## SSLC Examination March - 2019 .Mathematics

## English Version -Questions with Solutions.

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## Question.

1. In the figure O is the centre of the circle. $\angle \mathrm{AOC}=80^{\circ}$
(a) What is the measure of $\angle \mathrm{ABC}$ ?
(b) What is the measure of $\angle \mathrm{ADC}$ ?


## Solution.

Given $\angle \mathrm{AOC}=80^{\circ}$.
a) The measurement $\angle \mathrm{ABC}=\frac{1}{2} \times 80=40^{\circ}$.
b) The measurement $\angle \mathrm{ADC}=180-40=140^{\circ}$
( ABCD is a cyclic quadrilateral)
Question.
2. (a) Write the first integer term of the arithmetic sequence $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}$
(b) What is the sum of the first 7 terms of this sequence?

## Solution.

a) Given arithmetic sequence $=$

$$
\frac{1}{7}+\frac{2}{7}+\frac{3}{7}, \ldots \ldots \ldots . \quad d=2 / 7-1 / 7=1 / 7
$$

Hence the first integer term $=\frac{7}{7}=1$.
b) Sum $=\frac{n}{2}\left(x_{1}+x_{n}\right)=\frac{7}{2}\left(\frac{1}{7}+\frac{7}{7}\right)=4 . \quad\left[\frac{7}{2} \times \frac{8}{7}=4\right]$.

Hence the sum of the first 7 tearm be 4 .
Question.
3. (a) If $\mathrm{C}(-1, \mathrm{k})$ is a point on the line passing through the points $\mathrm{A}(2,4)$ and $\mathrm{B}(4,8)$ which
number is K ?
(b) What is the relation between the $x$ coordinate and the $y$ coordinate of any point on this line?

## Solution.

Given points be $\mathrm{A}(2,4), \mathrm{B}(4,8)$
Slope $=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}=8-4 / 4-2=4-2=2$.
a) ie., $\frac{k-8}{-1-4}=2$. Cross multiplay, we get

$$
\mathrm{k}=-2
$$

b) Let ( $\mathrm{x}, \mathrm{y}$ ) be any point on the slope ,

$$
\begin{aligned}
& \text { ie., } \frac{y-8}{x-4}=2 ; y-8=2(x-4) ; y-8=2 x-8 \\
& 2 x-y=0
\end{aligned}
$$

## Question.

4. (a) Find $\mathrm{P}(1)$ if $\mathrm{P}(x)=x^{2}+2 x+5$.
(b) If $(x-1)$ is a factor of $x^{2}+2 x+\mathrm{k}$, What number is k ?

## Solution.

Given polynomial , $\mathrm{P}(\mathrm{x})=\mathrm{x}^{2}+2 \mathrm{x}+5$
$P(1)=1^{2}+2 \times 1+5$
$=1+2+5$; then $\mathrm{k}=-8$.
b) Given $\mathrm{x}-1$ is a factor, polynomial , $\mathrm{P}(\mathrm{x})=\mathrm{x}^{2}+2 \mathrm{x}+\mathrm{k}$

$$
\begin{aligned}
\mathrm{P}(1) & =1^{2}+2 \times 1+\mathrm{k}=0 \\
& =1+2+\mathrm{k} ; \text { then } \mathrm{k}=-3 .
\end{aligned}
$$

## Question.

5. (a) What is the remainder on dividing the terms of the arithmetic sequence $100,107,114$
...... by 7 ?
(b) Write the sequence of all three digit numbers. Which leaves remainder 3 on division by 7 ? Which is the last term of this sequence?

## Solution.

Given sequence be $100,107,114$,
a) $\mathrm{d}=7$; Remainder $=100 / 7=2$.
b) $101,108,115$

Hence the last three digit term $=997$.

## Question.

6. $\quad \mathrm{AB}$ is the diameter of the circle. D is a point on the circle.

