ANNUAL EXAMINATION - ANSWER KEY -2019
II PUC – MATHEMATICS (held on 07-03-2019)

PART - A

1. A binary operation * on a set A is a function * : A $\times$ A $\rightarrow$ A . We denote * (a, b) by a * b.

OR

A binary operation * on a non empty set A is a rule that assigns to each ordered pair (a, b) of elements a and b of A, a unique element a*b $\in$A.

2. $\pi - \cos^{-1}\left(\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$.

3. Scalar matrix is a diagonal matrix in which all the diagonal elements are equal.

4. $3 - x^2 = 3 - 8 \Rightarrow x^2 = 8 \Rightarrow x = \pm\, 2\sqrt{2}$

5. $\frac{dy}{dx} = \cos\left(x^2+5\right)\,(2x)$

6. $\int (1-x)\sqrt{x}\,dx = \frac{2}{3} x^{\frac{3}{2}} - \frac{2}{5} x^{\frac{5}{2}} + c$

7. $\sqrt{x^2+x^2+x^2} = 1 \Rightarrow \sqrt{3x^2} = 1 \Rightarrow x = \pm\frac{1}{\sqrt{3}}$

8. The direction cosines are $\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$

9. A linear function Z = ax + by, where a, b are constants which can be optimized (maximized or minimized) in an LPP, is called objective function.

10. $P(F\,|\,E) = \frac{P(E\cap F)}{P(E)} = \frac{0.2}{0.6} = \frac{1}{3} = 0.34$

PART - B

11. Let $x_1, x_2 \in$ N,

$f(x_1) = f(x_2) \Rightarrow 2x_1 = 2x_2$

$x_1 = x_2$ $\quad\therefore$ f is one – one.

Let y $\in$ N and f(x) = y then

$f(x) = y \Rightarrow 2x = y \Rightarrow x = \frac{y}{2} \notin N$ (Domain) $\therefore$ f is not onto

OR

Range of f : { 2, 4, 6, ……..}

Co-domain = N = {1, 2, 3, 4, 5,……………}

$\therefore$ Range $\neq$ co-domain $\Rightarrow$ f is not onto

12. Let $\sin^{-1} x = y$,

then $x = \sin y = \cos\left(\frac{\pi}{2} - y\right)$

$\therefore \cos^{-1} x = \frac{\pi}{2} - y$; $y + \cos^{1} x = \frac{\pi}{2}$; $\therefore \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

13. Put $x = \sec\theta$

$\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \cot^{-1}\left(\frac{1}{\sqrt{\sec^2\theta - 1}}\right) = \cot^{-1}\left(\frac{1}{\sqrt{\tan^2\theta}}\right)$

$= \cot^{-1}\left(\frac{1}{\tan\theta}\right) = \cot^{-1}(\cot\theta) = \theta = \sec^{-1} x$.

14. Area of the triangle = $\frac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix} = \frac{1}{2}\left[2(1-8) - 7(1-10) + 1(8-10)\right]$

$= \frac{1}{2}\left[-14 + 63 - 2\right] = \frac{47}{2}$ sq. units.

15. $y = (\log x)^{\cos x}$

$\log y = \cos x \cdot \log(\log x)$

$\frac{1}{y}\frac{dy}{dx} = \cos x \frac{d}{dx}\left(\log(\log x)\right) + \log(\log x)\frac{d}{dx}(\cos x)$

$= \frac{\cos x}{x \log x} - \log(\log x)\sin x$

$\frac{dy}{dx} = y\left[\frac{\cos x}{x\log x} - \log(\log x)\sin x\right]$; $\frac{dy}{dx} = (\log x)^{\cos x}\left[\frac{\cos x}{x\log x} - \log(\log x)\sin x\right]$

16. $ax + by^2 = \cos y$

Differentiating w.r. to x

$a + 2by.\frac{dy}{dx} = (-\sin y)\frac{dy}{dx} \Rightarrow (2by + \sin y)\frac{dy}{dx} = -a$; $\frac{dy}{dx} = \frac{-a}{2by + \sin y}$

