ANSWERS KEY

CHEMISTRY

1. c	2. d	3. c	4. c	5. d	6. d	7. c	8. c	9. d	10. c	11. c	12. c	13. b
14. c	15. b	16. d	17. d	18. d	19. b	20. c	21. d	22. b	23. a	24. d	25. b	26. d
27. a	28. c	29. c	30. d									

PHYSICS

1. c	2. c	3. d	4. c	5. a	6. b	7. b	8. d	9. b	10. a	11. c	12. d	13. b
14. a	15. a	16. d	17. d	18. d	19. b	20. d	21. c	22. c	23. b	24. a	25. d	26. c
27. a	28. a	29. d	30. a									

MATHEMATICS

1. a	2. b	3. c	4. c	5. a	6. c	7. d	8. a	9. b	10. c	11. b	12. c	13. a
14. a	15. d	16. d	17. b	18. b	19. a	20. d	21. c	22. c	23. b	24. b	25. b	26. b
27. c	28. a	29. c	30. a									

HINTS AND EXPLANATIONS

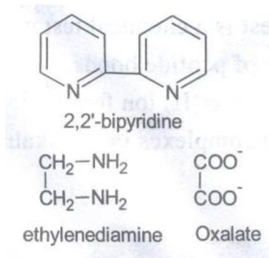
CHEMISTRY

Sol 1.

Lime acts as a flux and combines with silica (present as impurity) to form calcium silicate.

Sol 2.

All are bidentate ligands.



Sol 3.

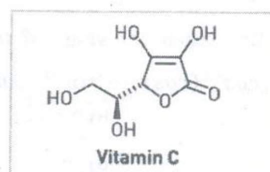
Chloromycetinis an antibiotic

Sol 4.

Silk is a protein based fibre. X-ray diffraction studies have shown that the silk is composed of long amino acid chains that form protein crystals. The majority of silk also contain beta-pleated sheet crystals that form from randomly repeated amino acid sequences rich in small amino acid residues.

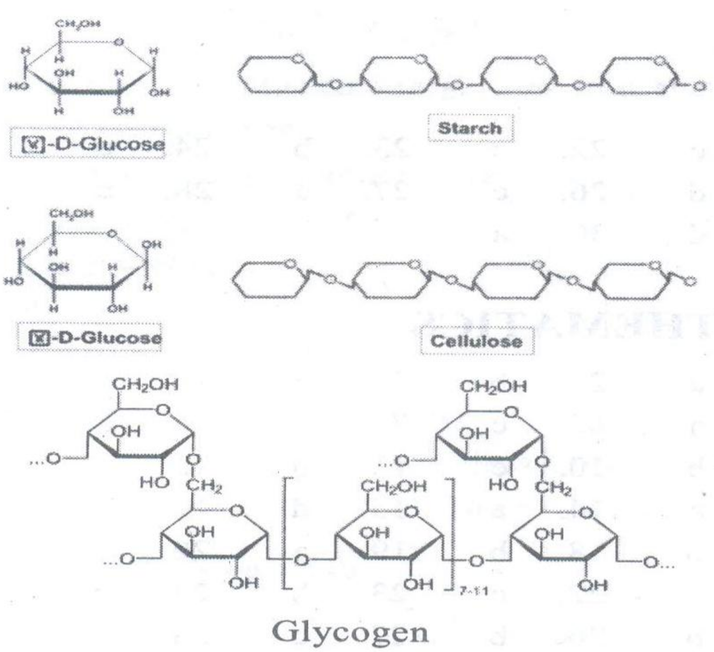
Sol 5.

Ascorbic acid is Vitamin C. Its structure is as follow.



Sol 6.

Cellulose. Starch and glycogen are polysaccharides of glucose.



Sol 7.

$NH_4OH \leftrightharpoons NH_4^+ + OH^-$

On adding NH_4CI equilibrium shifts in the backward direction due to common ion effect, i.e., conc of OH^- will decrease but due to added NH_4CI, conc. Of NH_4^+ ion will increase

Sol 8.

Acid Base

$M_1V_1 – M_2 V_2 = M_3 V_3$

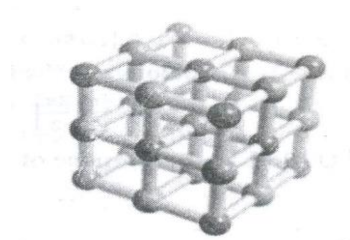
0.02 xl – $0.01 \times 1 = M_3 \times 2$ or

$M_3 = 0.01/2 = 0.005$

Sol 9.

