'A = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix}\begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$

$\begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix}\begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$

$= \begin{bmatrix} (\sin\alpha)(\sin\alpha) + (-\cos\alpha)(-\cos\alpha) & (\sin\alpha)(\cos\alpha) - (-\cos\alpha)(\sin\alpha) \\ (\cos\alpha)(\cos\alpha) + (\sin\alpha)(-\cos\alpha) & (\cos\alpha)(\cos\alpha) + (\sin\alpha)(\sin\alpha) \end{bmatrix}$

$= \begin{bmatrix} sin^2\alpha + cos^2\alpha & \sin\alpha\cos\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \sin\alpha\cos\alpha & cos^2\alpha + sin^2\alpha \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Hence, we have verified that $A'A = I$

#420574

Topic: Symmetric and Skew Symmetric Matrix

(i) Show that the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is a symmetric matrix

(ii) Show that the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is a skew symmetric matrix.

Solution

(i) We have :

$$A' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} = A$$

$\therefore A' = A$

Hence, A is a Symmetric matrix.

(ii) We have:

$$A' = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = -A$$

$\therefore A' = -A$

Hence, A is a skew symmetric matrix.

#420575

Topic: Symmetric and Skew Symmetric Matrix

For the matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that

(i) $\left(A + A'\right)$ is a symmetric matrix

(ii) $\left(A - A'\right)$ is a skew symmetric matrix.

Solution

$$A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$$

(i) $A + A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$

$\therefore \left(A + A'\right)' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} = A + A'$

Hence, $\left(A + A'\right)$ is a symmetric matrix.

(ii) \displaystyle A-{ A }^{ \prime }=\begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}-\begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}=\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}

\displaystyle \therefore \left(A-{ A }^{ \prime } \right)' =\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}=-\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}=-\left(A-{ A }^{ \prime } \right)

Hence, \displaystyle \left(A-{ A }^{ \prime } \right) is a skew symmetric matrix.

#420576

Topic: Transpose of a Matrix

Find \displaystyle \frac { 1 }{ 2 } \left(A+{ A }^{ T } \right) and \displaystyle \frac { 1 }{ 2 } \left(A-{ A }^{ T } \right) , when \displaystyle A=\left[\begin{matrix} 0 \\ -a \\ -b \end{matrix}\begin{matrix} a \\ 0 \\ -c \end{matrix}\begin{matrix} b \\ c \\ 0 \end{matrix} \right]

Solution

\displaystyle A=\left[\begin{matrix} 0 \\ -a \\ -b \end{matrix}\begin{matrix} a \\ 0 \\ -c \end{matrix}\begin{matrix} b \\ c \\ 0 \end{matrix} \right]

The given matrix is

\displaystyle A=\left[\begin{matrix} 0 \\ -a \\ -b \end{matrix}\begin{matrix} a \\ 0 \\ -c \end{matrix}\begin{matrix} b \\ c \\ 0 \end{matrix} \right]

\displaystyle \therefore { A }^{ T }=\left[\begin{matrix} 0 \\ a \\ b \end{matrix}\begin{matrix} -a \\ 0 \\ c \end{matrix}\begin{matrix} -b \\ -c \\ 0 \end{matrix} \right]

\displaystyle A+{ A }^{ T }=\left[\begin{matrix} 0 \\ -a \\ -b \end{matrix}\begin{matrix} a \\ 0 \\ -c \end{matrix}\begin{matrix} b \\ c \\ 0 \end{matrix} \right] +\left[\begin{matrix} 0 \\ a \\ b \end{matrix}\begin{matrix} -a \\ 0 \\ c \end{matrix}\begin{matrix} -b \\ -c \\ 0 \end{matrix} \right] =\left[\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right]

\displaystyle \therefore \frac { 1 }{ 2 } \left(A+{ A }^{ T } \right) =\left[\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right]

Now, \displaystyle A-{ A }^{ T }=\left[\begin{matrix} 0 \\ -a \\ -b \end{matrix}\begin{matrix} a \\ 0 \\ -c \end{matrix}\begin{matrix} b \\ c \\ 0 \end{matrix} \right] -\left[\begin{matrix} 0 \\ a \\ b \end{matrix}\begin{matrix} -a \\ 0 \\ c \end{matrix}\begin{matrix} -b \\ -c \\ 0 \end{matrix} \right] =\left[\begin{matrix} 0 \\ -2a \\ -2b \end{matrix}\begin{matrix} 2a \\ 0 \\ -2c \end{matrix}\begin{matrix} 2b \\ 2c \\ 0 \end{matrix} \right]

\displaystyle \therefore \frac { 1 }{ 2 } \left(A-{ A }^{ T } \right) =\left[\begin{matrix} 0 \\ -a \\ -b \end{matrix}\begin{matrix} a \\ 0 \\ -c \end{matrix}\begin{matrix} b \\ \\ 0 \end{matrix} \right]

#420577

Topic: Symmetric and Skew Symmetric Matrix

Express the following matrices as the sum of a symmetric and a skew symmetric matrix:

(i) \displaystyle \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}

(ii) \displaystyle \left[\begin{matrix} 6 \\ -2 \\ 2 \end{matrix}\begin{matrix} -2 \\ 3 \\ -1 \end{matrix}\begin{matrix} 2 \\ -1 \\ 3 \end{matrix} \right]

(iii) \displaystyle \left[\begin{matrix} 3 \\ -2 \\ -4 \end{matrix}\begin{matrix} 3 \\ -2 \\ -5 \end{matrix}\begin{matrix} -1 \\ 1 \\ 2 \end{matrix} \right]

(iv) \displaystyle \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}

Solution

(i) Let \displaystyle A=\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}, then \quad { A }^{ \prime }=\begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}

Now, \displaystyle A+{ A }^{ \prime }=\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}+\begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}=\begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}

Let \displaystyle P=\frac { 1 }{ 2 } \left(A+{ A }^{ \prime } \right) =\frac { 1 }{ 2 } \begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}=\begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}

Now, \displaystyle { P }^{ \prime }=\begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}=P

Thus, \displaystyle P=\frac { 1 }{ 2 } \left(A+{ A }^{ \prime } \right) is a symmetric matrix.

Now, \displaystyle A-{ A }^{ \prime }=\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}-\begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}=\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}

Let \displaystyle Q=\frac { 1 }{ 2 } \left(A-{ A }^{ \prime } \right) =\frac { 1 }{ 2 } \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}=\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}

Now, \displaystyle { Q }^{ \prime }=\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}=Q

Thus, \displaystyle Q=\frac { 1 }{ 2 } \left(A-{ A }^{ \prime } \right) is a skew-symmetric matrix.