17. $v = x^3$

$\delta v = \left(\frac{dv}{dx}\right)\delta x = 3x^2\,\delta x$

$\delta v = 3x^2 \times 0.02\,x = 0.06x^3\ m^3$

18. $I = \int \frac{\sec^2 x}{(1 - \tan x)^2\,dx}$

Put $1 - \tan x = t \quad \Rightarrow \quad \sec^2 dx = -dt$

$I = \int \frac{1}{t^2}(-dt) = \frac{1}{t} = \frac{1}{1 - \tan x} + c$

19. $\int \sin 2x . \cos 3x\ dx$

$= \frac{1}{2}\int(\sin 5x - \sin x)\,dx = \frac{1}{2}\left[\cos x - \frac{\cos 5x}{5}\right] + c$

20. Order = 2 , Degree is not defined.

21. $\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right)=8 \Rightarrow |\vec{a}|^2-|\vec{b}|^2=8 \Rightarrow 64\ \left|\vec{b}\right|^2-\left|\vec{b}\right|^2=8 \Rightarrow \left|\vec{b}\right|^2=\frac{8}{63} \Rightarrow \left|\vec{b}\right|=\sqrt{\frac{8}{63}}=\frac{2\sqrt{2}}{\sqrt{63}}$

22. $\vec{a}=2\hat{i}+3\hat{j}+2\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$

Projection of $\vec{a}$ on $\vec{b}$ is $\frac{\vec{a}\ .\ \vec{b}}{\left|\vec{b}\right|}=\frac{\left(2\hat{i}+3\hat{j}+2\hat{k}\right).\left(\hat{i}+2\hat{j}+\hat{k}\right)}{\sqrt{1^2+2^2+1^2}}=\frac{2+6+2}{\sqrt{6}}=\frac{10}{\sqrt{6}}=\frac{10\sqrt{6}}{6}=\frac{5\sqrt{6}}{3}$

23. The distance of a point (3, -2, 1) from the plane 2x - y + 2z + 3 = 0 is

$$p=\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|=\left|\frac{2.3-(-2)+2(1)+3}{\sqrt{4+1+4}}\right|=\frac{13}{3}$$

24. Let E : A solves the problem

F : B solves the problem

$\therefore$ E and F are independent events, then

$P(E\cap F)=P(E)\ .\ P(F)$

$P(E)=\frac{1}{2};P(F)=\frac{1}{3}$

$P(E\cap F)=\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$

Required probability $P(E \text{ or } F)=P(E\cup F)=P(E)+P(F)-P(E\cap F)=\frac{1}{2}+\frac{1}{3}-\frac{1}{6}=\frac{4}{6}=\frac{2}{3}$

PART - C

25. $R=\{(a, b) : a\le b^3\}$

R is not reflexive , for $\left(\frac{1}{2},\frac{1}{2}\right)\notin R\left(\because \frac{1}{2}\not\le\left(\frac{1}{2}\right)^3\right)$

R is not symmetric, for (a, b) $\in$ R, but (b, a) $\notin$ R

<u>For eg</u>: (a, b) = (1, 2) $\in$ R $(\because 1\le 2^3)$

But (b, a) = (2, 1) $\notin$ R $(\because 2\not\le 1^3)$

R is not transitive, for, (a, b) $\in$ R and (b, c) $\in$ R, but (a, c) $\notin$ R

<u>For eg:</u> (a, b) = (9 , 3) $\in$ R $(\because 9\le 3^3)$

(b, c) = (3, 2) $\in$ R $(\because 3\le 2^3)$

But (a, c) = (9, 2) $\notin$ R $(\because 9\not\le 2^3)$

26. $LHS=\cos^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{5}{12}\right)$

$$=\tan^{-1}\left(\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}}\right)=\tan^{-1}\left(\frac{56}{33}\right)=\cos^{-1}\left(\frac{33}{65}\right)=RHS$$

