## GATID 2018 <br> ELECTRICAL ENGINEERING

## Detailed Solution

## EXAM DATE: 09-02-2019 <br> AFTERNOON SESSION (02:30 PM-05:30 PM)

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## SECTION: GENERAL APTITUDE

1. The missing number in the given sequence 343 , 1331, $\qquad$ 4913 is
(a) 2744
(b) 2197
(c) 3375
(d) 4096

Ans. (b)
343, 1331, $\qquad$ 4913
$7^{3}, 11^{3}, \underline{13}^{3}, 17^{3}$
$\Rightarrow \quad 13^{3}=2197$
here, the given series is the cube of prime numbers.
2. The passengers were angry $\qquad$ the airline staff about the delay.
(a) with
(b) on
(c) about
(d) towards

Ans. (a)
3. I am not sure if the bus that has been booked will be able to $\qquad$ all the students.
(a) fill
(b) accommodate
(c) deteriorate
(d) sit

Ans. (b)
4. Newspapers are a constant source of delight and recreation for me. The $\qquad$ trouble is that I read $\qquad$ many of them.
(a) even, too
(b) only, quite
(c) even, quite
(d) only, too

Ans. (d)
5. It takes two hours for a person $X$ to mow the lawn. Y can mow the same lawn in four hours. How long (in minutes) will it take X and Y , if they work together to mow the lawn?
(a) 120
(b) 60
(c) 90
(d) 80

Ans. (d)
Rate of work of X persons $=\frac{\text { work }}{120}$

Rate of work of Y persons $=\frac{\text { work }}{240}$
$\Rightarrow\left(\frac{\text { work }}{120}+\frac{\text { work }}{240}\right) \mathrm{t}=$ work
$\Rightarrow \quad t=80$ minute
6. Given two sets $X=\{1,2,3\}$ and $Y=\{2,3,4\}$, we construct a set Z of all possible fractions where the numerators belong to set X and the denominators belong to set Y. The product of elements having minimum and maximum values in the set Z is $\qquad$ _.
(a) $1 / 8$
(b) $1 / 6$
(c) $3 / 8$
(d) $1 / 12$

Ans. (c)

$$
\left.\begin{array}{rl}
\mathrm{x} & =\{1,2,3\} \quad \mathrm{y}=\{2,3,4\} \\
\mathrm{z} & =\left\{\frac{\text { Numerator (from } \mathrm{x})}{\text { denominator }(\text { from } \mathrm{y})}\right\}
\end{array}\right\}, \begin{aligned}
& \mathrm{z}=\left\{\frac{1}{4}-----\frac{3}{2}\right\} \\
& \\
& \min \quad \max \\
& \Rightarrow \\
& \Rightarrow \frac{1}{4} \times \frac{3}{2}=\frac{3}{8}
\end{aligned}
$$

7. The ratio of the number of boys and girls who participated in an examination is 4.3. The total percentage of candidates who passed the examination is 80 and the percentage of girls who passed is 90 . The percentage of boys who passed is $\qquad$ _.
(a) 90.00
(b) 55.50
(c) 72.50
(d) 80.50

Ans. (c)
Participated $\frac{\text { Boys }}{\text { Girls }}=\frac{4}{3}$
Pass $\%=80 \%$
Let total students be x .
Passed students $=0.8 \mathrm{x}$
Boys $=\frac{4}{7} \mathrm{x}$

GATE 2019
Detailed Solution
09-02-2019 | AFTERNOON SESSION

Girls $=\frac{3}{7} x$
Passed girls $=0.9 \times \frac{3}{7} \mathrm{x}$
Let passed boys $=\mathrm{y}$

$$
\begin{aligned}
& \Rightarrow \quad \frac{0.9 \times \frac{3}{7} x+y}{x}=0.8 \\
& \Rightarrow \quad y=0.8 x-\frac{2.7}{7} x=\frac{2.9 x}{7}
\end{aligned}
$$

$\%$ Boys passed $=\frac{\frac{2.9 \mathrm{x}}{7}}{\frac{4}{7} \mathrm{x}} \times 100=72.5 \%$
8. How many integers are there between 100 and 1000 all of whose digits are even ?
(a) 60
(b) 80
(c) 100
(d) 90

Ans. (c)
No. of integers whole all digits are even

$$
\begin{aligned}
& 4 \boxed{4} \boxed{5} \\
& \Rightarrow 4 \times 5 \times 5=100
\end{aligned}
$$

9. An award winning study by a group of researchers suggests that men are as prone to buying on impulse as women but women feel more guilty about shopping.
(a) Many men and women indulge in buying on impulse
(b) All men and women indulge in buying on impulse
(c) Few men and women indulge in buying on impulse
(d) Some men and women indulge in buying on impulse
Ans. (d)
10. Consider five people -Mita, Ganga, Rekha, Lakshmi and Sana. Ganga is the taller than
both Rekha and Lakshmi. Lakshmi is taller than Sana. Mita is taller than Ganga.
11. Lakshmi is taller than Rekha
12. Rekha is shorter than Mita
13. Rekha is taller than Sana
14. Sana is shorter than Ganga
(a) 3 only
(b) 1 and 3
(c) 1 only
(d) 2 and 4

Ans. (d)

## SECTION: ELECTRICAL ENGINEERING

1. The output response of a system is denoted as $\mathrm{y}(\mathrm{t})$, and its Laplace transform is given by
$Y(s)=\frac{10}{s\left(s^{2}+\mathrm{s}+100 \sqrt{2}\right)}$
The steady state value of $y(t)$ is
(a) $\frac{1}{10 \sqrt{2}}$
(b) $10 \sqrt{2}$
(c) $100 \sqrt{2}$
(d) $\frac{1}{100 \sqrt{2}}$

Ans. (a)
Sol: Given transfer function
$\mathrm{y}(\mathrm{s})=\frac{10}{\mathrm{~s}\left(\mathrm{~s}^{2}+\mathrm{s}+100 \sqrt{2}\right)}$
The steady-state value of $y(t)=\underset{t \rightarrow \infty}{\operatorname{Lt}} y(t)$
According to final value theorem.
$\operatorname{Lt}_{\mathrm{t} \rightarrow \infty} \mathrm{y}(\mathrm{t})=\lim _{\mathrm{s} \rightarrow 0} \operatorname{Sy}(\mathrm{~s})$
$=\lim _{\mathrm{s} \rightarrow 0} \mathrm{~S}\left[\frac{10}{\mathrm{~s}\left(\mathrm{~s}^{2}+\mathrm{s}+100 \sqrt{2}\right)}\right]$
$=\frac{10}{100 \sqrt{2}}=\frac{1}{10 \sqrt{2}}$
$\operatorname{Lt}_{\mathrm{t} \rightarrow \infty} \mathrm{y}(\mathrm{t})=\frac{1}{10 \sqrt{2}}$

Hence, the steady-state value of $\mathrm{y}(\mathrm{t})$ is $\frac{1}{10 \sqrt{2}}$.
2. A coaxial cylindrical capacitor shown in Figure (i) has dielectric with relative permittivity $\epsilon_{\mathrm{r} 1}=2$. When one-fourth portion of the dielectric is replaced with another dielectric of relative permittivity $\epsilon_{r 2}$, as shown in Figure (ii), the capacitance is doubled. The value of $\epsilon_{\mathrm{r} 2}$ is
$\qquad$ —.


Figure (i)


Figure (ii)

Ans. (10)
Sol:


The capacitance for region $\left(\varepsilon_{r_{1}}\right)$

$$
\mathrm{C}_{1}=\frac{2 \varepsilon_{\mathrm{r}_{1}}}{\ln \frac{\mathrm{R}}{\mathrm{r}}}(\pi-\phi)
$$

and the capacitance for region $\left(\varepsilon_{r_{2}}\right)$

$$
\mathrm{C}_{2}=\frac{2 \varepsilon_{\mathrm{r}_{2}}}{\ln \frac{\mathrm{R}}{\mathrm{r}}} \phi
$$

here, $\phi=\frac{90^{\circ}}{2}=45^{\circ}=\frac{\pi}{4}$
$\Rightarrow \quad \mathrm{C}_{1}=\frac{2 \varepsilon_{\mathrm{r}_{1}}}{\ln \frac{\mathrm{R}}{\mathrm{r}}} \times \frac{3 \pi}{4}$
and $\quad \mathrm{C}_{2}=\frac{2 \varepsilon_{\mathrm{r}_{2}}}{\ln \frac{\mathrm{R}}{\mathrm{r}}} \times \frac{\pi}{4}$
As, $\mathrm{C}_{1}$ and $\mathrm{C}_{2}$ are in parallel

$$
\begin{aligned}
& \mathrm{C}_{\mathrm{eq}}=\mathrm{C}_{1}+\mathrm{C}_{2}=\frac{2 \varepsilon_{\mathrm{r}_{1}}}{\ln \frac{\mathrm{R}}{\mathrm{r}} \times \frac{3 \pi}{4}+\frac{2 \varepsilon_{\mathrm{r}_{2}}}{\ln \frac{\mathrm{R}}{\mathrm{r}}} \times \frac{\pi}{4}} \\
& =\frac{\pi}{2 \ln \frac{\mathrm{R}}{\mathrm{r}}}\left(3 \varepsilon_{\mathrm{r}_{1}}+\varepsilon_{\mathrm{r}_{2}}\right)
\end{aligned}
$$

For figure (i),

$$
\mathrm{C}=\frac{2 \pi \varepsilon_{\mathrm{r}_{1}}}{\ln \frac{\mathrm{R}}{\mathrm{r}}}
$$

Given that,
$\mathrm{C}_{\mathrm{eq}}=2 \mathrm{C}$

$$
\begin{aligned}
& \Rightarrow \quad \frac{\pi}{2 \ln \frac{\mathrm{R}}{\mathrm{r}}}\left(3 \varepsilon_{\mathrm{r}_{1}}+\varepsilon_{\mathrm{r}_{2}}\right)=2 \times \frac{2 \pi \varepsilon_{\mathrm{r}_{1}}}{\ln \frac{\mathrm{R}}{\mathrm{r}}} \\
& \Rightarrow \quad 3 \varepsilon_{\mathrm{r}_{1}}+\varepsilon_{\mathrm{r}_{2}}=8 \varepsilon_{\mathrm{r}_{1}} \\
& \Rightarrow \quad \varepsilon_{\mathrm{r}_{2}}=5 \varepsilon_{\mathrm{r}_{1}} \\
& \Rightarrow \quad \varepsilon_{\mathrm{r}_{2}}=5 \times 2=10 \\
& \Rightarrow \quad \varepsilon_{\mathrm{r}_{2}}=10
\end{aligned}
$$

3. A current controlled current source (CCCS) has an input impedance of $10 \Omega$ and output impedance of $100 \mathrm{k} \Omega$. When the CCCS is used in a negative feedback closed loop with a loop gain of 9 , the closed loop output impedance is
(a) $100 \mathrm{k} \Omega$
(b) $1000 \mathrm{k} \Omega$
(c) $100 \Omega$
(d) $10 \Omega$

Ans. (b)
Sol.
Current controlled current source (CCCS)

$$
\begin{aligned}
\mathrm{Z}_{\mathrm{in}} & =10 \Omega \\
\mathrm{Z}_{\mathrm{o} / \mathrm{p}} & =100 \mathrm{k} \Omega \\
\mathrm{~A} \beta & =9
\end{aligned}
$$



Output is series connection

$$
\begin{gathered}
Z_{o / p}=Z_{o}(1+A \beta)=100(1+9) \mathrm{k} \Omega \\
Z_{o / p}=1000 \mathrm{k} \Omega
\end{gathered}
$$

4. A $5 \mathrm{kVA}, 50 \mathrm{~V} / 100 \mathrm{~V}$, single-phase transformer has a secondary terminal voltage of 95 V when loaded. The regulation of the transformer is
(a) $9 \%$
(b) $5 \%$
(c) $1 \%$
(d) $4.5 \%$

Ans. (b)
Sol: Voltage regulation $=\frac{\mathrm{V}_{2 \mathrm{nl}}-\mathrm{V}_{2 \mathrm{fl}}}{\mathrm{V}_{2 \mathrm{nl}}} \times 100 \%$
Hence, $\mathrm{VR}=\frac{100-95}{100} \times 100 \%=5 \%$
[Note: For transformers, the voltage drop from no load to full load is given with respect to no load voltage as it is fixed by the power supply.