Representing A as sum of P and Q:

\displaystyle P+Q=\begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}+\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}=\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}=A

(ii) Let \displaystyle =\left[\begin{matrix} 6 \\ -2 \\ 2 \end{matrix}\begin{matrix} -2 \\ 3 \\ -1 \end{matrix}\begin{matrix} 2 \\ -1 \\ 3 \end{matrix} \right] , then \quad { A }^{ \prime }=\left[\begin{matrix} 6 \\ -2 \\ 2 \end{matrix}\begin{matrix} -2 \\ 3 \\ -1 \end{matrix}\begin{matrix} 2 \\ -1 \\ 3 \end{matrix} \right]

Now, \displaystyle A+{ A }^{ \prime }=\left[\begin{matrix} 6 \\ -2 \\ 2 \end{matrix}\begin{matrix} -2 \\ 3 \\ -1 \end{matrix}\begin{matrix} 2 \\ -1 \\ 3 \end{matrix} \right] +\left[\begin{matrix} 6 \\ -2 \\ 2 \end{matrix}\begin{matrix} -2 \\ 3 \\ -1 \end{matrix}\begin{matrix} 2 \\ -1 \\ 3 \end{matrix} \right] =\left[\begin{matrix} 12 \\ -4 \\ 4 \end{matrix}\begin{matrix} -4 \\ 6 \\ -2 \end{matrix}\begin{matrix} 4 \\ -2 \\ 6 \end{matrix} \right]

Let \displaystyle P=\frac { 1 }{ 2 } \left(A+{ A }^{ \prime } \right) =\frac { 1 }{ 2 } \left[\begin{matrix} 12 \\ -4 \\ 4 \end{matrix}\begin{matrix} -4 \\ 6 \\ -2 \end{matrix}\begin{matrix} 4 \\ -2 \\ 6 \end{matrix} \right] =\left[\begin{matrix} 6 \\ -2 \\ 2 \end{matrix}\begin{matrix} -2 \\ 3 \\ -1 \end{matrix}\begin{matrix} 2 \\ -1 \\ 3 \end{matrix} \right]

Now, \displaystyle { P }^{ \prime }=\left[\begin{matrix} 6 \\ -2 \\ 2 \end{matrix}\begin{matrix} -2 \\ 3 \\ -1 \end{matrix}\begin{matrix} 2 \\ -1 \\ 3 \end{matrix} \right] =P

Thus, \displaystyle P=\frac { 1 }{ 2 } \left(A+{ A }^{ \prime } \right) is a symmetric matrix.

Now, \displaystyle A-{ A }^{ \prime }=\left[\begin{matrix} 6 \\ -2 \\ 2 \end{matrix}\begin{matrix} -2 \\ 3 \\ -1 \end{matrix}\begin{matrix} 2 \\ -1 \\ 3 \end{matrix} \right] -\left[\begin{matrix} 6 \\ -2 \\ 2 \end{matrix}\begin{matrix} -2 \\ 3 \\ -1 \end{matrix}\begin{matrix} 2 \\ -1 \\ 3 \end{matrix} \right] =\left[\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right]

Let \displaystyle Q=\frac { 1 }{ 2 } \left(A-{ A }^{ \prime } \right) =\left[\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right]

Now, \displaystyle { Q }^{ \prime }=\left[\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right] =-Q

Thus, \displaystyle Q=\frac { 1 }{ 2 } \left(A-{ A }^{ \prime } \right) is a swek-symmetric matrix.

Representing A as sum of P and Q:

\displaystyle P+Q=\left[\begin{matrix} 6 \\ -2 \\ 2 \end{matrix}\begin{matrix} -2 \\ 3 \\ -1 \end{matrix}\begin{matrix} 2 \\ -1 \\ 3 \end{matrix} \right] +\left[\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right] =\left[\begin{matrix} 6 \\ -2 \\ 2 \end{matrix}\begin{matrix} -2 \\ 3 \\ -1 \end{matrix}\begin{matrix} 2 \\ -1 \\ 3 \end{matrix} \right] =A

(iii) Let \displaystyle A=\left[\begin{matrix} 3 \\ -2 \\ -4 \end{matrix}\begin{matrix} 3 \\ -2 \\ -5 \end{matrix}\begin{matrix} -1 \\ 1 \\ 2 \end{matrix} \right] , then \quad { A }^{ \prime }=\left[\begin{matrix} 3 \\ 3 \\ -1 \end{matrix}\begin{matrix} -2 \\ -2 \\ 1 \end{matrix}\begin{matrix} -4 \\ -5 \\ 2 \end{matrix} \right]

Now, \displaystyle A+{ A }^{ \prime }=\left[\begin{matrix} 3 \\ -2 \\ -4 \end{matrix}\begin{matrix} 3 \\ -2 \\ -5 \end{matrix}\begin{matrix} -1 \\ 1 \\ 2 \end{matrix} \right] +\left[\begin{matrix} 3 \\ 3 \\ -1 \end{matrix}\begin{matrix} -2 \\ -2 \\ 1 \end{matrix}\begin{matrix} -4 \\ -5 \\ 2 \end{matrix} \right] =\left[\begin{matrix} 6 \\ 1 \\ -5 \end{matrix}\begin{matrix} 1 \\ -4 \\ -4 \end{matrix}\begin{matrix} -5 \\ -4 \\ 4 \end{matrix} \right]

Let \displaystyle P=\frac { 1 }{ 2 } \left(A+{ A }^{ \prime } \right) =\frac { 1 }{ 2 } \left[\begin{matrix} 6 \\ 1 \\ -5 \end{matrix}\begin{matrix} 1 \\ -4 \\ -4 \end{matrix}\begin{matrix} -5 \\ -4 \\ 4 \end{matrix} \right] =\left[\begin{matrix} 3 \\ \frac { 1 }{ 2 } \\ -\frac { 5 }{ 2 } \end{matrix}\begin{matrix} \frac { 1 }{ 2 } \\ -2 \\ -2 \end{matrix}\begin{matrix} -\frac { 5 }{ 2 } \\ -2 \\ 2 \end{matrix} \right]

Now, \displaystyle P^{ \prime }=\left[\begin{matrix} 3 \\ \frac { 1 }{ 2 } \\ -\frac { 5 }{ 2 } \end{matrix}\begin{matrix} \frac { 1 }{ 2 } \\ -2 \\ -2 \end{matrix}\begin{matrix} -\frac { 5 }{ 2 } \\ -2 \\ 2 \end{matrix} \right] =P

Thus, \displaystyle P=\frac { 1 }{ 2 } \left(A+{ A }^{ \prime } \right) is a symmetric matrix.

