VIKAS PRE-UNIVERSITY COLLEGE, Mangaluru
Answer key – II PUC Basic Mathematics – 2019

Part A

1. $2A^{I} = 2\begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ -4 & 8 \end{bmatrix}$

2. (n-1)! = 9! = 362880

3. p:Oxygen is a gas
q: Gold is a compound
$\therefore$ The given proposition is $p \to q$

4. $5^3 : 4^3$ = 125 : 64

5. $\frac{5\times3600}{90} = 200$

6. sin5Acos3A = $\frac{1}{2}[\sin 8A + \sin 2A]$

7. (-g, -f) = (2, ½)

8. $\lim\limits_{x\to3}\left(\frac{x^2-4x}{x-2}\right) = \frac{3^2-4(3)}{3-2} = -3$

9. $\frac{d}{dx}\left(5e^x - \log_e x - 3\sqrt{x}\right) = 5e^x - \frac{1}{x} - \frac{3}{2\sqrt{x}}$

10. $I = \int (x^2 - \frac{6}{x} + 5e^x)dx = \frac{x^3}{3} - 6\log_e x + 5e^x + c$

Part B

11. $A - 3B = \begin{bmatrix} 1 & 3 & -1 \\ -1 & 0 & 2 \end{bmatrix} - 3\begin{bmatrix} 4 & -1 & 2 \\ 1 & 3 & -2 \end{bmatrix} = \begin{bmatrix} -11 & 6 & -3 \\ -4 & -9 & 8 \end{bmatrix}$

12. 2 R's together – 5 letters (CARROM)
5! = 120

13. P(Red or Green) = P(Red) + P(Green) = $\frac{8}{24} + \frac{6}{24} = \frac{14}{24} = \frac{7}{12}$

14. $p\to(q \wedge r)$; $T\to(T \wedge F)$; $T\to F=F$

15. If x is added to both numerator and denominator,
$\frac{5+x}{6+x} = \frac{8}{9}$
On solving x = 3

16. TD = 900, BG = 27 and hence BD = TD + BG =927
Face value = $\frac{BD \times TD}{BG} = \frac{927\times900}{27}$= Rs.30900

17. $\cos 15° = \cos(45°-30°) = \cos 45° \cos 30° + \sin 45° \sin 30°$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

18. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B} = \dfrac{\frac{1}{2}+\frac{1}{3}}{1-\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)} = \dfrac{\frac{5}{6}}{\frac{5}{6}} = 1$

19. Focus $\equiv (3,0)=(a,0) \Rightarrow a=3$

$\therefore$ Equation of the parabola is $y^2=4ax \Rightarrow y^2=12x$

20. i) At x=1, $f(x)=x^2-1 \Rightarrow f(1)=0$

ii) To find $\lim\limits_{x\to 1} f(x)$

L.H.L $= \lim\limits_{\substack{x\to 1^- \\ (x<1)}} (x^2-1) = 0$

R.H.L $= \lim\limits_{\substack{x\to 1^+ \\ (x>1)}} (-x^2-1) = -(1)^2 - 1 = -2$

$\therefore$ LHL $\neq$ RHL ; $\therefore \lim\limits_{x\to 1}$ does not exist ; $\therefore$ f(x) is not continuous at x=1

21. $y = (a^2-x^2)^{10}$

$$\frac{dy}{dx} = 10(a^2-x^2)^9(-2x) = -20x(a^2-x^2)^9$$

22. $C(x) = x^3-3x+7$

Average cost $(AC) = \dfrac{C(x)}{x} = \dfrac{x^3-3x+7}{x} = x^2 - 3 + \dfrac{7}{x}$

Marginal cost $(MC) = \dfrac{d(c(x))}{dx} = \dfrac{d(x^3-3x+7)}{dx} = 3x^2 - 3$

23. $I = \int \dfrac{4x+3}{2x^2+3x+5}dx = \int \dfrac{1}{t}dt$ | Put $2x^2+3x+5 = t$

$= \log_e(2x^2+3x+5) + c$ | $(4x+3)dx = dt$

24.

