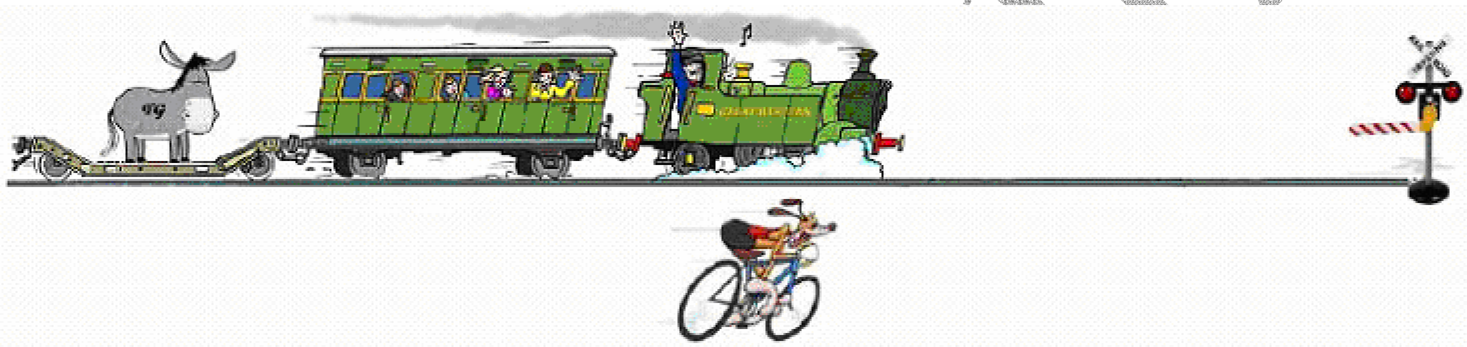
Total Gadha's Complete Book of Time, Speed and Distance



Time Speed and Distance

For an MBA aspirant, a problem on time, speed and distance means solving complex situations with the help of many equations. The chapter has both easy and tough aspects to it. The easy aspect is the formula; there is only one formula- Distance = speed × time- and it is known to every student. The tough aspect of course is the application of the formula; most students apply equations to solve the problems based on this formula. Let's have a look at a typical time, speed and distance problem:

Two trains start from stations A and B, respectively, towards each other at 12:00 noon. The trains take 7 hours and 13 hours, respectively, to cover the whole trip. What time do the two trains meet?

$$\boxed{\text{Distance} = \text{speed} \times \text{time}}$$

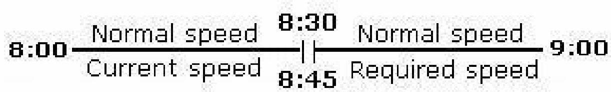
There are two special cases that are derived from this equation-

- **DISTANCE CONSTANT**- If the distance for travel is constant then the time taken for travel is inversely proportional to time, i.e.

$$\text{Time} \propto \frac{1}{\text{Speed}} \Rightarrow \frac{T_1}{T_2} = \frac{V_2}{V_1}$$

Therefore, if the speed of travel is doubled, the time of travel is halved. If the speed of travel is $\frac{1}{3}^{rd}$ of the original speed, the time of travel is thrice the original time. If the speed of travel is $\frac{2}{3}^{rd}$ of the original speed, the time of travel is $\frac{3}{2}$ times the original time. Let's put this principle into practice:

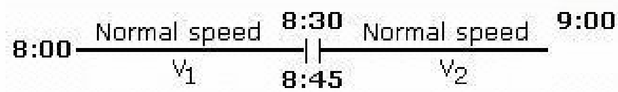
Examples:
I leave my home everyday at 8:00 am and reach my office at 9:00 am, not stopping anywhere along the way. One day, I left my home at my normal time and traveled the first half of the distance at $\frac{2}{3}^{rd}$ of my original speed. What should be my speed for the second half so that I reach my office in time?
Answer: If I travel with my normal speed, I take one hour to reach from my home to my office. Therefore, I take 30 minutes to reach halfway. Therefore, I reach the midpoint of my path at 8:30. If I travel with $\frac{2}{3}^{rd}$ of my original speed, the time to reach the midpoint of my path is $\frac{3}{2}$ of my original time. Therefore, I take $\frac{3}{2} \times 30 = 45$ minutes to reach halfway, as shown in the figure.



I reach midway at 8:45. To reach the office in time, I need to travel the second half of the distance in 15 minutes. With my normal speed, I travel the second half of the distance in 30 minutes. Therefore, to travel the same distance in 15 minutes, I will have to travel with a speed **double** of my normal speed.

In the previous question, I traveled the first half of the distance at $\frac{2}{3}^{rd}$ of my original speed and the second half of the distance at $\frac{3}{2}^{nd}$ of my original speed. At what time will I reach office?

Answer: As discussed in the previous question, I will reach the midpoint of the path at 8:45 with a speed $\frac{2}{3}^{rd}$ of my original speed.



If I travel the second half at $\frac{3}{2}$ of my original speed, my time becomes $\frac{2}{3}^{rd}$ of my original time, i.e. $\frac{2}{3} \times 30 = 20$ minutes. Therefore, the total time that I take is = 45 + 20 = 65 minutes. Therefore, I reach my office 5 minutes late.

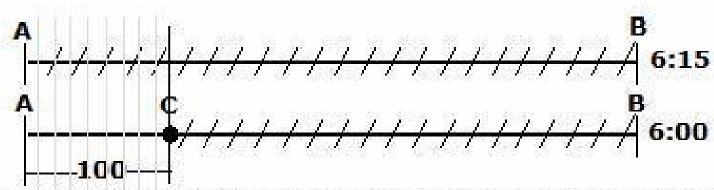
I go to my office everyday with the same speed. One day I traveled with a speed that was $\frac{3}{4}^{th}$ of my normal speed and I reached my office 15 minutes late. What time do I normally take to reach my office?

Answer: As distance is constant time taken will be inversely proportional to the speed. Since the speed becomes $\frac{3}{4}^{th}$ of the original speed, the time taken becomes $\frac{4}{3}^{rd}$ of the original time. Let the normal time taken everyday be t. Therefore, at $\frac{3}{4}^{th}$ of the normal speed, the time taken is $\frac{4t}{3}$.

Extra time taken = $\frac{4t}{3} - t = \frac{t}{3} = 15$ min $\Rightarrow t = 45$ min.

A train goes from station A to station B everyday. One day, it traveled with $\frac{4}{5}^{th}$ of its original speed because of engine trouble at A and reached station B at 6:15 pm. If the trouble had occurred after the train had traveled 100 km from A, the train would have reached B at 6:00 pm. What is the normal speed of the train?

Answer: The situation is summarized in the picture below. When the train travels the complete distance with speed $\frac{4}{5}^{th}$ of its original speed it reaches B at 6:15. When it travels the distance CB (where C is 100 km from A) with the speed $\frac{4}{5}^{th}$ of its original speed, it reaches B at 6:00.



The situation is similar to the previous question. From the figure we can see that the difference in arrival time in the two scenarios is occurring because of the difference in two speeds for the distance AC. Because the train is traveling the distance CB with the same speed in both the cases, this distance is not contributing to the change in arrival time.

For distance AC, the train is traveling with the normal speed in the second case and $\frac{4}{5}^{th}$ of the normal speed in the first case. Let the time to travel the distance AC with the normal speed be t. Therefore, in the first case, the time taken will be $\frac{5t}{4}$.

The extra time taken = $\frac{5t}{4} - t = \frac{t}{4} = 15$ minutes (the change in arrival time is 15 min) $\Rightarrow t = 60$ minutes.
Therefore, time taken by the train to travel 100 km = 1 h $\Rightarrow$ the speed of the train = 100 km/h

If a man cycles at 10 km/hr, then he arrives at a certain place at 1 p.m. If he cycles at 15 km/hr, he will arrive at the same place at 11 a.m. At what speed must he cycle to get there at noon? **(CAT 2004)**
1. 11 km/h 2. 12 km/h 3. 13 km/h 4. 14 km/h

Answer: The distance is constant in this case. Let the time taken for travel with a speed of 10 km/h be t. Now the speed of 15 km/h is $\frac{3}{2}$ times the speed of 10 km/h. Therefore, time taken with speed of 15 km/h will be $\frac{2t}{3}$.

Therefore, extra time taken $= t - \frac{2t}{3} = \frac{t}{3} = 2\text{ h} \Rightarrow t = 6\text{ h}$. Therefore, the distance = 10 × 6 = 60 km. To reach at noon, the man will take 5 h. Therefore, the speed = 60/5 = 12 km/h

Ram and Shyam run a race between points **A** and **B**, 5 km apart. Ram starts at 9 a.m. from **A** at a speed of 5 km/hr, reaches **B**, and returns to **A** at the same speed. Shyam starts at 9:45 a.m. from **A** at a speed of 10 km/hr, reaches **B** and comes back to **A** at the same speed. **(CAT 2005)**

At what time does Shyam overtake Ram?
1. 10:20 a.m. 2. 10:30 a.m. 3. 10:40 a.m. 4. 10:50 a.m.

Answer: Both Ram and Shyam are reaching B from A and then returning. When Shyam overtakes Ram, the distances traveled by both of them are equal. Therefore, the times taken are inversely proportional to their speeds. Let Ram take t minutes for travel Shyam overtakes him. As Shyam started 45 minutes after Ram he takes t – 45 minutes.

$\Rightarrow \frac{t}{t-45} = \frac{V_{Shyam}}{V_{Ram}} = 2 \Rightarrow t = 90\text{ min}$. Therefore, Shyam overtakes Ram at 10:30 am.

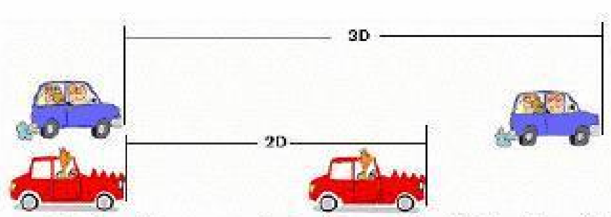
Arun, Barun and Kiranmala start from the same place and travel in the same direction at speeds of 30, 40 and 60 km per hour, respectively. Barun starts two hours after Arun. If Barun and Kiranmala overtake Arun at the same instant, how many hours Arun did Kiranmala start? **(CAT 2006)**
Answer: The ratio of speeds of Arun and Barun is 3: 4. Therefore, to travel the same distance, the times taken will be in the ratio 4: 3. Let Arun take time t. Therefore, Barun takes t – 2 as he started 2 hours after Arun.
1. 3 2. 3.5 3. 4 4. 4.5 5. 5

$\frac{t}{t-2} = \frac{4}{3} \Rightarrow t = 8$. Therefore Arun takes 8 hours. Now Kiranmala will take half that time as her speed is twice of Arun's speed. Therefore, she will take 4 hours for the same distance. Therefore, she should start 4 hours after Arun.

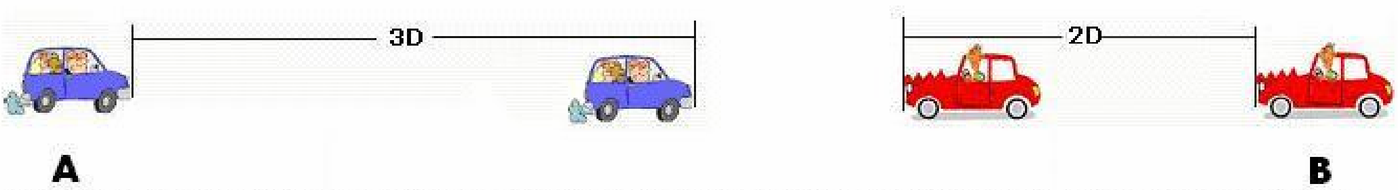
- **TIME CONSTANT**- If the time for travel is constant, then the distance traveled is directly proportional to the speed. i.e.

$$\text{Distance} \propto \text{Speed} \Rightarrow \frac{D_1}{D_2} = \frac{V_1}{V_2}$$

Therefore, if the speed of travel is doubled, the distance traveled in the same time will double. If the speed of travel is halved, the distance traveled in the same time will be halved... and so on.



In the figure above, two cars start from the same point with speeds of 40 km/h and 60 km/h. The ratio of their speed is 2: 3. Therefore, after any time interval t, the distance traveled by them would be in the ratio 2: 3, as shown in the figure.



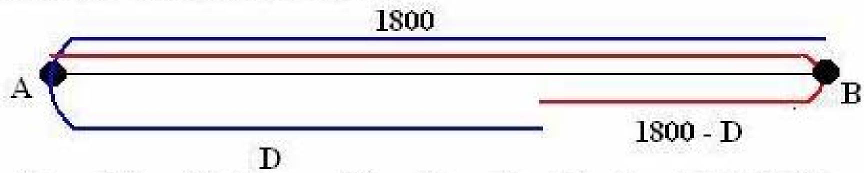
In the figure above, the two cars start simultaneously from two points, A and B, towards each other. The ratio of their speed is 2: 3. Therefore, after any time interval t, the distance traveled by them would be in the ratio 2: 3, as shown in the figure.

Let's put this principle into practise:

Examples:

Two cars simultaneously start traveling towards each other at 40 km/h and 50 km/h from two points A and B, respectively. The distance AB is 1800 km. At what distance from A do the cars meet for the first time?
Answer: Since the cars are taking equal time for travel, the distances traveled by them will be in the ratio 4: 5. The total distance is 1 800 km. Therefore, the distance traveled by the car from A will be equal to $\frac{4}{4+5} \times 1800 = 800$. Therefore, the cars will meet at a distance of 800 km from A.

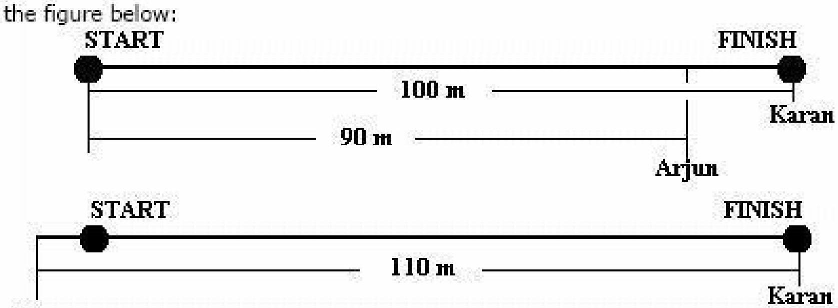
In the previous question, the two cars do not stop after meeting but continue on their way. After reaching the opposite end points, they again return along the path. At what distance from A will the cars meet for the second time?
Answer: The situation is summarized in the figure shown below:



Let the two cars meet at a distance D from A for the second time. The car from A has traveled a total distance of 1800 + 1800 – D. The car from B has traveled a total distance of 1800 + D.
Therefore, the total distance traveled by the two cars = 1800 + 1800 – D + 1800 + D = 3 × 1800 = 5400 km. Since the two cars are still traveling for the same time, the distances traveled will again be in the ratio of 4: 5. Therefore, the distance traveled by the car from B is equal to $\frac{5}{4+5} \times 5400 = 3000$. 3000 = 1800 + 1200. Therefore, the car from B travels 1800 from B to A and then a further distance of 1200 km. Therefore, the two cars meet at a distance of 1200 km from A.

Karan and Arjun run a 100-metre race, where Karan beats Arjun by 10 metres. To do a favour to Arjun, Karan starts 10 metres behind the starting line in a second 100-metre race. They both run at their earlier speeds. Which of the following is true in connection with the second race? **(CAT 2004)**
1. Karan and Arjun reach the finishing line simultaneously.
2. Arjun beats Karan by 1 metre.
3. Arjun beats Karan by 11 metres.
4. Karan beats Arjun by 1 metre.

Answer: The situation is shown in the figure below:



In the first case, Karan travels 100 m and Arjun travels 90 m in the same time. Therefore, the ratio of their speeds is 10: 9. In the second case, Karan is traveling 110 m to reach the finish line. In the same time, the distances traveled by both of them will again be in the same ratio as that of their speeds.

$$\frac{D_{Arjun}}{D_{Karan}} = \frac{V_{Arjun}}{V_{Karan}} \Rightarrow \frac{D_{Arjun}}{110} = \frac{9}{10} \Rightarrow D_{Arjun} = 99 \text{ m}$$

Therefore, Arjun has traveled only 99 m when Karan reaches the finish line. Hence, Karan beats Arun by 1 m.

Ram and Shyam run a race between points **A** and **B**, 5 km apart. Ram starts at 9 a.m. from **A** at a speed of 5 km/hr, reaches **B**, and returns to **A** at the same speed. Shyam starts at 9:45 a.m. from **A** at a speed of 10 km/hr, reaches **B** and comes back to **A** at the same speed. **(CAT 2005)**

At what time do Ram and Shyam first meet each other?

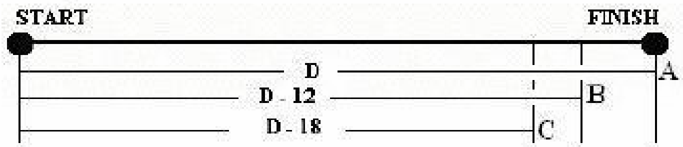
1. 10 a.m. 2. 10:10 a.m. 3. 10:20 a.m. 4. 10:30 a.m.

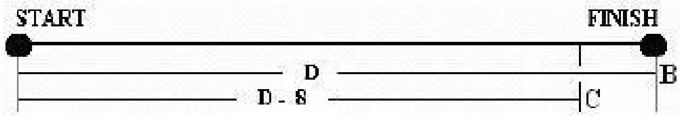
Answer: Ram takes one hour to travel 5 km and hence reaches **B** at 10:00 am. By 10:00 am, Shyam has traveled 2.5 km and is therefore, halfway through. After 10:00 am, Ram will start traveling back towards **A** and Shyam will be traveling towards **B.** Now, when they meet, they would have taken the same time for travel (after 10:00 am that is). Therefore, distance traveled by Shyam would be twice the distance traveled by Ram (Shyam's speed is twice that of Ram's speed). The initial gap between them at 10:00 am is half of **AB**. Now Shyam will travel 2/3rd of that distance and Shyam will travel 1/3rd of that distance. As Shyam travels half the distance in 15 minutes, he will travel 2/3rd of that distance in 2/3) × 15 = 10 min. Therefore, they will meet at 10:10 am.