Whereas, for alternators and transmission lines, the reference voltage is taken as full
load voltage.]
5. The output voltage of a single-phase full bridge voltage source inverter is controlled by unipolar PWM with one pulse per half cycle. For the fundamental rms component of output voltage to be $75 \%$ of DC voltage, the required pulse width in degrees (round off up to one decimal place) is $\qquad$ -
Ans. (112.88 ${ }^{\circ}$ )
Sol: Single phase full bridge VSI with PWM


Fig. (a) Output voltage waveform of PWM control (one pulse per half cycle)
$V_{\text {on }}=\sum_{\mathrm{n}=6 \mathrm{~K} \pm 1} \frac{4 \mathrm{~V}_{\mathrm{dc}}}{\mathrm{n} \pi} \sin \mathrm{nd} \cdot \sin \frac{\mathrm{n} \pi}{2} \cdot \sin \mathrm{n} \omega \mathrm{t}$
$\mathrm{V}_{01 \mathrm{rms}}=$ Fundamental rms output voltage
$\mathrm{V}_{01 \mathrm{rms}}=\frac{2 \sqrt{2}}{\pi} \mathrm{~V}_{\mathrm{dc}} \sin \mathrm{d} \cdot \sin \frac{\pi}{2}$
$\Rightarrow$ Given $\frac{\mathrm{V}_{01 \text { rms }}}{\mathrm{V}_{\mathrm{dc}}}=0.75$
$\Rightarrow 0.75=\sin \mathrm{d}\left(\frac{2 \sqrt{2}}{\pi}\right)$
$\Rightarrow \mathrm{d}=\sin ^{-1}\left[\frac{0.75}{0.9}\right]=56.44$
Hence, pulse width $2 \mathrm{~d}=2 \times 56.44=112.88^{\circ}$
6. The open loop transfer function of a unity feedback system is given by
$\mathrm{G}(\mathrm{s})=\frac{\pi \mathrm{e}^{-0.25 \mathrm{~s}}}{\mathrm{~s}}$
In G(s) plane, the Nyquist plot of G(s) passes through the negative real axis at the point.
(a) $(-0.75, \mathrm{j} 0)$
(b) $(-1.5, \mathrm{j} 0)$
(c) $(-1.25, \mathrm{j} 0)$
(d) $(-0.5, j 0)$


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Sol: Nyquist-plot cut the negative real axis at $\omega=\omega_{\mathrm{pc}} \quad$ (Phase cross-over frequency)
$G(j \omega)=\frac{\pi \mathrm{e}^{-0.25(\mathrm{j} \omega)}}{\mathrm{j} \omega}$
$\phi=-90^{\circ}-0.25 \omega \times \frac{180^{\circ}}{\pi}$

$\left.\phi\right|_{\omega=\omega_{\mathrm{pc}}}=-180^{\circ}=-90^{\circ}-\frac{\omega}{\pi}\left(45^{\circ}\right)$
$\Rightarrow \quad 90^{\circ}=\omega_{\mathrm{pc}}\left(\frac{45^{\circ}}{\pi}\right)$
$\Rightarrow \quad \omega_{\mathrm{pc}}=2 \pi$
Magnitude at cutting point
$\mathrm{a}=|\mathrm{GH}|_{\omega=\omega_{\mathrm{pc}}}$
$\mathrm{a}=\frac{\pi}{\omega_{\mathrm{pc}}}=\frac{\pi}{2 \pi}$

$$
\mathrm{a}=\frac{1}{2}
$$

Hence Nyquist-plot of G(s) passes through the negative used axis at the point $(-0.5, \mathrm{j} 0)$
7. The mean square of a zero mean random process is $\mathrm{kT} / \mathrm{C}$, where k is Boltzman's constant. T is the absolute temperature, and C is capacitance. The standard deviation of the random.
(a) $\frac{\mathrm{C}}{\mathrm{kT}}$
(b) $\sqrt{\frac{\mathrm{kT}}{\mathrm{C}}}$
(c) $\frac{\mathrm{kT}}{\mathrm{C}}$
(d) $\frac{\sqrt{\mathrm{kT}}}{\mathrm{C}}$

Ans. (b)
Sol: Given that,

$$
\sigma^{2}=\frac{\mathrm{KT}}{\mathrm{C}} \quad(\because \quad \text { Mean }=0)
$$

Hence, standard deviation

$$
\sigma=\sqrt{\frac{\mathrm{KT}}{\mathrm{C}}}
$$

8. If $f=2 x^{3}+3 y^{2}+4 z$, the value of line integral $\int_{C} \operatorname{grad} \mathrm{f} . \mathrm{dr}$ evaluated over contour C formed by the segments $(-3,-3,2) \rightarrow(2,-3,2) \rightarrow(2,6$, $2) \rightarrow(2,6,-1)$ is $\qquad$ .
Ans. (139)
Sol: $\quad \int_{\mathrm{c}} \operatorname{gradf} \cdot \mathrm{dr}=$ ?

$$
\overrightarrow{\mathrm{dr}}=\mathrm{dx} \hat{\mathrm{i}}+\mathrm{dy} \hat{\mathrm{j}}+\mathrm{dz} \hat{\mathbf{k}}
$$

and

$$
\operatorname{grad} \mathrm{f}=\frac{\mathrm{d}}{\mathrm{dx}}\left(2 \mathrm{x}^{3}+3 \mathrm{y}^{2}+4 \mathrm{z}\right) \hat{\mathrm{i}}
$$

$$
+\frac{d}{d y}\left(2 x^{3}+3 y^{2}+4 z\right) \hat{j}+\frac{d}{d z}\left(2 x^{3}+3 y^{2}+4 z\right) \hat{k}
$$

$$
\operatorname{grad} f=\left(6 x^{2}\right) \hat{i}+(6 y) \hat{j}+(4) \hat{k}
$$

So, $\quad \operatorname{grad} \mathrm{f} \cdot \overrightarrow{\mathrm{dr}}=6 \mathrm{x}^{2} \mathrm{dx}+6 \mathrm{ydy}+4 \mathrm{dz}$

$$
\int_{c} \operatorname{gradf} \cdot \mathrm{dr}=\int 6 \mathrm{x}^{2} \mathrm{dx}+\int 6 \mathrm{ydy}+\int 4 \mathrm{dz}
$$

Applying the limits

$$
\begin{aligned}
& \int_{c} \operatorname{gradf} \cdot \mathrm{dr}=\left[\int_{-3}^{2} 6 \mathrm{x}^{2} \mathrm{dx}+\int_{-3}^{-3} 6 \mathrm{ydy}+\int_{2}^{2} 4 \mathrm{dz}\right] \\
& +\left[\int_{2}^{2} 6 \mathrm{x}^{2} \mathrm{dx}+\int_{-3}^{6} 6 \mathrm{ydy}+\int_{2}^{2} 4 \mathrm{dz}\right]+ \\
& {\left[\int_{2}^{2} 6 \mathrm{x}^{2} \mathrm{dx}+\int_{6}^{6} 6 \mathrm{ydy}+\int_{2}^{-1} 4 \mathrm{dz}\right]}
\end{aligned}
$$

$$
\begin{aligned}
& \int_{\mathrm{c}} \mathrm{grad} \cdot \mathrm{fdr}=\left[\left(2 \mathrm{x}^{3}\right)_{-3}^{2}+0+0\right]+ \\
& {\left[\left(3 \mathrm{y}^{2}\right)_{-3}^{6}+0+0\right]+\left[0+0+(4 \mathrm{z})_{2}^{-1}\right]}
\end{aligned}
$$

$\int_{\mathrm{c}} \mathrm{gradf} \cdot \mathrm{dr}=[2(8+27)]+[3 \times(36-9)]+4[-1-2]$

$$
=70+81-12=139
$$

9. A six-pulse thyristor bridge rectifier is connected to a balanced three-phase, 50 Hz AC source. Assuming that the DC output current of the rectifier is constant, the lowest harmonic component in the AC input current is
(a) 100 Hz
(b) 150 Hz
(c) 300 Hz
(d) 250 Hz

Ans. (d)
Sol: Supply current as AC input current of 6 pulse thyristor bridge rectifier is quasi-square waveform.

$\mathrm{I}_{\mathrm{sn}}=\sum \frac{4 \mathrm{I}_{0}}{\mathrm{n} \pi} \sin \frac{\mathrm{n} \pi}{3} \sin \mathrm{n} \omega \mathrm{t}$
$\mathrm{n}=6 \mathrm{k} \pm 1$
Note: Due to symmetric waveform, even harmonic does not present.
Harmonic present $\mathrm{n}=6 \mathrm{k} \pm 1$
For lowest component, $\mathrm{k}=1, \mathrm{n}=5,7$
So, the lowest harmonic component in the AC input current is $\mathrm{nf}_{\text {in }}$
$=50 \times 5=250 \mathrm{~Hz}$
10. In the circuit shown below, the switch is closed at $t=0$. The value of $\theta$ in degrees which will give the maximum value of DC offset of the current at the time of switching is

(a) -30
(b) -45
(c) 60
(d) 90

Ans. (b)

## Sol:



By applying KVL in the loop, we get

$$
\mathrm{i}(\mathrm{t}) \cdot \mathrm{R}+\mathrm{L} \frac{\mathrm{di}(\mathrm{t})}{\mathrm{dt}}=\mathrm{V}(\mathrm{t})
$$

Solving this differential equation
$\mathrm{i}(\mathrm{t})=\binom{$ Complimentary }{ Integral }$+\binom{$ Particular }{ Integral }
For complimentary integral

$$
i(t) R+L \frac{d i(t)}{d t}=0
$$

so, we get $i(t)=\mathrm{A} \cdot \mathrm{e}^{-\mathrm{R} / / \mathrm{L}} \Rightarrow \mathrm{DC}$ offset.
For particular integral
where

$$
\mathrm{i}(\mathrm{t})=\frac{\mathrm{V}_{\mathrm{m}}}{\mathrm{z}} \sin (\omega \mathrm{t}+\theta-\phi)
$$

where

$$
\omega=377 \mathrm{rad} / \mathrm{sec}
$$

$$
\phi=\tan ^{-1}\left(\frac{\omega \mathrm{~L}}{\mathrm{R}}\right)
$$

$\mathrm{i}(\mathrm{t})=\mathrm{A} \cdot \mathrm{e}^{-\mathrm{Rt} / \mathrm{L}}+\frac{\mathrm{V}_{\mathrm{m}}}{\mathrm{z}} \sin (\omega \mathrm{t}+\theta-\phi)$
Applying boundary conditions
at $\mathrm{t}=0, \mathrm{i}(\mathrm{t})=0$

$$
\begin{gathered}
0=A \cdot e^{-0}+\frac{V_{m}}{z} \sin (\omega(0)+\theta-\phi) \\
A+\frac{V_{m}}{z} \sin (\theta-\phi)=0 \\
A=\frac{-V_{m}}{z} \sin (\theta-\phi)
\end{gathered}
$$

For maximum value of DC offset "A"

$$
\begin{gathered}
\theta-\phi=-90^{\circ} \\
\theta-\tan ^{-1}\left(\frac{\omega \mathrm{~L}}{\mathrm{R}}\right)=-90^{\circ} \\
\theta-\tan ^{-1}\left(\frac{377 \times 10 \times 10^{-3}}{3.77}\right)=-90^{\circ} \\
\theta-45^{\circ}=-90^{\circ} \\
\theta=-45^{\circ}
\end{gathered}
$$

11. A three-phase synchronous motor draws 200A from the line at unity power factor at rated load. Considering the same line voltage and load., the line current at a power factor of 0.5 leading is
(a) 400 A
(b) 300 A
(c) 200 A
(d) 100 A

Ans. (a)
Sol: Considering in per-unit system Initially,

$$
\begin{aligned}
\mathrm{V} & =\mathrm{V}_{\text {rated }}=1 \mathrm{pu} \\
\mathrm{I}=\mathrm{I}_{\text {rated }} & =1 \mathrm{pu} \quad\binom{\text { given } \mathrm{I}=200 \mathrm{~A}}{\text { consider as } \mathrm{I}_{\text {base }}} \\
\text { p.f } & =1 \text { (unity) }
\end{aligned}
$$

So,

$$
\begin{aligned}
& \mathrm{P}=\mathrm{VI} \cos \phi \\
& \mathrm{P}=1 \times 1 \times 1
\end{aligned}
$$

$$
\mathrm{P}=1 \mathrm{pu}
$$

Now,

$$
\mathrm{V}=\mathrm{V}_{\text {rated }}=1 \mathrm{pu}
$$

$$
\mathrm{I}=?
$$

$$
\mathrm{pf}=0.5
$$

and

$$
\mathrm{P}=1 \mathrm{pu}
$$

Since,

$$
\begin{aligned}
\mathrm{P} & =\mathrm{VI} \cos \phi \\
1 & =1 \times \mathrm{I} \times 0.5 \\
\mathrm{I} & =2 \mathrm{pu}
\end{aligned}
$$

So, line current in ampere is $\mathrm{I}=2 \times 200=$ 400A.
12. The partial differential equation
$\frac{\partial^{2} \mathbf{u}}{\partial \mathbf{t}^{2}}-\mathrm{C}^{2}\left(\frac{\partial^{2} \mathbf{u}}{\partial \mathbf{x}^{2}}+\frac{\partial^{2} \mathbf{u}}{\partial \mathbf{y}^{2}}\right)=0$; where $\mathbf{c} \neq 0$ is known as
(a) wave equation
(b) Laplace equation
(c) heat equation
(d) Poisson's equation

Ans. (a)
Sol: (i) Wave equation

$$
\frac{\partial^{2} \mathbf{u}}{\partial \mathbf{t}^{2}}=\mathbf{C}^{2}\left(\frac{\partial^{2} \mathbf{u}}{\partial \mathbf{x}^{2}}+\frac{\partial^{2} \mathbf{u}}{\partial \mathbf{y}^{2}}\right)
$$

(ii) Laplace equation

$$
\nabla^{2} \mathbf{u}=\frac{\partial^{2} \mathbf{u}}{\partial \mathbf{x}^{2}}+\frac{\partial^{2} \mathbf{u}}{\partial \mathbf{y}^{2}}=0
$$

(iii) Poisson's equation

$$
\nabla^{2} \mathrm{u}=\mathrm{f}
$$

(iv) Heat equation

$$
\frac{\partial \mathbf{u}}{\partial \mathrm{t}}-\alpha\left(\frac{\partial^{2} \mathbf{u}}{\partial \mathbf{x}^{2}}+\frac{\partial^{2} \mathbf{u}}{\partial \mathbf{y}^{2}}+\frac{\partial^{2} \mathbf{u}}{\partial \mathbf{z}^{2}}\right)=0
$$

13. The parameter of an equivalent circuit of a three-phase induction motor affected by reducing the rms value of the supply voltage at the rated frequency is
(a) magnetizing reactance
(b) rotor leakage reactance
(c) rotor resistance
(d) stator resistance

GATE 2019
Detailed Solution
09-02-2019 | AFTERNOON SESSION

Ans. (a)

Sol.