$$I = \int_1^2 xe^x dx = \left[x.e^x\right]_1^2 - \int_1^2 1.e^x dx$$

$$= (2e^2 - (1)e^1) - (e^x)_1^2 = 2e^2 - e - (e^2 - e^1) = e^2$$

Part C

25. .

$$AB = \begin{bmatrix} 16 & 16 \\ 10 & -18 \end{bmatrix} \Rightarrow (AB)^{I} = \begin{bmatrix} 16 & 10 \\ 16 & -18 \end{bmatrix}$$

$$B^{I} = \begin{bmatrix} -1 & 6 \\ 5 & 2 \end{bmatrix} \text{ and } A^{I} = \begin{bmatrix} 2 & -4 \\ 3 & 1 \end{bmatrix}$$

$$\therefore \mathbf{B}^{I}A^{I} = \begin{bmatrix} 16 & 10 \\ 16 & -18 \end{bmatrix} \Rightarrow (AB)^{I} = B^{I}A^{I}$$

26.

$$LHS = \begin{vmatrix} 1+a & b & c \\ a & 1+b & c \\ a & b & 1+c \end{vmatrix} = \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ a & b & 1+c \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ a & a+b & 1+c \end{vmatrix} = 1\begin{vmatrix} 1 & -1 \\ a+b & 1+c \end{vmatrix} = 1(1+c)-(-1)(a+b)$$

$$= 1+c+a+b = RHS$$

27. A particular boy is chosen hence only 11 students are to be chosen with the condition at least 5 boys out of 8 boys and at least 4 girls out of 7 girls.

Cases are

i. 5 boys and 6 girls, = ${}^{8}C_{5} \times {}^{7}C_{6}$ = 392

ii. 6 boys and 5 girls, = ${}^{8}C6_{6} \times {}^{7}C_{5}$ = 588

iii. 7 boys and 4 girls, = ${}^{8}C_{7} \times {}^{7}C_{4}$ = 280

Total Number of ways = 1260

28. A: Husband getting selected in an interview

B: Wife getting selected in an interview.

P(A) = 1/7 P(B) = 1/5

a. $P(A \cap B) = P(A)\,P(B) = 1/35$

b. P(A only) + P(B only) = (1/7 – 1/35) + (1/ 5 – 1/ 35) = 2/7

c. $P(A' \cap B') = P(A') \times P(B') = 6/7 \times 4/5 = 24/35$

29. Remaining Soldiers = 560 – 60 =500

Remaining Days = 70 – 20 =50

Soldiers	Days
560	50
500	x

It is inverse ratio. 560 : 500 = x : 50

x = 56 days

30. $P = \frac{F}{1+tr} = \frac{512.50}{1+(\frac{1}{2}\times 0.15)} = Rs. 476.44$

DV = F (1 – tr) = Rs.474.06

31.

Market value	Face Value
108	100
3240	x

Face Value = 3000

Market value	Face Value
104	100
x	3000

Market Value = 3120

Market value	Face Value
130	100
3120	x

Stock (Face value) = Rs.2400

32. Amount paid including ST = 2016

Cost after rebate = Rs. x ;   ST = 12%

2016 = (100 + ST)% of x

x =1800

Rebate = 2000 – 1800 = 200

Percentage Rebate = $\frac{200}{2000} \times 100 = 10\%$

33. Axis of the parabola is y-axis, hence the required equation of the parabola is $x^2=4ay$ ------(1)

Put (x,y) = (-1,-3) in eqn(1), we get,

$(-1)^2=4a(-3) \Rightarrow a = \frac{1}{-12}$

$\therefore x^2 = -\frac{1}{3}y \Rightarrow 3x^2 + y = 0$

34.

$$y = \frac{e^x - 1}{e^x + 1}$$

$$\frac{dy}{dx} = \frac{(e^x+1)(e^x) - (e^x-1)(e^x)}{(e^x+1)^2} = \frac{e^{2x} + e^x - e^{2x} + e^x}{(e^x+1)^2} = \frac{2e^x}{(e^x+1)^2}$$

35.