Insoluble ppt of $BaSO_4 \rightarrow BaSO_4 + 2NaCI$

Sol 10.



Each CI^- ion is surrounded by $6K^+$ ions as in NaCI crystal.

Sol 11.

Amalgam is an alloy of mercury, a solid dissolves in a liquid.

Sol 12.

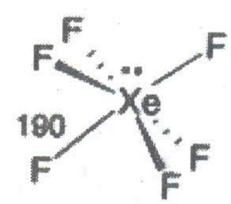
Carbon tetrachloride is non-polar, It is not miscible with polar water.

Sol 13.

For intravenous injections, saline water should be isotonic with blood. Normal saline (NS) is the commonly-used phrase for a solution of 0.90% w/v of NaCI, about 9.0 g per liter.

Sol 14.

Structure of XeF_6 is distorted octahedron due to presence of a lone pair of electron on Xe atom.



Sol 15.

Mn displays maximum number of oxidation states in its compounds.

Sol 16.

Due to high IE, Be does not give the flame test. Calcium gives a light yellow green colour, strontium – purple color and Ba – bluish green color.

Sol 17.

Reducing character of Group 16 hydrides increases down the group;

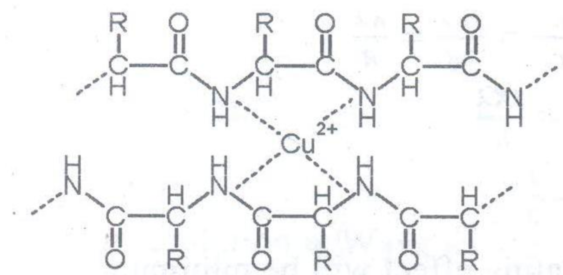
$H_2O < H_2S < H_2Se < H_2Te$.

Sol 18.

Both benzoic acid and naphthalene undergo sublimation and hence cannot be seprated from a mixture by this method. The best method for their separation is chromatography.

Sol 19.

The biuret test is a chemical test used for detecting the presence of peptide bonds. In the presence of peptides, a copper (II) ion forms violet-colored coordination complexes in an alkaline solution

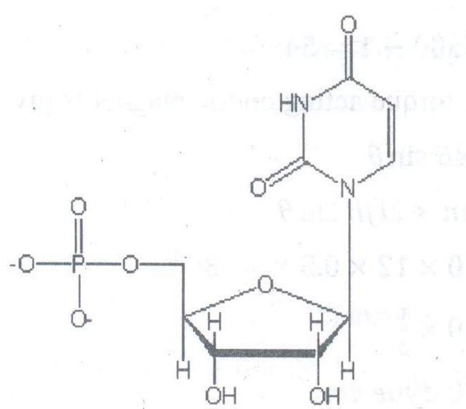


Sol 20.

$C_2H_5CN + KOH + H_2O \rightarrow C_2H_5COOK + NH_3$

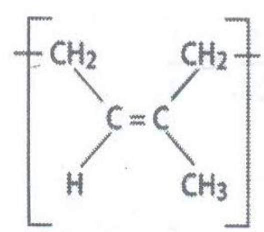
Sol 21.

The base molecule is attached to carbon 1 of sugar in RNA



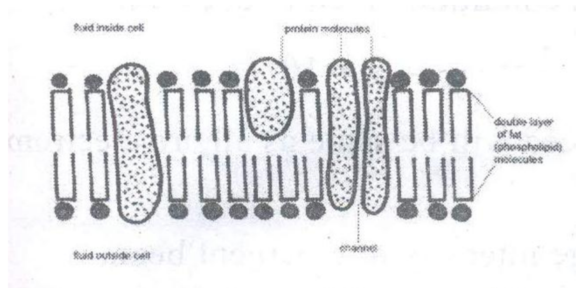
Sol 22.

Natural Rubber is formed from cis-polymerization of isoprene units.



Sol 23.

The basic function of the cell membrane is to protect the cell from its surroundings. It consists of the lipid bilayer with embedded proteins.



Sol 24.

On heating ammonium dichromate N_2 gas isevolved.