Now, \displaystyle \left(A-{ A }^{ \prime } \right) =\left[\begin{matrix} 3 \\ -2 \\ -4 \end{matrix}\begin{matrix} 3 \\ -2 \\ -5 \end{matrix}\begin{matrix} -1 \\ 1 \\ 2 \end{matrix} \right] -\left[\begin{matrix} 3 \\ 3 \\ -1 \end{matrix}\begin{matrix} -2 \\ -2 \\ 1 \end{matrix}\begin{matrix} -4 \\ -5 \\ 2 \end{matrix} \right] =\left[\begin{matrix} 0 \\ -5 \\ -3 \end{matrix}\begin{matrix} 5 \\ 0 \\ -6 \end{matrix}\begin{matrix} 3 \\ 6 \\ 0 \end{matrix} \right]

Let \displaystyle Q=\frac { 1 }{ 2 } \left(A-{ A }^{ \prime } \right) =\frac { 1 }{ 2 } \left[\begin{matrix} 0 \\ -5 \\ -3 \end{matrix}\begin{matrix} 5 \\ 0 \\ -6 \end{matrix}\begin{matrix} 3 \\ 6 \\ 0 \end{matrix} \right] =\left[\begin{matrix} 0 \\ -\frac { 5 }{ 2 } \\ -\frac { 3 }{ 2 } \end{matrix}\begin{matrix} \frac { 5 }{ 2 } \\ 0 \\ -3 \end{matrix}\begin{matrix} \frac { 3 }{ 2 } \\ 0 \\ 0 \end{matrix} \right]

Now, \displaystyle Q^{ \prime }=\left[\begin{matrix} 0 \\ -\frac { 5 }{ 2 } \\ -\frac { 3 }{ 2 } \end{matrix}\begin{matrix} \frac { 5 }{ 2 } \\ 0 \\ -3 \end{matrix}\begin{matrix} \frac { 3 }{ 2 } \\ 0 \\ 0 \end{matrix} \right] =Q

Thus, \displaystyle Q=\frac { 1 }{ 2 } \left(A-{ A }^{ \prime } \right) is a skew-symmetric matrix.

Representing A as the sum of P and Q:

\displaystyle P+Q=\left[\begin{matrix} 3 \\ \frac { 1 }{ 2 } \\ -\frac { 5 }{ 2 } \end{matrix}\begin{matrix} \frac { 1 }{ 2 } \\ -2 \\ -2 \end{matrix}\begin{matrix} -\frac { 5 }{ 2 } \\ -2 \\ 2 \end{matrix} \right] +\left[\begin{matrix} 0 \\ -\frac { 5 }{ 2 } \\ -\frac { 3 }{ 2 } \end{matrix}\begin{matrix} \frac { 5 }{ 2 } \\ 0 \\ -3 \end{matrix}\begin{matrix} \frac { 3 }{ 2 } \\ 0 \\ 0 \end{matrix} \right] =\left[\begin{matrix} 3 \\ -2 \\ -4 \end{matrix}\begin{matrix} 3 \\ -2 \\ -5 \end{matrix}\begin{matrix} -1 \\ 1 \\ 2 \end{matrix} \right] =A

(iv) Let \displaystyle A=\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}, then \quad { A }^{ \prime }=\begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}

Now, \displaystyle A+{ A }^{ \prime }=\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}+\begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}=\begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix}

Let \displaystyle P=\frac { 1 }{ 2 } \left(A+{ A }^{ \prime } \right) =\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}

Now, \displaystyle { P }^{ \prime }=\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}=P

Thus, \displaystyle P=\frac { 1 }{ 2 } \left(A+{ A }^{ \prime } \right) is a symmetric matrix.

Now, \displaystyle A-{ A }^{ \prime }=\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}+\begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}=\begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix}

Let \displaystyle Q=\frac { 1 }{ 2 } \left(A-{ A }^{ \prime } \right) =\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}

Now, \displaystyle { Q }^{ \prime }=\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}=Q

Thus, \displaystyle Q=\frac { 1 }{ 2 } \left(A-{ A }^{ \prime } \right) is a skew-symmetric matric.

Representing A as sum of P and Q:

\displaystyle P+Q=\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}+\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}=\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}=A

#420578

Topic: Symmetric and Skew Symmetric Matrix

If A, B are symmetric matrices of same order, then AB - BA is a ,

A Skew symmetric matrix

B Symmetric matrix

C Zero matrix

D Identity matrix

Solution

Given, A and B are symmetric matrices, therefore, we have:

$\displaystyle { A }^{ \prime }=A$ and $\displaystyle { B }^{ \prime }=B$.........(i)

Consider

$\displaystyle { \left(AB-BA \right) }^{ \prime }={ \left(AB \right) }^{ \prime }-{ \left(BA \right) }^{ \prime }, \left[\because { \left(A-B \right) }^{ \prime }={ A }^{ \prime }-{ B }^{ \prime } \right]$

$\displaystyle ={ B }^{ \prime }{ A }^{ \prime }-{ A }^{ \prime }{ B }^{ \prime },\quad \left[\because { \left(AB \right) }^{ \prime }={ B }^{ \prime }{ A }^{ \prime } \right]$

$\displaystyle =BA-AB$ [by (i)]

$\displaystyle =-(AB-BA)$

$\displaystyle \therefore { \left(AB-BA \right) }^{ \prime }=-\left(AB-BA \right)$

Thus, $\displaystyle \left(AB-BA \right)$ is a skew-symmetric matrix.