$$y = x^3 - 9x^2 + 15x - 1$$

$$\frac{dy}{dx} = 3x^2 - 18x + 15$$

$$\frac{dy}{dx} = 3x^2 - 18x + 15 = 0 \Rightarrow x = 1 \text{ or } x = 5 \text{ are the critical points}$$

Now $\frac{d^2y}{dx^2} = 6x - 18$

At x=1; $\frac{d^2y}{dx^2} = -12 < 0$

$\Rightarrow$x=1 is a point of maximum of the function $y=x^3-9x^2+15x-1$ and the maximum value is $y=1^3-9(1)^2+15(1)-1=6$

At x=5; $\frac{d^2y}{dx^2} = 12 > 0$

$\Rightarrow$x=5 is a point of minimum of the function y=x^3-9x^2+15x-1

and the minimum value is y =5^3-9$(5)^2$+15(5)-1=-26

36. $\frac{da}{dt} = 5\,cm\,/\,sec;\ \ \frac{dA}{dt} = ?,\ \ \frac{dp}{dt} = ?,\ \ a = 20$

Area of the square, A=a^2

$$\frac{dA}{dt} = 2a\frac{da}{dt} = 2(20)(5) = 200\ \ cm^2\ /\ sec$$

Perimeter of the square, p=4a

$$\therefore \frac{dp}{dt} = 4\frac{da}{dt} = 4x5 = 20\,cm\,/\,sec$$

37.

$$I = \int x^2 \sin x dx$$

$$= x^2\int \sin x dx - \int\left[\frac{d(x^{2)}}{dx}\int \sin x dx\right]dx$$

$$= x^2(-\cos x) - \int 2x(-\cos x)dx$$

$$= -x^2 \cos x + 2\int x\cos x dx$$

$$= -x^2 \cos x + 2\left[x\sin x - \int 1.\sin x dx\right]$$

$$= -x^2 \cos x + 2x\sin x + 2\cos x + c$$

38.

$$I = \int (2x+3)(x^2+3x+5)^{\frac{3}{2}}dx$$

$$= \int t^{\frac{3}{2}}dt = \frac{2}{5}(x^2+3x+5)^{\frac{5}{2}} + c$$

put $x^2 + 3x + 5 = t$

$(2x+3)dx = dt$

Part D

39. Middle term is $\frac{n}{2} + 1$ = 6^{th} term

r+1= 6 or r = 5, n = 10, x = $\frac{2x^2}{3}$, a = $-\frac{3}{2x}$

$T_{r+1} = {}^nC_r x^{n-r} a^r$

$T_6 = {}^{10}C_5\left(\frac{2x^2}{3}\right)^{10-5}, \left(-\frac{3}{2x}\right)^5 \ \ = - {}^{10}C_5(x)^5 = - 252\ (x)^5$

40. $\frac{x-1}{x(x+2)(x+4)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x+4}$

x-1 = A(x+2)(x+4) + B x (x+4) + C x (x+2)

Put x= -2, B = ¾

Put x – 4, C = - 5/8

Put x= 0, A = -1/8

Hence $\frac{x-1}{x(x+2)(x+4)} = \frac{-1}{8x} + \frac{3}{4(x+2)} - \frac{5}{8(x+4)}$

41.

p	q	r	~ p	~ p ∧ q	~ r	(~ p ∧ q) ∧ (~ r)
T	T	T	F	F	F	F
T	T	F	F	F	T	F
T	F	T	F	F	F	F
F	T	T	T	T	F	F
T	F	F	F	F	T	F
F	T	F	T	T	T	T
F	F	T	T	F	F	F
F	F	F	T	F	T	F

∴ (~ p ∧ q) ∧ (~ r) is neither.

42. Let distance from house to college = x Km

Time taken (at 4 km/h) = x/4

Time taken (at 5 km/h) = x/5

By given information, (converting minutes into hour)

$$\frac{x}{4}-\frac{5}{60}=\frac{x}{5}+\frac{2.5}{60}$$

On solving for x,

x=2.5Km

43.