By reducing the rms value of the supply voltage at rated frequency, magnetising current changes which changes the magnetizing reactance.
So, option (1).
14. The symbols, $\alpha$ and T, represent positive quantities, and $u(t)$ is the unit step function. Which one of the following impulse responses is NOT the output of a causal linear time-invariant system?
(a) $1+\mathrm{e}^{-\mathrm{at}} \mathrm{u}(\mathrm{t})$
(b) $\mathrm{e}^{-\mathrm{a}(\mathrm{t}-\mathrm{T})} \mathrm{u}(\mathrm{t})$
(c) $e^{+a t} u(t)$
(d) $e^{-a(t+T)} u(t)$

Ans. (a)
Sol.

$$
1+e^{-a t} u(t)
$$

Due to presence of 1, Given impulse response is non-cousal.
15. Which one of the following function is analytic in the region $|z| \leq 1$ ?
(a) $\frac{z^{2}-1}{z}$
(b) $\frac{\mathrm{z}^{2}-1}{\mathrm{z}+\mathrm{j} 0.5}$
(c) $\frac{\mathrm{z}^{2}-1}{\mathrm{z}-0.5}$
(d) $\frac{\mathrm{z}^{2}-1}{\mathrm{z}+2}$

Ans. (d)
Sol. In option 1, 2 and 3 singularities lying inside the contour but in option 4 singularity $\mathrm{z}=-$ 2 lying outside the contour $\mathrm{z}=1$.

So, the function given in option 4 is analytic in the region $|z| \leq 1$.
16. The inverse Laplace transform of $\mathrm{H}(\mathrm{s})=$ $\frac{s+3}{s^{2}+2 s+1}$ for $t \geq 0$ is
(a) $4 t \mathrm{e}^{-\mathrm{t}}+\mathrm{e}^{-\mathrm{t}}$
(b) $2 \mathrm{te}^{-\mathrm{t}}+\mathrm{e}^{-\mathrm{t}}$
(c) $3 \mathrm{te}^{-\mathrm{t}}+\mathrm{e}^{-\mathrm{t}}$
(d) $3 \mathrm{e}^{-\mathrm{t}}$

Ans. (b)
Sol: Given from function
$H(s)=\frac{s+3}{s^{2}+2 s+1} \quad t \geq 0$

## By using partial fraction

$\mathrm{H}(\mathrm{s})=\frac{\mathrm{s}+3}{(\mathrm{~s}+1)^{2}}=\frac{\mathrm{A}}{(\mathrm{s}+1)}+\frac{\mathrm{B}}{(\mathrm{s}+1)^{2}}$
$\mathrm{s}+3=\mathrm{A}(\mathrm{s}+1)+\mathrm{B}$
$\Rightarrow \quad \mathrm{s}+3=\mathrm{As}+\mathrm{A}+\mathrm{B}$
Equating coefficients, we get
$\Rightarrow \quad \mathrm{A}=1$
and

$$
\mathrm{B}=2
$$

$$
\mathrm{H}(\mathrm{~s})=\frac{\mathrm{A}}{(\mathrm{~s}+1)}+\frac{\mathrm{B}}{(\mathrm{~s}+1)^{2}}
$$

$$
\mathrm{H}(\mathrm{~s})=\frac{1}{(\mathrm{~s}+1)}+\frac{2}{(\mathrm{~s}+1)^{2}}
$$

By taking Inverse Laplace, we get

$$
h(t)=e^{-t}+\left(2 e^{-t}\right) t
$$

$$
\mathrm{h}(\mathrm{t})=2 \mathrm{te}^{-\mathrm{t}}+\mathrm{e}^{-\mathrm{t}}
$$

# MASTER TALENT REWARD EXAM (MTRE) A National Level Online Scholarship Test 

- Opportunity to get up to $100 \%$ off on tuition fee
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GATE 2019
17. The rank of the matrix, $M=\left[\begin{array}{lll}0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0\end{array}\right]$ is $\qquad$
Ans. (3)
Sol: Given data,

$$
\mathrm{M}=\left[\begin{array}{lll}
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 0
\end{array}\right]_{3 \times 3}
$$

## Determinant of M

$$
\begin{aligned}
|\mathrm{M}| & =\left|\begin{array}{lll}
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 0
\end{array}\right| \\
& =0(0-1)-1(0-1)+1(1-0) \\
& =1+1=2
\end{aligned}
$$

$\because \quad|M| \neq 0$
Hence, rank of the matrix is ' 3 '.
18. Given, $\mathrm{V}_{\mathrm{gs}}$ is the gate-source voltage, $\mathrm{V}_{\mathrm{ds}}$ is the drain source voltage, and $\mathrm{V}_{\text {th }}$ is the threshold voltage of an enhancement type NMOS transistor, the conditions for transistor to be biased in saturation are :
(a) $\mathrm{V}_{\mathrm{gs}}>\mathrm{V}_{\mathrm{th}} ; \mathrm{V}_{\mathrm{ds}} \leq \mathrm{V}_{\mathrm{gs}}-\mathrm{V}_{\mathrm{th}}$
(b) $\mathrm{V}_{\mathrm{gs}}>\mathrm{V}_{\mathrm{th}} ; \mathrm{V}_{\mathrm{ds}} \geq \mathrm{V}_{\mathrm{gs}}-\mathrm{V}_{\mathrm{th}}$
(c) $\mathrm{V}_{\mathrm{gs}}<\mathrm{V}_{\mathrm{th}} ; \mathrm{V}_{\mathrm{ds}} \geq \mathrm{V}_{\mathrm{gs}}-\mathrm{V}_{\mathrm{th}}$
(d) $\mathrm{V}_{\mathrm{gs}}<\mathrm{V}_{\mathrm{th}} ; \mathrm{V}_{\mathrm{ds}} \leq \mathrm{V}_{\mathrm{gs}}-\mathrm{V}_{\mathrm{th}}$

Ans. (b)
Sol.


Condition for Transistor in saturation
and

$$
\mathrm{V}_{\mathrm{GS}}>\mathrm{V}_{\mathrm{th}}
$$

$$
\mathrm{V}_{\mathrm{DS}}>\mathrm{V}_{\mathrm{oV}}
$$

$$
\mathrm{V}_{\mathrm{DS}}>\mathrm{V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{th}}
$$


19. The total impedance of the secondary winding, leads, and burden of a 5 ACT is $0.01 \Omega$. If the fault current is 20 times the rated primary current of the CT, the VA output of the CT is $\qquad$ .
Ans. (100)
Sol:


Secondary rated current of $\mathrm{CT}=5 \mathrm{~A}$
Secondary impedance $=0.01 \Omega$
CT Ratio $=\frac{\text { Primary current }}{\text { Secondary current }}$
Since, fault current is 20 times the rated primary current.

Hence, secondary current of CT will also be 20 times the rated secondary current.

Now, when fault occurs, the secondary current will be
$I_{s}=20 \times$ Rated secondary current of CT

$$
\begin{aligned}
& =20 \times 5 \mathrm{~A} \\
& =100 \mathrm{~A}
\end{aligned}
$$

Hence, VA output of the CT

$$
\begin{aligned}
& =\mathrm{EI} \\
& =(\mathrm{IZ}) \times \mathrm{I} \\
& =(100 \times 0.01) \times 100 \\
& =100 \mathrm{VA}
\end{aligned}
$$

20. The characteristic equation of a linear time-invariant (LTI) system is given by
$\Delta(\mathrm{s})=\mathrm{s}^{4}+3 \mathrm{~s}^{3}+3 \mathrm{~s}^{2}+\mathrm{s}+\mathrm{k}=0$
The system is BIBO stable if
(a) $\mathrm{k}>6$
(b) $0<\mathrm{k}<\frac{8}{9}$
(c) $0<\mathrm{k}<\frac{12}{9}$
(d) $\mathrm{k}>3$

Ans. (b)

## Sol: R-H criteria

| $S^{4}$ | 1 | 3 | k |
| :--- | :---: | :---: | :---: |
| $\mathrm{S}^{3}$ | 3 | 1 | 0 |
| $\mathrm{~S}^{2}$ | $\frac{8}{3}$ | k |  |
|  | $\mathrm{S}^{1}$ | $\frac{8 / 3-3 \mathrm{k}}{8 / 3}$ | 0 |
| $\mathrm{~S}^{0}$ | k |  |  |

For BIBO stable, all elements of first column have same sign,
$\mathrm{k}>0$ and $\frac{\frac{8}{3}-3 \mathrm{k}}{\frac{8}{3}}>0$
$\Rightarrow \quad \mathrm{k}<\frac{8}{9}$
Hence, $0<\mathrm{k}<\frac{8}{9}$
21. Five alternators each rated $5 \mathrm{MVA}, 13.2 \mathrm{kVA}$ with $25 \%$ of reactance on its own base are connected in parallel to a busbar. The short-circuit level in MVA at the busbar is $\qquad$ -.

Ans. (100)
Sol: According to the given condition.


For short circuit current $\left(I_{f}\right)$


$$
\begin{aligned}
& I_{\mathrm{f}}=\frac{\mathrm{V}_{\mathrm{pu}}}{\mathrm{X}_{\mathrm{pu}}}=\frac{1}{0.05} \\
& \mathrm{I}_{\mathrm{f}}=20 \mathrm{pu}
\end{aligned}
$$

So, short circuit level in MVA is $\left(\mathrm{I}_{\mathrm{f}}\right) \times($ MVA rating of alternator)
S.C (MVA) level $=20 \times 5(\mathrm{MVA})=100 \mathrm{MVA}$
22. A system transfer function is $\mathrm{H}(\mathrm{s})=$ $\frac{a_{1} s^{2}+b_{1} s+c_{1}}{a_{2} s^{2}+b_{2} s+c_{2}}$. If $a_{1}=b_{1}=0$, and all other coefficients are positive, the transfer function represents a
(a) high pass filter
(b) notch filter
(c) low pass filter
(d) band pass filter

Ans. (c)
Sol.

$$
\begin{aligned}
\left.\mathrm{H}(\mathrm{~s})\right|_{a_{1}=b_{1}=0} & =\frac{c_{1}}{\mathrm{a}_{2} \mathrm{~s}^{2}+\mathrm{b}_{2} \mathrm{~s}+\mathrm{c}_{2}} \\
\mathrm{H}(\mathrm{j} \omega) & =\frac{c_{1}}{j \omega \mathrm{~b}_{2}-\mathrm{a}_{2}(\omega)^{2}+\mathrm{c}_{2}}
\end{aligned}
$$

at $\omega=0$

$$
\mathrm{H}(\mathrm{j} \omega)=\mathrm{c}_{1} / \mathrm{c}_{2}=\mathrm{K}
$$

at $\omega=\infty$

$$
\mathrm{H}(\mathrm{j} \omega)=0
$$

So, given filter is a low pass filter.
23. The current I flowing in the circuit shown below in amperes (round off to one decimal place) is $\qquad$ -.


Ans. (1.4 amp)
Sol.


Current distribution is shown in the above diagram.
Applying KVL in the loop

$$
\begin{gathered}
20-2 \mathrm{I}-3(\mathrm{I}+2)-5 \mathrm{I}=0 \\
20-10 \mathrm{I}-6=0 \\
10 \mathrm{I}=14 \\
\mathrm{I}=1.4 \text { Ampere }
\end{gathered}
$$

24. $M$ is a $2 \times 2$ matrix with eigenvalues 4 and 9 The eigenvalues of $\mathrm{M}^{2}$ are
(a) -2 and -3
(b) 2 and 3
(c) 4 and 9
(d) 16 and 81

Ans. (d)
Sol: When $M$ is required, the eigen vectors remain unchanged whereas, the eigen values are squared.