#420579

Topic: Transpose of a Matrix

If $\displaystyle A=\begin{bmatrix} \cos { \alpha } & -\sin { \alpha } \\ \sin { \alpha } & \cos { \alpha } \end{bmatrix}$, then $\displaystyle A+{ A }^{ \prime }=I$, if the value of α is

A $\displaystyle \frac { \pi }{ 6 }$

B $\displaystyle \frac { \pi }{ 3 }$

C $\displaystyle n$

D $\displaystyle \frac {3 \pi }{ 2 }$

Solution

$\displaystyle A=\begin{bmatrix} \cos { \alpha } & -\sin { \alpha } \\ \sin { \alpha } & \cos { \alpha } \end{bmatrix}$

$\displaystyle \Rightarrow { A }^{ \prime }=\begin{bmatrix} \cos { \alpha } & \sin { \alpha } \\ -\sin { \alpha } & \cos { \alpha } \end{bmatrix}$

Now, $\displaystyle A+{ A }^{ \prime }=I$

$\displaystyle \therefore \begin{bmatrix} \cos { \alpha } & -\sin { \alpha } \\ \sin { \alpha } & \cos { \alpha } \end{bmatrix}+\begin{bmatrix} \cos { \alpha } & \sin { \alpha } \\ -\sin { \alpha } & \cos { \alpha } \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\displaystyle \Rightarrow \begin{bmatrix} 2\cos { \alpha } & 0 \\ 0 & 2\cos { \alpha } \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Comparing the corresponding elements of the two matrics, we have:

$\displaystyle 2\cos { \alpha } =1$

$\displaystyle \Rightarrow \cos { \alpha } =\frac { 1 }{ 2 } =\cos { \frac { \pi }{ 3 } }$ $\displaystyle \therefore \alpha =\frac { \pi }{ 3 }$

#420597

Topic: Properties of Matrices

Let $\displaystyle A=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, show that $\displaystyle { \left(aI+bA \right) }^{ n }={ A }^{ n }I+n{ a }^{ n-1 }bA$, where I is the identity matrix of order 2 and $\displaystyle n\in N$

Solution

It is given that \displaystyle A=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}

To show: \displaystyle P\left(n \right) :{ \left(aI+bA \right) }^{ n }={ A }^{ n }I+n{ a }^{ n-1 }bA,n\in N

We shall prove the result by using the principle of mathematical induction.

For \displaystyle n=1, we have:

\displaystyle P\left(I \right) :\left(aI+bA \right) =aI+b{ a }^{ 0 }A=aI+bA

Therefore, the result is true for \displaystyle n=k.

That is,

\displaystyle P\left(k \right) :{ \left(aI+bA \right) }^{ k }={ a }^{ k }I+k{ a }^{ k-1 }bA

Now, we prove that the result is true for \displaystyle n=k+1

Consider

\displaystyle { \left(aI+bA \right) }^{ k+1 }={ \left(aI+bA \right) }^{ k }{ \left(aI+bA \right) }

\displaystyle =\left({ a }^{ k }I+k{ a }^{ k-1 }bA \right) \left(aI+bA \right)

\displaystyle ={ a }^{ k+1 }I+k{ a }^{ k }bAI+{ a }^{ k }bIA+k{ a }^{ k-1 }{ b }^{ 2 }{ A }^{ 2 }

\displaystyle ={ a }^{ k+1 }I+\left(k+1 \right) { a }^{ k }bA+k{ a }^{ k-1 }{ b }^{ 2 }{ A }^{ 2 }...........(1)

Now,

\displaystyle { A }^{ 2 }=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}=0

From (1), we have:

\displaystyle { \left(aI+bA \right) }^{ k+1 }={ a }^{ k+1 }I+\left(k+1 \right) { a }^{ k }bA+O

\displaystyle ={ a }^{ k+1 }I+\left(k+1 \right) { a }^{ k }bA

Therefore, the result is true for \displaystyle n=k+1.

Thus, by the principle of mathematical induction, we have:

\displaystyle { \left(aI+nA \right) }^{ n }={ a }^{ n }I+n{ a }^{ n-1 }bA

where \displaystyle A=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},n\in N

#420598

Topic: Operations on Matrices

If \displaystyle A=\left[\begin{matrix} 1 \\ 1 \\ 1 \end{matrix}\begin{matrix} 1 \\ 1 \\ 1 \end{matrix}\begin{matrix} 1 \\ 1 \\ 1 \end{matrix} \right] , prove that \displaystyle { A }^{ n }=\left[\begin{matrix} { 3 }^{ n-1 } \\ { 3 }^{ n-1 } \\ { 3 }^{ n-1 } \end{matrix}\begin{matrix} { 3 }^{ n-1 } \\ { 3 }^{ n-1 } \\ { 3 }^{ n-1 } \end{matrix}\begin{matrix} { 3 }^{ n-1 } \\ { 3 }^{ n-1 } \\ { 3 }^{ n-1 } \end{matrix} \right] ,n\in N

Solution

It is given that $\displaystyle A=\left[\begin{matrix} 1 \\ 1 \\ 1 \end{matrix}\begin{matrix} 1 \\ 1 \\ 1 \end{matrix}\begin{matrix} 1 \\ 1 \\ 1 \end{matrix} \right]$

To show: $\displaystyle P\left(n \right) :{ A }^{ n }=\left[\begin{matrix} { 3 }^{ n-1 } \\ { 3 }^{ n-1 } \\ { 3 }^{ n-1 } \end{matrix}\begin{matrix} { 3 }^{ n-1 } \\ { 3 }^{ n-1 } \\ { 3 }^{ n-1 } \end{matrix}\begin{matrix} { 3 }^{ n-1 } \\ { 3 }^{ n-1 } \\ { 3 }^{ n-1 } \end{matrix} \right] ,n\in N$

We shall prove that the result by using the principle of mathematical induction .

For $\displaystyle n=1$, we have:

$\displaystyle P\left(1 \right) :\left[\begin{matrix} { 3 }^{ n-1 } \\ { 3 }^{ n-1 } \\ { 3 }^{ n-1 } \end{matrix}\begin{matrix} { 3 }^{ n-1 } \\ { 3 }^{ n-1 } \\ { 3 }^{ n-1 } \end{matrix}\begin{matrix} { 3 }^{ n-1 } \\ { 3 }^{ n-1 } \\ { 3 }^{ n-1 } \end{matrix} \right] =\left[\begin{matrix} { 3 }^{ 0 } \\ { 3 }^{ 0 } \\ { 3 }^{ 0 } \end{matrix}\begin{matrix} { 3 }^{ 0 } \\ { 3 }^{ 0 } \\ { 3 }^{ 0 } \end{matrix}\begin{matrix} { 3 }^{ 0 } \\ { 3 }^{ 0 } \\ { 3 }^{ 0 } \end{matrix} \right] =\left[\begin{matrix} 1 \\ 1 \\ 1 \end{matrix}\begin{matrix} 1 \\ 1 \\ 1 \end{matrix}\begin{matrix} 1 \\ 1 \\ 1 \end{matrix} \right] =A$

Therefore, the result is true for $\displaystyle n=1$.