27. A = IA

$$\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}A$$

$$\begin{bmatrix}1 & 2\\ 0 & -5\end{bmatrix}=\begin{bmatrix}1 & 0\\ -2 & 1\end{bmatrix}A \qquad \because R_2\to R_2-2R_1$$

$$\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\ \frac{2}{5} & -\frac{1}{5}\end{bmatrix} A \qquad \because R_2 \to \frac{1}{5}R_2$$

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}\frac{1}{5} & \frac{2}{5}\\ \frac{2}{5} & -\frac{1}{5}\end{bmatrix} A \qquad \because R_1 \to R_1 - 2R_2$$

$$\therefore A^{-1} = \begin{bmatrix}\frac{1}{5} & \frac{2}{5}\\ \frac{2}{5} & -\frac{1}{5}\end{bmatrix}$$

28. $x = a(\theta + \sin\theta) \Rightarrow \frac{dx}{d\theta} = a(1+\cos\theta) = 2a\cos^2\frac{\theta}{2}$

$y = a(1 - \cos\theta) \Rightarrow \frac{dy}{d\theta} = a(\sin\theta) = 2a\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2a\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2a\cos^2\frac{\theta}{2}} = \tan\frac{\theta}{2}$$

29. $f(x) = x^2 + 2x - 8$ is continuous in $[-4, 2]$

$f'(x) = 2x + 2 \Rightarrow f(x)$ is differentiable in $(-4, 2)$

$f(a) = f(-4) = 0$; $f(b) = f(2) = 0$

$f(-4) = f(2) = 0$

$\therefore$ Conditions of Rolle's Theorem are satisfied.

Now $f'(c) = 0 \Rightarrow 2c + 2 = 0$

$\therefore c = -1 \in (-4, 2)$

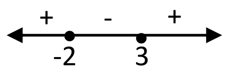
30. $f(x) = 2x^3 - 3x^2 - 36x + 7$

$f'(x) = 6x^2 - 6x - 36 = 6(x^2 - x - 6)$

$= 6((x + 2)(x - 3)) = 6(x-(-2))(x - 3) \Rightarrow x = -2, 3$

$f(x)$ is strictly increasing $\Rightarrow f'(x) > 0$

$\therefore x \in (-\infty, -2) \cup (3, +\infty)$.



31. $I = \int (\log_e x)\, x\, dx = (\log_e x)\frac{x^2}{2} - \int \frac{1}{x}.\frac{x^2}{2}dx \quad = (\log_e x)\frac{x^2}{2} - \frac{x^2}{4} + c$

OR

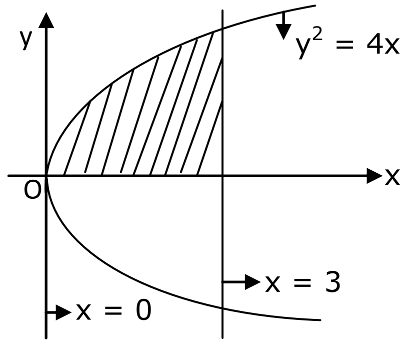
$I = \int (\log_e x)\, d\left(\frac{1}{2}x^2\right) = (\log_e x)\frac{1}{2}x^2 - \int \frac{1}{2}x^2.\frac{1}{x}\, dx$ $\qquad$ $\int u\,dv - uv - \int v\,dv$

$= (\log_e x)\frac{x^2}{2} - \frac{x^2}{4} + c$

32. $I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x}dx = \int_0^{\frac{\pi}{2}} \frac{-d(\cos x)}{1+\cos^2 x} = -\int_0^{\frac{\pi}{2}} \frac{1}{1+\cos^2 x}d(\cos x)$

$= \left[-\tan^{-1}(\cos x)\right]_0^{\frac{\pi}{2}} = -\tan^{-1}\left(\cos\frac{\pi}{2}\right) + \tan^{-1}(\cos(0)) = 0 + \frac{\pi}{4} = \frac{\pi}{4}$

33.