Hence, eigen values of $\mathrm{M}^{2}$ are 16 and 81.
Proof: Let $\lambda$ be the eigen value of $\mathrm{A}^{2}$, then

$$
\begin{gathered}
\left|A^{2}-\lambda I\right|=0 \\
\text { or } \quad(A-\sqrt{\lambda} I)(A+\sqrt{\lambda} I)=0
\end{gathered}
$$

hence, either $|A-\sqrt{\lambda I}|=0$
or $\quad|A+\sqrt{\lambda} I|=0$

Eigen values of A are $\sqrt{\lambda}$ or $-\sqrt{\lambda}$.
25. The $Y_{\text {bus }}$ matrix of a two-bus power system having two identical parallel lines connected between them in pu is given as
$Y_{\text {bus }}=\left[\begin{array}{ll}-j 8 & j 20 \\ j 20 & -j 8\end{array}\right]$
The magnitude of the series reactance of each line in pu (round off up to one decimal place) is
$\qquad$ -
Ans. (0.1)
Sol:


Let, $y$ be the admittance of identical lines and, $\mathrm{y}_{10}$ and $\mathrm{y}_{20}$ is the shunt admittance connected at bus-1 and bus-2 respectively.
Then, by using direct inspection method, $\mathrm{y}_{\mathrm{BUS}}$ is given as,

$$
\mathrm{y}_{\text {BUS }}=\left[\begin{array}{cc}
\mathrm{y}_{10}+2 \mathrm{y} & -2 \mathrm{y} \\
-2 \mathrm{y} & \mathrm{y}_{20}+2 \mathrm{y}
\end{array}\right]
$$

# CONVENTIONAL QUESTION PRACTICE PROGRAM for ESE-2019 Mains Exam 

## COMPLETE PACKAGE

## Classroom Program

includes Subject-wise tests

## Conventional Test Series

includes
11 Mixed Topic-wise \& 6 Full-length tests

## Classroom Program

$188^{\text {" } \mathrm{Feb}}$ Monday to Saturday
250-300 hrs 8:30 am to 2:30 pm
I Subject wise Practice, Discussion and Test
I Practice Booklets with Solution Outlines
\How to Write Answer- Test \& Counselling Session
$\$ Discussion and Practice Session of 250-300 hrs
I Under Guidance of Mr. Kanchan Kr. Thakur

Conventional Test Series
$17^{\text {th }}$ March $\mid$ Every Sunday

a) New Topics b) Revision Topics

1 Classroom Solutions + Discussion
IImprove Question Selection Ability
I Cover all Concepts in Various Topics
I Improve Time Management
1 Under Simulated Classroom Exam Env. Unique Approach : Test on New Topics + Revision Topics

| Course | Branch | Fees (₹) |
| :---: | :---: | :---: |
| Complete Package (Classroom Program + Conventional Test Series) for Non IES Master Students | CE | 18000/- |
| Complete Package (Classroom Program + Conventional Test Series) for Ex- IES Master Students | CE | 15000/- |
| Conventional Classroom Program (For Non IES Master Students) | CE | 15000/- |
| Conventional Classroom Program (For Ex- IES Master Students) | CE | 12000/- |
| Conventional Classroom Program (For Current year IES Master Classroom Program Students) | CE | 10000/- |
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## HOW TO APPLY

## GATE 2019

Detailed Solution
09-02-2019 | AFTERNOON SESSION

Comparing this matrix with given by $\mathrm{y}_{\text {BUS }}$.

$$
\begin{aligned}
& \mathrm{y}_{10}+2 \mathrm{y}=-\mathrm{j} 8 \\
& \mathrm{y}_{20}+2 \mathrm{y}=-\mathrm{j} 8 \\
& 2 \mathrm{y}=-\mathrm{j} 20 \\
\Rightarrow & \mathrm{y}=-\mathrm{j} 10
\end{aligned}
$$

Hence, series reactance $=\frac{1}{10}=0.10$
26. The closed loop line integral
$\oint_{|z|=5} \frac{z^{3}+z^{2}+8}{z+2} d z$
evaluated counterclockwise, is
(a) $-4 \mathrm{j} \pi$
(b) $+8 \mathrm{j} \pi$
(c) $-8 \mathrm{j} \pi$
(d) $+4 \mathrm{j} \pi$

Ans. (b)
Sol: According to Cauchy's integral formula
$f(a)=\frac{1}{2 \pi j} \int_{C} \frac{f(z)}{(z-a)} d z$
(if $f(z)$ is analytic within and on the closed curve C)

Hence,
$\int_{C} \frac{f(z)}{(z+2)}\left\{\right.$ where, $\left.f(z)=z^{3}+z^{2}+8\right\}$
$=2 \pi j f(-2) \quad\{\because \quad a=-2\}$
$=2 \pi j(-8+4+8)$
$=+j 8 \pi$
27. A DC-DC buck converter operates in continuous conduction mode. It has 48 V input voltage, and it feeds a resistive load of $24 \Omega$. The switching frequency of the converter is 250 Hz . If switch-on duration is 1 ms , the load power is
(a) 48 W
(b) 24 W
(c) 12 W
(d) 6 W

Ans. (d)
Sol. Given,

$$
\begin{aligned}
\mathrm{V}_{\mathrm{dc}} & =48 \mathrm{~V} \\
\mathrm{R} & =24 \Omega \\
\mathrm{f}_{\mathrm{s}} & =250 \mathrm{~Hz} \\
\mathrm{~T}_{\mathrm{on}} & =1 \mathrm{msec} \\
\mathrm{P}_{\mathrm{load}} & =?
\end{aligned}
$$


(Buck converter)
So, $\quad D=\frac{T_{\mathrm{ON}}}{\mathrm{T}}=\mathrm{T}_{\mathrm{on}} \mathrm{f}$
$\mathrm{D}=\left(1 \times 10^{-3}\right)(250)$

$$
\mathrm{D}=0.25
$$

Waveform


Current $I_{0}$ is ripple free due to high value of inductor present in Buck converter. But $\mathrm{V}_{\mathrm{c}}$ have some ripple.

Considering fourier transform of $\mathrm{I}_{0}$ and $\mathrm{V}_{\mathrm{c}}$

$$
\mathrm{I}_{0}=\mathrm{I}_{0 \mathrm{avg}}
$$

Power $=\left(\mathrm{V}_{0 \text { avg }} \times \mathrm{I}_{0 \text { avg }}\right)+($ harmonics power $)$
$\therefore$ Harmonics power $=0$; (Because no harmonics in current $\mathrm{I}_{0}$ )

$$
\begin{aligned}
\mathrm{P} & =\left(\mathrm{V}_{0 \text { avg }}\right)\left(\frac{\mathrm{V}_{0 \text { avg }}}{\mathrm{R}}\right) \\
& =\frac{\left(\mathrm{V}_{0 \text { avg }}\right)^{2}}{\mathrm{R}} \\
\mathrm{~V}_{0 \text { avg }} & =\alpha \mathrm{V}_{\mathrm{s}}=0.25 \times 48=12 \mathrm{~V} \\
\mathrm{P} & =\frac{(12)^{2}}{24} \\
\mathrm{P} & =6 \mathrm{~W}
\end{aligned}
$$

28. The voltage across and the current through a load are expressed as follows
$\mathrm{v}(\mathrm{t})=-170 \sin \left(377 \mathrm{t}-\frac{\pi}{6}\right) \mathrm{V}$
$\mathrm{i}(\mathrm{t})=8 \cos \left(377 \mathrm{t}+\frac{\pi}{6}\right) \mathrm{A}$
The average power in watts (round off to one decimal place) consumed by the load is $\qquad$
Ans. (588.9)
Sol: Given data,

$$
\begin{aligned}
\mathrm{v}(\mathrm{t}) & =-170 \sin \left(377 \mathrm{t}-\frac{\pi}{6}\right) \mathrm{V} \\
& =170 \cos \left(377 \mathrm{t}+\frac{\pi}{3}\right) \mathrm{V}
\end{aligned}
$$

and

$$
\mathrm{i}(\mathrm{t})=8 \cos \left(377 \mathrm{t}+\frac{\pi}{6}\right) \mathrm{A}
$$

The average power is given as,

$$
\mathrm{P}=\mathrm{VI} \cos \phi
$$

where, V and I are the rms values and $\phi$ is the phase angle difference or power factor angle between $\mathrm{v}(\mathrm{t})$ and $\mathrm{i}(\mathrm{t})$.

$$
\text { So, that, } \mathrm{V}=\frac{170}{\sqrt{2}} \mathrm{~V}, \mathrm{I}=\frac{8}{\sqrt{2}}, \phi=\frac{\pi}{6}
$$

$$
\Rightarrow \mathrm{P}=\frac{170}{\sqrt{2}} \times \frac{8}{\sqrt{2}} \times \cos \frac{\pi}{6}=588.9 \mathrm{Watts}
$$

29. A fully-controlled three-phase bridge converter is working from a $415 \mathrm{~V}, 50 \mathrm{~Hz} \mathrm{AC}$ supply. It is supplying constant current of 100 A at 40 V to a DC load. Assume large inductive smoothing and neglect overlap. The rms value of the AC line current in amperes (round off to two decimal places) is $\qquad$ _.
Ans. (81.649A)
Sol. For 3 phase 6 pulse AC to DC converter, source current is quasi square waveform.

SCR conducts for $(2 \pi / 3)$ in $2 \pi$ period.


$$
\begin{array}{ll}
\Rightarrow & \mathrm{I}_{\mathrm{Sr}}=\mathrm{I}_{\mathrm{o}} \sqrt{\frac{2 \pi / 3}{\pi}}=\mathrm{I}_{\mathrm{o}} \sqrt{\frac{2}{3}} \\
\Rightarrow & \mathrm{I}_{\mathrm{Sr}}=100 \times \sqrt{\frac{2}{3}}=81.649 \mathrm{~A}
\end{array}
$$

30. A 220 V (line), three-phase, Y-connected, synchronous motor has a synchronous impedance of $(0.25+j 2.5) \Omega /$ phase . The motor draws the rated current of 10 A at a 0.8 pf leading. The rms value of line-to-line internal voltage in volts (round off to two decimal places) is $\qquad$ .

## Ans. (245.34 volts)

Sol.

$$
\mathrm{z}_{\mathrm{s}}=(0.25+\mathrm{j} 2.5) \Omega / \mathrm{ph}
$$

$\mathrm{I}=10 \mathrm{~A}, 0.8 \mathrm{pf}$ leading


$$
\overrightarrow{\mathrm{V}}=\frac{220}{\sqrt{3}} \angle 0^{\circ}
$$

$$
\overrightarrow{\mathrm{E}}=\overrightarrow{\mathrm{V}}-\overrightarrow{\mathrm{I}} \overline{\mathrm{Z}}_{\mathrm{s}}
$$

$$
\begin{aligned}
& \overrightarrow{\mathrm{E}}=\frac{220}{\sqrt{3}} \angle 0^{\circ}-\left(10 \angle \cos ^{-1}(0.8)\right)(0.25+\mathrm{j} 2.5) \\
& \overrightarrow{\mathrm{E}}=127.01+13-21.5 \mathrm{j} \\
& \overrightarrow{\mathrm{E}}=140.01-21.5 \mathrm{j} \\
& \overrightarrow{\mathrm{E}}=141.65 \angle-8.736^{\circ} \text { volts (per phase) } \\
& \left|\mathrm{E}_{\mathrm{L}-\mathrm{L}}\right|=\sqrt{3} \times 141.65 \\
& \mid \mathrm{E}_{\mathrm{L}-\mathrm{L}}=245.345 \text { volts } \mid
\end{aligned}
$$

31. The probability of a resistor being defective is 0.02 . There are 50 such resistors in a circuit. The probability of two or more defective resistors in the circuit (round off to two decimal places) is $\qquad$ _.

Ans. (0.26)
Sol. Probability of a resistor being defective

$$
p(\text { def })=0.02
$$

Numbers of resistors $=50$
Approximated Poisson distribution will have mean $=\mu=n p$

$$
\Rightarrow \quad \mu=50 \times 0.02=1
$$

Poisson distribution,

$$
f(x)=\frac{e^{-\mu} \mu^{x}}{x!}
$$

$$
\mathrm{p}(0 \text { defective })=\frac{\mathrm{e}^{-\mu} \mu^{0}}{0!}=\mathrm{e}^{-1}
$$

$$
p(1 \text { defective })=\frac{\mathrm{e}^{-\mu} \mu^{1}}{1!}=\mathrm{e}^{-1}
$$

$p(2$ or more defective $)=1-p(0$ defective $)-$ $p$ (1 defective)
$\Rightarrow \quad \mathrm{p}$ (2 or more defective)

$$
\begin{aligned}
& =1-\mathrm{e}^{-1}-\mathrm{e}^{-1}=1-2 \mathrm{e}^{-1} \\
& =0.26
\end{aligned}
$$

32. The asymptotic Bode magnitude plot of a minimum phase transfer function G(s) is shown


Consider the following two statements.
Statement I : Transfer function G(s) has three poles and one zero
Statement II : At very high frequency ( $\omega \rightarrow \infty$ ), the phase angle $\angle \mathrm{G}(\mathrm{j} \omega)=-\frac{3 \pi}{2}$
Which of the following options is correct?
(a) Statement I is true and statement II is false.
(b) Both the statements are true
(c) Both the statements are false
(d) Statement I is false and statement II is true.

Ans. (d)
Sol.


Transfer function of given Bode plot is

$$
=\frac{K}{s\left(1+\frac{s}{1}\right)\left(1+\frac{s}{20}\right)}
$$

$$
G(s)=\frac{K(20)}{s(s+1)(s+20)}
$$

# M IES MASTER <br> Institute for Engineers (IES/GATE/PSUs) 

## ESE-2019 Conventional Test Schedule, Electrical Engineering

## Date

17th Mar 2019
24th Mar 2019

31st Mar 2019

07th Apr 2019

## 14th Apr 2019

21st Apr 2019

28th Apr 2019
05th May 2019
12th May 2019
19th May 2019
26th May 2019
02nd Jun 2019
09th Jun 2019
16th Jun 2019

Topic
N.T. : ECF-1, MC-1, MC-2, ADE-2
R.T. :
N.T. : ECF-2, MI-1, CS-1, CS-2
R.T. : ECF-1, MC-1, MC-2, ADE-2
N.T. : ECF-3, MI-2, MC-3, MC-4
R.T. : ECF-2, MI-1, CS-1, CS-2
N.T. : BEX-1, ADE-1, ADE-3
R.T. : ECF-3, MI-2, MC-3, MC-4
N.T. : EM-1, MATH-1, PS-1, SSP-1
R.T. : BEX-1, ADE-1, ADE-3
N.T. : CF-1, MATH-2, PS-2, PE-1
R.T. : EM-1, MATH-1, PS-1, SSP-1
N.T. : BEX-2, MI-3, CS-3, SSP-2
R.T. : CF-1, MATH-2, PS-2, PE-1
N.T. : EM-2, PS-3
R.T. : BEX-2, MI-1, MI-3,. CS-3, SSP-2, ADE-3, MC-1, MC-2
N.T. : CF-2, PE-2
R.T. : EM-2, ECF-1, ECF-3, MI-2, PS-2, PS-3, ADE-2, CS-2
N.T. : CF-3, MATH-3
R.T. : CF-2, ECF-2, MI-1, BEX-1, EM-1, CS-1, MI-3, CS-3, ADE-3, PE-2, SSP-1
N.T. :
R.T. : MATH-1, MATH-3, EM-1, EM-2, ECF-1, BEX-2, CF-3, ADE-2, CS-2, PS-1, PS-3 PE-1, SSP-2