$\angle \mathrm{ACB}+\angle \mathrm{ADB}+\angle \mathrm{AEB}=270^{\circ}$. Measure of one among $\angle \mathrm{ACB}, \angle \mathrm{ADB}, \angle \mathrm{AEB}$ is $110^{\circ}$. Write the measures of $\angle \mathrm{ADB}, \angle \mathrm{ACB}$, and $\angle \mathrm{AEB}$.

## Solution.

$\angle \mathrm{ADB}=90^{\circ}$ ( Measurement of semi circle angle)
$\angle \mathrm{ACB}+\angle \mathrm{ADB}+\angle \mathrm{AEB}=270^{\circ}($ Given $)$
ie., $\angle \mathrm{ACB}+90+\angle \mathrm{AEB}=270^{\circ}$

$$
\angle \mathrm{ACB}+\angle \mathrm{AEB}=270^{\circ}-90=180^{\circ} .
$$

The given condition any one of the $\angle \mathrm{ACB}, \angle \mathrm{AEB}$ be $110^{\circ}$.
take $\angle \mathrm{ACB}=110^{\circ}$
Hence $\angle \mathrm{AEB}=180-110=70^{\circ}$.
So the angles,
$\angle \mathrm{ADB}=90^{\circ}, \angle \mathrm{ACB}=110^{\circ}, \angle \mathrm{AEB}=70^{\circ}$.
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## Question.

7. If $x$ is a natural number
(a) What number is to be added to $x^{2}+6 x$ to get a perfect square ?
(b) If $x^{2}+\mathrm{a} x+16$ is a perfect square which number is ' a ' ?
(c) If $x^{2}+\mathrm{a} x+\mathrm{b}$ is a perfect square prove that $\mathrm{a}^{2}=4 \mathrm{~b}$.

## Solution.

Given $x^{2}+6 x$
a) $6 \mathrm{x}=2 \mathrm{ab} ; \mathrm{a}=\mathrm{x} ; \mathrm{b}=?$ ? $\quad \mathrm{b}=\frac{6 x}{2 x}=3$

Perfect square $=b^{2}=3^{2}=9$.
Hence 9 be added to them.
b) Given , $x^{2}+a x+16$ is perfect square

This is the form of $a^{2}+2 a b+b^{2}=(a+b)^{2}$ ie., $2 a b=a x ; a=x ; b^{2}=16 ; b=\sqrt{16}=4$

$$
\text { So, }(x+4)^{2}=x^{2}+a x+16
$$

Hence $\mathrm{a}=2 \mathrm{ab}=2 \times 4=8$.
c) Here we can see that $b=$ the square of the half of $a$
ie., $\mathrm{b}=\left(\frac{a}{2}\right)^{2} ; \mathrm{b}=\frac{a^{2}}{4}$
ie., $\mathrm{a}^{2}=4 \mathrm{~b}$. Hence proved.

## Question.

8. In the figure $\angle B=90^{\circ}, \angle C=44^{\circ}$

(a) What is the measure of $\angle A$ ?
(b) Which among the following is $\tan 44^{\circ}$ ?

$$
\left(\frac{A B}{B C}, \frac{A B}{A C}, \frac{B C}{A B}, \frac{B C}{A C}\right)
$$

(c) Prove that $\tan 44^{\circ} \times \tan 46^{\circ}=1$.

## Solution.

Gfiven $\angle \mathrm{B}=90^{\circ} ; \angle \mathrm{C}=44^{\circ}$
a) The measurement of $\angle \mathrm{A}=180-(90+44)=46^{\circ}$.
b) $\tan 44^{0}=\frac{\text { opposit e sid e }}{\text { adjacent sid } \mathrm{e}}=\frac{\mathrm{AB}}{\mathrm{BC}}$ (From the figure)
c) LHS $=\tan 44^{\circ} \times \tan 46^{\circ}$

$$
=\frac{\mathrm{AB}}{\mathrm{BC}} \times \frac{\mathrm{BC}}{\mathrm{AB}}=1=\mathrm{RHS}
$$

Hence proved.