Required area,

$$A = 2\int_0^3 y\,dx = 2\int_0^3 2x^{\frac{1}{2}}\,dx = 4\times\frac{2}{3}\left(x^{\frac{3}{2}}\right)_0^3$$

$$= \frac{8}{3}\left(3^{\frac{3}{2}}\right) = 8\sqrt{3} \text{ sq.units}$$

34. $y = ae^{3x} + be^{-2x}$ - (1)

Multiplying both sides by e^{2x}

$ye^{2x} = ae^{5x} + b$

Diff. w.r.t x, we get,

$ye^{2x}(2) + e^{2x}(y') = ae^{5x}(5)$

Multiplying both sides by e^{-5x}

$2ye^{2x}.e^{-5x} + y' e^{2x}e^{-5x} = 5ae^{5x}e^{-5x}$

$2ye^{-3X} + y' e^{-3x} = 5a$

$2ye^{-3x}(-3) + 2e^{-3x}(y') + y'(e^{-3x})(-3) + e^{-3x}(y'') = 0$

$e^{-3x}(-6y + 2y' - 3y' + y'') = 0$

$\therefore y'' - y' - 6y = 0$ $(\because e^{-3x} \neq 0)$

35. $\vec{a} + \vec{b} = (2, 3, 4)$; $\vec{a} - \vec{b} = (0, -1, -2)$

Required unit vectors perpendicular to both $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are

$$\hat{n} = \frac{\pm\left((\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})\right)}{\left|(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})\right|}$$

$$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix} = -2i + 4j - 2k$$

$$\therefore \left|(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})\right| = \sqrt{(-2)^2 + 4^2 + (-2)^2} = \sqrt{24} = 2\sqrt{6} \qquad \therefore \hat{n} = \frac{\pm(-2i + 4j - 2k)}{2\sqrt{6}} = \frac{\pm(-i + 2j - k)}{\sqrt{6}}$$

36. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (2,4,6) - (4,8,12) = (-2,-4,-6)$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OB} = (3,5,4) - (4,8,12) = (-1,-3,-8)$

$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = (5,8,5) - (4,8,12) = (1,0,-7)$

$$\therefore \overrightarrow{AB}.\left(\overrightarrow{AC} \times \overrightarrow{AD}\right) = \left[\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}\right] = \begin{vmatrix} -2 & -4 & -6 \\ -1 & -3 & -8 \\ 1 & 0 & -7 \end{vmatrix} = (-2)(21) + 4(15) - 6(0 + 3) = 0$$

$= -42 + 60 - 18 = 0$ $\therefore$ The points are coplanar

37. Required equation of the plane is $\left[\vec{r} - \vec{a}, \vec{b} - \vec{a}, \vec{c} - \vec{a}\right] = 0$

i.e $(\vec{r} - \vec{a}) . \left((\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})\right) = 0$

$$\left(\vec{b}-\vec{a}\right)\times\left(\vec{c}-\vec{a}\right)=\begin{vmatrix} i & j & k \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix}=16i+24j+32k=(16,24,32)$$

$\therefore\ \left(\vec{r}-(2,5,-3)\right).(16,24,32)=0$

$\vec{r}.(16,24,32)-(32+120-96)=0$

$\vec{r}.(16i+24j+32k)=56$

$\vec{r}.(2i+3j+4k)=7$

38. Let A be the event that one of the insured persons meets with an accident.
Total = 2000 + 4000 + 6000 = 12000
Let E_1, E_2, E_3 be the events that the insured scooter, car and truck drivers meet with an accident respectively.