Three runners A, B and C run a race, with runner A finishing 12 m ahead of runner B and 18 m ahead of runner C, while runner B finishes 8 m ahead of runner C. Each runner travels the entire distance at a constant speed. What was the length of the race? **(CAT 2001)**

1. 36 m 2. 48 m 3. 60 m 4. 72 m

Answer: The two cases are shown in the figure below:





When A reaches the finish line, B is D – 12 m behind and C is D – 18 m behind. Since the time taken is same the ratio of speeds of B and C is $\frac{D-12}{D-18}$. When B reaches the finish line, C is D – 8 m behind. The ratio of speeds of B and C is $\frac{D}{D-8}$.

$$\frac{D-12}{D-18} = \frac{D}{D-8} \Rightarrow D = 48 \text{ m}$$

A train approaches a tunnel AB. Inside the tunnel is a cat located at a point that is 3/8 of the distance AB measured from the entrance A. When the train whistles the cat runs. If the cat moves to the entrance of the tunnel A, the train catches the cat exactly at the entrance. If the cat moves to the exit B, the train catches the cat exactly at the exit. The speed of the train is greater than the speed of the cat by what order? **(CAT 2002)**

1. 3: 1 2. 4: 1 3. 5: 1 4. None of these

Answer: Let the train be at a distance y from A. Let the length of the tunnel AB be 8x. Therefore, the cat is at 3x from A. Now both the conditions given in the questions assume 'same time' scenario. Therefore, the ratio of the speeds of the cat and the train will be equal to the ratio of the distances traveled by them.

The ratio = $\frac{y}{3x} = \frac{y+8x}{5x} \Rightarrow y = 12x$. Therefore, the ratio of speeds $= \frac{y}{3x} = 4:1$

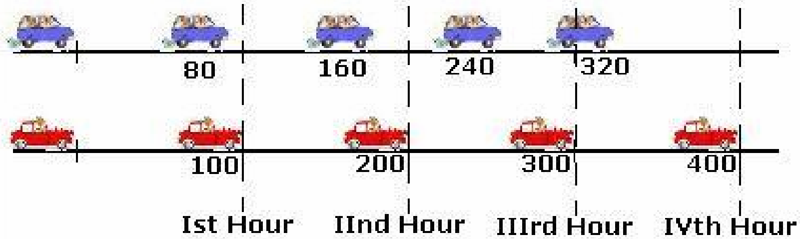
A train X departs from station A at 11.00 a.m. for station B, which is 180 km away. Another train Y departs from station B at 11.00 a.m. for station A. Tram X travels at an average speed of 70 km/hr and does not stop anywhere until it arrives at station B. Train Y travels at an average speed of 50 km/hr, but has to stop for 15 minutes at station C, which is 60 km away from station B enroute to station A. Ignoring the lengths of the trains, what is the distance, to the nearest km, from station A to the point where the trains cross each other? **(CAT 2001)**

1. 112 2. 118 3. 120 4. None of these

Answer: This question can be easily solved through a smart move. Train Y stops for 15 minutes. In 15 minutes it would have traveled 12.5 km. Therefore, let's us start train Y 12.5 km behind station B. Therefore, the total distance between the trains is 192.5 km. Now we can divide this distance in the ratio of their speeds, 7: 5. Therefore, the train X travels $\frac{7}{7+5} \times 192.5 \approx 112$ km

Relative Speed

In the figure given below, two cars start from the same point with speeds of 100 km/h and 80 km/h, respectively, in the same direction.



We can see that every hour the faster car covers 20 km more than the distance traveled by the slower car. Therefore, the distance between the slower car and the faster car increases by 20 km every hour. The gap between the cars is 20 km in one hour, 40 km in two hours, 60 km in three hours, and so on. Note that **the gap between the cars does not depend on their absolute speeds but the difference between their speeds**. The gap would still be increasing by 20 km every hour if the speeds of the cars were 90 km/h and 70 km/h. After four hours, the gap between the cars is 80 km.

If at this moment, we press the rewind button, the cars will start going backwards and the faster car will now be **chasing** the slower car. The initial distance between the faster car and the slower car is 80 km. The faster car will be reducing the gap between the cars by 20 km every hour. Therefore, to reduce the gap of 80 km, it will take 4 hours. From this analysis we deduce that

$$\text{Time taken} = \frac{\text{Initial gap}}{\text{Distance reduced per unit time}} = \frac{80}{20} = 4$$

Notice that the time taken for the faster car to catch up with the slower car from an initial distance of 80 km will be the same as when their speeds are 90 km/h and 70 km/h, 120 km/h and 100 km/h, or 160 km/h and 140 km/h.

In the figure given below, the two cars, traveling with 80 km/h and 100 km/h, are moving towards each other with an initial distance of 360 km between them. In one hour, the distance traveled by the first car is 80 km and the distance traveled by the second car is 100 km. Therefore, every hour, the two reduce the gap by 180 km.

In one hour, the distance traveled by the first car is 80 km and the distance traveled by the second car is 100 km. Therefore, every hour, the two reduce the gap by 180 km.

$$\text{Time taken} = \frac{\text{Initial gap}}{\text{Distance reduced per unit time}} = \frac{360}{180} = 2$$

Notice that the time taken for the two cars to meet from an initial distance of 360 km will be the same as when their speeds are 90 km/h and 90 km/h, 120 km/h and 60 km/h, or 160 km/h and 20 km/h.

Notice that the distance reduced per unit time is nothing but relative speed– the sum or difference of the speeds depending on the relative direction of travel of the two bodies. Therefore,

$$\text{Time taken} = \frac{\text{Initial gap}}{\text{Relative speed}}$$

Two boats, traveling at 5 and 10 km per hour, head directly towards each other. They begin at a distance of 20 km from each other. How far apart are they (in km) one minute before they collide?

Answer: As the boats are traveling towards each other, they are reducing the gap by 15 km every hour or 1/4 km every minute. Therefore, the gap before collision will be 1/4 km.

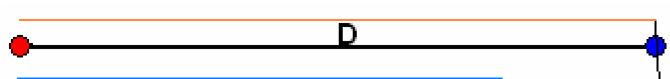
A thief ran out of a police station at 4:00 pm with a speed of 8 km/h. At 5:00 pm, a policeman ran after the thief with a speed of 10 km/h. At what time will the policeman catch the thief?

Answer: From 4:00 pm to 5:00 pm, the thief will cover a distance of 8 km. When the policeman starts running after the thief, he reduces the gap between them by 2 km per hour. Therefore, $\text{Time taken} = \frac{\text{Initial gap}}{\text{Distance reduced per unit time}} = \frac{8}{2} = 4\text{ h}$

In the above example, the policeman has a dog which runs up to the thief at 15 km/h, then returns to the policeman, again runs up to the thief and returns to the policeman, and so on, till the policeman catches the thief. Calculate

- The total distance traveled by the dog
- The total distance traveled by the dog in the forward direction.

Answer: The total distance traveled by the dog is easy to calculate. As the dog will be running for 4 h, the total distance it runs is equal to $4 \times 15 = 60$ km.



For the second part, let's see one cycle. Let the dog and the policeman be together at the red dot right now. Then the dog goes forward and catches the thief. Let it catch the thief after traveling a distance D from the red dot. Now the dog will return and

the policeman and the dog will meet somewhere in between. As the time of the travel is same the distance traveled by them will be in the ratio of their speed i.e. 10: 15 or 2: 3. The total distance covered is 2D, which we shall divide in the ratio 2: 3. Therefore, the dog travels 2D × 3/5 = 6D/5 and the policeman travels 2D × 2/5 = 4D/5. Therefore, they meet at a point which is 4/5 of the total distance. Therefore, the dog travels D in the forward direction and D/5 in the backward direction. Therefore, the ratio of the dog's forward distance to the backward distance is 5: 1.
Since the total distance traveled by the dog is 60 km, the distance in the forward direction is 60 × 5/6 = 50 km.

Average Speed

The formula for the average speed is Average speed = $\frac{\text{Total Distance}}{\text{Total Time}}$.

Let a body travel distances of d_1, d_2 and d_3 with speeds v_1, v_2 and v_3. Then the average speed would be equal to $\frac{d_1 + d_2 + d_3}{\frac{d_1}{v_1} + \frac{d_2}{v_2} + \frac{d_3}{v_3}}$.

If a body travels two equal distances with speeds v_1 and v_2, its average speed is equal to $\frac{2v_1v_2}{v_1 + v_2}$.

Cities A and B are in different time zones. A is located 3000 km east of B. The table below describes the schedule of an airline operating non-stop flights between A and B. All the times indicated are local and on the same day.

Departure		Arrival	
City	Time	City	Time
B	8:00 am	A	3:00 pm
A	4:00 pm	B	8:00 pm

Assume that planes cruise at the same speed in both directions. However, the effective speed is influenced by a steady wind blowing from east to west at 50 km per hour.

1. What is the time difference between A and B?

(1) 1 hour and 30 minutes (2) 2 hours (3) 2 hours and 30 minutes
(4) 1 hour (5) Cannot be determined

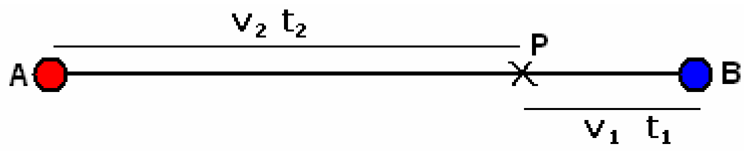
2. What is the plane's cruising speed in km per hour?

(1) 700 (2) 550 (3) 600 (4) 500 (5) Cannot be determined.

The flight leaves city B at 8:00 am and comes back at 8:00 pm taking 12 hours. Counting 1 hour of stoppage time (from 3:00 to 4:00), the flight takes 11 hours for the back and forth trip. Therefore, the average speed is 3000/11 km/h. Let the speed of the plane be v. Therefore, its speed from B to A and back are v – 50 and v + 50.

$\Rightarrow \frac{3000}{11} = \frac{2(v-50)(v+50)}{2v}$. Solving we get v = 550 km/h. Also the time difference between A and B = 1 hr.

Crossing of Two bodies (special case)



Let two bodies start from A and B simultaneously towards each other with speeds v_1 and v_2, respectively. After meeting at point P, they take time t_1 and t_2 to reach their destinations.
The distance AP = v_2t_2 and distance PB = v_1t_1.

Now since the bodies start simultaneously, they take the same time to meet at P.

$$\Rightarrow \frac{AP}{v_1} = \frac{PB}{v_2} \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{t_2}{t_1}}.$$

This is a special case to remember.

One day I started from A to B at exactly 12 noon. My friend started from B to A at exactly 2:00 pm. We met on the way at five past four and reached our destinations at exactly the same time. What time was it?

Answer: I have traveled for 245 minutes whereas my friend has traveled for 125 minutes when we meet. If we reverse the direction of travel, both my friend and I started at the same time, crossed each other and took 245 minutes and 125 minutes to reach our destinations. Therefore, $\frac{v_1}{v_2} = \sqrt{\frac{125}{245}} = \frac{5}{7}$. Therefore, the distance that I traveled in 245 minutes, my friend will travel in 245 × 5/7 = 175 minutes. Therefore, total time of travel for my friend = 125 + 175 = 300 minutes. Therefore, we reach our destinations at 7:00 pm.

Upstream and Downstream Cases/ Escalator

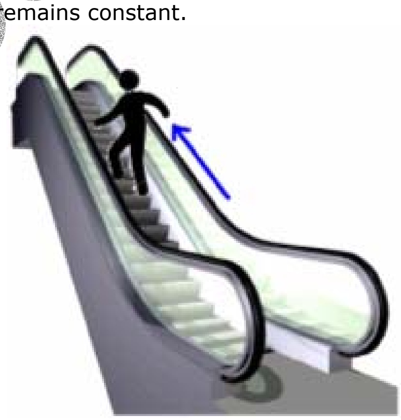
When a boat travels in the same direction as the river current, we say that it is traveling downstream. Thus if v is the speed of the boat in still water, and u is the speed of the current, then its total speed is
Downstream speed = v + u When a boat travels against the river current, it travels upstream. In this case, its total speed is
Upstream speed = v − u

A motorboat can go 10 miles downstream on a river in 20 mins. It takes 30 mins for this boat to go back at the same 10 miles. Find the speed of the current.
Answer: as the distance traveled is constant, the time taken is inversely proportional to speed.

$$\Rightarrow \frac{v+u}{v-u} = \frac{30}{20} \Rightarrow \frac{v}{u} = 5 \Rightarrow v + u = 6u = 10/20/60 = 30 \text{ m/h} \Rightarrow u = 5 \text{ m/h}.$$

Escalators

Escalators are simple application of 'upstream' and 'downstream' concept with a twist- the distance to be traveled, i.e. the total length of the escalator (measured in the number of stairs) remains constant.



If you think about it, a person will climb more stairs than the total length if he's climbing against the escalator movement and he'll climb less stairs than the total length if he's climbing in the same direction as the escalator movement. Let a person's climbing speed be v stairs/minute and the escalator speed be u stairs/minute. Let the person climb N stairs in total to cover the escalator. To climb N stairs he'll take time t = N/v minutes. In this time, escalator will throw out (N/v) × u stairs.

Case 1: Climbing against the escalator- Total length of escalator = Number of stairs climbed – stairs coming out of the escalator in the same time = N – (N/v) × u

Case 2: Climbing with the escalator- Total length of escalator = Number of stairs climbed + stairs coming out of the escalator in the same time = N + (N/v) × u

I am in the habit of walking up the subway escalator while it is running. I climb 20 steps at my normal pace and it takes me 60 seconds to reach the top, whereas my wife climbs only 16 steps and it takes her 72 seconds to reach the top. If the escalator broke down tomorrow, how many steps would I have to climb?

Answer: Let the speed of the escalator be u stairs/ sec. Since the length of the escalator is constant
20 + 60u = 16 + 72u ⇒ u = 1/3 stairs/ sec. Therefore, the total length of the escalator = 20 + 60u = 40 stairs.

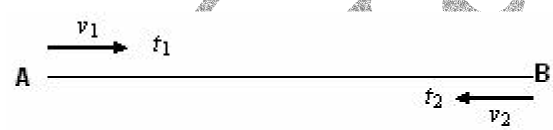
Use of Arithmetic and Harmonic Progression in Time, Speed and Distance

Here is a textbook situation in Time, Speed and Distance: A man goes from point A to point B with velocity v_1 and returns with velocity v_2. What is his average velocity?

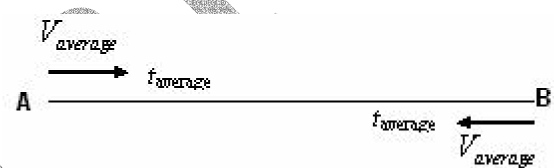
Using the formula $Average\,Velocity = \dfrac{Total\,Distance}{Total\,Time}$, the average velocity can be found to be $V_{average} = \dfrac{2v_1v_2}{v_1 + v_2}$. So far so good.

Ever wondered why we get this result? Why do we get the velocity as the harmonic mean of the two velocities? The answer lies in basics of arithmetic and harmonic progressions.

Let t_1 and t_2 be the time taken while going from A to B and coming back. The situation is shown below:



Now what will a hypothetical average velocity in this situation mean? It will mean that a person takes the same time, $t_{average}$, while going from A to B and coming back. The situation is summarized below:



The total time taken will be same as $t_1 + t_2$. or $2 \times t_{average} = t_1 + t_2$.

In another words, ***t_1*, *$t_{average}$*, and *t_2* will be in arithmetic progression**.

So how is this related to velocity?

Remember that when distance is constant velocity is inversely proportional to time?

I.e. V is proportional to 1/T or V = k/T.

If T_1, T_2, and T_3 were in **arithmetic progression**, then $1/T_1$, $1/T_2$, $1/T_3$ are in **harmonic progression** => k/T_1, k/T_2, k/T_3 are in **harmonic progression =>** V_1, V_2 and V_3 are in **harmonic progression!**

It can also be proved that if V_1, V_2 and V_3 are in **arithmetic progression**, then T_1, T_2, and T_3 are in **harmonic progression.**

So here's the rule:-

Given that the distance between two points is constant,

- If the velocities are in **arithmetic progression**, then the times taken are in **harmonic progression**.
- If the times taken are in **arithmetic progression**, then the velocities are in **harmonic progression**.

Now let's apply these rules in practical problems:-

1. If a man cycles at 10 km/hr, then he arrives at a certain place at 1 p.m. If he cycles at 15 km/hr, he will arrive at the same place at 11 a.m. At what speed must he cycle to get there at noon? **(CAT 2004)**

A. 11 km/h **B.** 12 km/h **C.** 13 km/h **D.** 14 km/h

Answer: Do you see that the times taken are in arithmetic progression with common difference of 1 hr?
Therefore, the velocities will be in harmonic progression.
$\Rightarrow$ the velocity needed to reach at 12:00 will be the harmonic mean of velocities needed to reach at 11:00 am and 1:00 pm.

$$\Rightarrow v = \frac{2 \times 10 \times 15}{10 + 15} = 12 \text{ km/h}$$

2. Arun, Barun and Kiranmala start from the same place and travel in the same direction at speeds of 30, 40 and 60 km per hour respectively. Barun starts two hours after Arun. If Barun and Kiranmala overtake Arun at the same instant, how many hours after Arun did Kiranmala start? **(CAT 2006)**

A. 3 **B.** 3.5 **C.** 4 **D.** 4.5 **E.** 5

Answer: Do you see that the speeds are in harmonic progression? Also, since Arun, Barun and Kiranmala are starting from a same point and meeting at a same point, they are traveling equal distances. Hence, time taken will be in arithmetic progression. Hence, Kiranmala will start 2 hours after Barun or 4 hours after Arun.

3. A boy is walking along the direction of two parallel railway tracks. On one of these tracks, trains are going in one direction at equal intervals. On the other track, trains are going in the opposite direction at the same equal intervals. The speed of every train is same. In one direction, a train crosses the boy every 20 minutes and, in the opposite direction, a train crosses the boy every 30 minutes. If the boy stands still beside the tracks, at intervals of how many minutes will two consecutive trains going in the same direction cross him?

A. 22 **B.** 24 **C.** 25 **D.** 26

Answer: Let the speed of a train be v and the walking speed of the boy be u. The train going in same direction as that of the boy will have a relative speed of $v - u$. A train going in the opposite direction as that of the boy will have a relative speed of $v + u$. If the boy stops moving, every train will have a speed of v relative to the boy.