## Question.

9. Draw a circle of radius 3 centimetres. Mark a point $P$ at a distance 6 centimetres from the centre of the circle. Draw tangents from P to the circle.

## Solution.

## Constriction

Draw a circle with radius 3 cm as center at
O. From the center draw op as 6 cm and draw perpendicular to OP and mark M on it.
Draw a circle with center At M and cut it


T and R respectively. Join PT and PR being the required tangents.

## Question.

10. (a) Find the coordinates of the point on $x$ axis, which is at a distance 4 units from $(3,4)$.
(b) Find the coordinates of the points on $x$ axis at a distance 5 units from $(3,4)$.

## Solution.

a) A point on $x$-axis $(x, 0)$
ie $(x-3)^{2}+(0-4)^{2}=4^{2}$
$(x-3)^{2}+4^{2}=16$
$(x-3)^{2}=0$
$\mathrm{x}-3=0$
$\mathrm{x}=3$
Point $(3,0)$ be from $(3,4)$ to the distance 4 unit.
b) $(x-3)^{2}+(0-4)^{2}=5^{2}$
$(x-3)^{2}+4^{2}=25$
$(x-3)^{2}=9$
$x-3= \pm 3$
$\mathrm{x}=6, \mathrm{x}=0$
Hence $(0,0),(6,0)$ be the point from $(3,4)$ at the distance 4 unit.

## Question.

11. The given figure is the lateral face of a square pyramid. $\mathrm{AB}=\mathrm{AC}=25$ centimetres and $\mathrm{BD}=\mathrm{DC}=15$ centimetres.
(a) What is the length of its base edge ?
(b) Find the lateral surface area of the pyramid.


## Solution.

a) From the figure base edge $=\mathrm{BD}+\mathrm{DC}=15+15=30 \mathrm{~cm}$.
b) $\mathrm{LSA}=2 \mathrm{al} ; \mathrm{a}=30 \mathrm{~cm}$; lateral edg (e) 25 cm .

$$
\begin{aligned}
l=? ; l & =\sqrt{\mathrm{e}^{2}-\left(\frac{a}{2}\right)^{2}} \\
& =\sqrt{25^{2}-\left(\frac{30}{2}\right)^{2}}=\sqrt{625-225}=\sqrt{400}=20
\end{aligned}
$$

Hence LSA $=2 \mathrm{al}=2 \times 30 \times 20=1200 \mathrm{~cm}^{2}$
$\qquad$

## Question.

12. In triangle $A B C, \angle A=30^{\circ}, \angle B=80^{\circ}$, circumradius of the triangle is 4 centumetres. vraw ule triangle. Measure and write the length of its smallest side.

## Solution.

## Constriction

Draw a circle with radius as 4 cm . And then make any radius with in Then draw the twice of the angle ,which will given the question.
Draw another two radii and then joint it to make a
$\triangle \mathrm{ABC}$
Measure the length of the

smaller side
It becomes 4 cm .
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## Question.

## 13. Find the following sums :

(a) $1+2+3+\ldots \ldots \ldots \ldots \ldots \ldots . . \ldots 100$
(b) $1+3+5+\ldots \ldots \ldots . . . . . . . .+99$
(c) $2+4+6+\ldots \ldots \ldots \ldots \ldots \ldots+100$
(d) $3+7+11+$ $+199$

## Solution.

a)Given $1+2+3+\ldots \ldots . . .+100$.