Let the result be true for $\displaystyle n=k$.

That

is $\displaystyle P\left(k \right) :{ A }^{ k }=\left[\begin{matrix} { 3 }^{ k-1 } \\ { 3 }^{ k-1 } \\ { 3 }^{ k-1 } \end{matrix}\begin{matrix} { 3 }^{ k-1 } \\ { 3 }^{ k-1 } \\ { 3 }^{ k-1 } \end{matrix}\begin{matrix} { 3 }^{ k-1 } \\ { 3 }^{ k-1 } \\ { 3 }^{ k-1 } \end{matrix} \right]$

Now, we prove that the result is true for $\displaystyle n=k+1$.

Now, $\displaystyle { A }^{ k+1 }=A.{ A }^{ k }$

$\displaystyle \left[\begin{matrix} 1 \\ 1 \\ 1 \end{matrix}\begin{matrix} 1 \\ 1 \\ 1 \end{matrix}\begin{matrix} 1 \\ 1 \\ 1 \end{matrix} \right] ^{ k }\left[\begin{matrix} { 3 }^{ k-1 } \\ { 3 }^{ k-1 } \\ { 3 }^{ k-1 } \end{matrix}\begin{matrix} { 3 }^{ k-1 } \\ { 3 }^{ k-1 } \\ { 3 }^{ k-1 } \end{matrix}\begin{matrix} { 3 }^{ k-1 } \\ { 3 }^{ k-1 } \\ { 3 }^{ k-1 } \end{matrix} \right]$

$\displaystyle =\left[\begin{matrix} 3.{ 3 }^{ k-1 } \\ 3.{ 3 }^{ k-1 } \\ 3.{ 3 }^{ k-1 } \end{matrix}\begin{matrix} 3.{ 3 }^{ k-1 } \\ 3.{ 3 }^{ k-1 } \\ 3.{ 3 }^{ k-1 } \end{matrix}\begin{matrix} 3.{ 3 }^{ k-1 } \\ 3.{ 3 }^{ k-1 } \\ 3.{ 3 }^{ k-1 } \end{matrix} \right]$

$\displaystyle =\left[\begin{matrix} { 3 }^{ \left(k+1 \right) -1 } \\ { 3 }^{ \left(k+1 \right) -1 } \\ { 3 }^{ \left(k+1 \right) -1 } \end{matrix}\begin{matrix} { 3 }^{ \left(k+1 \right) -1 } \\ { 3 }^{ \left(k+1 \right) -1 } \\ { 3 }^{ \left(k+1 \right) -1 } \end{matrix}\begin{matrix} { 3 }^{ \left(k+1 \right) -1 } \\ { 3 }^{ \left(k+1 \right) -1 } \\ { 3 }^{ \left(k+1 \right) -1 } \end{matrix} \right]$

Therefore, the result is true for $\displaystyle n=k+1$.

Thus by the principle of mathematical induction , we have:

$\displaystyle { A }^{ n }=\left[\begin{matrix} { 3 }^{ n-1 } \\ { 3 }^{ n-1 } \\ { 3 }^{ n-1 } \end{matrix}\begin{matrix} { 3 }^{ n-1 } \\ { 3 }^{ n-1 } \\ { 3 }^{ n-1 } \end{matrix}\begin{matrix} { 3 }^{ n-1 } \\ { 3 }^{ n-1 } \\ { 3 }^{ n-1 } \end{matrix} \right] ,n\in N$

#420599

Topic: Properties of Matrices

If $\displaystyle A=\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then prove $\displaystyle { A }^{ n }=\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ where n is any positive integer

Solution

It is given that $\displaystyle A=\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

To prove: $\displaystyle P\left(n \right) :{ A }^{ n }=\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix},n\in N$

We shall prove the result by using the principle of mathematical induction .

For $\displaystyle n=1$, we have:

$\displaystyle P\left(1 \right) :{ A }^{ 1 }=\begin{bmatrix} 1+2 & -4 \\ 1 & 1-2 \end{bmatrix}=\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}=A$

Therefore, the result is true for $\displaystyle n=1$.

Let the result be true for $\displaystyle n=k$.

That is ,

$\displaystyle P\left(k \right) :{ A }^{ k }=\begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix},n\in N$

Now, we prove that the result is true for $\displaystyle n=k+1$

Consider

$\displaystyle { A }^{ k+1 }={ A }^{ k }.A$

$\displaystyle =\begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

$\displaystyle =\begin{bmatrix} 3\left(1+2k \right) -4k & -4\left(1+2k \right) +4k \\ 3k+1-2k & -4k-1\left(1-2k \right) \end{bmatrix}$

$\displaystyle =\begin{bmatrix} 3+6k-4k & -4-4k \\ 3k+1-2k & -4k-1+2k \end{bmatrix}$

$\displaystyle =\begin{bmatrix} 3+2k & -4-4k \\ 1+k & -1-2k \end{bmatrix}$

$\displaystyle =\begin{bmatrix} 1+2\left(k+1 \right) & -4\left(k+1 \right) \\ 1+k & 1-2\left(k+1 \right) \end{bmatrix}$

Therefore, the result is true for $\displaystyle n=k+1$.

Thus, by the principle of mathematical induction , we have:

$\displaystyle { A }^{ n }=\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix},n\in N$

#420600

Topic: Symmetric and Skew Symmetric Matrix

If A and B are symmetric matrices, prove that $AB-BA$ is a skew symmetric matrix.

Solution

It is given that A and B are symmetric matrices.

Therefore, we have:

$\displaystyle { A }^{ \prime }=A$ and $\displaystyle { B }^{ \prime }=B$.............(1)

Now, $\displaystyle { \left(AB-BA \right) }^{ \prime }={ \left(AB \right) }^{ \prime }-{ \left(BA \right) }^{ \prime }, \quad \left[\because { \left(A-B \right) }^{ \prime }={ A }^{ \prime }-{ B }^{ \prime } \right]$

$\displaystyle ={ B }^{ \prime }{ A }^{ \prime }-{ A }^{ \prime }{ B }^{ \prime },\quad \left[\because { \left(AB \right) }^{ \prime }={ B }^{ \prime }{ A }^{ \prime } \right]$

$\displaystyle =BA-AB$ .... [using (1)]

$\displaystyle =-\left(AB-BA \right)$

$\displaystyle \therefore { \left(AB-BA \right) }^{ \prime }=-\left(AB-BA \right)$

Thus, $\displaystyle \left(AB-BA \right)$ is a skew-symmetric matrix.