Note that $v - u$, v, $v + u$ are terms of an arithmetic progression. Hence the times will be in harmonic progression. Hence, time interval when the boy stands still = harmonic mean of 20 and 30 = $\frac{2 \times 20 \times 30}{20 + 30}$ = 24 minutes.

4. A crew can row a certain course up stream in 84 minutes; they can row the same course down stream in 9 minutes less than they could row it in still water: how long would they take to row down with the stream?

Answer: Let the velocity of the boat be v and the velocity of the boat be u. Then, the up stream, still water, and down stream velocities will be $v - u$, v and $v + u$, respectively. As the velocities are in arithmetic progression and the distance is constant, the times will be in harmonic progression.

$\Rightarrow$ still water time = harmonic mean of up stream time and down stream time

Let the still water time be t. therefore, down stream time = $t - 9$.

$\Rightarrow t = \frac{2 \times 84 \times (t-9)}{84 + t - 9} \Rightarrow$ t = 72. Hence, down stream time = $t - 9 = 63$.

5. Climbing up an escalator going down, Ram takes 60 minutes to reach from ground floor to the first floor. Climbing down on the same escalator going down, Ram takes 30 minutes to reach the ground floor from the first floor. How many minutes will Ram take to reach the first floor from the ground floor if the stairs were still?

Don't you already know the answer? ☺

CLOCKS

The concept of relative speed can be applied to solve the problem of clocks. The two hands of a clock are nothing but two runners running around a circular track.

While solving the problem of clocks, we remember the following things:

- The distance is measured in degrees. There are 30 degrees between any two consecutive hour positions. At 4:00 o clock, the distance between the minute hand and the hour hand is 120°. At 7 o clock, the distance between the minute hand and hour hand is 210°, and so on.
- The speed is measured in degrees per minute. The minute hand rotates one full circle in one hour, or it travels 360° in 60 minutes. Therefore, the speed of the minute hand is 6°/min. The hour hand travels 30° (from one hour position to the next) in one hour. Therefore, the speed of the hour hand is 1/2°/min.
- The relative speed of the minute hand with respect to the hour hand is 6°/min – 1/2°/min = 11/2°/min. Therefore, the minute hand chases the hour hand around the circle with a relative speed of 11/2°/min

At what time between 4:00 pm and 5:00 pm, will the minute hand and the hour hand meet?
Answer: At 4:00 pm, the distance between the minute hand and the hour hand is 120°, as shown below:

Therefore, the minute hand is 120° behind at 4:00 pm and then it starts closing the gap by 11/2° every minute. Therefore, the time taken to catch up with hour hand completely = $\frac{\text{Initial gap}}{\text{Distance reduced per unit time}} = \frac{120}{\frac{11}{2}} = \frac{240}{11} = 21\frac{9}{11}$ minutes. Therefore, at $21\frac{9}{11}$ minutes past 4:00, the two hands will be together.

Chinmanbhai starts a trip when the hands of the clock are together between 8 am and 9 am. He arrives at his destination between 2 pm and 3 pm when the hands are exactly 180° apart. How long did the trip take?
Answer: At 8:00 am, the distance between the minute hand and the hour hand is 240°. It takes the minute hand $\frac{240}{\frac{11}{2}} = \frac{480}{11} = 43\frac{7}{11}$ min to catch up with the hour hand. Therefore, at $43\frac{7}{11}$ minutes past 8:00, the two hands are together. At 2:00 pm, the distance between minute hand and hour hand is 60°. Therefore, first the minute hand has to cover this gap of 60° and then get ahead by further 180° to make an angle of 180° with the hour hand. Therefore, the total relative distance it needs to travel = 240° with respect to the hour hand. Therefore, time taken = $\frac{240}{\frac{11}{2}} = \frac{480}{11} = 43\frac{7}{11}$ min. Therefore, at $43\frac{7}{11}$ minutes past 2:00, the two hands are 180° apart. Therefore, the total time taken for the trip = time taken from $8:43\frac{7}{11}$ to $2:43\frac{7}{11}$ = 6 hours.

What is the time interval between two successive meets of the minute hand and the hour hand?

Answer: Once the minute hand and the hour hand are together, the minute hand starts increasing the gap between the hour hand and itself by 11/2° every minute. Therefore, when it has increased the gap by 360°, it again meets the hour hand. The time taken to increase the gap by 360° = $\frac{360}{\frac{11}{2}} = \frac{720}{11} = 65\frac{5}{11}$ minutes. Therefore, the minute hand and the hour hand meet every $65\frac{5}{11}$ minutes.

TG left for Dagny's house between 5:00 pm and 6:00 pm and when he returned at some time between 8:00 pm and 9:00 pm, he noticed that the minute hand and hour hand had interchanged their positions. What time did TG leave?

Answer: The positions are shown in the figure below:

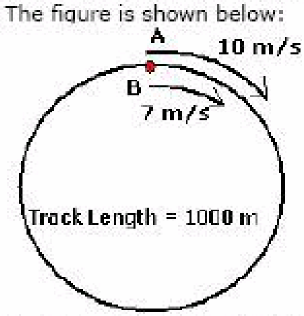
Let the angle between the minute hand and the hour hand be θ initially. To bring the hour hand from between 5 and 6 to between 8 and 9, the minute hand travels two complete circles and the angle 360 − θ. Therefore, the total distance traveled by the minute hand in degrees is $360 + 360 + 360 - \theta = 1080 - \theta$. As the speed of the minute hand is 6 degrees per minute, the time taken for travel = $\frac{1080-\theta}{6}$. In this time, the hour hand travels only θ. As the speed of the hour hand is 1/2 degrees per minute, the time taken for travel is 2θ.

$\Rightarrow \frac{1080-\theta}{6} = 2\theta \Rightarrow \theta = \frac{1080}{13}$. Therefore, the minute hand is 1080/13 degrees ahead of the hour hand between 5:00 and 6:00. At 5:00, the distance between the minute hand and the hour hand is 150°. Therefore, to get ahead by 1080/13 degrees, the minute hand will have to travel a relative distance of 150 + 1080/13 degrees = 3030/13 degrees. The time taken to get ahead = $\frac{\frac{3030}{13}}{\frac{11}{2}} = \frac{6060}{143} = 42\frac{54}{143}$ minutes.

Therefore, TG left at $5:42\frac{54}{143}$

CIRCULAR MOTION

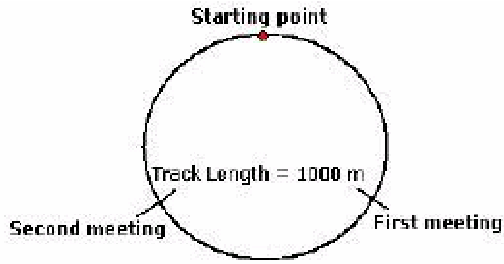
Let's examine two runners running on a circular track, of length 1000 m, in the same direction. The runners, A and B, start from the same point with speeds of 10 m/s and 7 m/s, respectively. The figure is shown below:



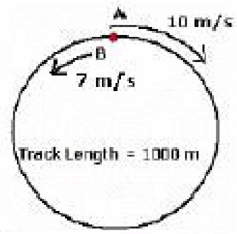
A completes the circle in 1000/10 s and B completes the circle in 1000/7 s. Therefore, A is at the starting point every 1000/10 = 100 s, i.e. A is at the starting point after 100 s, 200 s, 300 s... and so on. B is at the starting point after every 1000/7 s, i.e. B is at the starting point after 1000/7 s, 2000/7 s, 3000/7 s... and so on. **The LCM of 100 and 1000/7 is 1000. Therefore, both A and B are together at the starting point again after 1000 s.**

The moment A and B start from the starting point, A starts getting ahead of B. Every second, A increases the gap between B and him by 3 m. Therefore, the gap between A and B after is 3 m after 1 s, 6 m after 2s, 9 m after 3 s, and so on. When A increases the gap between B and him by 1000 m, he will catch up with B again. **Therefore, time taken for A to catch up with B again is =** $\frac{1000}{3}$ s.

In $\frac{1000}{3}$ s, A travels a distance of $10 \times \frac{1000}{3} = \frac{10000}{3}$ m and B travels a distance of $7 \times \frac{1000}{3} = \frac{7000}{3}$ m. Now, $\frac{1000}{3} = 1000 \times 3\frac{1}{3}$ and $\frac{7000}{3} = 1000 \times 2\frac{1}{3}$. Therefore, A takes $3\frac{1}{3}$ rounds and B takes $2\frac{1}{3}$ rounds (notice that A takes **exactly** one round more than B to catch B again, which is to be expected). Now, A keeps catching B after every 1000/3 s. **Therefore, A and B first meet after 1000/3 s at the first meeting point (shown in the figure), after 2000/3 s at the second meeting point (shown in the figure) and after 3000/3 = 1000 s at the starting point.** We have verified this result already, i.e. A and B meet at the starting point after 1000 s.



If A and B start running in the opposite directions from the starting point, they will reduce the gap between them by 17 m every second. Therefore, time taken for them to meet = 1000/17 s. Thus, A and B will meet every1000/17s. Note that, they would still meeting at the starting point every 1000 s.



TIME and WORK

Problems on Time and Work are a common feature in most of the standard MBA exams. If you are well versed with the basics and have practised these problems during your preparation, they give you an easy opportunity to score and also save time. Here, I will try and give you the basic fundas with the help of examples. Let us start with a very basic problem:

Problem 1: A takes 5 days to complete a piece of work and B takes 15 days to complete a piece of work. In how many days can A and B complete the work if they work together?

Standard Solution: Let us consider Work to be 1 unit. So if W = 1 Unit and A takes 5 days to complete the work then in 1 day A completes 1/5th of the work. Similarly B completes 1/15th of the work.
If they work together, in one day A and B can complete (1/5 + 1/15 = 4/15) of the work. So to complete 1 unit of work they will take 15/4 days.

New method: Let us assume W = 15 units, which is the LCM of 5 and 15.

Given that total time taken for A to complete 15 units of work = 5 days
--> A's 1 day work = 15/5 = 3 units
Given that total time taken for B to complete 15 units of work = 15 days
--> B's 1 day work = 15/15 = 1 unit
-->(A + B)'s 1 day work = 3 + 1 = 4 units
-->15 units of work can be done in 15/4 days.

Many solve Time and Work problems by assuming work as 1 unit (first method) but I feel it is faster to solve the problems by assuming work to be of multiple units (second method). This would be more evident when we solve problems which are little more complex than the above one.

Problem 2: X can do a work in 15 days. After working for 3 days he is joined by Y. If they complete the remaining work in 3 more days, in how many days can Y alone complete the work?

Solution: Assume W = 15 units.
(**Note:** You can assume work to be any number of units but it is better to take the LCM of all the numbers involved in the problem so that you can avoid fractions)
X can do 15 units of work in 15 days
-->X can do 1 unit of work in 1 day

(Note: If I had assumed work as 13 units for example then X's 1 day work would be 13/15, which is a fraction and hence I avoided it by taking work as 15 units which is easily divisible by 15 and 3)
Since X worked for 6 days, total work done by X = 6 days × 1 unit/day = 6 units.
Units of work remaining = 15 – 6 = 9 units.
All the remaining units of work have been completed by Y in 3 days
-->Y's 1 day work = 9/3 = 3 units.
If Y can complete 3 units of work per day then it would take 5 days to complete 15 units of work. So Y takes 5 days to complete the

work.

Problem 3: A, B and C can do a piece of work in 15 days. After all the three worked for 2 days, A left. B and C worked for 10 more days and B left. C worked for another 40 days and completed the work. In how many days can A alone complete the work if C can complete it in 75 days?

Solution: Assume the total work to be 600 units. (LCM of all the numbers)
Then C's 1 day work = 8 units.
-->(A + B + C)'s 1 day work = 40 units.
A, B, C work together in the first 2 days
-->Work done in the first 2 days = 40 × 2 = 80 units
C alone works during the last 40 days
-->Work done in the last 40 days = 40 × 8 = 320 units
Remaining work = 600 – (320 + 80) = 200 units
This work is done by B and C in 10 days.
-->(B + C)'s 1 day work = 20 units
-->A's 1 day work = (A + B + C)'s 1 day work – (B + C)'s 1 day work = 40 units – 20 units = 20 units
-->A can do the work of 600 units in 30 days.

Problem 4: Gerrard can dig a well in 5 hours. He invites Lampard and Rooney who can dig 3/4th as fast as he can to join him. He also invites Walcott and Fabregas who can dig only 1/5th as fast as he can (Inefficient gunners you see) to join him. If the five person team digs the same well and they start together, how long will it take for them to finish the job?

Solution: Let the work be 100 units.
Gerrard's 1 hour work = 100/5 = 20 units
Lampard and Rooney's 1 day work = 3/4 × 20 = 15 units.
Fabregas and Walcott's 1 day work = 1/5 × 20 = 4 units.
Þ In one day all five of them can do = 20 + 15 + 15 + 4 + 4 = 58 units of work. Hence they can complete the work in 100/58 days.

I hope you got the knack of it. Let us now see how to solve the second kind of problems in Time and Work – the **MANDAYS** problems.

In these kinds of problems we need to remember that the number of men multiplied by the number of days that they take to complete the work will give the number of mandays required to complete the work. The number of mandays required to complete a piece of work will remain constant. We will try and understand this concept by applying it to the next three problems.

A Very simple problem to start with:

Problem 5: If 10 men take 15 days to complete a work. In how many days will 25 men complete the work?

Solution: Given that 10 men take 15 days to complete the work. So the number of mandays required to complete the work = 10 × 15 mandays. So assume W = 150 mandays.

Now the work has to be done by 25 men and since W = 150 mandays, the number of days to complete the work would be 150/25 = 6 days.

Problem 6: A piece of work can be done by 8 boys in 4 days working 6 hours a day. How many boys are needed to complete another work which is three times the first one in 24 days working 8 hours a day?

Solution: Assume the first piece of work to be 8 × 4 × 6 = 192 boy-day-hours.
The second piece of work = 3 (The first piece of work) = 3 × 192 = 576 boy-day-hours. So W = 576 boy-day-hours.
If this work has to be completed in 24 days by working 8 hours a day the number of boys required would be 576/(24 × 8) = 3 boys.

Problem 7: X can do a piece of work in 20 days working 7 hours a day. The work is started by X and on the second day one man whose capacity to do the work is twice that of X, joined. On the third day another man whose capacity is thrice that of X, joined and the process continues till the work is completed. In how many days will the work be completed, if everyone works for four hours a day?

Solution: Since X takes 20 days working 7 hours a day to complete the work, the number of day-hours required to complete this work would be 140 day-hours. Like in the two problems above, this is going to be constant throughout. So, W = 140 day-hours.
Amount of work done in the 1st day by X = 1day × 4 hours = 4 day-hours
2nd day, X does again 4 day-hours of work. The second person is twice as efficient as X so he will do 8 day-hours of work. Total work done on second day = 8 + 4 = 12 day-hours. Amount of work completed after two days = 12 + 4 = 16 day-hours.
3rd day, X does 4 day-hours of work. Second Person does 8 day-hours of work. Third person who is thrice as efficient as X does 12 day-hours of work. Total work done on 3rd day = 4 + 8 + 12 = 24 day-hours
Amount of work completed after 3 days = 16 + 24 = 40 day-hours
Similarly on 4th day the amount of work done would be 4 + 8 + 12 + 16 = 40 day-hours
Work done on the 5th day = 4 + 8 + 12 + 16 + 20 = 60 day-hours
Total work done after 5 days = 4 + 12 + 24 + 40 + 60 = 140 day-hours = W.

So it takes 5 days to complete the work.

Remember that whenever there is money involved in a problem, the money earned should be shared by people doing the work together in the ratio of total work done by each of them. Again I will explain this with the help of an example:

Problem 8: X can do a piece of work in 20 days and Y can do the same work in 30 days. They finished the work with the help of Z in 8 days. If they earned a total of Rs. 5550, then what is the share of Z?

Solution: Let work W = 120 units. (LCM of 20, 30 and 8)
X's 1 day work = 6 units
Y's 1 day work = 4 units
(X + Y + Z)'s 1 day work = 15 units.

So Z's 1 day work = 15 – (6 + 4) = 5 units
In 8 days Z would have completed 5 units/day × 8 days = 40 units of work
Since Z does 40/120 = 1/3rd of the work, he will receive 1/3rd of the money, which is 1/3 x 5550 = Rs. 1850.

Pipes and Cisterns

Problem 9: There are three hoses, A, B and C, attached to a reservoir. A and B can fill the reservoir alone in 20 and 30 mins, respectively whereas C can empty the reservoir alone in 45 mins. The three hoses are kept opened alone for one minute each in the the order A, B and C. The same order is followed subsequently. In how many minutes will the reservoir be full?

Solution: These kinds of problems can be solved in the same way as we solve problems where one or more men are involved. A, B and C are equivalent to three people trying to complete a piece of work.
The amount of work to be done would be the capacity of the reservoir. Lets assume capacity of the reservoir = W = 180 (LCM of 20, 30, 45) litres.
A can fill the reservoir in 20 mins Þ In 1 min A can fill 180/20 = 9 L. B can fill 180/60 = 6 L in a minute.
In one minute C can empty 180/45 = 4 L from the reservoir.
1^{st} Minute => A is opened => fills 9 L
2^{nd} Minute => B is opened =>fills another 6 L
3^{rd} Minute => C is opened => empties 4 L
Hence every 3 minutes => (9 + 6 – 4 =) 11 litres are filled into the reservoir.
So in 45 minutes (11 × 15 =) 165 litres are filled.
In the 46^{th} minute A is opened and it fills 9 litres. In the 47^{th} minute B is opened and it fills 6 litres.
Hence the reservoir will be full in 47 minutes.

Problem 10: There is an empty reservoir whose capacity is 30 litres. There is an inlet pipe which fills at 5 L/min and there is an outlet pipe which empties at 4 L/min. Both the pipes function alternately for 1 minute. Assuming that the inlet pipe is the first one to function, how much time will it take for the reservoir to be filled up to its capacity?
Solution: The work to be done = Capacity of reservoir = W = 30 litres
1^{st} Minute => inlet pipe opened => 5l filled
2^{nd} minute => inlet pipe closed; outlet pipe opened => 4l emptied
In 2 minutes (5 litres -4 litres =) 1l is filled into the reservoir.
It takes 2 minutes to fill 1l => it takes 50 minutes to fill 25 litres into the tank.
In the 51^{st} minute inlet pipe is opened and the tank is filled.