Sum $=\frac{n}{2}[n+1]=\frac{100}{2}[100+1]=50 \times 101=5050$
b) Given $1+3+5+\ldots \ldots . . .+99$.

$$
\begin{aligned}
& \mathrm{x}_{1}=1 ; \mathrm{d}=3-1=2 ; \mathrm{x}_{\mathrm{n}}=99 ; \mathrm{n}=? \\
& \mathrm{n}=\frac{x_{n}-x_{1}}{d}+1=\frac{99-1}{2}+1=50 .
\end{aligned}
$$

Sum $=\frac{n}{2}\left[x_{1}+x_{n}\right]=\frac{50}{2}[1+99]=25 \times 100=2500$.
c) Given $2+4+6+\ldots \ldots \ldots .+100$.

$$
\mathrm{x}_{1}=2 ; \mathrm{d}=4-2=2 ; \mathrm{x}_{\mathrm{n}}=100 ; \mathrm{n}=\text { ? }
$$

$\mathrm{n}=\frac{x_{n}-x_{1}}{d}+1=\frac{100-2}{2}+1=50$.
Sum $=\frac{n}{2}\left[x_{1}+x_{n}\right]=\frac{50}{2}[2+100]=25 \times 102=2550$.
d) Given 3+7+11+.........+ 199
$\mathrm{x}_{1}=3 ; \mathrm{d}=7-3=4 ; \mathrm{x}_{\mathrm{n}}=199 ; \mathrm{n}=$ ?
$\mathrm{n}=\frac{x_{n}-x_{1}}{d}+1=\frac{199-3}{4}+1=50$.

Sum $=\frac{n}{2}\left[x_{1}+x_{n}\right]=\frac{50}{2}[3+199]=25 \times 202=5050 .$. .drvsr

## Question.

14. A box contains some green and blue balls. 7 red balls are put into it. Now the probability of getting a red ball from the box is $\frac{7}{24}$ and that of a blue ball is $\frac{1}{3}$.
(a) How many balls are there in the box?
(b) How many of them are blue?
(c) What is the probability of getting a green ball from the box?

## Solution.

Given probability of red ball $=\frac{7}{24}$
a) Hence total ball = 24 ( Favorable / Total number)
b) Let the blue ball be 'b’

Probability of blue ball $=\frac{1}{3}$
ie., $\frac{b}{24}=\frac{1}{3} ; b=8$.
Number of blue ball $=8$.
c) Number of green balls $=24-(8+7)=24-15=9$ (F)

Probability of green ball $P($ Green $)=\frac{F}{N}=\frac{9}{24}=\frac{3}{8}$.

## Question.

15. Land is acquired for road widening from a square ground, as shown in the figure. The width of the acquired land is 2 metres. Area of the remaining ground is 440 square metres.

(a) What is the shape of the remaining ground?
(b) What is the length of the remaining ground ?

## Solution.

a) Shape of the remaining ground be Rectangle.
b) Let the length be ' $x$ '

## Breadth $=x-2$.

Area $=440 \mathrm{~m}^{2}$. (Given)
ie., $x(x-2)=440$.
$x^{2}-2 x=440$
$x^{2}-2 x-440=0$ (Solve by any method)
$(x-22)(x+20)=0$
$\mathrm{x}-22=0$ or $\mathrm{x}+20=0$
Hence $x=22$ or -20 ; - 20 rejectable ; so $\mathrm{x}=22$
Hence the length $=22 \mathrm{~m}$; Breadth $=22-2=20 \mathrm{~m}$.
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## Question.

16. In the figure $P$ is the centre of the circle. $A, B$ and $D$ are points on the circle. $\angle \mathrm{P}=90^{\circ}, \mathrm{AD}=5$ centimetres.

(a) What is the measure of $\angle \mathrm{A}$ ?
(b) What is the area of triangle APD ?
(c) Find the area of the parallelogram $A B C D$.

## Solution.

a) In $\triangle \mathrm{ABD}$,
$\angle \mathrm{D}=90^{\circ}$ (Angle in the semi circle )
$\angle \mathrm{P}=90^{\circ}$, so that $\angle \mathrm{ADP}=45^{\circ}$.
Hence $\angle \mathrm{A}=45^{\circ}$.
b) In isosceles right angled $\triangle \mathrm{APD}, \mathrm{AD}=5 \mathrm{~cm}$.