$(NH_4)_2Cr_2O_7 \rightarrow Cr_2O_3 + 4H_2O + N_2$

Sol 25.

The green house effect is caused by CO_2

Sol 26.

None of the given compounds undergo Friedel Craft reaction. Aromatic rings substituted with electron withdrawing group ($N_{2+}COOH$) do not electron with electron withdrawing groups (NO_2, COOH) do not give Friedel Craft reactions. Aniline (Lewis base) combines with $AICI_3$ (Lewis acid) used in Friedel Craft reaction.

Sol 27.

$CH_3COOH + SOCI_2 \rightarrow$

$CH_3COCI + SO_2 + HCI$

Sol 28.

For preparation of butyl methyl ether, the alkyl halide should be primary as tertiary alkyl halides give elimination reaction preferably. $\Longrightarrow CH_3CI + NaOC(CH_3)_3 \rightarrow$

$CH_3OC(CH_3)_3 + NaCI$

$(CH_3)_3\ C\text{-}CI + NaOCH_3 \rightarrow (CH_3)_2\ C = CH_2 + NaCI$

Sol 29.

Formic acid decomposes sodium carbonate and reacts with Tollen's reagent to formblack ppt.

$2HCOOH + Na_2CO_3 \rightarrow$

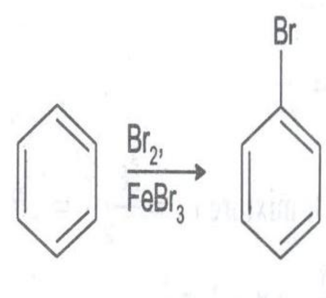
$2HCOONa + CO_2 + H_2O$

$HCOOH + 2[Ag\ (NH_3)_2]\ OH \rightarrow$

$2Ag + CO_2 + 2H_2O + 4NH_3$

Sol 30.

Formation of bromobenzene from benzene requires and a Lewis acid catalyst.



PHYSICS

Sol 1.

Making use formula

$g = \pi^2 r \frac{l}{T^2}$ we get

$\frac{\Delta g}{g}$x 100 = $\frac{\Delta l}{l}$ x 100 + 2 $\frac{\Delta T}{T}$ x 100

= 1 + (2 x 3) = 7%

Sol 2.

Both P and Q will reach the ground at the same time because both P and Q have same intial velocity and same acceleration due to gravity.

Sol 3.

Because F = μmg

$\Rightarrow \mu = \frac{F}{mg} = \frac{\propto}{ß\, x\ 9.8} = \frac{\propto}{9.8\ ß}$

Sol 4.

As potential energy = $\frac{1}{2}$ Kx²

$\Rightarrow$ P.E. $\propto$ K

$(P.E.)_1 \propto K_1$ and $(P.E.)_2 \propto K_2$

$\therefore \frac{(P.E.)_1}{(P.E.)_2} = \frac{K_1}{K_2}$

Sol 5.

The centre of gravity of a solid body always lies within the body is a correct statement while all other are incorrect.

Sol 6.

As latitude λ increases, the acceleration due to gravity, g also increase

Sol 7.

Using $\theta = \frac{dr}{l} = \frac{0.2x45}{100} = 0.09°$

Sol 8.

The maximum energy wavelengths of two stars are given

$\lambda_m = 3.6 \text{ x } 10^{-7}$ m $\lambda_m = 4.8 \text{ x } 10^{-7}$ m Using Wein's displacement of two stars

$\Rightarrow \frac{T}{T'} = \frac{\lambda_m'}{\lambda_m} = \frac{4.8x\ 10^{-7}}{3.6\ x\ 10^{-7}} = \frac{4.8}{3.6} = \frac{4}{3}$

Sol 9.

For mixture, using the relations

$\frac{c_p}{c_v} = \gamma$ and $C_p - C_v = R$

we get $c_v = \frac{R}{\gamma - 1}$

But here for mixture $c_v = \frac{\frac{3}{2}R + \frac{5}{2}R}{2} = 2R \Rightarrow 2R = \frac{R}{\gamma - 1}$ i.e. $\gamma - 1 = \frac{1}{2}$

Or$\gamma = 1 + \frac{1}{2} = 1.5$

Sol 10.

Logarithmic decrement $\theta = KT$

K is damping factor which depends upon (Resistance of medium/2x mass)

∴For smaller mass i.e. for pendulum A damping factor is more hence logarithmic decrement is more for pendulum A.