#420601

Topic: Symmetric and Skew Symmetric Matrix

Show that the matrix $\displaystyle { B }^{ \prime }AB$ is symmetric or skew symmetric according as A is symmetric or skew symmetric.

Solution

We suppose that A is a symmetric matrix, then $\displaystyle { A }^{ \prime }=A$.........(1)

Consider $\displaystyle { \left({ B }^{ \prime }AB \right) }^{ \prime }={ \left\{ { B }^{ \prime }\left(AB \right) \right\} }^{ \prime }$

$\displaystyle ={ \left(AB \right) }^{ \prime }{ \left({ B }^{ \prime } \right) }^{ \prime }\quad \left[{ \left(AB \right) }^{ \prime }={ B }^{ \prime }{ A }^{ \prime } \right]$

$\displaystyle ={ B }^{ \prime }{ A }^{ \prime }\left(B \right) \quad \left[{ \left({ B }^{ \prime } \right) }^{ \prime }=B \right]$

$\displaystyle ={ B }^{ \prime }\left({ A }^{ \prime }B \right)$

$\displaystyle ={ B }^{ \prime }\left(AB \right)$ [using (1)]

$\displaystyle \therefore { \left({ B }^{ \prime }AB \right) }^{ \prime }={ B }^{ \prime }\left(AB \right)$

Thus, if A is a symmetric matrix, then $\displaystyle { B }^{ \prime }AB$ is a symmetric matrix.

Now, we suppose that A is a skew-symmetric matrix.\displaystyle

Then, $\displaystyle { A }^{ \prime }=-A$

Consider

$\displaystyle { \left({ B }^{ \prime }AB \right) }^{ \prime }={ \left[{ B }^{ \prime }\left(AB \right) \right] }^{ \prime }={ \left(AB \right) }^{ \prime }{ \left({ B }^{ \prime } \right) }^{ \prime }$

$\displaystyle =\left({ B }^{ \prime }{ A }^{ \prime } \right) B={ B }^{ \prime }\left(-A \right) B$

$\displaystyle =-{ B }^{ \prime }AB$

$\displaystyle \therefore { \left({ B }^{ \prime }AB \right) }^{ \prime }=-{ B }^{ \prime }AB$

Thus, if A is a skew-symmetric matrix, then $\displaystyle { B }^{ \prime }AB$ is a skew-symmetric matrix.

Hence, if A is a symmetric or skew-symmetric matrix , then $\displaystyle { B }^{ \prime }AB$ is a symmetric or skew-symmetric matrix accordingly.

#420603

Topic: Operations on Matrices

For what values of x.

$\begin{bmatrix} 1 &2&1\end{bmatrix}\begin{bmatrix} 1&2&0\\2&0&1\\1&0&2\end{bmatrix}\begin{bmatrix}0\\2\\x \end{bmatrix} = 0$?

Solution

We have:

$\displaystyle \left[\begin{matrix} 1 & 2 & 1 \end{matrix} \right] \left[\begin{matrix} 1 \\ 2 \\ 1 \end{matrix}\begin{matrix} 2 \\ 0 \\ 0 \end{matrix}\begin{matrix} 0 \\ 1 \\ 2 \end{matrix} \right] \left[\begin{matrix} 0 \\ 2 \\ x \end{matrix} \right] =0$

$\displaystyle \Rightarrow \left[\begin{matrix} 1+4+1 & 2+0+0 & 0+2+2 \end{matrix} \right] \left[\begin{matrix} 0 \\ 2 \\ x \end{matrix} \right] =0$

$\displaystyle \Rightarrow \left[\begin{matrix} 6 & 2 & 4 \end{matrix} \right] \left[\begin{matrix} 0 \\ 2 \\ x \end{matrix} \right] =0$

$\displaystyle \Rightarrow \left[6\left(0 \right) +2\left(2 \right) +4\left(x \right) \right] =0$

$\displaystyle \Rightarrow \left[4+4x \right] =\left[0 \right]$

$\displaystyle \therefore 4+4x=0$

$\therefore x=-1$

#420604

Topic: Properties of Matrices

If $\displaystyle A=\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $\displaystyle { A }^{ 2 }-5A+7I=0$

Solution

It is given that $\displaystyle A=\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$\displaystyle \therefore { A }^{ 2 }=A.A=\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$\displaystyle =\begin{bmatrix} 3\left(3 \right) +1\left(-1 \right) & 3\left(1 \right) +1\left(2 \right) \\ -1\left(3 \right) +2\left(-1 \right) & -1\left(1 \right) +2\left(2 \right) \end{bmatrix}$

$\displaystyle =\begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}=\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$\displaystyle \therefore L.H.S. ={ A }^{ 2 }-5A+7I$

$\displaystyle =\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}-5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}+7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\displaystyle =\begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}-\begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}+\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$\displaystyle =\begin{bmatrix} -7 & 0 \\ 0 & -1 \end{bmatrix}+\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$\displaystyle =\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}=0=$ R.H.S.

$\displaystyle \therefore { A }^{ 2 }-5A+7I=0$

#420605

Topic: Operations on Matrices

Find x, if $\displaystyle \begin{bmatrix} x&-5&-1 \end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3 \end{bmatrix}\begin{bmatrix}x\\4\\1\end{bmatrix}=0$

Solution

We have :

$\displaystyle \left[\begin{matrix} x & -5 & -1 \end{matrix} \right] \left[\begin{matrix} 1 \\ 0 \\ 2 \end{matrix}\begin{matrix} 0 \\ 2 \\ 0 \end{matrix}\begin{matrix} 2 \\ 1 \\ 3 \end{matrix} \right] \left[\begin{matrix} x \\ 4 \\ 1 \end{matrix} \right] =0$

$\displaystyle \Rightarrow \left[\begin{matrix} x+0-2 & 0-10+0 & 2x-5-3 \end{matrix} \right] \left[\begin{matrix} x \\ 4 \\ 1 \end{matrix} \right] =0$

$\displaystyle \Rightarrow \left[\begin{matrix} x-2 & -10 & 2x-8 \end{matrix} \right] \left[\begin{matrix} x \\ 4 \\ 1 \end{matrix} \right] =0$

$\displaystyle \Rightarrow \left[x\left(x-2 \right) -40+2x-8 \right] =0$

$\displaystyle \Rightarrow \left[{ x }^{ 2 }-2x-40+2x-8 \right] =\left[0 \right]$

$\displaystyle \Rightarrow \left[{ x }^{ 2 }-48 \right] =\left[0 \right]$

$\displaystyle \therefore { x }^{ 2 }-48=0$

$\displaystyle \Rightarrow { x }^{ 2 }=48$

$\displaystyle \Rightarrow x=\pm 4\sqrt { 3 }$

#420606

Topic: Properties of Matrices

A manufacturer produces three products x, y, z which he sells in two markets. Annual sales are indicated below:

Market Products

I	10000	2000	18000
II	6000	20000	8000

(a) If unit sale prices of x, y and z are Rs. 2.50, Rs. 1.50 and Rs. 1.00, respectively, find the total revenue in each market with the help of matrix algebra..