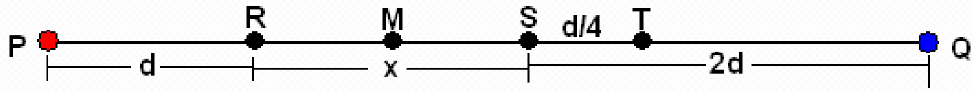
Problem 11: Sohan can work for three hours non-stop but then needs to rest for half an hour. His wife can work for two hours but rests for 15 min after that, while his son can work for 1 hour before resting for half an hour. If a work takes 50 man-hours to get completed, then approximately how long will it take for the three to complete the same? Assume all of them all equally skilled in their work.
(a) 15 (b) 17 (c) 20 (d) 24
Solution: W = 50 man-hours
Since all of them are equally skilled; in 1 hour they can do 3 man-hours of work if no one is resting. It will take them 50/3 = 16.6 hours to complete the work if they work continuously.
But, since they take breaks the actual amount of time would > 17 hours.
Option (a) and (b) are ruled out.
Now let us calculate the amount of work done in 20 hours.
Sohan does 3 man-hours in every 3.5 hours (because he takes rest for half an hour on the 4^{th} hour)
In 20 hours (3.5 × 5 + 2.5) Sohan completes => 3 × 5 + 2.5 = 17.5 man-hours ---- (**1**)
His wife completes 2 man-hours every 2.25 hours (because she rests on the 3^{rd} hour)
In 20 hours (2.25 × 8 + 2) she completes => 2 × 8 + 2 = 18 man-hours. ---- (**2**)
Child completes 1 man-hours every 1.5 hour.
In 20 hours (1.5 × 13 + 0.5) he completes 1 × 13 + 0.5 = 13.5 man-hours of work. ------ (**3**)
Adding **1, 2 & 3**
In approximately 20 hours 49 man-hours will be completed; so the work can be completed in 20^{th} hour.

SOLVED PROBLEMS

Two cars A and B start simultaneously from two points P and Q with certain speeds towards each other. After reaching a point R, speed of A decreases by 1/3. It then meets B at a point S, where SQ = 2PR. If the speed of A had become 1/3 less at the mid point of RS, the cars would have met at T where ST = PR/4. Find RS: PR.

A. 5: 1 B. 6: 1 C. 8: 1 D. 10: 1

Answer: The figure is shown below. M is the midpoint of RS with RS = x. Let the distance PR = d, SQ = 2d and ST = d/4



A starts traveling with $\frac{2}{3}^{rd}$ of his usual speed after point R and then travels a distance of x. If he had traveled with his usual speed, he would have traveled a distance of $\frac{3x}{2}$ in the same time. Since the time of travel of both A and B is the same,

When A travels d + $\frac{3x}{2}$ B travels 2d. Therefore, ratio of speeds = $\frac{d+\frac{3x}{2}}{2d} = \frac{2d+3x}{4d}$.

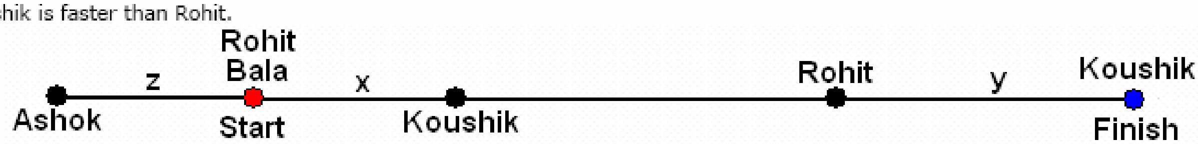
In the second case, A starts traveling with $\frac{2}{3}^{rd}$ of his usual speed after point M and then travels a distance of $\frac{x}{2}+\frac{d}{4}$. If he had traveled with his usual speed, he would have traveled a distance of $\frac{3}{2}\left(\frac{x}{2}+\frac{d}{4}\right) = \frac{6x+3d}{8}$ in the same time. The total distance traveled = d + $d+\frac{x}{2}+\frac{6x+3d}{8} = \frac{10x+11d}{8}$ In this time, B travels a distance of $2d-\frac{d}{4} = \frac{7d}{4}$. Therefore, ratio of speeds = $\frac{10x+11d}{14d}$.

$\Rightarrow \frac{10x+11d}{14d} = \frac{2d+3x}{4d} \Rightarrow$ x = 8d. Therefore, RS: PR = 8: 1

Bala and Rohit start at the starting line of a 100m track, while Koushik is given a head start of 'x' m. Ashok starts from point which is 'z' m behind the starting line. Rohit is beaten by Koushik by 'y' m. In a 100m race, Ashok can beat Bala by 'y' m. y > x and z = (100y)/(100-y). If Ashok and Koushik finished the race together, who runs the least distance of the four?

A. Koushik B. Rohit C. Bala D. Cannot be determined.

Answer: The known situation has been summarized in the figure given below. Since Koushik beats Rohit by a distance greater than the lead (y > x), Koushik is faster than Rohit.



In a 100 m race Ashok can beat Bala by y m $\Rightarrow$ When Ashok travels 100 m, Bala travels 100 – y metres.
Therefore, in this race when Ashok travels 100 + z metres, Bala will travel

$\frac{100-y}{100}\times(100+z)$ metres = $\frac{100-y}{100}\times\left(100+\frac{100y}{100-y}\right) = \frac{100-y}{100}\times\left(\frac{10000}{100-y}\right) = 100$ metres . Therefore, Ashok and Bala finish together. And Koushik finishes with them.

Koushik travels 100 – x, Rohit travels 100 – y, Bala travels 100 and Ashok travels 100 + z. The least distance is traveled by Rohit.

"When I take my dog for a walk," said a friend. "He frequently supplies me with some interesting puzzle to solve. One day he waited to see which way I should go, and when I started he raced along to the end of the road, immediately returning to me again racing to the end of the road and again returning. He did this four times in all, at a uniform speed, and then ran at my side the remaining distance, which according to my paces measured 81 metres. I afterwards measured the distance from my door to the end of the road and found it to be 625 metres. I walk at the rate of 4 km per hour.

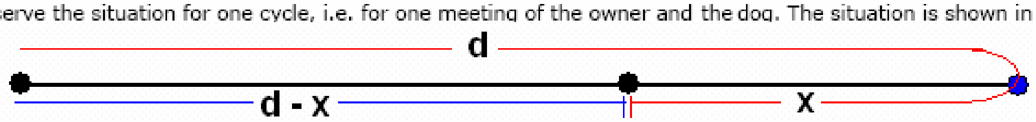
What is the speed of my dog when racing to and fro?

A. 48 kmph B. 4 kmph C. 16 kmph D. 8 kmph

The distance traveled by the dog till the time of the first reunion with me is equal to

A. 1000 m B. 850 m C. 1200 m D. 1250 m

Answer: Let's observe the situation for one cycle, i.e. for one meeting of the owner and the dog. The situation is shown in the image below:



Let the ratio of the speed of the owner to that of his dog be k. The dog travels the whole distance d and comes back a distance x to meet its owner. The owner meanwhile travels a distance equal to d – x. Therefore, the ratio of the speed = $\frac{d+x}{d-x} = k \Rightarrow x = \frac{d(k-1)}{k+1}$

For the second cycle, x would be the total distance. Therefore, the remaining distance after the second cycle $x_1 = \frac{x(k-1)}{k+1} = d\left(\frac{k-1}{k+1}\right)^2$

The remaining distance after the 3rd cycle = $x_2 = d\left(\frac{k-1}{k+1}\right)^3$

The remaining distance after the 4th cycle = $x_3 = d\left(\frac{k-1}{k+1}\right)^4$

Now, d = 625, and the remaining distance after the 4th cycle = 81.

Therefore, $625\left(\frac{k-1}{k+1}\right)^4 = 81 \Rightarrow k = 4$. Therefore, the ratio of the speeds = 4: 1

⇒ The speed of the dog = 4 × 4 = 16 kmph

The total distance traveled by the dog in the first cycle = d + x = $d + \frac{d(k-1)}{k+1} = \frac{2dk}{k+1} = \frac{2 \times 625 \times 4}{5} = 1000$ m

Mr. Sharma starts from his house to office 30 min late then his usual time. So, he increases his speed by 25% over his usual speed but still reaches 15 min late. Another day he starts 15 min later then his usual time and increases his speed by 50% comparatively to the previous day speed and reaches 20 min before the office time. What is the distance between Mr. Sharma's house and office?

A. 10Km B. 15 Km C. 5 Km D. 20Km E. 25 Km

Answer: Let me Sharma take time t to reach his office after getting 30 min late. This time will be his normal time he takes daily to reach his office. Since he increases his speed by 25% ($\frac{1}{4}$) his traveling speed becomes $\frac{5}{4}^{th}$ of his usual speed. Therefore, the time taken becomes $\frac{4}{5}t$.

Therefore, time saved = $t - \frac{4}{5}t = \frac{t}{5} = 15 \text{ min} \Rightarrow t = 75 \text{ min}$

The second information is redundant as it will give the same result of usual time taken = 75 min. I don't think distance can be calculated. Does anyone have the answer?

Angle between hour and minute hands is exactly 1 degree. The time is an integral number n of minutes after noon (0< n <720).Total possible values of n are

A. 3 B. 2 C. 1 D. 4

Answer: The relative speed of the minute hand is $\frac{11}{2}$ degrees / minute. Therefore, in one minute, the minute hand gets ahead of the hour hand by $\frac{11}{2}$ degrees. The minute hand takes $\frac{2}{11}$ minutes to gain/cover one degree on the hour hand.

To meet the hour hand again, the minute hand takes $\frac{360}{\frac{11}{2}} = \frac{720}{11}$ minutes. $\frac{2}{11}$ minutes before meeting, the minute hand would be 1 degree behind. Therefore, time taken = $\frac{720}{11} - \frac{2}{11} = \frac{718}{11}$. After meeting minute hand would take $\frac{2}{11}$ minutes to get ahead by 1 degree. Therefore, time taken = $\frac{720}{11} + \frac{2}{11} = \frac{722}{11}$

Therefore after every $\frac{720k}{11} - \frac{2}{11}$ minutes, the minute hand is one degree behind the hour hand and after every $\frac{720k}{11} + \frac{2}{11}$ minutes, the minute hand is one degree ahead of the hour hand. We need to find integral values for these times. For the first condition, we get k = 7 and n = 458 and for the second condition, we get k = 4 and n = 262. Therefore, we have two integral values of n below 720.

Without stoppage a train travels at average speed of 75 km/h and with stoppage it covers the same distance at 60 km/h. How many minutes per hour does the train stop?

Answer: Let the speed of the train be 75 km/h and the distance to be traveled be also 75 km. Therefore, in the first case, the train takes 1 h to cover the distance. In the second case, the average speed of the train = 60 km/h.

Therefore, the train travels 60 km in one hour. To travel 60 km, the train takes $\frac{60}{75} \times 60 = 48$ min. Therefore, the train stos for 60 – 48 = 12 min every hour.

In a race of 600m, Ajay beats Vijay by 60m and in a race of 500 m Vijay beats Anjay by 25m. By how many metres will Ajay beat Anjay in a 400m race?

Answer:

When Ajay travels 600m, Vijay travels 540m. Therefore, when Ajay travels 400m, Vijay travels $\frac{540}{600} \times 400 = 360\text{m}$.

When Vijay travels 500m, Anjay travels 475m. Therefore, when Vijay travels 360m, Anjay travels $\frac{475}{500} \times 360 = 342\text{m}$. Therefore, Ajay beats Anjay by 400 – 342 = 58m.

R walked down a descending escalator and took 40 steps to reach the bottom. S started simultaneously from the bottom, taking 2 steps for every 1 step taken by R. Time taken by R to reach the bottom from the top is same as time taken by S to reach the top from the bottom.

How many steps more than R did S take before they crossed each other?
If R were to walk at the speed of S, what percentage of the initial time would he be able to save?

Answer:
Not many of you realize that escalator problems are nothing but "upstream/downstream" problems with the river replaced with the escalator.

Let the speed of R be v, speed of S be 2v and speed of escalator = u
R and S are taking the same time to cover the same distance
$\Rightarrow$ v + u (downstream) = 2v - u (upstream) $\Rightarrow$ v = 2u

Therefore, let the escalator come out at the speed of 1 stair/minute. R and S cover stairs at the rate of 2 stairs/minute and 4 stairs/minute, respectively.

When R covers 40 stairs, escalator gives out 20 stairs $\Rightarrow$ total stairs = 40 + 20 = 60.

When S goes up by 4 stairs, escalator brings him down by 1 stairs. Therefore, his upstream speed is 3 stairs/minutes. They meet midway as their upstream and downstream speeds are equal. Therefore, both have covered 30 stairs.

To cover 30 stairs R covers 20 stairs and 10 stairs are given out by the escalator. S covers twice of R, i.e. 40 stairs. **Therefore, S covers 20 stairs more than R.**

Right now, R takes (60 stairs)/(3 stairs/minute) = 20 minutes. When he climbs 4 stairs/minute (Speed of S), his downstream speed will be 5 stairs/minute and time taken = 60/5 = 12 minutes.

Percentage of time saved = (8/20) × 100 = 40%

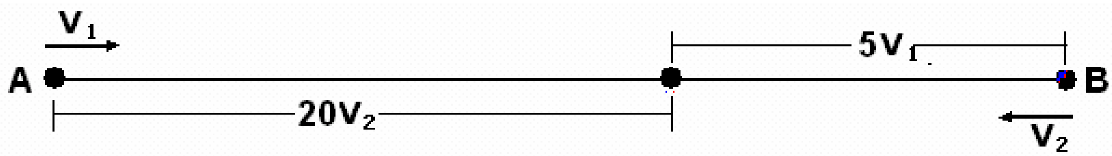
If a person increases his usual speed by 15km/h, he reaches his destination one hour earlier than his usual time. If he decreases his speed by 10km/h, he will be late by one hour. The distance traveled by him is equal to
A. 420 km B. 360 km C. 480 km D. 300 km

Answer: **1st METHOD:**
I will use the funda of arithmetic and harmonic progression here. Read about the funda in Total Gadha's Quant lessons. Let the normal time taken for travel be t. Therefore, the times taken in two cases are t – 1 and t +1. As the time taken are in arithmetic progression (t – 1, t, t + 1), the speeds (v + 15, v, v – 10) will be in harmonic progression for a fixed distance. Therefore, v would be harmonic mean of v + 15 and v – 10.
$\Rightarrow v = \frac{2(v+15)(v-10)}{2v+5} \Rightarrow 2v^2 + 5v = 2v^2 + 10v - 300 \Rightarrow v = 60\text{km/h}$. Now, 75 × (t – 1) = 50 × (t + 1) $\Rightarrow$ t = 5h.
Therefore, distance = 60 × 5 = 300km.

2nd METHOD:
Let the usual speed be v and the normal time taken be t. Distance is constant in each case.
(v + 15) × (t – 1) = vt $\Rightarrow$ 15t – v = 15 --- (1)
(v – 10) × (t + 1) = vt $\Rightarrow$ v – 10t = 10 --- (1)
Solving (1) and (2) we get v = 50 and t = 5. Therefore, distance = 300km.

Two trains start at the same time from two stations A and B towards each other. They arrive at B and A respectively in 5 hours and 20 hours after they have passed each other. If the speed of the train that started from A is 56 kmph, then the speed of the second train is equal to
A. 26kmph B. 28kmph C. 24kmph D. 30kmph

Answer: The situation is shown in the image given below. Let the speed of trains from A and B be V_1 and V_2, respectively. After assign each other they cover a distance of $5V_1$ and $20V_2$ to reach B and A, respectively, as shown.



When they meet, they have taken the same time for travel. In other words, the train from A has taken the same time to cover $20V_2$ as the train from B has taken to cover $5V_1$.

$\Rightarrow \frac{20V_2}{V_1} = \frac{5V_1}{V_2} \Rightarrow \frac{V_1}{V_2} = 2 \Rightarrow V_2 = \frac{V_1}{2} = 28\text{kmph}$

If a person increases his usual speed by 20%, he reaches his office 15 minutes early. By how many minutes will he be late to his office, if he reduces his usual speed by 20%

A. 10 B. 20 C. 15.5 D. 22.5

Answer: If the person increases his speed by 20% ($\frac{1}{5}$), his speed becomes $\frac{6}{5}$ times his original speed and therefore, his time for travel becomes $\frac{5}{6}$ times his original time. Therefore, $\frac{1}{6}^{th}$ time is saved. This is equal to 15 min. therefore, original time taken = 15 × 6 = 90 min.

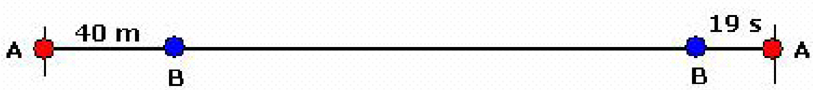
If the person decreases his speed by 20% ($\frac{1}{5}$), his speed becomes $\frac{4}{5}$ times his original speed and therefore, his time for travel becomes $\frac{5}{4}$ times his original time = $\frac{5}{4} \times 90 = 112.5\text{min}$. Therefore he will be late by 112.5 – 90 = 22.5 min.

In a kilometer race, if A gives B a head start of 40m, then A wins by 19 seconds. If A gives B a head start of 30 seconds, then B wins by 40m. The times taken by each of them to run a kilometer race (in seconds) are

A. 105, 125 B. 125, 145 C. 135, 150 D. 125, 150

I found out a great way to solve this:

Notice the first case. When A gives a lead of 40 m, he wins by 19 s.



Let's see the first case from the perspective of the finish point. When A reaches the finish point B is 19 s behind. If at this moment, I press the rewind button, both A and B will start going toward the start point with B having a lead of 19 s. And then A will beat B by 40 m. The situation can be shown below:

Therefore, when A gives a lead of 19 s to B, he beats B by 40 m. Therefore when A travels 1000 m, B travels 960 – 19v m, where v is B's speed. The speed ratio = $\frac{960 - 19v}{1000}$

The second case can be shown as below:

When A travels 960 m, B travels 1000 – 30v m. The speed ratio = $\frac{1000 - 30v}{960}$

$\frac{1000 - 30v}{960} = \frac{960 - 19v}{1000} \Rightarrow v = 40/6 \Rightarrow \text{time taken} = 150 \text{ s}$

While two trains are crossing each other, a person sitting in the slower train observes that the faster train crossed him in 24 seconds. If the speeds of the two trains are 80km/hr and 65km/hr, what is the length of the faster train?