Sol 11.

As $f \propto \sqrt{T}$

$\therefore f_1 : f_2 : f_3 : f_4 = \sqrt{1} : \sqrt{2} : \sqrt{9} : \sqrt{16}$

$= 1 : 2 : 3 : 4$

Sol 12.

$E_1 = \frac{2k\lambda}{2R} = \frac{k\lambda}{R}$ and $E_2 = \frac{2K\lambda}{R}$

$\Rightarrow \frac{E_1}{E_2} = \frac{1}{2}$

Sol 13.

The heating effect will be minimum

Sol 14.

Given $\square_r = 5500$

using relation $\square_r = 1 + x$

$x = \square_r - 1$

$= 5500 - 1 = 5499$

Sol 15.

The torque acting on the magnet is given as

$= MB \sin\theta = (m \times 2l)\, B \sin\theta$

$= 10 \times 12 \times 0.5 \times \sin 30°$

$= 60 \times \frac{1}{2} = 30$ dyne cm

Sol 16.

$\text{Cos}\, \emptyset = \frac{R}{\tau} = \frac{R}{[R^2 + (\sqrt{3}R^2)]^{1/2}}$

$= \frac{R}{(R^2 + 3R^2)^{1/2}} = \frac{R}{2R} = \frac{1}{2}$

$\Rightarrow \emptyset = 60°$

Or $\emptyset = \frac{\pi}{3}$

Sol 17.

Using $T = \frac{1}{v} = \frac{1}{50}$

For the condition given in question

$T = \frac{1}{4} \times \frac{1}{5} = \frac{1}{200} = 5 \times 10^{-3}$ s

Sol 18.

The speed will be same as all are electromagnetic waves

Sol 19.

A verage intensity of emergent beam

$I = I_0 / 2$

Where I_0 is the intensity of incident light

$I_0 = \frac{Energy}{Area\ x\ Time} = \frac{Power}{Area}$

$= \frac{10^{-3}}{3x\ 10^{-4}} = \frac{10}{3}$ Wm^{-2}

$\Rightarrow I = \frac{1}{2} \text{ x } \frac{10}{3} = \frac{5}{3}$ Wm^{-2}

Sol 20.

Energy of light passing through polarizer E = IAT

in which T is the Time period of one revolution

$T = \frac{2\pi}{\omega} = 2 \text{ x } \frac{3.14}{31.4} = \frac{1}{5} \text{ s} \Rightarrow E = \frac{5}{3} \text{ x } (3 \text{ x } 10^{-4}) \text{ x } \frac{1}{5}$

$E = 10^{-4}$ J

Sol 21.

As limit of resolution ∝ Wave length

∴New limit of resolution

$= \frac{new\ \ wave\ \ lengt\ h}{previos\ \ wave\ \ lengt\ h}$ x previous limit of resolution

$= \frac{4800}{6000} \text{ x } 0.1 = 0.8$ nm

Sol 22.

Any particle in motion is accompanied by matter waves

Sol 23.

Using $E = mc^2$

$= 10^{-8} \text{ x } (3 \text{ x } 10^8)^2$

$E = 9 \text{ x } 10^8$ J

Sol 24. Both the statements are self explanatory

Sol 25.

$53 = 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 32 + 16 + 4 + 1$

$= 11\ 01\ 01$

Sol 26.

As $L = \frac{\lambda}{2} = \frac{c}{v x 2} = \frac{3x\ 10^8}{(5\ x\ 10^8) x\ 2} = \frac{c}{10}$

$L = 0.3$ m

Sol 27.

As $\mu mg \cos\theta > mg \sin\theta$

$\Rightarrow$ Force of friction, $f = mg \sin\theta$

Sol 28.

$P = P_1 + P_2 = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{0.4} - \frac{1}{0.25} = -1.50$

Sol 29.

If Q is released from a point not very far from the origin on x-axis, only then motion is simple harmonic. The motion will be periodic but not simple harmonic otherwise.

Sol 30.