Full Length-1 (Test Paper-1 + Test Paper-2)
Full Length-2 (Test Paper-1 + Test Paper-2)
Full Length-3 (Test Paper-1 + Test Paper-2)

| Test Type | Timing |
| :--- | :--- |
| Conventional Test | 10:00 A.M. to 1:00 P.M. |
| Conventional Full Length Test Paper-1 | 10:00 A.M. to 1:00 P.M. $\quad$ Sunday |
| Conventional Full Length Test Paper-2 | Sunday |

Note : The timing of the test may change on certain dates. Prior information will be given in this regard.
*N.T. : New Topic. *R.T. : Revision Topic
Call us : 8010009955, 011-41013406 or Mail us : info@iesmaster.org

Subject Code Details

| Engineering Mathematics (MATH) | MATH-1 | MATH-2 |  | MATH-3 |
| :---: | :---: | :---: | :---: | :---: |
|  | - Linear Algebra Complex Variables <br> - Transform Theory | - Calculus Differential Equations |  | - Probability and Statistics <br> - Numerical Methods |
| Electrical Materials (EM) | EM-1 |  | EM-2 |  |
|  | - Crystal Structures \& Solid State Band Theory $\bullet$ Dielectrics - Magnetic materials |  | - Conductive materials $\bullet$ Photo conductivity $\bullet$ Nano materials - Superconductors |  |
| Electric Circuits \& Fields (ECF) | ECF-1 | ECF-2 |  | ECF-3 |
|  | - Circuit Elements -3-phase Circuits - Network Graphs Transient and steady state Response | - Magnetically Coupled Circuits <br> - Network Theorems Two-port networks <br> - Resonance Basic Filters |  | - Electrostatics and Magneto statics <br> - Time varying fields \& Maxwell's Equations |
| Electrical \& Electronic Measurements | MI-1 | MI-2 |  | MI-3 |
|  | - Errors, Units, Dimensions \& standards <br> - Galvanometers Types of Instruments - Measurement of Power | - Measurement of Energy Measurement of resistance Potentiometers $\bullet$ AC bridges $\bullet$ CRO $\vee$-meter |  | - Electronic Instrumentation <br> - Data Acquisition System <br> - Transducers |
| Computer Fundamentals (CF) | CF-1 | CF-2 |  | CF-3 |
|  | - Architecture, CPU, I/O, Memory, Peripheral devices Boolean algebra <br> - Number system arithmetic functions | - Basic of OS, Virtual memory <br> - File system * Networking |  | - Data Representation and Programming, Programming languages |
| Basic <br> Electronics <br> Engineering (BEX) | BEX-1 |  | BEX-2 |  |
|  | - Basics of diodes, BJT, FET, MOSFET |  | - Transistor amplifiers - equivalent circuits \& frequency response <br> - Oscillators, Feedback amplifiers |  |
| Analog Digital Electronics (ADE) | ADE-1 | ADE-2 |  | ADE-3 |
|  | - OPAMP <br> - Multivibrator, Sample and Hold circuits <br> - Filters | - Digital Electronics * Microprocessors |  | - Communications |
| Systems and Signal Processing (SSP) | SSP-1 |  | SSP-2 |  |
|  | - Continuous \& discrete-time signals - Shifting and scaling <br> - Linear, time-invariant and causal system <br> - Laplace \& Z-transform |  | - Fourier series $\downarrow$ Discrete Fourier Transform <br> $\bullet$ FFT $\bullet$ FIR and IIR Filters •Bilinear Transformation |  |
| Control System (CS) | CS-1 | CS-2 |  | CS-3 |
|  | - Basics Block diagram Algebra <br> - Signal flow <br> - Mathematical Modeling | - Time Response Analysis <br> - Stability $\leqslant$ Root Locus |  |  <br> Compensators © State Variable Analysis <br> - Frequency Response \& its stability |
| Electrical Machines (MC) | MC-1 $\quad$ MC-2 |  | MC-3 | MC-4 |
|  |  | Polyphase Induction Machines <br> - Single Phase motors | *DC Machin | s Synchronous Machines |
|  | PS-1 | PS-2 |  | PS-3 |
| Power System (PS) | - Electric Power Sources-Thermal, Hydro <br> Nuclear, Wind \& Solar <br> - Performance of lines \& cables <br> - HVDC \& Corona <br> - Smart Grid; Environment Implications | - Symmetrical Components \& Fault Analysis <br> - Power System stability \& dynamics <br> - Load flow; Matrix Representation |  |  <br> Power Economics Load Frequency control Voltage Control \& Compensation <br> - FACTS Power System Protection - Solid state Relays |
|  | PE-1 |  | PE-2 |  |
| Electronics and Drivers (PE) | - Power Semiconductor Devices * High Frequency Inductors \& transformers <br> - Diode Rectifiers $\leqslant$ Phase Controlled Rectifiers |  | - Choppers; DC-DC switched mode converters $\downarrow$ Inverters; DC-AC switched mode converters <br> - AC Voltage Controllers Cycloconverters Electric Drives <br> - Resonant Converters |  |

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$$
\phi(\mathrm{j} \omega)=-90^{\circ}-\tan ^{-1} \omega-\tan ^{-1}\left(\frac{\omega}{20}\right)
$$

Transfer function $G(s)$ has only three Poles. So, statement I is false.
and $\left.\quad \phi(\mathrm{j} \omega)\right|_{\omega \rightarrow \infty}=-90-90-90=-270$

$$
=-\frac{3 \pi}{2}
$$

So, statement II is true.

## Option (d)

33. Consider a $2 \times 2$ matrix $\mathrm{M}=\left[\begin{array}{ll}\mathrm{v}_{1} & \mathrm{v}_{2}\end{array}\right]$ where, $\mathrm{v}_{1}$ and $v_{2}$ are the column vectors. Suppose $\mathrm{M}^{-1}=\left[\begin{array}{l}\mathbf{u}_{1}^{\mathrm{T}} \\ \mathbf{u}_{2}^{\mathrm{T}}\end{array}\right]$, where $\mathrm{u}_{1}^{\mathrm{T}}$ and $\mathrm{u}_{2}^{\mathrm{T}}$ are the row vectors. Consider the following statements:
Statement I: $u_{1}^{T} v_{1}=1$ and $u_{2}^{T} v_{2}=1$
Statement II : $u_{1}^{T} v_{2}=0$ and $u_{2}^{T} v_{1}=0$
(a) Statement 1 is true and statement 2 is false
(b) Both the statements are false
(c) Statement 2 is true and statement 1 false
(d) Both the statements are true.

Ans. (4)
Sol. Let

$$
\begin{aligned}
\mathrm{M} & =\left[\begin{array}{ll}
\mathrm{a} & \mathrm{~b} \\
\mathrm{c} & \mathrm{~d}
\end{array}\right]=\left[\begin{array}{ll}
\mathrm{V}_{1} & \mathrm{~V}_{2}
\end{array}\right], \\
\mathrm{V}_{1} & =\left[\begin{array}{l}
\mathrm{a} \\
\mathrm{c}
\end{array}\right], \mathrm{V}_{2}=\left[\begin{array}{l}
\mathrm{b} \\
\mathrm{~d}
\end{array}\right] \\
\mathrm{M}^{-1} & =\frac{1}{\mathrm{ad}-\mathrm{bc}}\left[\begin{array}{cc}
\mathrm{d} & -\mathrm{b} \\
-\mathrm{c} & \mathrm{a}
\end{array}\right]=\left[\begin{array}{l}
\mathrm{U}_{1}^{\mathrm{T}} \\
\mathrm{U}_{2}^{\mathrm{T}}
\end{array}\right] \\
\mathrm{U}_{1}^{\mathrm{T}} & =\left[\begin{array}{ll}
\frac{\mathrm{d}}{\mathrm{ad}-\mathrm{bc}} & \frac{-\mathrm{b}}{\mathrm{ad}-\mathrm{bc}}
\end{array}\right] \\
\mathrm{U}_{2}^{\mathrm{T}} & =\left[\begin{array}{ll}
\frac{-c}{\mathrm{ad}-\mathrm{bc}} & \frac{\mathrm{a}}{\mathrm{ad}-\mathrm{bc}}
\end{array}\right]
\end{aligned}
$$

Checking given statement

## Statement-I:

$\mathrm{U}_{1}^{\mathrm{T}} \mathrm{V}_{1}=\left[\begin{array}{ll}\frac{d}{a d-b c} & \frac{-b}{a d-b c}\end{array}\right]\left[\begin{array}{l}\mathrm{a} \\ \mathrm{c}\end{array}\right]=1$
$=\frac{a d-b c}{a d-b c}=1$
$\mathrm{U}_{2}^{\mathrm{T}} \mathrm{V}_{2}=\left[\begin{array}{ll}\frac{-c}{\mathrm{ad}-\mathrm{bc}} & \frac{\mathrm{a}}{\mathrm{ad}-\mathrm{bc}}\end{array}\right]\left[\begin{array}{l}\mathrm{b} \\ \mathrm{d}\end{array}\right]$
$=\frac{-b c+a d}{a d-b c}=1$
So, statement 1 true.

## Statement-II:

$$
\begin{aligned}
\mathrm{U}_{1}^{\mathrm{T}} \mathrm{~V}_{2} & =\left[\begin{array}{ll}
\frac{d}{\mathrm{ad}-\mathrm{bc}} \frac{-\mathrm{b}}{\mathrm{ad}-\mathrm{bc}}
\end{array}\right]\left[\begin{array}{l}
\mathrm{b} \\
\mathrm{~d}
\end{array}\right] \\
& =\frac{\mathrm{bd}}{\mathrm{ad}-\mathrm{bc}}-\frac{\mathrm{bd}}{\mathrm{ad}-\mathrm{bc}}=0 \\
\mathrm{U}_{2}^{\mathrm{T}} \mathrm{~V}_{1} & =\left[\begin{array}{ll}
\frac{-c}{\mathrm{ad}-\mathrm{bc}} & \frac{\mathrm{a}}{\mathrm{ad}-\mathrm{bc}}
\end{array}\right]\left[\begin{array}{l}
\mathrm{a} \\
\mathrm{c}
\end{array}\right] \\
& =\frac{-\mathrm{ac}}{\mathrm{ad}-\mathrm{bc}}+\frac{\mathrm{ac}}{\mathrm{ad}-\mathrm{bc}}=0
\end{aligned}
$$

Statement-II ture.
34. The transfer function of a phase lead compensator is given by
$\mathrm{D}(\mathrm{s})=\frac{3\left(\mathrm{~s}+\frac{1}{3 \mathrm{~T}}\right)}{\left(\mathrm{s}+\frac{1}{\mathrm{~T}}\right)}$.
The frequency (in $\mathrm{rad} / \mathrm{sec}$ ), at which $\angle \mathrm{D}(\mathrm{j} \omega)$ is maximum is
(a) $\sqrt{\frac{1}{3 \mathrm{~T}^{2}}}$
(b) $\sqrt{3 \mathrm{~T}}$
(c) $\sqrt{\frac{3}{\mathrm{~T}^{2}}}$
(d) $\sqrt{3 \mathrm{~T}^{3}}$

Ans. (a)
Sol. Given phase lead compensator transfer function

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$$
\mathrm{D}(\mathrm{~s})=\frac{3\left(\mathrm{~S}+\frac{1}{3 \mathrm{~T}}\right)}{\left(\mathrm{S}+\frac{1}{\mathrm{~T}}\right)}
$$

The Frequency (in rad/sec) at which $\left.\angle \mathrm{D}(\mathrm{j} \omega)\right|_{\text {max }}$

$$
\begin{aligned}
& \omega_{\mathrm{m}}=\sqrt{\text { zero } \times \text { pole }} \\
& \omega_{\mathrm{n}}=\sqrt{\left(\frac{-1}{3 \mathrm{~T}}\right) \times\left(\frac{-1}{\mathrm{~T}}\right)}=\sqrt{\frac{1}{3 \mathrm{~T}^{2}}} \\
& \omega_{\mathrm{m}}=\sqrt{\frac{1}{3 \mathrm{~T}^{2}}}
\end{aligned}
$$

35. A single-phase controlled thyristor converter is used to obtain an average voltage of 180 V with 10 A constant current to feed a DC load. It is fed from single-phase AC supply of $230 \mathrm{~V}, 50$ Hz . Neglect the source impedance. The power factor (round off to two decimal places) of AC main is $\qquad$ —.