We know that $45: 45: 90$
ie., $\mathrm{AP}=\mathrm{PD}=\frac{5}{\sqrt{2}}$
Area of the $\triangle \mathrm{APD}=\frac{1}{2} \times b h=\frac{1}{2} \times \mathrm{AP} \times \mathrm{PD}$

$$
=\frac{1}{2} \times \frac{5}{\sqrt{2}} \times \frac{5}{\sqrt{2}}=\frac{25}{4}=6.25 \mathrm{~cm}^{2} .
$$

c) Area of the parallelogram $\mathrm{ABCD}=\mathrm{bh}$

$$
\text { Base } A B=\frac{5}{\sqrt{2}}+\frac{5}{\sqrt{2}}=\frac{10}{\sqrt{2}}=5 \sqrt{2} \mathrm{~cm}
$$

$$
\mathrm{h}(\mathrm{PD})=\frac{5}{\sqrt{2}} \mathrm{~cm}
$$

Hence the area $=5 \sqrt{2} \times \frac{5}{\sqrt{2}}=25 \mathrm{~cm}^{2}$.

## Question.

17. (a) Draw the coordinate axes and mark the points $A(1,1), B(7,1)$.
(b) Draw an isosceles right triangle ABC with AB as hypotenuse.
(c) Write the coordinates of C .

## Solution.

a) and b) see the figure.
c) $(4,4)$


## Question.

18. In the figure chord $B C$ is extended to $P$. Tangent from $P$ to the circle is $P A . A Q$ is the bisector of $\angle B A C$.

(a) Write one pair of equal angles from the figure.
(b) If $\angle \mathrm{PAC}=x$ and $\angle \mathrm{PCA}=y$ prove that $\angle \mathrm{BAC}=y-x$.
(c) Prove that $\angle \mathrm{PAQ}=\frac{y+x}{2}$.

## Solution.

a) A pair of equale angles $=\angle \mathrm{ABC}=\angle \mathrm{PAC}$
b) Given $\angle \mathrm{PAC}=\mathrm{x} ; \angle \mathrm{PCA}=\mathrm{y}$

Given $\angle \mathrm{PAC}=\mathrm{x}$ ie., $\angle \mathrm{ABC}$ is also be x
Given $\angle \mathrm{PCA}=\mathrm{y}$ ie., $\angle \mathrm{ACB}-180-\mathrm{y}$.
$\angle \mathrm{BAC}=180-(\angle \mathrm{ABC}+\angle \mathrm{ACB})$
$=180-(\mathrm{x}+180-\mathrm{y})$
$=180-x-180+y$
$=-x+y=y-x$
Hence proved.
c) $\mathrm{LHS}=\angle \mathrm{PAB}=\angle \mathrm{PAC}+\angle \mathrm{CAQ}$

$$
\begin{aligned}
& =\mathrm{x}+\angle \mathrm{BAC} / 2 \\
& =x+\frac{y-x}{2}=\frac{2 x+y+x}{2}=\frac{x+y}{2}=\text { RHS }
\end{aligned}
$$

Hence proved.

## Question.

19. If $x-1$ is a factor of the second degree polynomial $\mathrm{P}(x)=a x^{2}+\mathrm{b} x+\mathrm{c}$ and $\mathrm{P}(0)=-5$.
(a) What is the value of $c$ ?
(b) Prove that $\mathrm{a}+\mathrm{b}=5$.
(c) Write a second degree polynomial whose one factor is $x-1$.

## Solution.

Given polynomial $=\mathrm{ax}^{2}+\mathrm{bx}+\mathrm{c}$ and
$x-1$ is a factor. $P(0)=-5$.
a) If $P(0)=-5$, ie., $c=-5$.
b) $\mathrm{P}(1)=\mathrm{a}(1)^{2}+\mathrm{b}(1)+\mathrm{c}=0$

$$
=a+b+-5=0
$$

$=\mathrm{a}+\mathrm{b}=5$. Hence proved.
c) In $\mathrm{ax}^{2}+\mathrm{bx}+\mathrm{c}, \mathrm{a}+\mathrm{b}=5$ and $\mathrm{c}=-5$ will be satisfied So we can write more than one polynomials .Such that $3 x^{2}+2 x-5$ or $2 x^{2}+3 x-5$ or $4 x^{2}+x-5$.