In a non uniform magnetic field, the needle will experience both and torque

MATHEMATICS

Sol 1.

$$\begin{vmatrix} e^a & e^{2a} & e^{3a}-1 \\ e^b & e^{3b} & e^{3b}-1 \\ e^c & e^{2c} & e^{3c}-1 \end{vmatrix} = \begin{vmatrix} e^a & e^{2a} & e^{3a} \\ e^b & e^{2b} & e^{3b} \\ e^c & e^{2c} & e^{3c} \end{vmatrix} -$$

$$\begin{vmatrix} e^a & e^{2a} & 1 \\ e^b & e^{2b} & 1 \\ e^c & e^{2c} & 1 \end{vmatrix} = e^a e^b e^c \begin{vmatrix} 1 & e^a & e^{2a} \\ 1 & e^b & e^{2b} \\ 1 & e^c & e^{2c} \end{vmatrix} - \begin{vmatrix} 1 & e^a & e^{2a} \\ 1 & e^b & e^{2b} \\ 1 & e^c & e^{2c} \end{vmatrix}$$

$$= (e^{a+b+c} - 1) \begin{vmatrix} 1 & e^a & e^{2a} \\ 1 & e^b & e^{2b} \\ 1 & e^c & e^{2c} \end{vmatrix} = (e^0 - 1) \begin{vmatrix} 1 & e^a & e^{2a} \\ 1 & e^b & e^{2b} \\ 1 & e^c & e^{2c} \end{vmatrix} = 0$$

$$\begin{bmatrix} \because a, b, c \text{ are cube roots of unity} \\ \therefore a + b + c = 0 \end{bmatrix}$$

Sol 2.

Given that one root of

$x^2 - \lambda x + 12 = 0$ is even prime

$\therefore x = 2$ is root of $x^2 - \lambda x + 12 = 0$

$\therefore (2)^2 - \lambda(2) + 12 = 0$

$4 - 2\lambda + 12 = 0$

$2\lambda = 16 \Rightarrow \lambda = 8$ Given that $x^2 + \lambda x + \mu = 0$ has equal roots

$\therefore$Disc. = 0

$\therefore \lambda^2 - 4 \times 1 \times$ ⍰ $= 0$

$(8)^2 - 4$⍰ $= 0$

$64 = 4$⍰ $\Rightarrow$ ⍰ $= 16$

Sol 3.

The given series is

$30 + 28 + 26 + 24 + \ldots\ldots. + 0$

$T_n = a + (n - 1)\, d$

$0 = 30 + (n - 1)(-2)$

$30 - 2n + 2 = 0$

$-2n = -32 \Rightarrow n = 16$

$\therefore$SumofntermsS$_n = \frac{n}{2}$ $[a + 1] = \frac{16}{2}[30 + 0]$

$= 16 \times 15 = 240$

Sol 4.

Total number of arrangement of the word

BANANA $= \frac{6!}{3!2!}$ Number of arrangement of word BANANA in which 2N's comes together $= \frac{5!}{3!}$

Required number of arrangement of the word BANANA in which the two N's do not appear adjacently

$= \frac{6!}{3!2!} - \frac{5!}{3!} = 60 - 20 = 40$

Sol 5.

Given $\left(x+\frac{1}{x}\right)^n = \frac{(1+x)^n}{x^n}$

$(1 + x)^n$ contains (n + 1) terms

∴ (n + 1) = 101⇒ n = 50

Sol 6.

$\begin{vmatrix} \lambda & -1 & -2 \\ 2 & -3 & \lambda \\ 3 & -2 & 1 \end{vmatrix} = 0 \Rightarrow 2\lambda^2 - 6\lambda - 8 = 0$

⇒ (λ – 4) (λ + 1) = 0 ∴ λ = 4, - 1

Sol 7.

In a skew symmetric matrix, all the diagonal elements are zero

Sol 8.

Let A = 2

Sol 7.

In a skew symmetric matrix, all the diagonal elements are zero

Sol 8.

Let A = 2^{30}

Log_{10} A = 30 log_{10} 2 = 3 x 0.301 = 9.03

Number of digits in 2^{30} = + 1 = 20

Sol 9.

The probability of hitting the target 4^{th} time at the 8^{th} throw

$= \left(\begin{matrix} Probability\ of\ hitting\ the\ target\ 3\ times\ in \\ the\ first\ 7\ throws \end{matrix}\right)$ x

$$\left(\begin{matrix} Probability\ of\ hitting\ the\ target\ at\ the \\ 8th\ throw \end{matrix}\right)$$

$= 7_{c_3}\left(\frac{1}{3}\right)^3\left(1-\frac{1}{3}\right)^4\ x\,\frac{1}{3}$

$= \frac{7.6.5}{3.2.1}\left(\frac{1}{3}\right)^3\left(\frac{4}{3}\right)^4 x\,\frac{1}{3} = \frac{35(4)^4}{(3)^8}$

Sol 10.