A. 50m B. 150m C. 100m D. 450m

Answer: Relative speed of the faster train with respect to the slower train = 80 – 65 = 15 km/h = $\frac{25}{6}\text{m/s}$.

Time taken = Time taken = $\frac{\text{Relative distance}}{\text{Relative speed}} = \frac{\text{Length of the faster train}}{\text{Relative speed}} \Rightarrow$ Length of the faster train $= 24 \times \frac{25}{6} = 100$m

Two clocks are set to show the correct time at 7.00pm on a day. One clock loses two minutes in an hour and the other clock gains 3 minutes in a hour. Exactly, after how many days, will both the watches show the correct time?

A. 10 B. 15 C. 30 D. 60

Answer: Every clock will show the correct time when it is late or fast by hours which are multiples of 12. The first clock loses 2 min in 1 hour. Therefore, to lose 12 h it will take $\frac{12 \times 60}{2} = 360$ h = 15 days. The second clock gains 3 minutes in 1 h. therefore, to gain 12 h it will take $\frac{12 \times 60}{3} = 240$ h = 10 days. Therefore, both the clocks will show correct time in 30 days (LCM of 15 and 10).

A person left point A for point B . Two hours later, another person left A for B and arrived at B at the same time as the first person .Had both started simultaneously from A and B traveling towards each other , they would have met in 80 minutes .How much time did it take the faster person to travel from A to B .

Answer: Let the time taken by the second person be t minutes. Therefore, the time taken by the first person is t + 120. Let the total distance be d. The second person travels d in t minutes. Therefore he will travel $\frac{80d}{t}$ in 80 minutes. Similarly, the first person will travel $\frac{80d}{t+120}$ in 80 mins.

$$\frac{80d}{t} + \frac{80d}{t+120} = d \Rightarrow t = 120 \text{ mins} = 2 \text{ h.}$$

In a 4000 meter race around a circular stadium having a circumference of 1000 meters, the fastest runner and slowest runner reach the same point at the end of 5th minute, for the first time after the start of the race. All the runners have the same starting pt. and each runner maintains a uniform speed throughout the race. If the fastest runner runs at twice the speed of slowest runner, what is the time taken by the fastest runner to finish the race?

A. 20 min B. 15 min C. 10 min D. 5 min

Answer: As the speed of fastest runner is twice the speed of the slowest runner, the fastest runner travels twice the distance as that traveled by the slowest runner in the same time. Also, to catch the slowest runner again, the fastest runner will have to travel one round extra than the distance traveled by the slowest runner. Therefore, it can be reasoned out that the slowest runner is taking one round whereas the fastest runner is taking two rounds in 5 min. therefore, to take 4 rounds, the fastest runner will take 10 min. (**Please work out the logic. It's fun**)

A man builds $\frac{1}{8}^{\text{th}}$ of a wall everyday. Out of the length of the wall built per day, 20% falls off each day(including last day's work).In how many days he can complete the wall?

Answer: The wall would never be completed. The moment the length of the wall reaches 62.5%, 20% of the wall = 12.5% would be demolished. Therefore, the new added length would be equal to the demolished length.

A man starts a piece of work. Starting from 2nd day onwards, everyday a new man joins. With every man joining the work that each man can do per day doubles. The work is completed in 5 days. On which day they would have completed the work if the work each one can do per day remained constant.

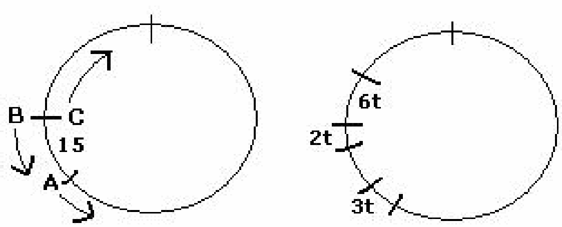
Answer: Let the initial units of work that a man can do on the first day be w. Therefore, the units of work that each man can do on 2nd, 3rd, 4th and 5th day would be 2w, 4w, 8w and 16w, respectively.
Therefore, total units of work done = w + 2 × 2w + 3 × 4w + 4 × 8w + 5 × 16w = 129w.

If the work each man could remained constant the units of work done in n days = w(1 + 2 + 3 + ...+ n) = $\frac{n(n+1)w}{2}$

$$\frac{n(n+1)w}{2} = 129w \Rightarrow n \approx 16\text{days.}$$

A and B are running in a circular track in direction opposite to which C is running, in fact running at twice and thrice the speed of A and B, respectively, and on the same track. They start running from same point. It is known that A's average speed is 3m/s and track is 120 meter in length. When will B, after start, find himself equidistant between A and C for first time?

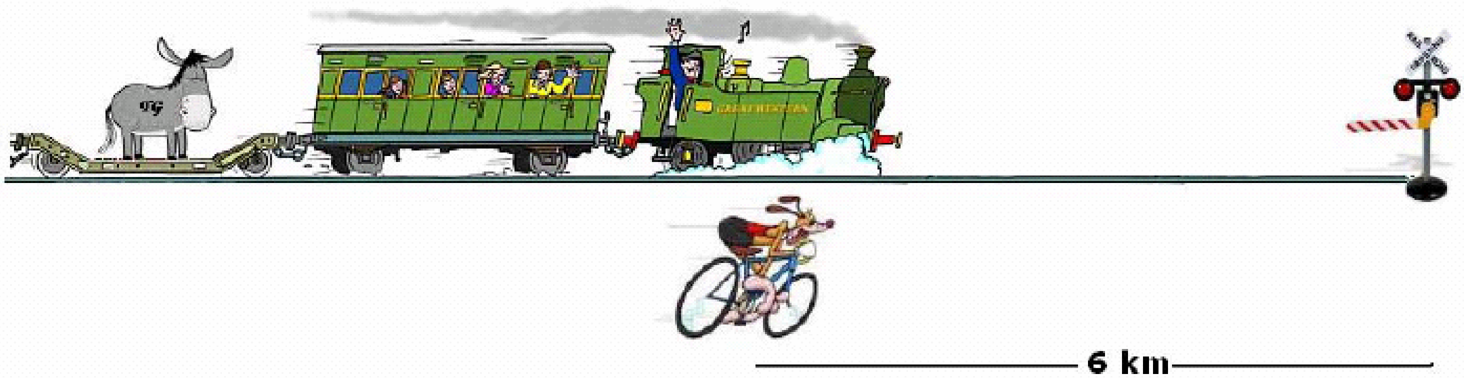
Answer:
See the figure below:



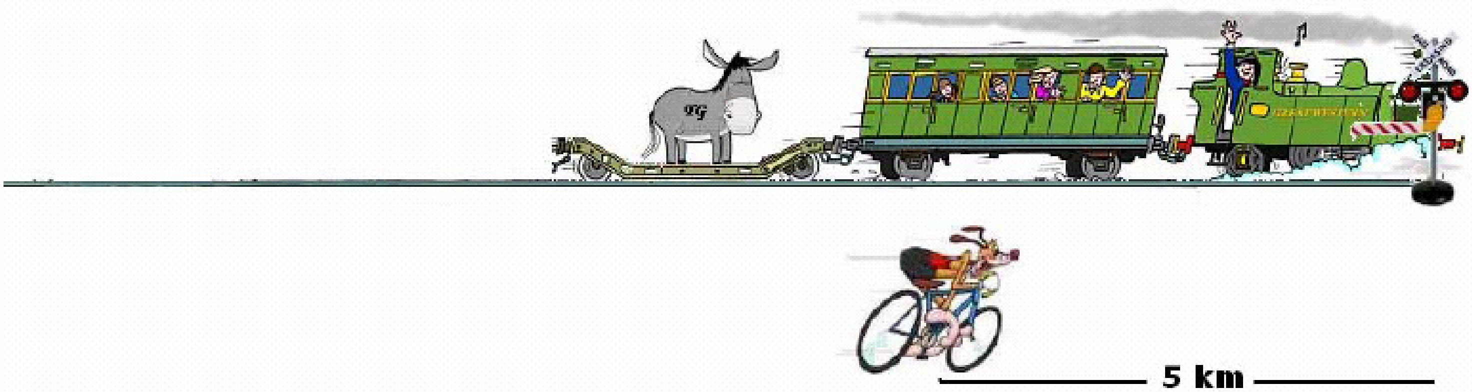
The speeds of A, B, and C are 3, 2 and 6. C will meet A first and then meet B after 120/(2 + 6) = 15 s. In 15 s A will travel 45 m and B will travel 30 m, as shown in the first figure.
Let after next t seconds, B is between A and C. The distance between B and C is 2t + 6t. The distance between A and B is 15 - 2t + 3t = 15 + t.
2t + 6t = 15 + t --> t = 15/7 s.
Total time = 15 + 15/7 = 120/7 s

A railway track runs parallel to a road and a cyclist, whose speed is 12 km/h, meets a train at the crossing everyday at the same time. One day, the cyclist started 25 min late and met the train 6 km ahead of the railway crossing. What is the speed of the train?

The first thing to note is that the train and the cyclist are traveling in the same direction. If the cyclist is 25 min late than the normal time he would be $\frac{25}{60} \times 12 = 5\text{km}$ away from the crossing when the train reaches the crossing. Therefore, if the cyclist and train were traveling in opposite direction, the train would have met the cyclist closer than 5 km from the crossing.



Now the train meets the cyclist, both train and the cyclist traveling in the same direction, when the cyclist is 6 km away from the crossing. When the cyclist is 5 km away from the crossing the train would have reached the crossing.



Therefore, while the cyclist travels 1 km, the train has traveled 6 km. Hence, the speed of the train is 6 times the speed of the cyclist. The speed of the train = 72 km/h.

Assignment

1. A train moves 59.4 km in 66 minutes. What is its speed in m/s?
a. 14 m/s b. 15 m/s c. 16 m/s d. 17 m/s e. None of these

2. If Mayank goes from Delhi to Gurgaon at a speed of 20 m/s and comes back at a speed of 48 km/h, then what his average speed during the entire journey?
a. 57.6 km/h b. 59.5 km/h c. 15 m/s d. 16.5 m/s e. None of these

3. An airplane flying 400 km covers the first 100 km at the rate of 100 km/h, the second 100 km at 200 km/h, the third 100 km at the rate of 300 km/h and the last 100 km at the rate of 400 km/h. If it returns with the same average speed as it had during its onward flight what is the time taken for the round trip?
a. 2.08 hrs b. 3.17 hrs c. 4.17 hrs d. 5 hrs e. None of these

4. If Rajesh drives his bike at a speed of 40 km/h from his office, then he reaches his home 10 min late. If he drives at 60 km/h, then he reaches 10 min early. Find the distance between his office and home?
a. 25 Km b. 30 Km c. 35 Km d. 40 Km e. None of these

5. In the above problem, Find out the speed of Rajesh if he wish to reach his home in time?
a. 24 km/h b. 30 km/h c. 38 km/h d. 40 km/h e. None of these

6. Pinku starts from a place at 3 pm at 2.5 km/h. Tinku follows him from the same place at 5 pm, walking at 3 km/h. If Pinku takes some rest on the way and is overtaken by Tinku after he has covered 24 km, then pinku took rest for
a. 20 mins b. 24 mins c. 30 mins d. 36 mins e. None of these

7. Driving at $5/8^{th}$ of his usual rate, Sanju reaches the market 18 minutes late. Find his usual time taken to reach the market?
a. 20 mins b. 24 mins c. 30 mins d. 36 mins e. None of these

8. A train running between two cities arrives at its destination 16 minutes late when it goes at 50 km/h and 10 minutes late when it goes at 75km/h. Determine the distance between the two cities?
a. 12 km b. 15 km c. 18 km d. 20 km e. None of these

9. A car has 310 km to run. After going $1/5^{th}$ of the distance, its carburetor starts troubling and it can only run the remaining part of the journey at $4/5^{th}$ of the original speed. If it arrives 2 hours late, what was its original speed?
a. 30 km/h b. 31 km/h c. 32 km/h d. 60 km/h e. None of these

10. A distance is covered at a certain speed in a certain time. If the triple of this distance is covered in four times the time, then what is the ratio of the two speeds?
a. 4 : 3 b. 2 :3 c. 3 : 5 d. 3 : 7 e. None of these

11. A man traveled a distance of 95 km in 12 hours partly on foot at the rate of 5 km/h and partly on bicycle at the rate of 12 km/h. Find the distance traveled on foot?
a. 60 km b. 55 km c. 45 km d. 40 km e. None of these

12. A train, 150 meter long, runs at the rate of 54 km/h. How long will it take to pass a platform of length 90 meter.
a. 16 sec b. 10 sec c. 12 sec d. 18 sec e. None of these

13. The distance between two towns P & Q is 200 km. A motorcyclist starts from P towards B at 8 am at a speed of 40 km/h. Another motorcyclist starts from Q towards P at 9 am at a speed of 35 km/h. At what time will they cross each other?
a. 10:48 am b. 11:00 am c. 11:08 am d. 11:38 am e. None of these

14. Two trains starts at the same time from two stations and proceed towards each other at the rate of 40 km/h and 50 km/h respectively. When they meet, it is found that one train has traveled 80 km more than the other. Find the distance between the two stations?
a. 630 km b. 650 km c. 700 km d. 720 km e. None of these

15. The distance between two towns is 200 km. Raj starts from town A towards town B at 9:00 am at a speed of 20 km/h and Abhi starts from town B towards town A at the same time at a speed of 30 km/h. At what time they cross each other?
a. 1:00 am b. 12:00 pm c. 1:00 pm d. 2:00 pm e. None of these

16. In the above problem, Find out the location of the point where they cross each other for the first time?
a. 80 km from town A
b. 100 km from town A
c. 120 km from town B

d. both (a) & (c)
e. None of the above

17. In the above problem, after crossing each other Raj continued towards town B and Abhi continued towards town A. After reaching their destination they returned towards the town from which they started and crossed each other again. Find out the location of the point where they cross each other for the second time.
a. 40 km from point A
b. 60 km from point A
c. 100 km from point A
d. 120 km from point A
e. None of the above

18. A ship is rowed 28 km down a river in 4 hours and 12 km up in 6 hours. Find the speed of the ship in still water?
a. 4.5 km/h b. 5 km/h c. 5.5 km/h d. 2.5 km/h e. None of these

19. In the above problem, determine the speed of the river?
a. 4.5 km/h b. 5 km/h c. 5.5 km/h d. 2.5 km/h e. None of these

20. A boat goes 30 km upstream and 42 km downstream in 8 hours. It also goes upstream 42 km and downstream 56 km in 11 hours. Determine the speed of the boat and the stream?
a. 8 km/h, 4 km/h
b. 10 km/h, 4 km/h
c. 10 km/h, 6 km/h
d. 8 km/h, 6 km/h
e. None of these

21. Raju takes 4 hours less to row down a 12 km stream than he takes to row up, For this 24 km roundtrip, if he double his rowing speed, he would take half an hour less to row downstream than to row upstream. Find the speed of the stream in km/h?
a. 0 km/h b. 2 km/h c. 4 km/h d. 6 km/h e. None of these

22. A man takes 7 hrs in walking to a certain place and driving back. He would have gained 3 hrs by driving both ways. How long would he take to walk both ways?
a. 6 hrs b. 8 hrs c. 10 hrs d. 12 hrs e. None of these

23. At 7:00 am, Ravish started from A towards B and Amit started from B towards A. At 9:00 am they crossed each other and continued towards their respective destination. If the total time taken by Ravish to reach his destination is three hours more than that taken by Amit. Find out the ratio of Ravish's speed to Amit's speed?
a. 1 : 2
b. 2 : 3
c. 2 : 5
d. 3 : 2
e. Cannot be determined

Directions for questions 24 and 25: Answer the questions on the basis of the information given below.

There are three cities P, Q and R, not on the same straight road. Two buses X and Y starts simultaneously from P and Q respectively towards R. By the time Y reaches R, X is exactly half way to R. Immediately after Y reaches R, it starts travelling towards P and it crosses X at a point 165 km from P. The ratio of the speeds of X and Y is 3 : 5. Assume that the roads joining P to R, Q to R and Q to P are all straight roads.

24. If initially, instead of moving towards R, X and Y starts moving towards each other, which of the following cannot be a possible distance from P at which they will cross each other?
a. 20 km b. 45 km c. 120 km d. 180 km e. None of these

25. If Q is twice as far as from P as it is from R, and X takes 18 hours and 40 minutes to cover the distance from P to Q, how much time would Y take to cover the distance from R to P?
a. 2 hours and 24 minutes
b. 3 hours
c. 3 hours and 36 minutes
d. 4 hours
e. None of these

Chetan is returning home from his friend's house and his dog is with him. It takes Chetan 45 minutes to reach his home. His dog runs twice as fast as Chetan walks. The moment Chetan and his dog leave his friend's house the dog runs ahead and reaches home before Chetan, and returns to meet his master. After meeting Chetan, the dog again runs towards home. The moment the dog reaches home it returns again to meet Chetan.

26. After how many minutes from the start of the trip does Chetan meet his dog for the second time?

a. 30 min
b. 40 min
c. 25 min
d. 33 min
e. None of these

Use the following information to answer the next question.

Kipper and Tipper are moving along two straight roads that intersect perpendicularly at X. When Kipper is at X, Tipper is 500m from X. After 2 minutes both Kipper and Tipper are at equal distance from X, and after 8 more minutes they are again at equal distance from X.

27.What is the ratio of Kipper's speed to Tipper's speed?

a .4: 5
b. 2: 3
c. 5: 8
d. 1: 2
e. None of these

Ranbir, the forest ranger, started on a four-hour drive one day on the safari. When he started the jeep his jeep's milometer reading was 29792, a palindrome. (A palindrome is a number that reads the same forward and backward.) At his destination inside the forest, the milometer reading was another palindrome.