$\therefore\ P(E_1)=\frac{2000}{12000}=\frac{1}{6}$; $P(E_2)=\frac{4000}{12000}=\frac{1}{3}$; $P(E_3)=\frac{6000}{12000}\ \frac{1}{2}$

Now $P(A|E_1)=0.01$; $P(A|E_2)=0.03$; $P(A|E_3)=0.15$

Required = $P(E_1|A)=\dfrac{P(E_1)\,P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+P(E_3)P(A|E_3)}$

$$=\frac{\frac{1}{6}\times 0.01}{\frac{1}{6}\times 0.01+\frac{1}{3}\times 0.03+\frac{1}{2}\times 0.15}$$

$$=\frac{0.01}{0.52}=\frac{1}{52}$$

PART - D

39. Let $x_1, x_2 \in N$, Now, $f(x_1)=f(x_2)$

$\Rightarrow 4x_1+3=4x_2+3 \Rightarrow 4x_1=4x_2 \Rightarrow x_1=x_2 \therefore$ f is one – one

Let $y\in Y$ and $f(x)=y$

$4x+3=y \Rightarrow 4x=y-3$

$x=\frac{y-3}{4}\in N$ (domain) $\quad\therefore$ f is onto

Let $f^{-1}(y)=x\Rightarrow y=f(x)$

$y=4x+3\Rightarrow x=\frac{y-3}{4};\quad f^{-1}(y)=\frac{y-3}{4}$

40.

$$A=\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$$A^2=AxA=\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}=\begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}$$

$$A^3=AxA^2=\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}\begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}=\begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$$

$$A^3 - 23A - 40I = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - 23\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} - 40\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

41. Let $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$

$|A| = 3(2-3) + 2(4+4) + 3(-6-4) = -17$

Co - factor matrix of $A = \begin{bmatrix} -1 & -8 & -10 \\ -5 & -6 & 1 \\ -1 & 9 & 7 \end{bmatrix}$ $\quad \therefore$ $\text{Adj}(A) = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$

$X = A^{-1}B$, where, $A^{-1} = \dfrac{\text{Adj}A}{|A|}$

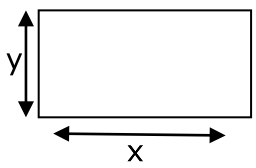
$$A^{-1} = \frac{-1}{17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}\begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix} = \frac{-1}{17}\begin{bmatrix} -8 + (-5) + (-4) \\ -64 + (-6) + 36 \\ -80 + 1 + 28 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow x = 1, \; y = 2, \; z = 3$$

42. $y = \sin^{-1} x \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\sqrt{1 - x^2}}$

$$\Rightarrow \sqrt{1 - x^2}\,\frac{dy}{dx} = 1 \;\Rightarrow\; \sqrt{1 - x^2}\,\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot\frac{1(-2x)}{2\sqrt{1 - x^2}} = 0 \Rightarrow (1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$$

43. $\dfrac{dx}{dt} = -3\text{cm}/\text{min}$ and $\dfrac{dy}{dt} = 2\,\text{cm}/\text{min}$



(i) The perimeter $P = 2(x + y)$

$$\frac{dP}{dt} = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$$
$$= 2(-3 + 2) = -2\,\text{cm}/\text{min}$$

(ii) The area A of the rectangle is $A = x.y$

$$\frac{dA}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt} = 10(2) + (6)(-3) = 2\,\text{cm}^2/\text{min}$$

44. $\dfrac{1}{x^2 - a^2} = \dfrac{1}{(x - a)(x + a)}$

$$= \frac{1}{2a}\left[\frac{(x + a) - (x - a)}{(x - a)(x + a)}\right] = \frac{1}{2a}\left[\frac{1}{x - a} - \frac{1}{x + a}\right]$$