Ans. (0.7826)
Sol. Given:
Single phase full controlled converter
$\mathrm{V}_{0}=180$ volt $=$ average output voltage

$$
\mathrm{V}_{\mathrm{ac}}^{\mathrm{rms}}, ~=230 \mathrm{volt}
$$

Input power factor $=\frac{\mathrm{V}_{0} \mathrm{I}_{0}}{\mathrm{~V}_{\mathrm{sr}} \mathrm{I}_{\mathrm{sr}}}=\frac{180 \times 10}{230 \times 10}$

$$
\operatorname{IPF}=\left(\frac{180}{230}\right)=0.7826
$$

36. The line currents of a three-phase four wire system are square waves with amplitude of 100 A . These three currents are phase shifted by $120^{\circ}$ with respect to each other. The rms value of neutral current is
(a) 300 A
(b) 0 A
(c) $\frac{100}{\sqrt{3}} \mathrm{~A}$
(d) 100 A

Ans. (d)
Sol. Line current of a $3 \phi-4$ wire system are square wave and phase shifted by $120^{\circ}$ with respect to each other


We get neutral current a square wave form of time period $2 \pi / 3$.
The neutral current can be expressed at

$$
\mathrm{i}_{\mathrm{n}}=\left\{\begin{array}{cc}
100 \mathrm{~A}, & 0 \leq \mathrm{t}<\frac{\pi}{3} \\
-100 \mathrm{~A}, & \frac{\pi}{3} \leq \mathrm{t}<\frac{2 \pi}{3}
\end{array}\right.
$$

This square waveform can be split into

$$
\mathrm{i}_{\mathrm{n} 1}=100 \mathrm{~A} ; \quad 0 \leq \mathrm{t}<\frac{\pi}{3}
$$

## M

$$
\mathrm{i}_{\mathrm{n} 2}=-100 \mathrm{~A} ; \quad \frac{\pi}{3} \leq \mathrm{t}<\frac{2 \pi}{3}
$$

For rms value,

$$
\begin{aligned}
\left.\mathrm{i}_{\mathrm{n} 1}\right|_{\mathrm{rms}} & =100 \sqrt{\frac{\mathrm{t}_{1}}{\mathrm{~T}}} \\
\left.\mathrm{i}_{\mathrm{n} 2}\right|_{\mathrm{rms}} & =\left(\frac{1}{\mathrm{~T}} \int_{\mathrm{t}_{1}}^{\mathrm{T}}(-100)^{2} \cdot \mathrm{~d}(\omega \mathrm{t})\right)^{1 / 2} \\
& =100 \sqrt{\frac{\mathrm{~T}-\mathrm{t}_{1}}{\mathrm{~T}}}
\end{aligned}
$$

where, $\mathrm{t}_{1}=\frac{\pi}{3}$ and $\mathrm{T}=\frac{2 \pi}{3}$
Now, $\left.\mathrm{i}_{\mathrm{n}_{1}}\right|_{\mathrm{rms}}=100 \sqrt{\frac{\pi / 3}{2 \pi / 3}}=\frac{100}{\sqrt{2}}$

$$
\left.\mathrm{i}_{\mathrm{n} 2}\right|_{\mathrm{rms}}=100 \sqrt{\frac{\pi / 3}{2 \pi / 3}}=\frac{100}{\sqrt{2}}
$$

Now, in $\left.\right|_{\mathrm{rms}}=\sqrt{\left(\left.\mathrm{i}_{\mathrm{n}_{1}}\right|_{\mathrm{rms}}\right)^{2}+\left(\left.\mathrm{i}_{\mathrm{n}_{2}}\right|_{\mathrm{rms}}\right)^{2}}$

$$
\begin{aligned}
& =\sqrt{\frac{100^{2}}{2}+\frac{100^{2}}{2}} \\
& =100 \mathrm{~A}
\end{aligned}
$$

37. In the circuit shown below, $X$ and $Y$ are digital inputs, and Z is a digital output. The equivalent circuit is a

(a) XOR gate
(b) NOR gate
(c) NAND gate
(d) XNOR gate

Ans. (a)
Sol:

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$Z=\bar{X} Y+X \bar{Y}$

$$
\mathrm{Z}=\mathrm{X} \oplus \mathrm{Y}
$$

It is an XOR gate.
38. A moving coil instrument having a resistance of $10 \Omega$ gives a full scale deflection when the current is 10 mA . What should be the value of the series resistance, so that it can be used as voltmeter for measuring potential difference up to 100 V ?
(a) $990 \Omega$
(b) $9990 \Omega$
(c) $99 \Omega$
(d) $9 \Omega$

Ans. (b)
Sol. Given:
PMMC instrument


Hence, $\quad 100=10 \times 10^{-3} \times\left(10+R_{\text {se }}\right)$

$$
\begin{array}{lr}
\Rightarrow & 10+\mathrm{R}_{\mathrm{se}}=\frac{100}{10^{-2}}=10000 \\
\Rightarrow & \mathrm{R}_{\mathrm{se}}=10000-10=9990 \Omega
\end{array}
$$

i.e. series resistance $=9990 \Omega$
39. A $30 \mathrm{kV}, 50 \mathrm{~Hz}, 50 \mathrm{MVA}$ generator has the positive, negative, and zero sequence reactances of

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$0.25 \mathrm{pu}, 0.15 \mathrm{pu}$, and 0.05 pu , respectively. The neutral of the generator is grounded with a reactance so that the fault current for a bolted LG fault and that of a bolted three-phase fault at the generator terminal are equal. The value of grounding reactance in ohms (round off to one decimal place) is $\qquad$ -
Ans. (1.8)
Sol: Fault current for single line to ground fault

$$
\begin{equation*}
\mathrm{I}_{\mathrm{F}_{\mathrm{LG}}}=\frac{3 \mathrm{~V}}{\left(\mathrm{x}_{0}+\mathrm{x}_{1}+\mathrm{x}_{2}+3 \mathrm{x}_{\mathrm{n}}\right)} \tag{i}
\end{equation*}
$$

$\mathrm{x}_{0}=$ Zero sequence reactance
$\mathrm{x}_{1}=$ Positive sequence reactance
$\mathrm{x}_{2}=$ Negative sequence reactance
$\mathrm{x}_{\mathrm{n}}=$ Neutral reactance of generator
Fault current for $3 \phi$ fault

$$
I_{f 3 \phi}=\frac{V}{x_{1}}
$$

Given, $\mathrm{I}_{\mathrm{fLG}}=\mathrm{I}_{\mathrm{f} 3 \phi}$

$$
\text { so, } \begin{aligned}
& \frac{3 V}{x_{0}+x_{1}+x_{2}+3 x_{n}}=\frac{V}{x_{1}} \\
& 3 x_{1}=x_{0}+x_{1}+x_{2}+3 x_{n} \\
& x_{n}=\frac{2 x_{1}-x_{0}-x_{2}}{3} \\
& x_{n}=\frac{2(0.25)-0.05-0.15}{3} \\
& x_{n}=0.1 \mathrm{pu} \\
& \mathrm{x}_{\mathrm{n}}(\text { in } \Omega)=(0.1) \times\left(\frac{(30)^{2}}{50}\right)
\end{aligned}
$$

$$
\mathrm{x}_{\mathrm{n}}(\text { in } \Omega)=1.8 \Omega
$$

40. In the circuit below, the operational amplifier is ideal. If $V_{1}=10 \mathrm{mV}$ and $\mathrm{V}_{2}=50 \mathrm{mV}$, the output voltage $\left(\mathrm{V}_{\text {out }}\right)$ is

(a) 100 mV
(b) 600 mV
(c) 400 mV
(d) 500 mV

Ans. (c)
Sol: Sources are connected on both terminals, so we applied Superposition theorem,


$$
\begin{aligned}
\mathrm{V}_{\text {out }}=\left(-\frac{\mathrm{R}_{2}}{\mathrm{R}_{1}}\right) \mathrm{V}_{1}+ & \left(1+\frac{\mathrm{R}_{2}}{\mathrm{R}_{1}}\right) \mathrm{V} \\
& \left\{\begin{array}{c}
\mathrm{R}_{1}=10 \mathrm{k} \Omega \\
\mathrm{R}_{2}=100 \mathrm{k} \Omega \\
\mathrm{R}_{\mathrm{a}}=10 \mathrm{k} \Omega \\
\mathrm{R}_{\mathrm{b}}=100 \mathrm{k} \Omega
\end{array}\right.
\end{aligned}
$$

$$
\text { and } \quad \mathrm{V}=\left(\frac{\mathrm{R}_{\mathrm{b}}}{\mathrm{R}_{\mathrm{a}}+\mathrm{R}_{\mathrm{b}}}\right) \mathrm{V}_{2}
$$

So, $\mathrm{V}_{\text {out }}=\left(-\frac{\mathrm{R}_{2}}{\mathrm{R}_{1}}\right) \mathrm{V}_{1}+\left(1+\frac{\mathrm{R}_{2}}{\mathrm{R}_{1}}\right)\left(\frac{\mathrm{R}_{\mathrm{b}}}{\mathrm{R}_{\mathrm{a}}+\mathrm{R}_{\mathrm{b}}}\right) \mathrm{V}_{2}$
$\mathrm{V}_{\text {out }}=\left(-\frac{100}{10}\right)(10)+\left(1+\frac{100}{10}\right)\left(\frac{100}{10+100}\right)(50)$

$$
\mathrm{V}_{\text {out }}=400 \mathrm{mV}
$$

41. The magnitude circuit shown below has uniform cross-sectional area and air gap of 0.2 cm . The mean path length of the core is 40 cm . Assume that leakage and fringing fluxes are negligible. When the core relative permeability is assumed to be infinite, the magnetic flux density computed in the air gap is 1 tesla. With same Ampere-turns, if the core relative permeability is assumed to be 1000 (linear), the flux density in tesla (round off to three decimal places) calculated in the air gap is $\qquad$ _.


Ans. (0.834)
Sol.


Let $\mathrm{a}=$ uniform x -sectional area We know that

$$
\begin{array}{r}
\phi=\text { flux }=\frac{\mathrm{MMF}}{\text { Total reluctance }}=\frac{\mathrm{NI}}{\mathrm{~S}_{\mathrm{T}}} \\
\mathrm{~S}_{\mathrm{T}}=\text { Sairgap }+ \text { Score }
\end{array}
$$

$$
\begin{aligned}
& =\left[\frac{l_{\text {air }}}{\mu_{0}(1) \mathrm{a}}+\frac{l_{\text {core }}}{\mu_{0} \cdot \mu_{\mathrm{r}} \cdot \mathrm{a}}\right] \\
\mathrm{S}_{\mathrm{T}} & =\frac{1}{\mu_{0} \mathrm{a}}\left[l_{\mathrm{air}}+\frac{l_{\text {core }}}{\mu_{\mathrm{r}}}\right]
\end{aligned}
$$

Case 1: When $\mu_{\text {rcore }} \rightarrow \infty, \mathrm{B}=1 \mathrm{~T}$

$$
\begin{align*}
& \Rightarrow \quad \mathrm{MMF}=\mathrm{NI}=\mathrm{B}_{1}(\mathrm{a})\left[l_{\mathrm{air}}+\frac{l_{\text {core }}}{\mu_{\mathrm{r}} \rightarrow \infty}\right] \frac{1}{\mu_{0} \mathrm{a}} \\
& \Rightarrow \quad \mathrm{NI}=1(\mathrm{a})\left[l_{\mathrm{air}}+0\right] \times \frac{1}{\mu_{\mathrm{o}} \mathrm{a}}=\frac{l_{\text {air }}}{\mu_{0}} \\
& \Rightarrow \quad \mathrm{NI}_{1}=\frac{l_{\text {air }}}{\mu_{0}} \tag{i}
\end{align*}
$$

Case 2 :

$$
\mu_{\mathrm{r}}=1000
$$

mmf $=$ same as in Case-1
$\Rightarrow \quad \mathrm{mmf}=\mathrm{NI}_{1}=\mathrm{B}_{2}(\mathrm{a})\left[l_{\text {air }}+\frac{l_{\text {core }}}{\mu_{\mathrm{r}}}\right] \times \frac{1}{\mu_{0} \mathrm{a}}$
Put $\mathrm{NI}_{1}$ value from equation (i)

$$
\begin{array}{ll}
\Rightarrow & \frac{l_{\text {air }}}{\mu_{0}}=\mathrm{B}_{2} \frac{1}{\mu_{0}}\left[l_{\text {air }}+\frac{l_{\text {core }}}{1000}\right] \\
\Rightarrow & 0.2=\mathrm{B}_{2}\left[0.2+\frac{39.8}{100}\right] \\
\Rightarrow & \mathrm{B}_{2}=\frac{0.2}{0.2+0.0398}=\frac{0.2}{0.2398} \\
\Rightarrow & \mathrm{~B}_{2}=0.834 \text { Tesla }
\end{array}
$$

42. A $0.1 \mu \mathrm{~F}$ capacitor charged to 100 V is discharged through a $1 \mathrm{k} \Omega$ resistor. The time in ms (round off to two decimal places) required for the voltage across the capacitor to drop to 1 V is $\qquad$ -.
Ans. (0.46)
Sol. Initially, $\mathrm{V}(0)=100 \mathrm{~V}$

$$
\begin{aligned}
& \mathrm{C}=0.1 \mu \mathrm{~F} \\
& \mathrm{R}=1 \mathrm{k} \Omega
\end{aligned}
$$

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So,

$$
\mathrm{V}(\mathrm{t})=\mathrm{V}(\infty)+[\mathrm{V}(0)-\mathrm{V}(\infty)] \mathrm{e}^{-\mathrm{t} / \tau}
$$

Here, $\mathrm{V}(\infty)=0$
and

$$
\begin{aligned}
\tau & =\mathrm{RC} \\
\tau & =1000 \times 0.1 \times 10^{-6} \\
\tau & =10^{-4} \mathrm{sec}
\end{aligned}
$$

So,

$$
V(t)=0+[100-0] \mathrm{e}^{-t / 10^{-4}}
$$

$$
\mathrm{V}(\mathrm{t})=100 \mathrm{e}^{-\mathrm{t} / 10^{-4}}
$$

Now, voltage drops to 1 V ,

$$
\begin{aligned}
& 100 e^{-t / 10^{-4}}=1 \\
& e^{t / 10^{-4}}=100
\end{aligned}
$$

Taking log on both side

$$
\begin{aligned}
\frac{\mathrm{t}}{10^{-4}} & =\ln (100) \\
\mathrm{t} & =4.6 \times 10^{-4} \mathrm{sec} \\
\mathrm{t} & =0.46 \mathrm{msec}
\end{aligned}
$$

43. The current I flowing in the circuit shown below in amperes is $\qquad$ _.


Ans. (0A)
Sol: According to Milliman's Theorem, the equivalent circuit of the given circuit is


Where, $\mathrm{E}_{\mathrm{eq}}=\frac{\frac{\mathrm{E}_{1}}{\mathrm{R}_{1}}+\frac{\mathrm{E}_{2}}{\mathrm{R}_{2}}+\frac{\mathrm{E}_{3}}{\mathrm{R}_{3}}+\frac{\mathrm{E}_{4}}{\mathrm{R}_{4}}}{\frac{1}{\mathrm{R}_{1}}+\frac{1}{\mathrm{R}_{2}}+\frac{1}{\mathrm{R}_{3}}+\frac{1}{\mathrm{R}_{4}}}$
$=\frac{\frac{200}{50}+\frac{160}{40}+\left(\frac{-100}{25}\right)+\left(-\frac{80}{20}\right)}{\frac{1}{50}+\frac{1}{40}+\frac{1}{25}+\frac{1}{20}}$
$=\frac{4+4-4-4}{\left(\frac{1}{50}+\frac{1}{40}+\frac{1}{25}+\frac{1}{20}\right)}$
$=0 \mathrm{~V}$
So, the current ' I ' flowing in the circuit is 0 A .
44. A single-phase transformer of rating 25 kVA , supplies a 12 kW load at power factor of 0.6 lagging. The additional load at unity power factor in kW (round off to two decimal places) that may be added before this transformer exceeds its rated kVA is $\qquad$ —.