Since $0 < \{x\} < 1$

$\Rightarrow \tan 0 < \tan\{x\} < \tan 1$

$\Rightarrow 0 < \tan\{x\} < \tan 1$

$\therefore \frac{1}{\tan\{x\}} > 0$

Then $\left[\frac{1}{\tan\{x\}}\right] = 1, 2, 3, \ldots\ldots..$

$\therefore$ Range of function f(x) is N, the set of natural numbers.

Sol 11.

1 – cosx cos4x cos5x

$= \sin^2 x + \cos^2 x - \cos x \cos 4x \cos 5x$

$= \sin^2 x + \cos x (\cos x - \cos 4x \cos 5x)$

$= \sin^2 x + \cos x \begin{bmatrix} \cos(5x - 4x) \\ \cos 4x \ cos5s \end{bmatrix}$

$= \sin^2 x + \cos x [\sin 5x \sin 4x]$

$\mathrm{Lt}_{x \to 0} \frac{1 - cosx \ cos 4x \ cos 5x}{\sin^2 x}$

$= \mathrm{Lt}_{x \to 0} \frac{\sin^2 x + cosx \ (\sin 5x \ sin 4x)}{\sin^2 x}$

$\mathrm{Lt}_{x \to 0}\ 1 + \frac{cosx \frac{sin\,5x}{5x}\frac{sin\,4x}{4x}.20}{\left(\frac{sinx}{x}\right)^2}$

$= 1 + \frac{(1)(1)(1)(20)}{(1)^2} = 1 + 20 = 21$

Sol 12.

Given $f(x) \frac{1}{(x-3)(x-5)}$ and $g(x) = \frac{1}{x}$

$f\,[g(x)] = \frac{1}{\left(\frac{1}{x} - 3\right)\left(\frac{1}{x} - 5\right)} = \frac{x^2}{(1-3x)(1-5x)}$

$\therefore f\,[g(x)]$ is discontinuos at $x = \frac{1}{3}, x = \frac{1}{5}$

Sol 13.

Given $x^y = e^{x+y}$

$Y \ln x = (x + y)$

$Y(\ln x - 1) = x$

$y = \frac{x}{\ln x - 1}$

$\frac{dy}{dx} = \frac{(\ln x - 1)1 - x\left(\frac{1}{x}\right)}{(\ln x - 1)^2}$

$\frac{dy}{dx} = \frac{\ln x - 2}{(\ln x - 1)^2}$

Sol 14.

The equation of tangent at (x_1, y_1) to the curve $y = \cos x$ is

$y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1, y_1)} (x - x_1)$

$y - y_1 = (-\sin x)_{(x_1, y_1)} (x - x_1)$

∴ Equation of tangent through (0,0) is $\Longrightarrow y - 0 = 0 \Rightarrow y = 0$

Sol 15.

If $f(x) = \begin{cases} \frac{1}{3} - x, & x < \frac{1}{3} \\ \left(\frac{1}{3} - x\right)^2, & x \geq \frac{1}{3} \end{cases}$

$Lf'\left(\frac{1}{3}\right) = -1$

$Rf'\left(\frac{1}{3}\right) = 2\left(\frac{1}{3} - \frac{1}{3}\right) = 0$

$Lf'\left(\frac{1}{3}\right) \neq Rf'\left(\frac{1}{3}\right)$ ∴ f is not differentiable at $x = \frac{1}{3} \in (0,1)$

∴ Lagrange mean value theorem is not applicable to f (x) in [0,1]

Sol 16.

$f'(x) = x + 1 + \cos x > 0 \ \forall x \in R$

∴ f (x) is an increasing function $= \left[\frac{t^2}{2} + t + \sin t\right]_1^2 = (2 + 2 + \sin 2) - \left(\frac{1}{2} + 1 \sin 1\right)$

$= \frac{5}{2} + (\sin 2 - \sin 1)$

Sol 17.