## Question.

20. A circular sheet of paper is divided into two sectors. Central angle of one of them is $160^{\circ}$.
(a) What is the central angle of the remaining sector ?
(b) These sectors are bent into cones of maximum volume. If the radius of the small cone is 8 centimetres, what is the radius of the other ?
(c) What is the slant height of the cones ?

## Solution.

Given center angle $=160^{\circ}$. Radius of the small cone $(\mathrm{r})=$ 8 cm
a) Center angle of the second sector $=360-160=200^{\circ}$.
b) $\frac{r}{l}=\frac{x^{0}}{360}$ (Formula)

$$
\frac{8}{l}=\frac{160}{360} ; l=\frac{360 \times 8}{160}=18 \mathrm{~cm}
$$

Here sector's radii are equal
ie., $\frac{r}{l}=\frac{x^{0}}{360}=\frac{r}{18}=\frac{200}{360} ; r=\frac{200 \times 18}{360}=10 \mathrm{~cm}$.
c) Slant height ( l ) = Radius of the sector

$$
=18 \mathrm{~cm} .
$$

## Question.

21. Equation of the line $A B$ is $3 x-2 y=6$. $P$ is a point on the line. The line intersects the $y$-axis at $A$ and the $x$-axis at $B$.
(a) What is the $x$ coordinate of A ?
(b) What is the length of OA ?
(c) What is the length of $O B$ ?
(d) The $x$ coordinate and the $y$ coordinate of $P$ are same. Find the coordinates of $P$.


## Solution.

Given the equation of $\mathrm{AB}=3 \mathrm{x}-2 \mathrm{y}=6$.
a) x - coordinate of $\mathrm{A}=0$.
b) Put $x$ - coordinate be 0 in the equation $3 x-2 y=6$.
ie., $3 \times 0-2 y=6$. ; ie., $-2 y=6 ; y=6 /-2=-3$.
The coodinate of $\mathrm{A}=(0,-3)$.
The length of $\mathrm{OA}=3$ unit.
c) $y$ - coordinate of $x$-axis be 0 .
ie., Put $y=0$ in the equation $3 x-2 y=6$
ie., $3 x-2 \times 0=6 ; 3 x=6 ; x=6 / 3=2$.
The coordinate of $\mathrm{B}=(2,0)$
The length of $\mathrm{OB}=2$ unit.
d) Let $y=x$ in tne equation $3 x-2 y=6$.
ie., $3 x-2 x=6 ; x=6$.
So, the coordinate of $P$ be $(6,6)$.

## Question.

22. If the terms of the arithmetic sequence $\frac{2}{9}, \frac{3}{9}, \frac{4}{9}, \frac{5}{9} \ldots \ldots . . .$. are represented as $x_{1}, x_{2}, x_{3} \ldots \ldots$. then
(a) $x_{1}+x_{2}+x_{3}=$ $\qquad$
(b) $x_{4}+x_{5}+x_{6}=$ $\qquad$
(c) Find the sum of first 9 terms.
(d) What is the sum of first 300 terms?