28. If Ranbir never exceeded the safe speed limit of 55 miles/hr, which of the following was his greatest possible average speed?

a. 40 mph
b. 50 mph
c. 205/4 mph
d. 211/4 mph
e. None of these

Appu and Gappu both start running around a circular racetrack in the same direction, each of them going at a constant speed. Appu is on bicycle and Gappu is on foot. Appu goes all the way around the track and catches up with Gappu. The moment he meets Gappu, he turns around and heads back towards the starting point while Gappu keeps going on his path. When Appu reaches the starting point he again meets Gappu who is just finishing his first round.

29. The ratio of speeds of Appu and Gappu is

a. $1 + \sqrt{2}: 1$
b. 2: 1
c. 3: 1
d. $4 - \sqrt{2}: 1$
e. None of these

If Roshan and Swati leave their houses at 10:00, walking directly towards each other, they meet at 10:10. If Roshan leaves his house at 10:00 and Swati leaves her house at 10:03, again walking towards each other, they meet at 10:11.

30. At what time will Swati reach Roshan's house if she leaves her house at 10:00?

a.10: 15
b.10: 20
c.10: 24
d.10: 30
e. None of these

Ghanshyam and Sartaj leave Mathland simultaneously to go to Algebratown. Ghanshyam is on bicycle while Sartaj is on a bike that is six times faster than Ghanshyam's bicycle. Halfway between Mathland and Algebratown, Sartaj had an accident and his bike breaks down. Luckily, a farmer gave him a lift on his wagon that goes half as fast as Ghanshyam's bicycle.

31. Who reaches Algebratown first?

a. Ghanshyam
b. Sartaj
c. Both reach at the same time
d. Cannot be determined

In a kilometer race, in which A, B and C are running, A beats B by 100 meters, and B beats C by 100 meters. In the next kilometer race, C starts at the starting line, B starts 100m behind the starting line and A starts 100m behind B.

32. Who wins the race and by how many meters is the winner ahead of the person just behind him?
    a. A, 20 m
    b. B, 88 m
    c. C, 1 m
    d. It's a tie
    e. Cannot be determined

Four vehicles are traveling on a straight highway with constant speeds. The truck overtook the car at 2:00pm, and then met the scooter at 4:00pm and the motorcycle at 6:00pm. The motorcycle met the car at 7:00pm then it overtook the scooter at 8:00.

33. At what time scooter and the car meet?
    a. 5:00 pm
    b. 5:20 pm
    c. 5:30 pm
    d. 5:40 pm
    e. None of these

A column of soldiers is marching at a constant speed. A soldier at the last row of the column was commanded to go to the head of the column, deliver a message there, and return to his position at the last row of the column. After carrying out the order, when the soldier returned to his former position at the last row, he noticed that the end of the column was in the same position that the head of the column had occupied when he first left his own position.

34. If the soldier did not stop anywhere in between leaving and rejoining his position what is the ratio of the soldier's speed to that of the column?
    a. $1 + \sqrt{2}: 1$
    b. 2: 1
    c. 3: 1
    d. $4 - \sqrt{2}: 1$
    e. None of these

35. Two friends go for a Sunday bike ride. First, they averaged 12 miles per hour and then stopped to rest. On the way back, they only averaged 8 miles per hour over the same distance. Excluding resting time, what was the average speed of the riders?

36. Kate runs twice as fast as she walks. On her way to school, she walks for twice the length of time as she runs. In this way she takes 20 minutes to get there. On her way home, she runs for twice the length of time as she walks. How long does she take to get home?

37. A man rows upstream in a river. When he covered 1 km from starting point his hat fell down and started to move downstream the man kept swimming upstream for 5 minutes but realised he has dropped his hat then rows down stream to catch the hat. Finally the hat and man meet at start. Find the speed of flow of river.

38. Two cars A and B drive straight towards the same point P with speeds a and b respectively. At the start, A, B and P form an equilateral triangle. After some time A and B move to the new positions, B covers 80 miles, and ABP becomes a right triangle. At the moment when A arrives at P, B is 120 miles away from P. What is the distance between A and B in the beginning?

39. Rajesh went on a 20-mile test drive on his new Honda. He started with a certain speed. After covering each mile, he decreased his speed by 25%. If he took 8 mins and 6 seconds to cover the first 4 miles, find the time he would take to cover the next 4 miles.

40. Three cars started from the same point at the same time in three different directions. The first two cars move in a straight line. It is noticed that after 2 hours, all the cars are at the same distance from the starting point. If the distance b/w the two cars which travelled in a straight line is 170 Kms, how far from the 2nd car is the 3rd car, given that the distance b/w 1st and 3rd car is 136 Kms?

41. At his normal speed, Prashant can travel 18 miles downstream in a fast flowing stream in 9 hours less than what he takes to travel the same distance upstream. The time for the downstream trip would take 1 hour less than the upstream trip provided that he doubles his rate of rowing. What is the speed of the stream in miles/hr?

42. Crime master Gogo, riding on a motorcycle, starts at the back of a 2 km train as its front enters a 4 km tunnel. Both GoGo and the train travel at constant speed and GoGo exits the tunnel just as the train is entirely in the tunnel. When the front of the train emerges from the tunnel, Gogo turns instantly and heads back toward the train. How many meters from the tunnel does Gogo meet the front of the train?

43. A woman is walking down a downward-moving escalator and steps down 10steps to reach the bottom. Just as she reaches the bottom of the escalator, a sale commences on the floor above. She runs back up the downward moving escalator at a speed five times that which she walked down. She covers 25 steps in reaching the top. How many steps are visible on the escalator when it is switched off?

44. There is a escalator and 2 persons move down it. A takes 50 steps and B takes 75 steps while the escalator is moving down. Given that the time taken by A to take 1 step is equal to time taken by B to take 3 steps, find the no. of steps in the escalator while it is stationary?

45. Ram and Shyam are moving upwards on a moving escalator. When Ram takes 2 steps Shyam moves 3 steps in the same time. Shyam reaches the top taking 25 steps while Ram takes 20 steps to reach the top. If escalator's movement is ceased, then how many steps are needed to be taken by shyam to reach the top?

46. Joe and Charlie went on an excursion. At the end of their hike they arrived at a highway and decided to take a bus. Joe continued to walk forward to the next bus-stop at a speed of 4 km/h, while Charlie assumed that the previous stop was closer by, and thus headed in the opposite direction at a speed of 6 km/h. They each arrived just on time to catch the bus. Find out if Charlie's assumption was right, given that the bus travelled at a speed of 60 km/h.

47. In a swimming pool, 6 swimmers have to swim such that 3 swimmers start from end A at intervals of 1 minute and the remaining 3 start from end B at intervals of 2 minutes where A and B are opposite ends of the pool & length of the pool is 120 meters. The speed of each swimmer is 20m/min. Whenever two swimmers meet, each of them reverse their direction & start swimming in the opposite direction without any time delay. Each swimmer stops when he reaches one of the ends. If the first swimmer from each start simultaneously, after how many minutes from that time will there be no swimmer in the pool?

48. At 10:00 am, Jinny starts writing consecutive natural numbers, starting with 1, in a row from left to right at a rate of 60 digits per minute. At 10:15 am, Johnny starts rubbing out digits from left to right, starting from the first digit, at a rate of 90 digits per minute.

(a) What is the difference between the rightmost and the leftmost digit one minute before Johnny catches up with Jinny?

(b)If Jinny stops the moment Johnny catches up with her, what is the last digit to be erased?

49. In a city a local shuttle train starts from and arrive at the station at fixed intervals and run at uniform speed. A boy was walking down the railway track at certain speed. Every 1/2 an hour a local train overtook him and every 20 min, a local train passed him in the opposite direction. Find the time interval between a local train passing a certain point on the railway track and the immediately next local train passing that point in same direction.

50. At what time between 7:00 and 8:00 does the minute hand makes an angle of 10 degrees with the hour hand?

51. A and B working together can do a piece of work in 18 days, B and C in 24 days, C and A in 36 days. How many days will it take if all of them work together?

52. Arun and Barun start moving from the same point on a circular track in opposite directions. Arun can complete 1 round in 3 min whereas Barun can do the same in 5 min. After how much time do they meet for the first time?

53. John and Mary run in opposite directions on a circular track, starting at diametrically opposite points. They first meet after John has run 100 meters. They next meet after Mary has run 150 meters past their first meeting point. Each person runs at a constant speed. What is the length of the track in meters?

54. Two pipes can fill a cistern in 14 hours and 16 hours respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom, 32 minutes extra are taken for the cistern to be filled up. When the cistern is full in what time will the leak empty it?

55. A, B and C star running simultaneously from the same point around a circular track of length 1200 m. A and B are running in the same direction with speeds 4 m/s and 6 m/s whereas C is running in the opposite direction with 10 m/s. When will A, B and C be together for the first time after the start?

56. A and B can do a job in 10 days working together whereas B and C can finish the same job in 15 days working together. If A works for 5 days and B works for 8 days C takes 9 days to finish the job. How much time will C take to finish the complete job alone?

57. A and B can do a piece of work in 21 and 24 days respectively. They started the work together and after some days A leaves the work and B completes the remaining work in 9 days. After how many days did A leave?

58. A person leaves home between 1 and 2 and comes back at 6 and 7. He finds that the minutes and hours hand exchanged positions. Find the time at which he left.

59. Angle between hour and minute hands is exactly 1 degree. The time is an integral number n of minutes after noon ($0 < n < 720$). Total possible values of n are

60. A can complete a piece of work in 4 days. B takes double the time taken by A. C takes double that of B, and D takes double that of C to complete the same task. They are paired in groups of two each. One pair takes two-thirds the time needed by the second pair to complete the work. Which is the first pair?
1. A, B 2. A, C 3. B, C 4. A, D

61. A can give B a 40m headstart and C a 90m headstart in a km race. How many meters of headstart can B give give to c in a km race?

62. There are 12 pipes attached to a tank. Some of them are fill pipes and some are drain pipes. Each of the fill pipes can fill the tank in 12 hours, while each of the drain pipes will take 24 hours to drain a full tank completely. If all the pipes are kept open when the tank was empty, it takes 2 hours for the tank to overflow. How many of these pipes are drain pipes?

63. In a km race, if A gives B a headstart of 40m, then A wins by 19 seconds, If A gives B a headstart of 30 seconds, then B wins by 40m, Find the time taken by each to finish the race.

64. Divya and Raveena can do a work alone exactly in 20 and 25 days respectively. However, when they work together, they do 25% more work than is expected. If they work for a few days alone and for few days together (both being integers only), then the work could not have been completed in exactly
(1) 10 days (2) 14 days (3) 16 days (4) 17 days (5) either none or atleast 2 of these

65. "The hour hand on the my watch moves at the correct speed, but the minute hand moves one and a half times as fast as it should. Yesterday, it showed the correct time at 3 p. m. When did it next show the correct time?"

66. If it takes two men two hours to dig a hole three meters long, three meters wide, and three meters deep, how long would it take the same two men to dig a hole six meters long, six meters wide, and six meters deep.

67. At what time instant after 8 o'clock do the short and long hands of a clock enclose the equal angles with the horizontal?

68. A group of men working at the same rate can finish a job in 45 hours. However, the men report to work, one at a time, at equal intervals over a period of time. Once on the job, each man stays until the job is finished. If the first man works five times as many hours as the last man, find
the number of hours the first man works
the total number of men in the group

69. A pipe P can fill an empty tank in 12 min and another pipe Q, can empty same tank in 20 min. Pipe P is configured to close automatically when the water level reaches in the tank reaches 3/4th of the height of the tank and simultaneously the pipe Q opens and operates until the water level fall to exactly 1/4th of the height of the tank after which it closes and simultaneously P opens. If this process continues indefinitely, what portion of the tank is filled after exactly one hour?

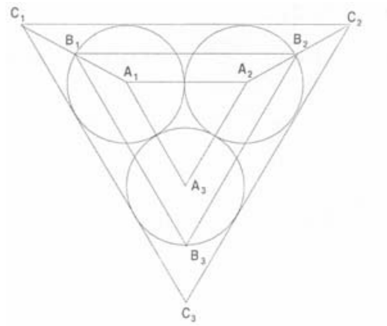
70. Two friends A and B run around a circular track of length 510 metres, starting from the same point, simultaneously and in the same direction. A who runs faster laps B in the middle of the 5^{th} round. If A and B were to run a 3 km race long race, how much start, in terms of distance, should A give B so that they finish the race in a dead heat?

71. Oarsman A rows on a river, x miles with the current and x miles against the current. Oarsman B rows 2x miles on a lake where there is no current. Assuming the rowing strength of both the oarsmen to be the same, who takes more time, A or B?

72. Motorboat M leaves shore A as N leaves B; they move across a lake at constant speed. They meet for the first time 500 yards from A. Each returns from the opposite shore without halting, and they meet 300 yards from B. How long is the lake?

73. Jack London raced from Skagway in a sled pulled by 5 huskies to reach the camp where a comrade was dying. For 24 hours the huskies pulled the sled at full speed. Then 2 dogs ran off with a pack of wolves. London, left with 3 dogs, was slowed down proportionally. He reached camp 48 hours later than he had planned. If the runaway huskies had stayed in harness for 50 more miles, he would have been only 24 hours late. How far is the camp from Skagway?

Consider three circular parks of equal size with centers at A_1, A_2, and A_3 respectively. The parks touch each other at the edge as shown in the figure (not drawn to scale). There are three paths formed by the triangles $A_1A_2A_3$, $B_1B_2B_3$, and $C_1C_2C_3$, as shown. Three sprinters A, B, and C begin running from points A_1, B_1, and C_1 respectively. Each sprinter traverses her respective triangular path clockwise and returns to her starting point.



74. Let the radius of each circular park be *r*, and the distances to be traversed by the sprinters A, B and C be *a*, *b* and *c*, respectively. Which of the following is true?

a. $b-a=c-b=3\sqrt{3}\ r$

b. $b-a=c-b=\sqrt{3}\ r$

c. $b=\dfrac{a+c}{2}=2\left(1+\sqrt{3}\right)r$

d. $c=2b-a=\left(2+\sqrt{3}\right)r$

75. Sprinter A traverses distances A_1A_2, A_2A_3, and A_3A_1 at average speeds of 20, 30, and 15, respectively. B traverses her entire path at a uniform speed of $(10\sqrt{3}+20)$. C traverses distances C_1C_2, C_2C_3, and C_3C_1 at average speeds of $\dfrac{40}{3}\left(\sqrt{3}+1\right), \dfrac{40}{3}\left(\sqrt{3}+1\right),$ and 120, respectively. All speeds are in the same unit. Where would B and C be respectively when A finishes her sprint?

a. B_1, C_1
b. B3, C_3
c. B_1, C_3
d. B_1, Somewhere between C_3 and C_1

76. Sprinters A, B and C traverse their respective paths at uniform speeds of u, v and w respectively. It is known that $u^2:v^2:w^2$ is equal to Area A: Area B: Area C, where Area A, Area B and Area C are the areas of triangles $A_1A_2A_3$, $B_1B_2B_3$, and $C_1C_2C_3$, respectively. Where would A and C be when B reaches point B_3?

a. A_2, C_3
b. A_3, C_3
c. A_3, C_2
d. Somewhere between A_2 and A_3, Somewhere between C_3 and C_1.

77. Two straight roads R1 and R2 diverge from a point A at an angle of 120°. Ram starts walking from point A along R1 at a uniform speed of 3 km/hr. Shyam starts walking at the same time from A along R2 at a uniform speed of 2 km/h. They continue walking for 4 hours along their respective roads and reach points B and C on R1 and R2, respectively. There is a straight line path connecting B and C. Then Ram returns to point A after walking along the line segments BC and CA. Shyam also returns to A after walking along line segments CB and BA. Their speeds remain unchanged. The time interval (in hours) between Ram's and Shyam's return to the point A is

a. $\dfrac{10\sqrt{19}+26}{3}$

b. $\dfrac{2\sqrt{19}+10}{3}$

c. $\dfrac{\sqrt{19}+26}{3}$

d. $\dfrac{\sqrt{19}+10}{3}$

78. A train X departs from station A at 11.00 a.m. for station B, which is 180 km away. Another train Y departs from station B at 11.00 a.m. for station A. Tram X travels at an average speed of 70 kms/hr and does not stop anywhere until it arrives at station B. Train Y travels at an average speed of 50 kms/hr, but has to stop for 15 minutes at station C, which is 60 kms away from station B enroute to station A. Ignoring the lengths of the trains, what is the distance, to the nearest km, from station A to the point where the trains cross each other?

a. 112
b. 118
c. 120
d. none of these

79. Two motorcyclists started at the same time, covered the same distance, and returned home at the same time. But one rode twice as long as the other rested on his trip, and the other rode three times as long as the first one rested on his trip. Who rode faster?

80. Two candles have different lengths and thicknesses. The long one can burn $3\frac{1}{2}$ hours; the short one, 5 hours. After 2 hours burning, the candles are equal in length. Two hours ago, what fraction of the long candle's height gave the short candle's height?

81. Everyday at noon a ship leaves Le Havre for New York and another ship leaves New York for Le Havre. The trip lasts 7 days and 7 nights. How many New York-Le Havre ship will the ship leaving Le Havre today meet during the journey to New York?

82. My watch is 1 second fast per hour and Mogli's watch is $1\frac{1}{2}$ seconds slow per hour. They are both set at 12:00 noon. When will they both show the same correct time again?

83. An engineer goes every day by train to the city where he works. At 8:30 a.m., as soon as he gets off the train, a car picks him up and takes him to the plant. One day the engineer takes the train arriving at 7:00 a.m., and starts walking toward the plant. One the way, the car picks him up and he arrives at the plant 10 minutes early. How long was the engineer walking?

84. Two cars leave simultaneously from points A and B on the same road in opposite directions. Their speeds are constant, and in the ratio 5 to 4, the car leaving at A being faster. The cars travel to and fro between A and B. They meet for the second time at the 145th milestone and for the third time at the 201st. What milestones are A and B?

85. A train leaves a station precisely on the minute, and after having traveled 8 miles, the driver consults his watch and sees the hour-hand is directly over the minute-hand. The average speed over the 8 miles is 33 miles per hour. At what time did the train leave the station?

86. Andy leaves at noon and drives at constant speed back and forth from town A to town B. Bob also leaves at noon, driving at 40 km per hour back and forth from town B to town A on the same highway as Andy. Andy arrives at town B twenty minutes after first passing Bob, whereas Bob arrives at town A forty-five minutes after first passing Andy. At what time do Andy and Bob pass each other for the third time?