$|lnx| = \ln x \because 1 < x < \infty$

$\int |\ln x|\, dx = \int \ln dx$

$= \int \ln x.\, 1\, dx$

$= \ln x.x \int \frac{1}{x},\, x dx$

$= x|\ln x| - x + c$

Sol 18.

$I = \int_{-\pi/2}^{\pi/2} \frac{e^x \sec^2 x}{e^x - 1}\, dx$

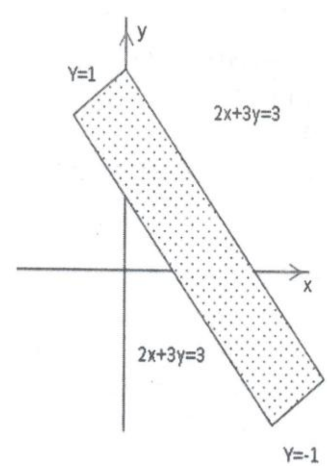
Given $f\ (x) = \frac{e^x \sec^2 x}{e^x - 1}$

$f(-x) = \frac{e^{-x} \sec^2(-x)}{e^{-x} - 1} = \frac{\sec^2 x}{1 - e^x} = -\frac{\sec^2 x}{e^x - 1} = -f(x)$

$\therefore$ f (x) is odd function

$\therefore \int_{-\pi/2}^{\pi/2} f\ (x) dx = 0$

Sol 19.



Given curve is $|x + y - 1| + |x + 2y - 1| = 1$

If both the terms are positive, then $(x + y - 1) + (x + 2y - 1) = 1$

$\Rightarrow 2x + 3y = 3$ If first term is +ve and second term is – ve, then

$(x + y - 1) - (x + 2y - 1) = 1 \Rightarrow -y = 1 \Rightarrow y = -1$

If first term is –ve and second term is +ve, then

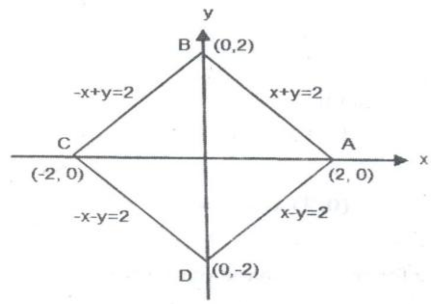
$-(x + y - 1) + (x + 2y - 1) = 1 \quad \Rightarrow y = 1$

If both the terms are –ve, then

$-(x + y - 1) - (x + 2y - 1) = 1 \Rightarrow -2x - 3y + 1 = 0 \Rightarrow 2x + 3y = 1$

Required area $= \int_{-1}^{1} \left[\left(\frac{3-3y}{2}\right) - \left(\frac{1-3y}{2}\right)\right] dy = \int_{-1}^{1} dy = [y]_{-1}^{1} = 2$ sq. units

Sol 21.



$d(x, y) = 2 \Rightarrow |x| + |y| = 2$ The graph of which is shown in the figure

The graph is a square $AB = BC = CD = DA = 2\sqrt{2}$

Area = AB x AD = $2\sqrt{2} \times 2\sqrt{2} = 8$ sq. units

Sol 22.

$\because (x - 2)(x + m) = -1$ has integral roots

$\therefore$ either $x - 2 = +1$ and $x + m = -1$

$x = 3$ and $3 + m = -1$

$\therefore m = -4$ Or $x - 2 = -1$ and $x + m = 1$

$x = 1$ and $1 + m = 1$

$\therefore m = 0$

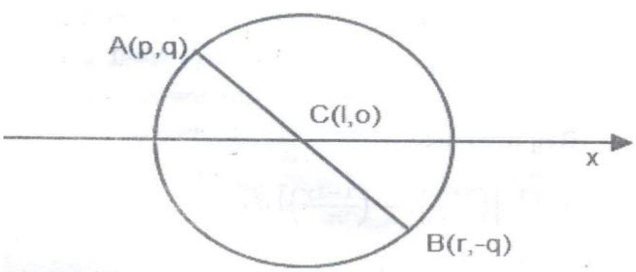
Hence joint equation of lined through the origin having slopes – 4 and 0 is

$[y - (-4)][y - 0x] = 0 \Rightarrow (y + 4x) y = 0$

$\Rightarrow y^2 + 4xy = 0$

Sol 23.