(b) If the unit costs of the above three commodities are Rs. 2.00, Rs. 1.00 and 50 paise respectively. Find the gross profit.

Solution

(a) The unit sale prices of x, y and z are respectively Rs. 2.50, Rs. 1.50 and Rs. 1.00.

Total revenue in market I can be represented as:

$$\displaystyle \left[\begin{matrix} 10000 & 2000 & 18000 \end{matrix} \right] \left[\begin{matrix} 2.50 \\ 1.50 \\ 1.00 \end{matrix} \right]$$

$$\displaystyle =10000\times 2.50+2000\times 1.50+18000\times 1.00$$

$$\displaystyle =25000+3000+18000$$

$$\displaystyle =46000$$

Total revenue in market II can be represented as:

$$\displaystyle \left[\begin{matrix} 6000 & 20000 & 8000 \end{matrix} \right] \left[\begin{matrix} 2.50 \\ 1.50 \\ 1.00 \end{matrix} \right]$$

$$\displaystyle =6000\times 2.50+20000\times 1.50+8000\times 1.00$$

$$\displaystyle =15000+30000+8000$$

$$\displaystyle =53000$$

So, the total revenue in market I is Rs 46000 and in market II is Rs. 53000.

(b) The unit cost prices of x,y and z are respectively given as Rs. 2.00, Rs. 1.00 and 50 paise.

So, the total cost prices of all the products in market I can be represented as:

$$\displaystyle \left[\begin{matrix} 10000 & 2000 & 18000 \end{matrix} \right] \left[\begin{matrix} 2.00 \\ 1.00 \\ 0.50 \end{matrix} \right]$$

$$\displaystyle =10000\times 2.00+2000\times 1.00+18000\times 0.50$$

$$\displaystyle =20000+2000+9000$$

$$\displaystyle =31000$$

Since, the total revenue in market I is Rs. 46000.

So, the gross profit in this market is Rs 46000-Rs 31000 =Rs 15000.

The total cost prices of all the products in market II can be represented as:

$$\displaystyle \left[\begin{matrix} 6000 & 20000 & 8000 \end{matrix} \right] \left[\begin{matrix} 2.00 \\ 1.00 \\ 0.50 \end{matrix} \right]$$

$$\displaystyle =6000\times 2.00+20000\times 1.00+8000\times 0.50$$

$$\displaystyle =12000+20000+4000$$

$$\displaystyle =Rs36000$$

Since, the total revenue in market II is Rs. 53000.

So, the gross profit in this market is Rs.53000 - Rs.36000 =Rs.17000.

#420607

Topic: Operations on Matrices

Find the matrix X so that $\displaystyle X\left[\begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{matrix} \right] =\left[\begin{matrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{matrix} \right]$

Solution

It is given that:

$\displaystyle X\left[\begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{matrix} \right] =\left[\begin{matrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{matrix} \right]$

The matrix given on the R.H.S. of the equation is a 2×3 matrix and the one given on the L.H.S. of the equation is a 2×3 matrix . Therefore, X has to be a 2×2 matrix.

Now, let $\displaystyle X=\begin{bmatrix} a & c \\ b & d \end{bmatrix}$

Therefore, we have:

$\displaystyle \begin{bmatrix} a & c \\ b & d \end{bmatrix}\left[\begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{matrix} \right] =\left[\begin{matrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{matrix} \right]$

$\displaystyle \Rightarrow \left[\begin{matrix} a+4c & 2a+5c & 3a+6c \\ b+4d & 2b+5d & 3b+6d \end{matrix} \right] =\left[\begin{matrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{matrix} \right]$

Equating the corresponding elements of the two matrices, we have:

$\displaystyle \begin{matrix} a+4c=-7 & 2a+5c=-8 & 3a+6c=-9 \\ b+4d=2 & 2b+5d=4 & 3b+6d=6 \end{matrix}$

Now, $\displaystyle a+4c=-7\Rightarrow -7-4c$

$\displaystyle \therefore 2a+5c=-8\Rightarrow -14-8c+5c=-8$

$\displaystyle \Rightarrow -3c=6 \Rightarrow c=-2$

$\displaystyle \therefore a=-7-4\left(-2 \right) =-7+=1$

Now, $\displaystyle b+4d=2\Rightarrow b=2-4d$

$\displaystyle \therefore 2b+5d=4\Rightarrow 4-8d+5d=4$

$\displaystyle \Rightarrow -3d=0$

$\displaystyle \Rightarrow d=0$

$\displaystyle \therefore b=2-4\left(0 \right) =2$

Thus, $a=1$, $b=2$, $c=-2$, $d=0$

Hence, the required matrix X is $\displaystyle \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

#420608

Topic: Properties of Matrices

If A and B are square matrices of the same order such that $AB = BA$, then prove by induction that $\displaystyle { AB }^{ n }={ B }^{ n }A$. Further, prove that $\displaystyle \left({ AB }\right) ^n={ A }^{ n }{ B }^{ n }$ for all $\displaystyle n\in N$.

Solution

A and B are square matrices of the same order such that $AB = BA$

To prove: $\displaystyle P\left(n \right) :{ AB }^{ n }={ B }^{ n }A,n\in N$

For $n = 1$, we have:

$\displaystyle P(1):AB=BA$ [Given]

$\displaystyle \Rightarrow { AB }^{ 1 }={ B }^{ 1 }A$

Therefore, the result is true for $n=1$.

Let the result be true for $n=k$.

$\displaystyle P\left(k \right) :{ AB }^{ k }={ B }^{ k }A..........(1)$

Now, we prove that the result is rue for $n=k+1$.