No. of units	Total Units produced	Cumulative average Hours	Total time
1	1	1000	1000
1	2	900	1800
2	4	810	3240
4	8	729	5832
8	16	656.1	10497.6
16	32	590.49	18895.68

Total labour hours = 18895.68

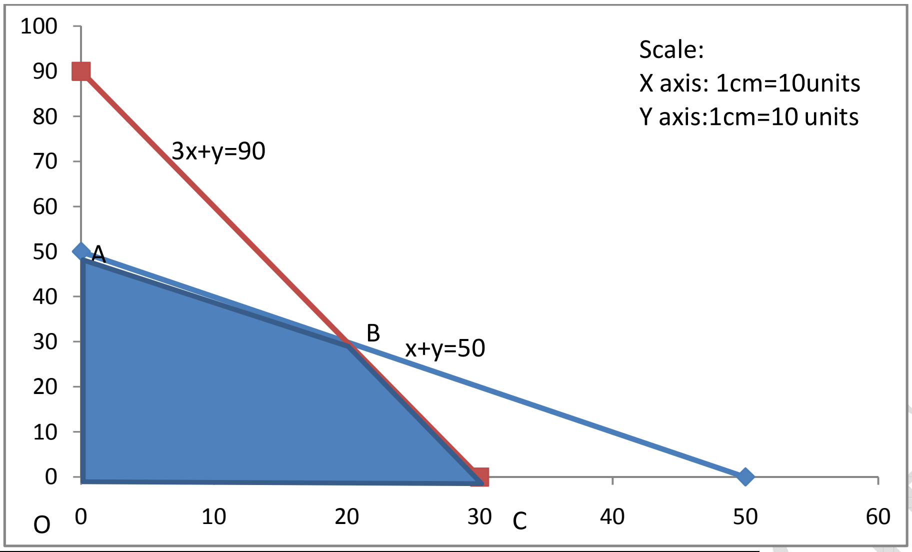
Total labour cost = 15× 18895.68 = Rs.283435.20

44. x + y = 50

x	0	50
y	50	0

3x + y = 90

x	0	30
y	90	0



Corner Points	Z=60x + 15y
O(0, 0)	0
A(0, 50)	750
B(20, 30)	1650
C(30, 0)	**1800 (Maximum)**

Z is maximum at C(30, 0)

x = 30, y = 0 and Z = 1800

45. LHS

=Sin2A+sin2B+sin2C

$$= 2\sin\left(\frac{2A+2B}{2}\right)\cos\left(\frac{2A-2B}{2}\right)+\sin 2C$$

=2sinCcos(A-B)+2sinCcosC

=2sinC(cos(A-B)+cos(180°-(A+B))

=2sinC(cos(A-B)-cos(A+B))

=2sinC(2sinAsinB)

=4sinAsinBsinC

=RHS

46. Equation of the circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

Passing through (5, 3) ⇒ 34 + 10g + 6f + c = 0

Passing through (1, 5) ⇒ 26 + 2g + 10f + c = 0

Passing through (3, -1) ⇒10 + 6g – 2f + c = 0

On solving, g = –2 , f = –2, c = –2

Hence the required equation of the circle is $x^2 + y^2 – 4x – 4y – 2 = 0$

47. $y = \log_e(x + \sqrt{x^2+1})$

$$y_1 = \frac{1}{(x+\sqrt{x^2+1})}\left[1 + \frac{1}{2\sqrt{x^2+1}}(2x)\right]$$

$$y_1 = \frac{1}{\sqrt{x^2+1}} \Rightarrow \left(\sqrt{x^2+1}\right)y_1 = 1$$

$$\left(\sqrt{x^2+1}\right)y_2 + y_1\frac{1}{2\sqrt{x^2+1}}(2x) = 0$$

$\Rightarrow (x^2+1)y_2+xy_1 =0$

48. $y^2=4x$ -----(1)