Suppose chord AB bisect at C (1,0) then other end point of chord B (r, -q), where $l = \frac{p+q}{2}$



which lies on $x^2 + y^2 = px + qy$

$\Rightarrow r^2 + q^2 = pr - q^2 \Rightarrow r^2 - pr + 2q^2 = 0$

For the two chords

$B^2 - 4AC > 0$

$\Rightarrow (-p)^2 - 4(1)(2q^2) > 0 \Rightarrow p^2 > 8q^2$

Sol 24.

The given parabola is

$y^2 - 12x - 4y + 4 = 0$

$\Rightarrow (y - 2)^2 = 12x$

Its vertex is (0,2) and a = 3

Its focus is (3, 2).

Hence for required parabola vertex is (3,2) and focus is (3,3) $\therefore a = 1$

Hence equation of parabola is

$(x - 3)^2 = 4\ (1)\ (y - 3)$

$\Rightarrow x^2 - 6x - 4y + 21 = 0$

Sol 25.

Given that y is major axis. Therefore

$f(3a) < f\ (a^2 - 4)$

$\Rightarrow 3a > a^2 - 4$ ($\because$ f is decreasing)

$\Rightarrow a^2 - 3a - 4 < 0$

$\Rightarrow (a + 1)(a - 4) < 0$

$\Rightarrow a + 1 > 0$ and $a - 4 < 0$

$\Rightarrow -1 < a$ and $a < 4$

$\Rightarrow a \in (-1, 4)$

Sol 26.

The given condition gives

$\frac{5+5\sin^2\alpha}{5} = 3\frac{25-25\sin^2\alpha}{25} \Rightarrow \sin^2\alpha = \frac{1}{2}$

$\Rightarrow \sin\alpha = \pm\frac{1}{\sqrt{2}}$

$\therefore \alpha = \frac{\pi}{4}, \frac{5\pi}{4}$

Sol 27.

Let P (x_1, y_1, z_1) be any point on the given line ln + my + nz = p

$\therefore lx_1 + my_1 + nz_1 = p$ (i)

Let Q be (a, b, c) and Q, P, Q are collinear

So $\frac{x_1}{a} = \frac{y_1}{b} = \frac{z_1}{c} = k$ (say) (ii)

Now 0P. 0Q = $p^2 \sqrt{x_1^2 + y_1^2 + z_1^2}\sqrt{a^2 + b^2 + c^2} = p^2$

$\Rightarrow k\sqrt{a^2 + b^2 + c^2} = p^2$ (iii)

From (i) & (ii)

k (al + bm + cn) = p (iv)

From (iii) & (iv)

p (al + bm + cn) = $a^2 + b^2 + c^2$

∴ Locus of Q (a, b, c) is

p (lx + my + nz) = $x^2 + y^2 + z^2$

Sol 28.

Given that $\frac{\pi}{2} \leq \theta \leq \pi$

∴ |sin θ| = sinθ& |cos θ| = - cos θ

∴ x = sin θ |sin θ| = $\sin^2 \theta$

y = cos θ |cos θ| = - $\cos^2 \theta \Rightarrow x - y = \sin^2 \theta + \cos^2 \theta = 1$

Sol 29.

Given that

|cotx + cosecx| = |cotx| + |cosecx|

$|f(x) + g(x)| = |f(x)| + |g(x)| \Rightarrow \cot x \operatorname{cosec} x \geq 0$

$\Rightarrow \frac{cosx}{\sin^2 x} \geq 0 \Rightarrow \cos x \geq 0$ and $\sin x \neq 0$

$\Rightarrow x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

$\therefore x \in \left[-\frac{\pi}{2}, 0\right) \cup \left(x, \frac{\pi}{2}\right]$

Sol 30.

Given that $x \in \left(\frac{3\pi}{2}, 2\pi\right)$

$\therefore \cos^{-1}(\cos x) = 2\pi - x$

and $\sin^{-1}(\sin x) = x - 2\pi$

$\therefore \cos^{-1}(\cos x) + \sin^{-1}(\sin x) = 0$

$\Rightarrow \sin\{\cos^{-1}(\cos x) + \sin^{-1}(\sin x)\} = 0$

$\Rightarrow \cos^{-1}\{\sin\{\cos^{-1}(\cos x) + \sin^{-1}(\sin x)\}\}$

$= \cos^{-1} 0 = \frac{\pi}{2}$