Ans. (7.20 kW)
Sol: 12 kW load at 0.6 pf
So, $S_{\text {Load }}=12+j 16$
Now, P is added extra
$\mathrm{S}_{\text {Load }}=(\mathrm{P}+12)+\mathrm{j} 16$
$\left|\mathrm{S}_{\mathrm{Load}}\right|=\sqrt{(\mathrm{P}+12)^{2}+(16)^{2}}$
$\left|\mathrm{S}_{\text {Load }}\right|=25$
$\sqrt{(\mathrm{P}+12)^{2}+(16)^{2}}=25$
$(\mathrm{P}+12)^{2}+16^{2}=625$
$(P+12)= \pm 19.2$
$\mathrm{P}=7.2,-31.2$
P is positive value
So, 7.20 kW extra load at unity pf can be added.
45. If $A=2 x i+3 y j+4 z k$ and $u=x^{2}+y^{2}+z^{2}$, then $\operatorname{div}(u A)$ at $(1,1,1)$ is $\qquad$
Ans. (45)
Sol. $u A=\left(2 x^{3}+2 x y^{2}+2 x z^{2}\right) \hat{i}+\left(3 x^{2} y+3 y^{3}\right.$

$$
\begin{aligned}
& \left.+3 y z^{2}\right) \hat{j}+\left(4 x^{2} z+4 y^{2} z+4 z^{3}\right) \hat{k} \\
& \begin{aligned}
& \operatorname{div}(u A)=\frac{d}{d x}\left(2 x^{3}+2 x y^{2}+2 x z^{2}\right)+\frac{d}{d y} \\
&\left(3 x^{2} y+3 y^{3}+3 y z^{2}\right)+\frac{d}{d z}\left(4 x^{2} z+4 y^{2} z+4 z^{3}\right) \\
& \operatorname{div}(u A)=\left(6 x^{2}+2 y^{2}+2 z^{2}\right)+\left(3 x^{2}+9 y^{2}+3 z^{2}\right) \\
& \quad+\left(4 x^{2}+4 y^{2}+12 z^{2}\right) ; A t(1,1,1) \\
& \operatorname{div}(u A)=(6+2+2)+(3+9+3)+(4+4+12) \\
& \operatorname{div}(u A)=45
\end{aligned}
\end{aligned}
$$

46. A periodic function $\mathrm{f}(\mathrm{t})$, with a period of $2 \pi$, is represented as its Fourier series,
$f(t)=a_{0}+\sum_{n=1}^{\infty} a_{n} \cos n t+\sum_{n=1}^{\infty} b_{n} \sin n t$
$\mathrm{f}(\mathrm{t})= \begin{cases}\mathrm{A} \sin \mathrm{t}, & 0 \leq \mathrm{t} \leq \pi \\ 0, & \pi<\mathrm{t}<2 \pi\end{cases}$
the Fourier series coefficients $a_{1}$ and $b_{1}$ of $f(t)$ are
(a) $\mathrm{a}_{1}=0 ; \mathrm{b}_{1}=\frac{\mathrm{A}}{2}$
(b) $\mathrm{a}_{1}=\frac{\mathrm{A}}{2} ; \mathrm{b}_{1}=0$
(c) $\mathrm{a}_{1}=\frac{\mathrm{A}}{\pi} ; \mathrm{b}_{1}=0$
(d) $\mathrm{a}_{1}=0 ; \mathrm{b}_{1}=\frac{\mathrm{A}}{\pi}$

Ans. (a)
Sol. $T=2 \pi \omega=(2 \pi) / T=1$


$$
\left.\mathrm{a}_{1}\right|_{\mathrm{T}=2 \pi} ^{\omega=1}=\frac{2}{2 \pi} \int_{0}^{2 \pi} \mathrm{~A} \sin t \cos t d t
$$

$$
=\frac{\mathrm{A}}{\pi} \int_{0}^{\pi} \sin \mathrm{t} \cdot \cos \mathrm{tdt}
$$

$$
a_{1}=\frac{A}{\pi} \int_{0}^{\pi} \frac{\sin 2 t}{2}=\frac{A}{2 \pi}\left[\frac{-\cos 2 t}{2}\right]_{0}^{\pi}
$$

$$
a_{1}=\frac{A}{4 \pi}[\cos 0-\cos 2 \pi]=0
$$

$$
a_{1}=0
$$

$$
\mathrm{b}_{\mathrm{n}}=\frac{2}{\mathrm{~T}} \int_{0}^{\mathrm{T}} \mathrm{x}(\mathrm{t}) \sin \mathrm{n} \omega \mathrm{t} \mathrm{~d}(\omega \mathrm{t})
$$

$$
\mathrm{b}_{1}=\frac{2}{2 \pi} \int_{0}^{\pi} \mathrm{A} \sin \mathrm{t} \cdot \sin \mathrm{tdt}
$$

$$
=\frac{\mathrm{A}}{\pi} \int_{0}^{\pi} \sin ^{2} \mathrm{t} d \mathrm{t}
$$

$$
=\frac{\mathrm{A}}{\pi} \int_{0}^{\pi} \frac{(1-\cos 2 \mathrm{t})}{2} \mathrm{dt}
$$

$$
=\frac{\mathrm{A}}{\pi} \int_{0}^{\pi}\left[\frac{1}{2}-\frac{\cos 2 \mathrm{t}}{2}\right] \mathrm{dt}
$$

$$
=\frac{\mathrm{A}}{\pi}\left[\left|\frac{1}{2}(\mathrm{t})-\frac{\sin 2 \mathrm{t}}{4}\right|_{0}^{\pi}\right]
$$

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$$
\mathrm{b}_{1}=\frac{\mathrm{A}}{\pi} \times \frac{\pi}{2}=\frac{\mathrm{A}}{2}
$$

47. The enhancement type MOSFET in the circuit below operates according to the square law, $\mu_{\mathrm{n}} \mathrm{C}_{\mathrm{ox}}=100 \mu \mathrm{~A} / \mathrm{V}^{2}$, the threshold voltage ( $\mathrm{V}_{\mathrm{T}}$ ) is 500 mV . Ignore channel length modulation. The output voltage $\mathrm{V}_{\text {out }}$ is

(a) 500 mA
(b) 2 V
(c) 100 mV
(d) 600 mV

Ans. (d)

Sol. $\mathrm{I}_{\mathrm{D}}=\frac{1}{2}\left(\mu_{\mathrm{n}} \mathrm{C}_{\mathrm{ox}}\right)\left(\frac{\omega}{\mathrm{L}}\right)\left[\mathrm{V}_{\mathrm{gs}}-\mathrm{V}_{\mathrm{T}}\right]^{2}$

$$
\begin{gathered}
5 \times 10^{-6}=\left(\frac{1}{2}\right)\left(100 \times 10^{-6}\right)(10)\left[\mathrm{V}_{\text {out }}-0.5\right]^{-2} \\
{\left[\mathrm{~V}_{\text {out }}-0.5\right]^{-2}=0.01} \\
\mathrm{~V}_{\text {out }}-0.5= \pm 0.1 \\
\mathrm{~V}_{\text {out }} \quad=0.6 \mathrm{~V} \text { and } 0.4 \mathrm{Volts}
\end{gathered}
$$

So, $\mathrm{V}_{\text {out }}=600 \mathrm{mV}$
Option (d) is correct
48. In the single machine infinite bus system shown below, the generator is delivering the real power of 0.8 pu and 0.8 power factor lagging to the
infinite bus. The power angle of the generator in degree (round off to one decimal place) is


Ans. (20.51 ${ }^{\circ}$ )
Sol: Equivalent circuit of the network can be drawn as


Given,

$$
\begin{aligned}
\mathrm{P} & =0.8 \mathrm{pu} \\
\mathrm{pf} & =0.8 \mathrm{pu} \text { (lagging) } \\
\mathrm{V} & =1 \mathrm{pu} \\
\mathrm{P} & =\mathrm{VI} \cos \phi \\
0.8 & =1 \times \mathrm{I} \times 0.8
\end{aligned}
$$

so, $\quad \mathrm{I}=1 \mathrm{pu}$

$$
\mathrm{I}=1 \angle-\cos ^{-1}(0.8) \mathrm{pu}
$$

Applying KVL to determine $\mathrm{E} \angle \delta$
(generator voltage)

$$
\overline{\mathrm{E}}=\overline{\mathrm{V}}+\overline{\mathrm{I}} \overline{\mathrm{X}}_{\mathrm{eq}}
$$

$\left.\mathrm{E} \angle \delta=1 \angle 0^{\circ}+\left[(0.25+0.2+0.2) \angle 90^{\circ}\right)\right]\left[1 \angle-\cos ^{-1}(0.8)\right]$
$\mathrm{E} \angle \delta=1+0.65 \angle 53.13^{\circ}$
$\Rightarrow \quad \mathrm{E} \angle \delta=1.484 \angle 20.51^{\circ}$
So, $\delta=20.51^{\circ}$
49. The output expression for the Karnaugh map shown below is

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|  |  | 01 | 11 | 10 |
| :---: | :---: | :---: | :---: | :---: |
| 00 | 0 | 1 | 1 | 0 |
| 01 | 1 | 1 | 1 | 1 |
| 11 | 1 | 1 | 1 | 1 |
| 10 | 0 | 0 | 0 | 0 |

(a) $\mathrm{Q} \overline{\mathrm{R}}+\overline{\mathrm{S}}$
(b) $\mathrm{Q} \overline{\mathrm{R}}+\mathrm{S}$
(c) $\mathrm{QR}+\overline{\mathrm{S}}$
(d) $\mathrm{QR}+\mathrm{S}$

Ans. (b)
Sol:

$$
\begin{aligned}
& \mathrm{f}(\mathrm{P}, \mathrm{Q}, \mathrm{R}, \mathrm{~S})=\mathrm{S}+\mathrm{Q} \overline{\mathrm{R}}
\end{aligned}
$$

50. In a DC-DC boost converter, the duty ratio is controlled to regulate the output voltage at 48 V . The input DC voltage is 24 V . The output power is 120 W . The switching frequency is 50 kHz . Assume ideal components and a very large output filter capacitor. The converter operates at the boundary between continuous and discontinuous conduction mode. The value of the boost inductor (in $\mu \mathrm{H}$ ) is $\qquad$
Ans. (24)
Sol. Given Boost converter
Output voltage

$$
\mathrm{V}_{0}=48 \text { Volt }
$$

Input DC voltage

$$
\mathrm{V}_{\mathrm{s}}=24 \mathrm{volt}
$$

Output power

$$
P_{0}=120 \text { watt }
$$


(a)

$$
\begin{aligned}
(1-\alpha) & =\frac{\mathrm{V}_{\mathrm{S}}}{\mathrm{~V}_{0}}=\frac{24}{48}=\frac{1}{2} \\
\alpha & =\frac{1}{2}=\text { Duty ratio }
\end{aligned}
$$

Assuming losses switch/converter.

$$
\begin{aligned}
\mathrm{P}_{\mathrm{in}} & =\mathrm{P}_{0} \\
\mathrm{~V}_{\mathrm{S}} \mathrm{I}_{\mathrm{S}} & =120 \\
\mathrm{I}_{\mathrm{S}} & =120 / 24=5 \mathrm{Amp}
\end{aligned}
$$



Fig. (b): Inductor current waveform.
At the boundary between contineous and discontineous conduction mode.

$$
\begin{aligned}
& \mathrm{I}_{\mathrm{mn}}=0 \\
& \mathrm{I}_{\mathrm{mn}}=0 \\
& \mathrm{I}_{\mathrm{Lay}}=\frac{\Delta \mathrm{I}_{\mathrm{L}}}{2}=\mathrm{I}_{\mathrm{S}} \\
& \Delta \mathrm{I}_{\mathrm{L}}=2 \times 5=10 \mathrm{Amp}
\end{aligned}
$$

During $\mathrm{T}_{\mathrm{ON}}$ :-

$$
\begin{aligned}
\mathrm{V}_{\mathrm{s}} & =\mathrm{V}_{\mathrm{L}}=\mathrm{L} \frac{\Delta \mathrm{I}_{\mathrm{L}}}{\mathrm{~T}_{\mathrm{on}}} \\
\Delta \mathrm{I}_{\mathrm{L}} & =\frac{\alpha \mathrm{V}_{\mathrm{S}}}{\mathrm{f}_{\mathrm{LC}}} \\
\mathrm{~L}_{\mathrm{C}} & =\frac{\alpha \mathrm{V}_{\mathrm{S}}}{\mathrm{f} \Delta \mathrm{I}_{\mathrm{L}}}
\end{aligned}
$$