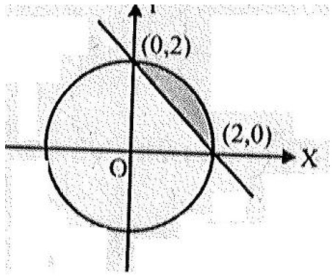
$$\therefore \int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\left[\int \frac{dx}{x - a} - \int \frac{dx}{x + a}\right] = \frac{1}{2a}\left[\log|(x - a)| - \log|(x + a)|\right] + c = \frac{1}{2a}\log\left|\frac{x - a}{x + a}\right| + c$$

$$\int \frac{dx}{x^2 - 16} = \int \frac{dx}{x^2 - 4^2} = \frac{1}{8}\log_e\left|\frac{x - 4}{x + 4}\right| + c$$

45. $x^2+y^2=4 \Rightarrow y=\sqrt{4-x^2}$

$x+y=2 \Rightarrow y=2-x$

$\text{Required area} = \int_0^2 \left[\sqrt{4-x^2}-(2-x)\right]dx$



$= \left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}-2x+\frac{x^2}{2}\right]_0^2$

$= \left[0+2\sin^{-1}1-4+2\right]-\left[0\right]$

$= 2\left(\frac{\pi}{2}\right)-2=(\pi-2)\text{ sq .units}$

46. Given, $\frac{dy}{dx}+y\sec x=\tan x \Rightarrow P = \sec x, Q=\tan x$

$I.F = e^{\int pdx} = e^{\int \sec x dx} = e^{\log_e(\sec x+\tan x)} = \sec x+\tan x$

$\therefore$ The general solution is

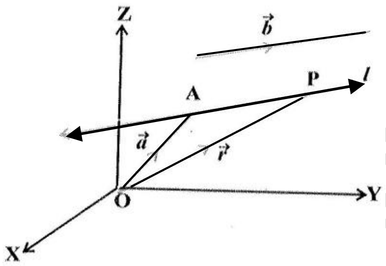
$y\,(I.F)=\int Q(I.F)dx+c$

$y(\sec x+\tan x)=\int \tan x(\sec x+\tan x)dx+c$

$= \int \tan x.\sec x dx+\int \tan^2 x\,dx+c \Rightarrow \sec x+\int(\sec^2 x-1)dx+c$

$y(\sec x+\tan x)=\sec x+\tan x-x+c$

47.



Let 'l' be the line which passes through the point A and is parallel to a given vector $\vec{b}$.

Let $\vec{r}$ be the position vector of any arbitrary point P on the line 'l'

Then $\overrightarrow{AP}$ is parallel to $\vec{b}$

That is $\overrightarrow{AP}=\lambda\vec{b}$, where λ is some real number

$\overrightarrow{OP}-\overrightarrow{OA}=\lambda\vec{b}$

$\vec{r}=\vec{a}+\lambda\vec{b}$ is the vector form.

Let $A(x_1, y_1, z_1)$ be the given point and $P(x, y, z)$ be the arbitrary point on the line.

Let a, b, c be the direction ratios of $\vec{b}$

$\therefore \overrightarrow{OA}=\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$

$\overrightarrow{OP}=\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

and $\vec{b}=a\hat{i}+b\hat{j}+c\hat{k}$

Substituting these values in vector equation,

we get $x\hat{i}+y\hat{j}+z\hat{k}=x_1\hat{i}+y_i\hat{j}+z_1\hat{k}+\lambda\left(a\hat{i}+b\hat{j}+c\hat{k}\right)$

Equating the coefficients of $\hat{i}, \hat{j}, \hat{k}$

we get $x - x_1 = \lambda a$, $y - y_1 = \lambda b$; $z - z_1 = \lambda c$

By eliminating λ, we get Cartesian form equation of the line.

$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$.