87. Tortoise travels uniformly 20 km a day. The hare, starting from the same point three days later to overtake the tortoise, travels at the uniform rate of 15 km the first day, at a uniform rate of 19 km the second day, and so forth in arithmetic progression. After how many days does the hare catches up with the tortoise?

88. Shailendra and Ashu are running in opposite direction on a circular track of 117 m length, with shailendra in the clockwise direction. They start from point O with the speed of 5 m/s and 8 m/s respectively. Every time they meet they interchange their direction and speed. If they continue doing so, find out where they will meet for the 13th time.

Three Problems from our CAT CBT Club (Find the solutions in our CAT CBT Club only)

89. A raft and a motorboat left town A simultaneously and traveled downstream to town (The raft always moves at the same speed as the current, which is constant.) The motorboat arrived at town B, immediately turned back, and encountered the raft two hours after they had set out from A. How much time did it take the motorboat to go from A to B? (Assume that it travels at a constant speed)

90. Angad and Bali traveled on the same road and at the same rate from Delhi to Bhatinda. At the 50th milestone from Bhatinda, Angad overtook a herd of goats which were going at the rate of 1.5 miles per hour. Two hours after that Angad met a wagon which was moving at the rate of 2.25 miles per hour. Bali overtook the same herd of goats at the 45th milestone from Bhatinda, and met the wagon exactly 40 minutes before he came to the 31st milestone. How far away from Bhatinda was Bali when Angad reached Bhatinda?

91. One of the ski lifts at Kulu Manali climbs the mountain in a line that parallels one of the ski runs. Both the run and the lift are two km long, and a chair passes the starting point of the lift every 10 seconds. A skier starts her run just as the chair arrives at the top of the lift

and another chair starts back down. She arrives at the bottom just as a chair is starting up the mountain and another chair is completing its descent. Counting these two chairs and the two at the start of her run, she sees 97 chairs on their way up the mountain and 61 chairs on their way down. If the chair that started down the slope at the same time that the skier did is still on its way down when the skier reaches the bottom of the slope, what is the average speed of the skier?

TSD-Solutions

1. Speed in meter's = $\frac{59400}{66 \times 60}$ = 15m/s. Hence (b)

2. For average speed in kmph 20m/s = 72km/h.

So average = $\frac{2 \times 72 \times 48}{72 + 48} = 57.6 km / h$

Hence answer (a).

3. Total time taken= $2\left[\frac{100}{100} + \frac{100}{200} + \frac{100}{300} + \frac{100}{400}\right]$

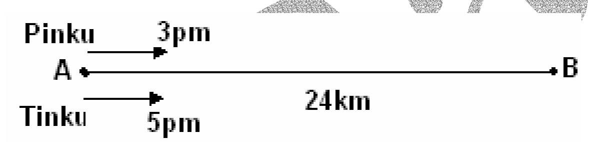
= 4.17 hrs. Hence answer (c).

4. Let the distance between his home & office is 'd' kms.

Then $\frac{d}{40} - \frac{20}{60} = \frac{d}{60} \Rightarrow$ d = 40 kms. Hence answer (d).

5. To reach on time (i.e.) in 50 min. his speed should be $\frac{40}{50} \times 60$ =48kmph.

Hence answer (c).



6. Let they started from pt. A & Pinku was overtaken at B. Tinku must have reached B after $\frac{24}{3}$ = 8hrs. Pinku should take $\frac{24}{2.5}$ = 9hrs. 36 mins. To reach thus.

But as the reached with B at 1 am i.e. he took rest for 24 mins.

Hence answer (b)

7. $A + \frac{5^{th}}{8}$ of usual rate, time taken in $\frac{8}{5}$ times of original time. Let original time taken is t min $\Rightarrow \frac{8}{5} + -t = 18 \Rightarrow$ t=30 min. Hence answer (c)

8. As the distance is constant $\frac{d_1}{d_2} = \frac{t_2}{t_1} \Rightarrow \frac{50}{75} = \frac{t+10}{t+16} \Rightarrow$ t = 2 min . Distance = 75 × 12/60 = 15 km.

9. Let the original speed is 's' $\left[\frac{62}{s} + \frac{248}{\frac{4}{5}s} = t + 2\right]$

$\Rightarrow$ To cover 248kms, $\left(\frac{4^{th}}{5} of\ 310\right)$ the time taken is $\frac{5}{4}$ times of the usual time.

$\Rightarrow \quad \frac{5}{4}t - t = 2 \qquad \Rightarrow$ t=8hrs.

$\Rightarrow$ Usual speed = $\frac{248}{8} = 31 kmph.$

10. Let the normal speed = s= $\frac{d}{t}$

speed how = $\frac{3d}{4t} = s_1$

s: $s_1 = \frac{d}{t} : \frac{3d}{4t} \Rightarrow 4:3$. Hence answer (a).

11. Let the distance traveled on foot = x kms.

$\frac{x}{5} + \frac{95 - x}{12} = 12$

⇒ x = 35 kms. Hence answer (e).

12. Speed in m/s = $54 \times \frac{5}{18} = 15 m/s.$

Time taken to pass the platform = $\frac{150 + 90}{15} = 16 \sec s.$

Hence answer (a).

13. Till 9am distance traveled by the motorcyclist who started from P = 40kms.
Distance between them at 9am = 200 - 40 = 160 kms.

Time taken to meet = $\frac{160}{40 + 35} = 2 hrs. 8 \min s.$

Hence they'll meet at 11:08 pm. Hence answer (c).

14. Every hour the faster train travels 10 kms more than the other one i.e. they both traveled for 8 hrs. Distance traveled by them in 8 hours = 8(40 + 50) =720kms. Hence answer (d).

15. $\frac{200}{30 + 20}$ = 4hrs. So they will meet after 1:00pm. Hence answer (c).

16. In 4hrs. Raj must have traveled 80 kms. Hence they'll meet 80 kms. Form A. Hence answer (e).

17. When they meet for the 2nd time, they must have traveled 400 kms combinely and Raj must have traveled 160 kms. More the 1st starting point. So hence answer (e).

18. Downs team speed = $\frac{28}{4} = 7 kmph.$

Ups team speed = $\frac{12}{6} = 2 kmph.$

Speed of the ship = $\frac{7 + 2}{2} = 4.5 kmph.$

19. Speed of the river = $\frac{7 - 2}{2} = 2.5 kmph.$

20. Let 'x' & 'y' kmph be the speed of the boat and the river.

$$\Rightarrow \quad \frac{30}{x - y} + \frac{42}{x + y} = 8$$

$$\frac{42}{x - y} + \frac{56}{x + y} = 11$$

solving, we get x = 10 kmph. Y= 4 kmph. Hence answer (b).

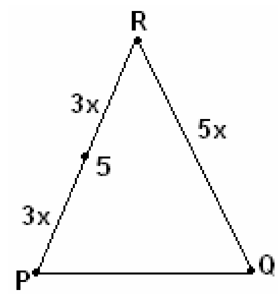
21. In the first case

$$\frac{12}{v-u} - \frac{12}{v+u} = 4 \Rightarrow \frac{12 \times 2u}{v^2 - u^2} = 4 \text{ and}$$
$$\frac{12}{2v-u} - \frac{12}{2v+u} = 0.5 \Rightarrow \frac{12 \times 2u}{4v^2 - u^2} = 0.5$$

Dividing one equation by the other and solving we get u = 8

22. For one side, instead of walking if he drives he saves 3hrs. So, in the first case had he not been driving while coming back he would have taken 3hrs. more i.e. (7+3) hrs. = 10hrs. Hence answer (c).

23. The distances each traveled in 2 hr was traveled by the other in t and t + 3 hours. Equating the speed ratio for both the distances 2/t and (t + 3)/2 we get t (t + 3) = 4 ⇒ t = 1. Ratio = 1 : 2



24 & 25. Let the distance between R & Q is 5x. So, by the time Y travels 5x, X will travel 3x. Now when X is at S and Y is at R. They start moving towards each other. Before meeting, X would have traveled $3x \times \frac{5}{8} = \frac{15x}{8}$ $\Rightarrow 3x + \frac{9x}{8} = 165$

$\Rightarrow x = 40$

⇒ Distance between P & R is 240kms.
Distance between R & Q is 200kms.

24. In ∆PQR, PQ + QR > PR.
⇒ PQ = 20kms. is not possible. Hence answer (a)

25. PQ = 400kms. (given)

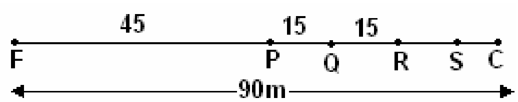
X travels 400kms in $18\frac{2}{3} = \frac{56}{3}$ *hours*

⇒ Speed of X= $\frac{400}{50} \times 3kmph.$

⇒ Speed of Y= $\frac{400}{56} \times 3 \times \frac{5}{3} = \frac{2000}{56} kmph.$

Y will cover PR (=240kms) in $\frac{240}{2000} \times 56 hrs.$

i.e. 6.72hrs. Hence answer (e)



26. Let Chetan start from point F toward C. Let the initial distance between F & C is 90m and hence speed of Chetan & his dog are 2 and 4 meters/min respectively. Let chetan is at 'P' when the dog reaches at 'C' for the first time.
⇒ FP = 45m.

(∴ dog will travel 90m in $\frac{90}{4}$ min & in the same time chetan will travel $\frac{90}{4} \times 2 = 45m$).

Now, when the dog turns back, let they meet at point Q. They will travel PC distance in a ratio of 1:2. $\Rightarrow$

PQ= $\frac{1}{3}\times 45 = 15m.$

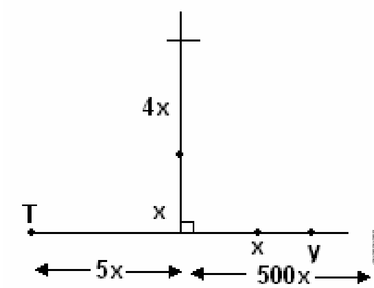
||ly now, R will be the midpoint of QC.

$\Rightarrow$ QR = 15m.

and again RS= $\frac{1}{3}\times RC = 5m.$

$\Rightarrow$ Chetan had traveled PS= 80m., when he met the dog for the second time. So the time taken by him was $\frac{80}{2} = 40\,\text{min}$.

Hence answer (b).



27. Let initially Tipper was at Y.

$\Rightarrow$ XY = 500m.

Let the distance traveled by kipper in 2min was 'x'.

$\Rightarrow$ Distance Traveled by Tipper in 2 min was (500-x)m. In 8 more minutes kipper will travel 4x more i.e. he will be 5x away from point X. So, it means Tipper will also be at point T, 5x away from X. $\Rightarrow$ Speed of Tipper = $\frac{500-x}{2} = \frac{6x}{8}$

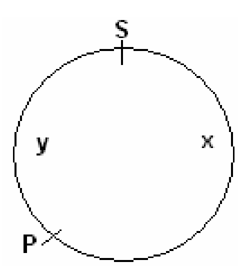
$\Rightarrow$ x = 200m.

$\Rightarrow$ Ratio of speed of kipper : Tipper = 200: (500-200) = 2:3. Hence answer (b).

28. As Ranbir's maximum Possible speed was 55mph he traveled a maximum of (55 $\times$ 4) = 220miles. The palindromes after 29792 are 29892, 29992, 30003, 30103… But the maximum possible reading can be (29792 + 220) = 30012 only.

$\Rightarrow$ The reading was 30003.

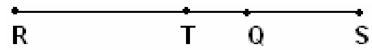
$\Rightarrow$ His speed was $\frac{30003-29792}{4} = \frac{211}{4}mph.$ Hence answer (d).



29. Let S is the starting point. Let the Smaller distance between SP is y & larger one is x. $\Rightarrow \frac{x}{2x+y} = \frac{y}{x} \Rightarrow x^2 = 2xy + y^2$

Divid y the equation by y^2, we get $\left(\frac{x}{y}\right)^2 = \frac{2x}{y}+1$

$\Rightarrow \frac{x}{y} = 1\pm\sqrt{2}$. Hence answer (a).



30. Let Roshan & Swati start from their houses R & S respectively. When they both started the same time i.e. 10:00, the meet at T at 10:10.

Let RT=x, TS=y

Now, when Swati leaves at 10:03, they meet at 10:11 i.e. somewhere between T & S, lets say at Q.

$\Rightarrow$ RQ= 1.1x (as Roshan took 11min. instead of 10).

As, Swati took 8 minutes to reach Q.

$\Rightarrow$ SQ = 0.8 y

Now, RT + TS = RQ + QS

x + y = 1.1x + 0.8y

0.2y = 0.1x $\quad \Rightarrow \dfrac{x}{y} = \dfrac{2}{1} \Rightarrow x = 2y$

$\Rightarrow$ RT = 2TS.

If Swati travels TS in 10min. she will take 20min. move to travel RT.

$\Rightarrow$ She will reach Roshan's house at 10:30.

31. Let the time taken by Sartaj to reach halfway between Math land & Algebra town is t. So, time taken by Ghaushyam to cover the same is 6t. ||ly Ghaushyam will travel the remaining half in 6t, where Sartaj will take 12t now. So total time taken by Sartaj & Ghaushyam are 13t & 12t respectively. Hence Ghaushyam will reach first. Answer (a)

32. A : B = 1000 : 900 = 10 : 9

B : C = 1000 : 900 = 10 : 9

$\Rightarrow$ A : B : C = 100 : 90 : 81

Now, A & B are 200 & 100 meters behind the starting line respectively. So, by the time A would have traveled 1200m, B & C will have traveled 1080 & 972m. respectively.

$\Rightarrow$ A won the race & B was 20m behind. Answer (a).

33. The trick is that scooter and motorcycle are moving in same direction opposite to that of car and truck. After this one can easily form 4 equations and from them can calculate the time taken for scooter and car to meet. The answer is 5:20 PM.

Situation at 2:00 PM:

Let the distance between truck (T) and scooter(S) be D_1 and between truck and motorcycle (M) be D_2.

$(T + S) \times 2 = D_1$, $(T+ M) \times 4 = D_1 + D_2$, $(M + C) \times 5 = D_1 + D_2$, $(M - S) \times 6 = D_2$.

Solve for D_1/C+S which comes out to be 10/3 hours.

34. Let the length of column is l and speed of the soldier & Column is 's' & 'c' respectively .

$$\Rightarrow \frac{l}{s-c} + \frac{l}{s+c} = \frac{l}{c}$$

$$\frac{2s}{s-c^2} = \frac{1}{c}$$

$$\Rightarrow 2sc = s^2 - c^2$$

Dividing by $c^2, \dfrac{2s}{c} = \left(\dfrac{s}{c}\right)^2 - 1$

Putting $\dfrac{s}{c} = t, 2t = t^2 - 1$

$$\Rightarrow t^2 - 2t - 120$$

$\Rightarrow t = 1 + \sqrt{2}.$ So, $\dfrac{s}{c} = \left(1+\sqrt{2}\right):1$. Hence answer (a).

35. Average Speed = $\dfrac{2 \times 12 \times 8}{12 + 8} = 9.6 kmph.$

36. Let by walking she covers d distance in 1min.

$\Rightarrow$ by walking she covers 2d distance in 1min.

In first case she walks for $\frac{40}{3}\text{min}$ & run for $\frac{20}{3}\text{min}$

$\Rightarrow$ Distance between home & school is $\frac{40}{3}d + \frac{40}{3}d = \frac{80}{3}d$.

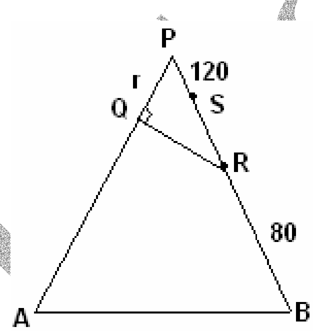
Let she ran for 2t min & walked for t min while coming back.

$\Rightarrow 2t.2d + t.d = \frac{80}{3}d$

$\Rightarrow t = \frac{16}{3}\text{min}$.

37. Had the speed of stream be zero the man would have got the hat after 10minutes. But because the hat was 1km behind that point, it means the steam covered 1kms in 10minutes.

So, the speed of stream= $\frac{1000}{10 \times 60} = \frac{5}{3} m/s$.



38. Let PQ=x

$\Rightarrow$ in 30-60-90 triangle PQR

PR = 2x

When A reaches at P, let B is at S. $\Rightarrow$ Ps=120

$\Rightarrow$ RS = 2x-120

So, PB = 80+2x $\Rightarrow$ PA = 80+2x

Hence AQ= 80+2x-x = 80+x

Ratio of the speeds A & B is

$$\frac{80+x}{80} = \frac{x}{2x-120}$$

$\Rightarrow$ x=80kms.

So, Distance between AB = 80+160=240kms.

39. After covering each mile the speed reduced of $\frac{3}{4}$ times hence the time will increase to $\frac{4}{3}$ times.

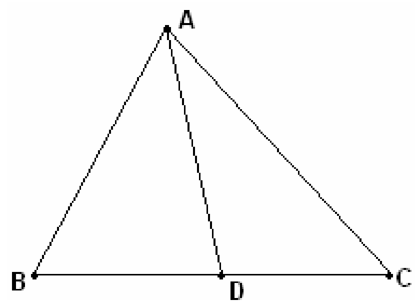
$\Rightarrow$ time taken for 1st 4 miles

$$= t + \frac{4}{3}t + \left(\frac{4}{3}\right)^2 t + \left(\frac{4}{3}\right)^3 t = 486\sec$$

time taken for the next 4 miles

$$= \left(\frac{4}{3}\right)^3 t + \left(\frac{4}{3}\right)^5 t + \left(\frac{4}{3}\right)^6 t + \left(\frac{4}{3}\right)^7 t$$

$$= \left(\frac{4}{3}\right)^4 \left[t + \frac{4}{3}t + \left(\frac{4}{3}\right)^2 t + \left(\frac{4}{3}\right)^3 t\right] = \left(\frac{4}{3}\right)^4 \times 486 = 1536 \sec s = 25 \min s 36 \sec s.$$



40. Let after 2hrs, the cars are equidistant from the starting pt. D i.e. at B, A & C.
$\Rightarrow$ AD is the median of ΔABC.
Where BC=170kms, AB=136kms. And as BD= AD = DC $\Rightarrow$ AD= 85kms.
Using Appolonius theorem

$$AB^2 + AC^2 = 2\left(AD^2 + BD^2\right)$$

$$(136)^2 + AC^2 = 2\left(85^2 + 85^2\right)$$

$$AC^2 = 4046 \Rightarrow AC = \sqrt{4046} kms.$$

41. We make the two equations in both the cases

$$\frac{18}{v-u} - \frac{18}{v+u} = 9 \Rightarrow \frac{18 \times 2u}{v^2 - u^2} = 9$$

$$\frac{18}{2v-u} - \frac{18}{2v+u} = 1 \Rightarrow \frac{18 \times 2u}{4v^2 - u^2} = 1$$

Solving we get u = 20/3

42. Gogo exits the tunnel i.e. covers 6kms (2kms + 4kms) just as the train is completely inside i.e. the train traveled 2kms in the same time.
$\Rightarrow$ Speed of Gogo : Speed of train = 2:6 = 1:3
when the train will emerging from the tunnel, Gogo must have traveled 6kms. More now, the train and Gogo will travel towards each other at a relative speed of the sum of their speeds.
$\Rightarrow$ They will travel 6kms in a ratio of 1:3.