By putting all value

$$
\begin{gathered}
\mathrm{L}_{\mathrm{C}}=\frac{\frac{1}{2} \times 24}{50 \times 10^{3} \times 10}=24 \times 10^{-6} \mathrm{H} \\
\mathrm{~L}_{\mathrm{C}}=24 \mu \mathrm{H}
\end{gathered}
$$

Hence, value of the boost inductor is $24 \mu \mathrm{H}$
51. A delta-connected, $3.7 \mathrm{~kW}, 400 \mathrm{~V}$ (line), 1 threephase, $4-$ pole, $50-\mathrm{Hz}$ squirrel-cage induction motor has the following equivalent circuit parameters per phase referred to the stator: $\mathrm{R}_{1}=5.39 \Omega, \quad \mathrm{R}_{2}=5.72 \Omega, \quad \mathrm{X}_{1}=\mathrm{X}_{2}=8.22 \Omega$. Neglect shunt branch in the equivalent circuit. The starting line current in amperes (round off to two decimal places) when it is a connected to a 100 V (line), 10 Hz , three-phase AC source is

Ans. (14.95)
Sol. A delta connected $3.1 \mathrm{KW}, 400 \mathrm{~V}$ For $50 \mathrm{~Hz}, 400 \mathrm{~V}$ (line); given parameters are:
$\mathrm{R}_{1}=5.39 \Omega, \mathrm{R}_{2}=5.72 \Omega, \mathrm{X}_{1}=\mathrm{X}_{2}=8.22 \Omega$


At starting $\mathrm{V}=100 \mathrm{~V}, \mathrm{f}=10 \mathrm{~Hz}$
So, $\mathrm{X}_{1}$ and $\mathrm{X}_{2}$ changes due to frequency change

$$
\begin{aligned}
& \mathrm{X}_{1 \text { new }}=\mathrm{X}_{2 \text { new }}=\left(\frac{10}{50}\right)(8.22) \\
& \mathrm{X}_{1 \text { new }}=\mathrm{X}_{2 \text { new }}=1.644 \Omega \\
& \text { So, } \mathrm{I}_{\mathrm{ph}} \\
&=\frac{\mathrm{V}_{\mathrm{ph}}}{\mathrm{Z}} \\
& \mathrm{I}_{\mathrm{ph}}=\frac{100}{(5.39+5.72)+\mathrm{j}(1.644+1.644)} \\
& \mathrm{I}_{\mathrm{ph}}=\frac{100}{(11.11+\mathrm{j} 3.288)}
\end{aligned}
$$

$$
\left|\mathrm{I}_{\mathrm{ph}}\right|=8.63 \mathrm{Amp}
$$

But line to line current, $\mathrm{I}_{\mathrm{L}-\mathrm{L}}=\sqrt{3} \times 8.63$

$$
=14.95 \mathrm{Amp}
$$

52. A three-phase $50 \mathrm{~Hz}, 400 \mathrm{kV}$ transmission line is 300 km long. The line inductance is $1 \mathrm{mH} /$ km per phase and the capacitance is $0.01 \mu \mathrm{~F} / \mathrm{km}$ per phase. The line is under open circuit condition at the receiving end and energized with 400 kV at the sending end, the receiving end line voltage in kV (roundoff to two decimal places) will be $\qquad$ _.
Ans. (418.59)
Sol. Given,
3 -phase, $50 \mathrm{~Hz}, 400 \mathrm{kV}, 300 \mathrm{~km}$ long line
$\mathrm{L}=1 \mathrm{mH} / \mathrm{Km}$ per phase
$\mathrm{C}=0.01 \mu \mathrm{~F} /$ Km per phase
Hence,

$$
\begin{aligned}
Z & =j \omega L=j\left(2 \pi \times 50 \times 10^{-3} \times 300\right) \\
& =j(30 \pi) \Omega
\end{aligned}
$$

and

$$
\begin{aligned}
Y & =j \omega C \\
& =j\left(2 \pi \times 50 \times 0.01 \times 10^{-6} \times 300\right) \\
& =j\left(3 \pi \times 10^{-4}\right) s
\end{aligned}
$$

Approximate ABCD parameters,

$$
\begin{aligned}
\mathrm{A} & =\mathrm{D}=1+\frac{\mathrm{YZ}}{2}, \mathrm{~B}=\mathrm{Z}\left[1+\frac{\mathrm{YZ}}{6}\right] \\
\mathrm{C} & =\mathrm{Y}\left[1+\frac{\mathrm{YZ}}{6}\right] \\
\because \quad \mathrm{V}_{\mathrm{s}} & =\mathrm{AV}_{\mathrm{r}}+\mathrm{BI}_{\mathrm{r}}
\end{aligned}
$$

$$
\text { for open circuit, } I_{r}=0
$$

$$
\Rightarrow \quad \mathrm{V}_{\mathrm{r} 0}=\frac{\mathrm{V}_{\mathrm{s}}}{\mathrm{~A}}
$$

$$
\Rightarrow \quad V_{r 0}=\frac{400}{1+\frac{Y Z}{2}}=\frac{400}{1+\frac{(\mathrm{j} 30 \pi)\left(\mathrm{j} 3 \pi \times 10^{-4}\right)}{2}}
$$

## 7.

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$$
\begin{aligned}
& =\frac{400}{1-0.044} \\
& =\frac{400}{0.956}=418.59 \mathrm{kV}
\end{aligned}
$$

53. In a 132 kV system, the series inductance up to the point of circuit breaker location is 50 mH . The shunt capacitance at the circuit breaker terminal is $0.05 \mu \mathrm{~F}$. The critical value of the resistance in ohms required to be connected across the circuit breaker contacts which will give no transistor oscillation is $\qquad$ _.

Ans. (500)
Sol: Given data,

$$
\mathrm{L}=50 \mathrm{mH}, \mathrm{C}=0.05 \mu \mathrm{~F}
$$

now, the critical resistance to avoid current chopping or transient oscillations will be given as,

$$
\begin{aligned}
\mathrm{R} & =\frac{1}{2} \sqrt{\mathrm{~L} / \mathrm{C}} \\
\mathrm{R} & =\frac{1}{2} \sqrt{\frac{50 \times 10^{-3}}{0.05 \times 10^{-6}}} \\
& =500 \Omega
\end{aligned}
$$

54. A 220 V DC shunt motor takes 3 A at no-load. It draws 25 A when running at full-load at 1500 rpm. The armature and shunt resistances are $0.5 \Omega$ and $220 \Omega$, respectively. The no-load speeding rpm (round off to two decimal places) is $\qquad$ .

Ans. (1579.33)
Sol. DC shunt motor

$$
\mathrm{V}_{\mathrm{dc}}=220 \mathrm{~V}, \mathrm{R}_{\mathrm{a}}=0.5 \Omega, \mathrm{R}_{\mathrm{sh}}=220 \Omega
$$



$$
\begin{aligned}
& \mathrm{I}_{\mathrm{f}}=\text { Field current } \\
& \mathrm{I}_{\mathrm{f}}=\frac{\mathrm{V}_{\mathrm{dc}}}{\mathrm{R}_{\mathrm{sh}}}=\frac{220}{220}=1 \Omega
\end{aligned}
$$

## Case 1:At No-load

$$
\begin{aligned}
\mathrm{I}_{\mathrm{L}} & =3 \mathrm{~A}, \quad \mathrm{I}_{\mathrm{f}}=1 \mathrm{~A} \\
\mathrm{I}_{\mathrm{a} 0} & =\mathrm{I}_{\mathrm{L}}-\mathrm{I}_{\mathrm{f}}=2 \mathrm{~A} \\
\epsilon_{\mathrm{b} 0} & =\text { back emf at No-load } \\
& =\mathrm{V}_{\mathrm{dc}}-\mathrm{I}_{\mathrm{a}_{0}} \cdot \mathrm{r}_{\mathrm{a}} \\
\epsilon_{\mathrm{b} 0} & =220-(2 \times 0.5)=219 \mathrm{~V}
\end{aligned}
$$

Case 2 : At Full-load

$$
\begin{aligned}
\mathrm{I}_{\mathrm{L}} & =25 \mathrm{~A}, \mathrm{I}_{\mathrm{f}}=1 \mathrm{~A}, \mathrm{~N}_{\mathrm{f}}=1500 \mathrm{rpm} \\
\mathrm{I}_{\mathrm{af}} & =25-1=24 \mathrm{~A} \\
\epsilon_{\mathrm{bf}} & =\mathrm{V}_{\mathrm{dc}}-\mathrm{I}_{\mathrm{af}} \mathrm{r}_{\mathrm{a}}=220-(24 \times 0.5) \\
\epsilon_{\mathrm{bf}} & =208 \mathrm{~V}
\end{aligned}
$$

As we know, $\epsilon_{b} \propto \mathbf{N} \phi \propto \mathbf{N}$
[ $\phi$ is constant, as V is constant]
So, $\quad \frac{\mathrm{N}_{0}}{\mathrm{~N}_{\mathrm{f}}}=\frac{\epsilon_{\mathrm{b} 0}}{\epsilon_{\mathrm{bf}}}$

$$
\begin{aligned}
& \mathrm{N}_{0}=\text { No load speed }=1500 \times \frac{219}{208} \\
& \mathrm{~N}_{0}=1579.33 \mathrm{rpm}
\end{aligned}
$$

55. Consider a state-variable model of a system
$\left[\begin{array}{l}x_{1} \\ x_{2}\end{array}\right]=\left[\begin{array}{cc}0 & 1 \\ -\alpha & -2 \beta\end{array}\right]+\left[\begin{array}{l}0 \\ \alpha\end{array}\right] r$
$\mathrm{y}=\left[\begin{array}{ll}1 & 0\end{array}\right]\left[\begin{array}{l}\mathrm{x}_{1} \\ \mathrm{x}_{2}\end{array}\right]$
where $y$ is the output, and $r$ is the input. The damping ratio $\xi$ and the undamped natural frequency $\omega_{\mathrm{n}}(\mathrm{rad} / \mathrm{sec})$ of the system are given by
(a) $\xi=\sqrt{\beta} ; \omega_{\mathrm{n}}=\sqrt{\alpha}$
(b) $\xi=\frac{\sqrt{\alpha}}{\beta} ; \omega_{\mathrm{n}}=\sqrt{\beta}$
(c) $\xi=\frac{\beta}{\sqrt{\alpha}} ; \omega_{\mathrm{n}}=\sqrt{\alpha}$
(d) $\xi=\sqrt{\alpha} ; \omega_{\mathrm{n}}=\frac{\beta}{\sqrt{\alpha}}$

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Ans. (c)
Sol.
Given data,

$$
\begin{aligned}
{\left[\begin{array}{l}
\dot{x}_{1} \\
\dot{x}_{2}
\end{array}\right] } & =\left[\begin{array}{ll}
0 & 1 \\
-\alpha & 2 \beta
\end{array}\right]\left[\begin{array}{l}
\mathrm{x}_{1} \\
\mathrm{x}_{2}
\end{array}\right]+\left[\begin{array}{l}
0 \\
\alpha
\end{array}\right] \mathrm{r} \\
\mathrm{y} & =\left[\begin{array}{ll}
1 & 0
\end{array}\right]\left[\begin{array}{l}
\mathrm{x}_{1} \\
\mathrm{x}_{2}
\end{array}\right]
\end{aligned}
$$

Comparing these state equation with

$$
\begin{aligned}
\dot{x} & =A x+B u \\
y & =C x+D u
\end{aligned}
$$

One gets,

$$
\begin{aligned}
& A=\left[\begin{array}{ll}
0 & 1 \\
-\alpha & -2 \beta
\end{array}\right], B=\left[\begin{array}{l}
0 \\
\alpha
\end{array}\right] \\
& C=\left[\begin{array}{ll}
1 & 0
\end{array}\right], D=0
\end{aligned}
$$

Now transfer function is given as,

$$
\begin{array}{rlrl}
\mathrm{T} . \mathrm{F} & =\mathrm{C}[\mathrm{SI}-\mathrm{A}]^{-1} \mathrm{~B}+\mathrm{D} \\
\because & {[\mathrm{SI}-\mathrm{A}]} & =\left[\begin{array}{ll}
\mathrm{s} & -1 \\
\alpha & \mathrm{~s}+2 \beta
\end{array}\right]
\end{array}
$$

$\therefore \quad[\mathrm{SI}-\mathrm{A}]^{-1}=\frac{1}{\left(\mathrm{~s}^{2}+2 \beta \mathrm{~s}+\alpha\right)}\left[\begin{array}{ll}\mathrm{s}+2 \beta & 1 \\ -\alpha & \mathrm{s}\end{array}\right]$
Now,
$\mathrm{C}[\mathrm{SI}-\mathrm{A}]^{-1} \cdot \mathrm{~B}$
$=\left[\begin{array}{ll}1 & 0\end{array}\right] \times \frac{1}{\left(s^{2}+2 \beta s+\alpha\right)}\left[\begin{array}{ll}s+2 \beta & 1 \\ -\alpha & s\end{array}\right] \times\left[\begin{array}{l}0 \\ \alpha\end{array}\right]$
$=\frac{\alpha}{\mathrm{s}^{2}+2 \beta \mathrm{~s}+\alpha}$
Hence, characteristics equation is,

$$
s^{2}+2 \beta s+\alpha=0
$$

Comparing the characteristics equation with
$\mathrm{s}^{2}+2 \xi \omega_{\mathrm{n}} \mathrm{s}+\omega_{\mathrm{n}}^{2}=0$
One gets,

$$
\begin{aligned}
\omega_{\mathrm{n}} & =\sqrt{\alpha}, 2 \xi \omega_{\mathrm{n}}=2 \beta \\
\Rightarrow \quad \xi & =\frac{\beta}{\sqrt{\alpha}}
\end{aligned}
$$