48. Let 'X' denote the number of spade cards among the 5 cards drawn.

In a well shuffled deck of 52 cards, there are 13 spade cards.

p = P(success) = P (a spade card drawn)

$= \frac{13}{52} = \frac{1}{4}$ and $q = 1 - \frac{1}{4} = \frac{3}{4}$ and n = 5

$P(X = r) = {}^nC_r P^r q^{n-r}$, where r = 0, 1, 2, 3, 4, 5

$P(X = r) = {}^5C_r \left(\frac{1}{4}\right)^r \left(\frac{3}{4}\right)^{5-r}$

(i) P(all the five cards are spades)

$P(X = 5) = {}^5C_5\, p^5 q^0 = \left(\frac{1}{4}\right)^5 = \frac{1}{1024}$

(ii) P(only 3 cards are spades) = P(X=3)

$= {}^5C_3\, p^3 q^2 = \frac{5 \times 4 \times 3}{3!}\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2 = \frac{60}{1 \times 2 \times 3} \times \frac{3^2}{4} = \frac{90}{1024} = \frac{45}{512}$

(iii) P(none is a spade) = P(X = 0)

$= {}^5C_0 p^0 q^5 = \left(\frac{3}{4}\right)^5 = \frac{243}{1024}$

PART – E

49. a) Consider $I = \int_0^a f(x)dx$

put a – x = t

dx = -dt . When x = 0, t = a and when x = a, t = 0

$\therefore \int_0^a f(a - x)\,dx = \int_a^o f(t)(-dt) = \int_o^a f(t)dt$

$= \int_o^a f(x)\,dx \quad \therefore \int_o^a f(x)\,dx = \int_o^a f(a - x)\,dx$

Let $I = \int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\sin^5 x + \cos^5 x}\,dx$ -----(1)

$I = \int_0^{\frac{\pi}{2}} \frac{\cos^5\left(\frac{\pi}{2} - x\right)}{\sin^5\left(\frac{\pi}{2} - x\right) + \cos^5\left(\frac{\pi}{2} - x\right)}\,dx$

$= \int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\cos^5 x + \sin^5 x}\,dx$ -----(2)

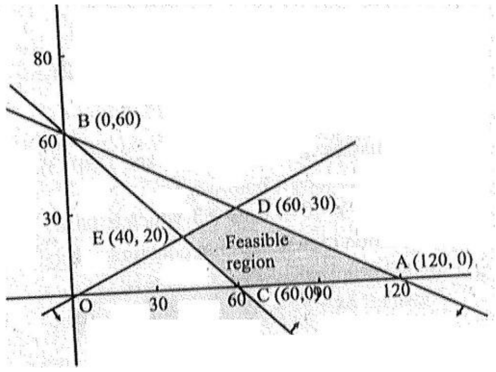
(1) + (2) gives ⇒ $2I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x + \cos^5 x}{\sin^5 x + \cos^5 x}\,dx = \int_0^{\frac{\pi}{2}} dx = \frac{\pi}{2} \quad \therefore I = \frac{\pi}{4}$

b) $\Delta = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = \begin{vmatrix} a & -b & 0 \\ 0 & b & -c \\ 1 & 1 & 1+c \end{vmatrix}$ $\quad \begin{bmatrix} R_1 = R_1 - R_2 \\ R_2 = R_2 - R_3 \end{bmatrix}$

$\Delta = a(b + bc + c) - (-b)(0 + c) + 0$

$= ab + abc + ac + bc = abc\left(1 + \frac{ab}{abc} + \frac{bc}{abc} + \frac{ac}{abc}\right) = abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$

50. a)



Solution lies in region CEDA

Sl .No	Corner point	Value of Z = 5x + 10 y
1	C (60, 0)	300 ← minimum
2	E (40, 20)	400
3	D (60, 30)	600 ← maximum
4	A (120, 0)	600 ← maximum

Minimum value of Z is 300 which occurs at (60, 0) and maximum value of Z is 600 which occurs at all points on the line segment joining the points A(120, 0) and D (60, 30).

b) Given function is continuous at x = 5 and f(5) = 5k + 1

$\therefore \lim_{x \to 5^-} f(x) = f(5) = \lim_{x \to 5^+} f(x)$

$\Rightarrow 5k + 1 = (3 \times 5) - 5 \Rightarrow 5k + 1 = 10 \Rightarrow 5k = 9 \Rightarrow k = \frac{9}{5}$

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