$y=x$.........2

Put y=x in eqn 1

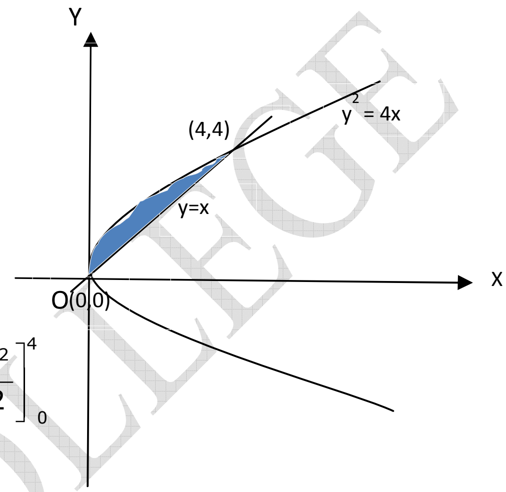
$X^2-4x=0 \Rightarrow x=0$ or $x=4$

$\therefore y = 0$ or $y = 4$

The points of intersection are

(x,y) = (0,0) and (x,y) = (4,4)

$\therefore$ They lie on the same side of x-axis



$\therefore$ Required Area A= $\int_0^4 (2x^{\frac{1}{2}} - x)dx = \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{x^2}{2}\right]_0^4$

$$= \frac{4}{3}(4)^{\frac{3}{2}} - \frac{4^2}{2} = \frac{32}{3} - 8 = \frac{8}{3}\text{sq.units}$$

Part E

49. (a)

$9x + 10y + 2z = 780$

$15x + 5y + 4z = 900$

$6x + 10y + 3z = 820$

$\therefore AX=D \Rightarrow X=A^{-1}D$ where $A^{-1}=\frac{1}{|A|}.adj.(A)$

$$|A| = \begin{vmatrix} 9 & 10 & 2 \\ 15 & 5 & 4 \\ 6 & 10 & 3 \end{vmatrix} = 9(-25) - 10(21) + 2(120) = -195$$

$$\text{Adj.}(A) = \begin{bmatrix} -25 & -10 & 30 \\ -21 & 15 & -6 \\ 120 & -30 & -105 \end{bmatrix}$$

$$X = \frac{1}{-195}\begin{bmatrix} -25 & -10 & 30 \\ -21 & 15 & -6 \\ 120 & -30 & -105 \end{bmatrix}\begin{bmatrix} 780 \\ 900 \\ 820 \end{bmatrix} = \frac{-1}{195}\begin{bmatrix} -3900 \\ -7800 \\ -19500 \end{bmatrix} = \begin{bmatrix} 20 \\ 40 \\ 100 \end{bmatrix}$$

x=20, y=40, z=100

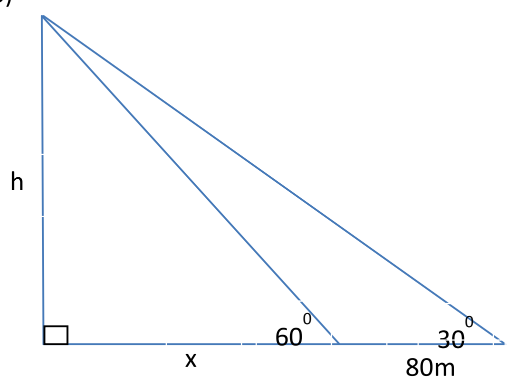
b. $(1+0.2)^5 = 1^5 + {}^5C_1\,(0.2) + {}^5C_2\,(0.2)^2 + {}^5C_3\,(0.2)^3 + {}^5C_4\,(0.2)^4 + {}^5C_5\,(0.2)^5$

$= 1 + 5(0.2) + 10(0.04) + 10\,(0.008) + 5\,(0.0016) + 1\,(0.00032)$

$= 1 + 1 + 0.4 + 0.08 + 0.008 + 0.00032 = 2.48832 \approx 2.4883$

50. a) Standard Proof

b)



$\tan 60^0 = \sqrt{3} = \frac{h}{x} \implies h = \sqrt{3}x$ ---------(1)

$\tan 30^0 = \frac{1}{\sqrt{3}} = \frac{h}{x+80} \Rightarrow h = \frac{x+80}{\sqrt{3}}$

Hence $\frac{x+80}{\sqrt{3}} = \sqrt{3}x \Rightarrow x = 40$m (width of the river)

Substituting in (1), h = $40\sqrt{3}$m (height of the tree)

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