$\displaystyle { AB }^{ k+1 }={ AB }^{ k }.B$

$\displaystyle =\left({ B }^{ k }A \right) B$ [By(1)]

$\displaystyle ={ B }^{ k }\left(AB \right)$ [Associative law]

$\displaystyle ={ B }^{ k }\left(BA \right)$ [AB=BA (given)]

$\displaystyle =\left({ B }^{ k }B \right) A$ [Associative law]

$\displaystyle ={ B }^{ k+1 }A$

Therefore, the result is true for $n=k+1$.

Thus, by the principle of mathematical induction , we have $\displaystyle { AB }^{ n }={ B }^{ n }A,n\in N$.

Now, we prove that $\displaystyle \left({ AB }^{ n } \right) ={ A }^{ n }{ B }^{ n }$ for all $\displaystyle n\in N$

For $n=1$, we have"

$\displaystyle { \left(AB \right) }^{ 1 }={ A }^{ 1 }{ B }^{ 1 }=AB$

Therefore, the result is true for $n=1$.

Let the result be true for $n=k$.

$\displaystyle { \left(AB \right) }^{ k }={ A }^{ k }{ B }^{ k }...........(2)$

Now we prove that the result is true for $n=k+1$.

$\displaystyle { \left(AB \right) }^{ k+1 }={ \left(AB \right) }^{ k }.{ \left(AB \right) }$

$\displaystyle =\left({ A }^{ k }{ B }^{ k } \right) .\left(AB \right)$ [By (2)]

$\displaystyle ={ A }^{ k }\left({ B }^{ k }A \right) B$ [Associative law]

$\displaystyle ={ A }^{ k }\left({ A }^{ k }B \right) B\quad \left[{ AB }^{ n }={ B }^{ n }Afor\quad all\quad n\in N \right]$

$\displaystyle =\left({ A }^{ k }A \right) .\left({ B }^{ k }B \right)$ [Associative law]

$\displaystyle ={ A }^{ k+1 }{ B }^{ k+1 }$

Therefore, the result is true for $n=k+1$.

Thus, by the principle of mathematical induction , we have $\displaystyle \left({ AB } \right) ^n={ A }^{ n }{ B }^{ n }$, for all natural numbers.

#420609
Topic: Operations on Matrices

Choose the correct answer in the following questions:

If $\displaystyle A=\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $\displaystyle { A }^{ 2 }=I$ then

A $\displaystyle 1+{ \alpha }^{ 2 }+\beta \gamma =0$

B $\displaystyle 1-{ \alpha }^{ 2 }+\beta \gamma =0$

C $\displaystyle 1-{ \alpha }^{ 2 }-\beta \gamma =0$

D $\displaystyle 1+{ \alpha }^{ 2 }-\beta \gamma =0$

Solution

$\displaystyle A=\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$

$\displaystyle \therefore { A }^{ 2 }=A.A=\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$

$\displaystyle =\begin{bmatrix} { \alpha }^{ 2 }+\beta \gamma & \alpha \beta -\alpha \beta \\ \alpha \gamma -\alpha \gamma & \beta \gamma +{ \alpha }^{ 2 } \end{bmatrix}$

$\displaystyle =\begin{bmatrix} { \alpha }^{ 2 }+\beta \gamma & 0 \\ 0 & \beta \gamma +{ \alpha }^{ 2 } \end{bmatrix}$

Now, $\displaystyle { A }^{ 2 }=I\Rightarrow \begin{bmatrix} { \alpha }^{ 2 }+\beta \gamma & 0 \\ 0 & \beta \gamma +{ \alpha }^{ 2 } \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

On comparing the corresponding elements, we have:

$\displaystyle { \alpha }^{ 2 }+\beta \gamma =1$

$\displaystyle \Rightarrow { \alpha }^{ 2 }+\beta \gamma -1=0$

$\displaystyle \Rightarrow 1-{ \alpha }^{ 2 }-\beta \gamma =0$

#420610

Topic: Symmetric and Skew Symmetric Matrix

If the matrix A is both symmetric and skew symmetric, then

A A is a diagonal matrix

B (boxed) A is a zero matrix

C A is a square matrix

D None of these

Solution

If A is both symmetric and skew symmetric, then we should have

$\displaystyle { A }^{ \prime }=A$ and $\displaystyle { A }^{ \prime }=-A$

$\displaystyle \Rightarrow A=-A$

$\displaystyle \Rightarrow A+A=0$

$\displaystyle \Rightarrow 2A=0$

$\displaystyle \Rightarrow A=0$

Therefore, A is a zero matrix.

#420611

Topic: Properties of Matrices

If A is square matrix such that $\displaystyle { A }^{ 2 }=A$, then $\displaystyle { \left(I+A \right) }^{ 3 }-7A$ is equal to

A A

B $I-A$

C (boxed) I

D $3A$

Solution

$\displaystyle { \left(I+A \right) }^{ 3 }-7A={ I }^{ 3 }+{ A }^{ 3 }+3{ I }^{ 2 }A+3{ A }^{ 2 }I-7A$

$\displaystyle =I+{ A }^{ 3 }+3A+3{ A }^{ 2 }-7A$

$\displaystyle =I+{ A }^{ 2 }.A+3A+3A-7A, \quad \left[\because { A }^{ 2 }=A \right]$

$\displaystyle =I+A.A-A$

$\displaystyle =I+{ A }^{ 2 }-A$

$\displaystyle =I+A-A =I$

$\displaystyle \therefore { \left(I+A \right) }^{ 3 }-7A=I$

#427121

Topic: Properties of Matrices

For the matrix $A=\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the numbers a and b such that ${ A }^{ 2 }+aA+bI=0$

Solution

Given $A=\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

Also, ${ A }^{ 2 }+aA+bI=O$ ....(1)

Now, ${ A }^{ 2 }=\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow { A }^{ 2 }=\begin{bmatrix} 9+2 & 6+2 \\ 3+1 & 2+1 \end{bmatrix}$

$\Rightarrow { A }^{ 2 }=\begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$

Substituting the values in (1), we get

$\begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}+a\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}+b\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}+\begin{bmatrix} 3a & 2a \\ a & a \end{bmatrix}+\begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 11+3a+b & 8+2a \\ 4+a & 3+a+b \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

On comparing corresponding elements of both matrix, we get

$a+4=0$

$\Rightarrow a=-4$

Also, $3+a+b=0$

$\Rightarrow 3-4+b=0$

$\Rightarrow b=1$

Hence, the values of a and b are -4,1 respectively.