$\Rightarrow$ Distance traveled by train= $6 \times \frac{1}{4} = \frac{3}{2} kms$

So, they will meet 1500 meters away from tunnel.

43. Let the speed of escalator is 's' steps per minute.
Let the speed of woman is 1 step per minute.
Hence 10 + 10s = 25 – 5s
15 s = 15
$\Rightarrow$ s = 1
So, total no of steps in the escalator are 10 + 10 $\times$ 1 = 20 steps.

44. Let A takes 1 step in t minutes
$\Rightarrow$ B takes 3 step in t minutes
Lets the escalator takes s steps per minutes

50 + 50t.s = $75 + \frac{75.t}{3}.s$

50 + 50st = 75 + 25st
st = 1
$\Rightarrow$ No. of steps in escalator = 50 + 50 $\times$ 1 = 100 steps.

45. Let Ram takes 2 steps in t min.

$\Rightarrow$ Shyam takes 3 steps in t min.

Let speed of the escalator is 's' steps/min.

$$\frac{25}{3}t.s+25=\frac{20}{2}t.s+20$$

$$50st+150=60st+120$$

$$10st=30\Rightarrow st=3$$

$\Rightarrow$ length of the escalator = 50 steps.

46. Let Charlie reached the bus stop in exactly one hour i.e. the bus stop to which Charlie went was 6kms, away. At that time, Joe was 10kms away from this bus stop. Now, the bus will cover this 10kms with a relative speed of (60 - 4) = 56kms in $\frac{10}{56}=10\frac{5}{7}\text{min}$. So Joe's bus stop was $\left(4+4\times\frac{75}{7}\times\frac{1}{60}\right)=\frac{33}{7}kms$ away from the starting pt. It means that Charlie's assumption was wrong.

47. As each of them have same speed, so after meeting reversing their direction does not create any difference. Any swimmer will reach the other end in 6mintues. The last swimmer will start from end B 4min. after the first swimmer. So after (4 + 6) = 10min. the pool will be empty. Hence answer 10min.

48. By 10:15, Jinny would have written 15 $\times$ 60 = 900digits. Now the relative speed of Jinny & Johny is 60 – 90 = -30 digits/minutes i.e. Johny will erase 30 digits more than Jinny writes every minutes. So they will meet after $\frac{900}{30}=30\min utes$ i.e. at 10:45 am.

a) Till 10:44 Jinny must have written 44$\times$60 digits i.e. 2640 digits. So the last digit should be 6. ||ly Johny must have erased 29 $\times$ 90 = 2610 digits so the left most digit should be 8. Hence answer 2.

b) Johny catches up with Jinny at 10:45 am. So the last digit to be erased was 2700[th] digit. Hence last digit is 6. Hence answer 6.

$\frac{d}{v-u}=30$ and

$\frac{d}{v+u}=20$. Solving we get $\frac{d}{v}=24$

50. The angle between the two hands at 7:00 is 210.

The two hands will make an angle of 10° when either the min hand is 10° behind the hour hand or 10° ahead of the hour hand

i.e. at $7hr.\frac{200}{5.5}\text{min}\&\,7hr.\frac{220}{5.5}\text{min}$.

i.e. at $7:\frac{400}{11}or7:\frac{440}{11}$.

51. Work done by A & B in 1 day = $\frac{1}{18}$

Similarly work done by B & C in 1 day = $\frac{1}{24}$

Work done by A & C in 1 day = $\frac{1}{36}$

$\Rightarrow$ Work done by A, B & C in 2 days = $\frac{1}{18}+\frac{1}{24}+\frac{1}{36}$

So they will complete the job in 16days.

52. Let the length of track = 15 meters
So speed of Arun & Barun in meters per minutes is 5 & 3 respectively.

So they will meet after = $\frac{15}{5+3} = \frac{15}{8} \min utes.$

53. To cover half of the track John traveled 100 meters, So to cover full track he must have traveled 200 meters, whereas Mary traveled 150 meters. Hence length of track is 350 meters.

54. Work done by the toe pipes in 1 hour = $\frac{1}{14} + \frac{1}{16} = \frac{15}{112}.$

$\Rightarrow$ they can fill the tank in $\frac{112}{15}$ hours.

Let the work done by leak in 1 hour is x.

$$\Rightarrow \frac{1}{\frac{15}{112} - x} = \frac{1}{\frac{15}{112}} + \frac{32}{60}$$

Solving for x, we get x = $\frac{1}{112}.$

So, the leak alone can empty it in 112hrs.

55. A & B will meet after every $\frac{1200}{6-4} = 600 \sec s.$

B & C will meet after every $\frac{1200}{6+10} = 75 \sec s.$

So all A, B & C will meet after L.C.M (600, 75)
i.e. 600secs.

56. A + B = $\frac{1}{10}$

B + C = $\frac{1}{15}$

5(A) + 8(B) + 9(C) = 1
5A + 5B + 3B + 3C + 6C = 1
5(A+B) + 3(B+C) + 6C = 1

$$5.\frac{1}{10} + 3.\frac{1}{15} + 6C = 1$$

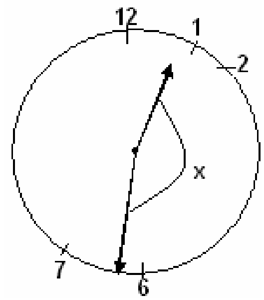
$$\Rightarrow C = \frac{1}{20}$$

So C, can complete the job in 20 days.

57. Let A left after 'x' days

So, $x\left(\frac{1}{21} + \frac{1}{24}\right) + 9\left(\frac{1}{24}\right) = 1$

Solving we get, x = T.



58. Let, at the time he left the minute hand was x ahead of the hour hand.

Now by the time, hour hand traveled x°,the minute hand traveled $360 = x^\circ + 1440^\circ$

$\Rightarrow$ 1440 + 360 – x = 12x

$\Rightarrow \quad x = \dfrac{1800}{13}$

So the time when he left was

$$= 1 : \frac{\frac{1800}{3} + 30}{5.5}$$

59. Every hour, twice the hands of the clock will make an angle of 1°., e.g. between 2 & 3 it will make twice i.e. at

$2hr \dfrac{60-1}{5.5} \min \& \ 2hr \dfrac{60+1}{5.5} \min.$

$\Rightarrow$ the minutes count can be of the type $\dfrac{30x+1}{5.5} \& \dfrac{30x-1}{5.5}$

Where x is a whole no. from 0 to 11. Only 2 values of x gives an integral solutions of these equation i.e. at x = 4 and 7.

60. A, B, C & D can complete the job in 4, 8, 16, 32 days respectively.

A & D combinely can do the job in $\dfrac{32}{9}$ *days.*

B & C combinely can do the job in $\dfrac{32}{6}$ *days.*

Hence answer A & D.

61. A : B = 1000: 960 = 25:24

A : C = 1000:910 = 100:91

$\Rightarrow$ A : B : C = 2500 : 2400 : 2184

= 625 : 600 : 546

$\Rightarrow$ When B travels 1000 m C will travel $1000 \times \dfrac{546}{600 = 947.92}$ *meters.*

$\Rightarrow$ B can give C a head start of 52.09 meters.

62. Each fill pipe can fill $\dfrac{1}{12}$ of the tank in an hour.

Each drain pipe can drain $\dfrac{1}{24}$ of the tank in an hour.

Let, there are 'x' drain pipes.

$$\Rightarrow 2\left[(12-x)\frac{1}{12}+x.\frac{1}{24}\right]=1$$

$\Rightarrow$ x = 12 Hence answer 12 pipes.

63. Let A complete 1000 meter in t secs.
$\Rightarrow$ B complete 960 meter in (t+19) secs.

A will complete 960 meter in $\frac{960}{1000}\times t=\frac{24}{25}t\sec s.$

B complete 1000 meter in $\frac{24}{25}t+30\sec s.$

Comparing the speeds of B, we get $\frac{960}{1000}=\frac{t+19}{\frac{24}{25}t+30}$

Solving, we get, t = 125 secs.
$\Rightarrow$ A completes 1000 meter in 125 secs.
B completes 960 meter in 150 secs.

64. Let the total work in of 100 units. So, Divya & Raveena alone can make 5 & 4 units per day. So if they work together they will make 9 × 1.25 = 11.25 units in a day. Let the Divya, Raveena & together work on x, y & z days respectively x, y, z being integers.
$\Rightarrow$ 5x + 4y + 11.25z = 100
$\Rightarrow$ z = either 0 or 4 or 8
if z = 0, no. of days would be between 20 & 25.
If z = 4, x & y can be (11, 0) (7, 5) & (3, 10).
If z = 8, s & y can be (2, 0) only.
So all but 14 days is possible. Hence answer (b).

65. Every hour the minute hand traveled 90 minutes.
$\Rightarrow$ When the distance between the original time & the defective watch will be 60 min, it will show the same time i.e. after 2hrs. Hence answer 5 p.m.

66. As all the dimensions are doubled, so the volume will become 8 times. So, time taken will also be 8 times of original time i.e. 16 hrs.

67. Whenever the two hands will be in a straight line, they will make equal angles with the horizontal. i.e. at $8:\frac{240-180}{5.5}$ i.e.

$$8:\frac{120}{11}hrs.$$

68. Let each person makes 1 unit per hour. Let the last person worked for d hours. Hence first person worked for 5d hours and there are (4d + 1) men.
Now, 5d + (5d-1) + (5d-2) +….d = 45 (4d+1)

$$\frac{5d(5d+1)}{2}-\frac{(d-1)d}{2}=45(4d+1)$$

$$25d^2+5d-d^2+d=360d+90$$

Solving for d we get, d = 15 or $-\frac{1}{4}(rejected)$

Hence the first man worked for 75 hours. And total no. of men in the group = 4×15+1=61.

69. Let the capacity of tank is 60 ltrs.

$\Rightarrow$ P can fill 5 ltrs per minutes & Q can empty 3 ltrs per minutes. $\frac{3^{th}}{4}$ of the tank will get filled in 9 minutes.

To reach $\frac{1^{th}}{4}$ height of the tank. Q will have to empty half of the tank i.e. Q will take 10 minutes.

Now, P will have to fill half of the tank i.e. 6 minutes.

$\Rightarrow$ after 25 minutes tank is at $\frac{3^{th}}{4}$ height, 41 minutes and 57 minutes is again at $\frac{3^{th}}{4}$ height. Q will empty $\frac{3'}{20}$ of the tank in 3 minutes. Hence height at the end of 1 hour = $\frac{3^{th}}{5}$ of the tank.

70. A overtakes B for the first time in the middle of 5th round i.e. when A had traveled 4.5 rounds B had traveled 3.5 rounds.

So, Ratio of their speeds = $\frac{510 \times 4.5}{510 \times 3.5} = 9:7$

So in a 3 km race, when A will travel 3000 meter, B will travel $3000 \times \frac{7}{9} = \frac{7000}{3} m.$

So A can give B a start of $3000 - \frac{7000}{9} = 666.66 meters.$

71. Time taken by the Oarsman A = $\frac{x}{v+u} + \frac{x}{v-u} = \frac{2xu}{v^2 - u^2} = \frac{2x}{v - \frac{u}{v}}$

Time taken by the Oarsman B = $\frac{2x}{v}$

We can see that $v > u - \frac{u}{v}$ $\quad \therefore$ B taken more time

72. Total distance traveled by the first meeting = length of the lake = d (say)
Total distance traveled by the second meeting = 3d
$\therefore$ initial distance traveled will also become three times
$\therefore$ distance traveled by motorboat M by the second meeting = 500 × 3 = 1500
1500 = d + 300 $\Rightarrow$ d = 1200 m

73. Speed after two dogs leave = $\frac{3}{5}$th of the original speed

Now the difference of 24 hours comes because of 50 miles distance $\Rightarrow$ difference of 48 hours will come because of 100 miles

Therefore he traveled 100 miles after the dogs run off. Speed becomes $\frac{3}{5}$ $\Rightarrow$ time becomes $\frac{5}{3}$ of the original $\Rightarrow$ excess time = $\frac{2}{3}$ of the original time $\Rightarrow$ $\frac{2}{3}t$ = 48 h $\rightarrow$ t = 72 hours (for 100 miles)

Speed = $\frac{100}{72} = \frac{25}{18}$ miles/hour

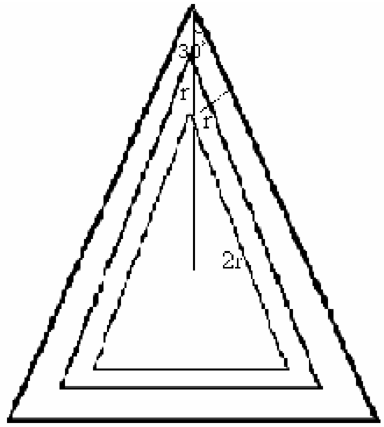
Since he traveled for 24 hours initially

Distance = $24 \times \frac{25}{18} = \frac{100}{3}$ miles

Total distance = $\frac{100}{3} + 100 = \frac{400}{3}$ miles

74, 75 and 76

The lengths of the sides of two triangles can be found to be $2r, r\left(2+\sqrt{3}\right)$, and $2r\left(1+\sqrt{3}\right)$ rest can be calculated



74. 1
75. 3
76. 2

77. 2
78. Since train y rests for 15 minutes in between
Let train x travel for 15 min first and then lets start train y.

In 15 min train x will travel $\frac{70}{u} = 17.5\text{km}.$

Left distance = 180 – 17.5 = 162.5
This distance can be divided in the ration 70:50 = 7 : 5

Distance from $A = 17.5 + 162.5 \times \frac{7}{2} \approx 112\text{km}$

79. Let the travel and resting time of first be x and y and of that of the second be a and b, respectively,
x = 2b a = 3y Time taken is same

$\Rightarrow x + y = a + b$ $\Rightarrow x + y = 3y + \frac{x}{2}$

$\frac{x}{2} = 2y$ x= 4y

so first motorcyclist is traveling and resting for 4t and t hour, whereas the second is traveling for 3t hours. Since second in traveling for shorter time, it is traveling faster.

80. The first one burns in $\frac{7}{2}$hours and the second in 5 hours.

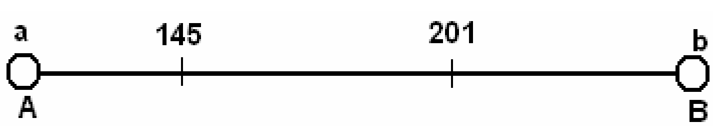
In two hours they will burn $\frac{4}{7}$ and $\frac{2}{5}$ of their length. Let their lengths be x and y $\Rightarrow \frac{3}{7}x = \frac{3}{5}y$ $\Rightarrow \frac{x}{y} = \frac{7}{5}$ $\Rightarrow y = \frac{5}{7}x$

81. Let a ship be leaving Le Havre on 10th of the month. At that time, a ship which left New York on 3rd would be arriving. Also, the ship leaving on 10th would arrive at New York on 17th. Therefore, it would meet every ship leaving New york from 3rd to 17th. These are 15 ships in all.

82. When both the clocks are slow or fast by an integral multiple of 12 hours, they will show the correct time simultaneously. This will happen first time after 3600 days.

83. The car was scheduled to reach the station at 8:30 am. When it met the engineer it saved 10 minutes- 5 minutes to get to the station and five minutes to come back to the meeting point. That means that the car met the engineer at 8:25 am. Therefore, the engineer was walking for 1 hour 25 minutes.

84. Let the milestones of A and B be a and b, respectively. The distances traveled till the 2nd and 3rd meeting will always be in the ratio 5: 4



$\frac{b-a+b-145}{b-a+145-a}=\frac{5}{4}$ and $\frac{2(b-a)+201-a}{2(b-a)+b-201}=\frac{5}{4}$. Solve to get a = 103, b = 229.

85. Let the driver leave at some minutes past N O clock. At N:00, the gap between minute and hour hand is 30N. The minute hand and hour hand are together after $\frac{30N}{\frac{11}{2}}=\frac{60N}{11}$ minutes. The driver left at $\frac{60N}{11}-\frac{8}{33}\times 60$ minutes ago. This should be an integer. This is an integer for N = 10. Therefore, the driver left at 10:40.

86. Let their speeds be V_a and V_b. They take 20 and 45 minutes after crossing each other to reach their destinations.
$\Rightarrow \frac{V_a}{V_b}=\sqrt{\frac{45}{20}}=\frac{3}{2}$ and time taken for the first meeting t = $\sqrt{20\times 45}=30$ min. The total time for travel for each of them is 60 min and 75 min for one way. After the first meeting, they travel twice the distance for every subsequent meet. Therefore, they take 30 × 2 = 60 minutes for each subsequent meet. Time taken for the 3rd meet = 30 + 60 × 2 = 150 minutes and they meet at B.

87. After 10 days Tortoise has traveled 200 km and the hare has traveled 189 km. Sometime during the eleventh day Tortoise and hare meet. Let x be the number of hours on the eleventh day when the meeting occurs.
$\Rightarrow \frac{x}{24}\times 43=11+\frac{x}{24}\times 20 \Rightarrow x=\frac{11}{23}\times 24$. Therefore days = $10\frac{11}{23}$

88. Interchanging speeds and directions just means that the two persons are crossing each other. They meet at the starting point.