## Solution.

Given sequence $=2 / 9,3 / 9,4 / 9,5 / 9$
a) $\mathrm{x}_{1}+\mathrm{x}_{2}+\mathrm{x}_{3}=\frac{2}{9}+\frac{3}{9}+\frac{4}{9}=\frac{9}{9}=1$.
b) $x_{4}+x_{5}+x_{6}=\frac{5}{9}+\frac{6}{9}+\frac{7}{9}=\frac{18}{9}=2$.
c) $\mathrm{x}_{1}=2 / 9 ; \mathrm{x}_{\mathrm{n}}=10 / 9 ; \mathrm{n}=9$

Sum of the first 9 terms $=\frac{n}{2}\left(x_{1}+x_{n}\right)=\frac{9}{2}(2 / 9+10 / 9)$

$$
=\frac{9}{2} \times \frac{12}{9}=6
$$

d) $\mathrm{x}_{1}=2 / 9 ; \mathrm{x}_{\mathrm{n}}=301 / 9 ; \mathrm{n}=300$

Sum of the first 300 terms $=\frac{n}{2}\left(x_{1}+x_{n}\right)$

$$
=\frac{300}{2}(2 / 9+301 / 9)=\frac{300}{2} \times \frac{303}{9}=5050 .
$$

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## Question.

23. Draw a rectangle of area 12 square centimetres. Draw a square having the same area.

## Solution.

## Construction

Draw a rectangle ABCD , as AB $=4 \mathrm{~cm}$ and
$\mathrm{BC}=\mathrm{AD}=3 \mathrm{~cm}$. AB line be produced to E such that $\mathrm{BC}=\mathrm{BE}$

. Make O as the midpoint of AE
and draw a semi circle, $O E$ as the radius with the center at O. . BC produced and cut at G on the semi circle and draw a rectangle with the side as BG . This is the required measured square.
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## Question.

24. A boy standing at one bank of a river sees the top of a tree on the other bank directly opposite to the boy at an elevation of $60^{\circ}$. Stepping 40 metres back, he sees the top at an elevation of $30^{\circ}$.
(a) Draw a rough figure and find the height of the tree.
(b) What is the width of the river?

## Solution.

a)

In the figure
Tree $=\mathrm{AB}$
Let C be the first and P be the second place of the boy respectively.
$\mathrm{PC}=40 \mathrm{~m} ; \angle \mathrm{BPC}=30^{\circ}$; $\angle \mathrm{BCA}=60^{\circ}$.
In $\triangle \mathrm{PCB}$,

$\angle \mathrm{PBC}=\angle \mathrm{BCA}-\angle \mathrm{BPC}$

$$
=60-30=30^{\circ} .
$$

$\triangle \mathrm{PCD}$ be an isosceles triangle. So, $\mathrm{BC}=\mathrm{PC}=40 \mathrm{~m}$.
In the right triangle ABC The angles are 30, 60, 90
ie., $1: \sqrt{3}: 2$.
$\mathrm{AB}=\frac{\mathrm{BC}}{2} \times \sqrt{3}=\frac{40}{2} \times \sqrt{3}=20 \sqrt{3} \mathrm{~m}$
$\mathrm{AC}=\frac{\mathrm{BC}}{2}=\frac{40}{2}=20 \mathrm{~m}$.
Hence the height of the tree $=\mathrm{AB}=20 \sqrt{3} \mathrm{~m}$.
b) The width of the river $=A C=20 \mathrm{~m}$.
$\qquad$

## Question.

25. Circle with centre 0 touches the sides of the triangle at $P, Q$ and $R, A B=A C$, $A Q=4$ centimetres and $C Q=6$ centimetres.
$\Lambda$

(a) What is the length of CP ?
(b) Find the perimeter and the area of the triangle.
(c) What is the radius of the circle?

## Solution.

Given, $\mathrm{AB}=\mathrm{AC}, \mathrm{AQ}=4 \mathrm{~cm} . \mathrm{CQ}=6 \mathrm{~cm}$
a) $\mathrm{CP}=\mathrm{CQ}=6 \mathrm{~cm}$ ( tangents are equal)
b) $\mathrm{AC}=\mathrm{AQ}+\mathrm{CQ}=4+6=10 \mathrm{~cm}$.
$A B=A C=10 \mathrm{~cm}$.
$A Q=A R=4 \mathrm{~cm} . ; B R=A B-A R=10-4=6 \mathrm{~cm}$.
$B P=B R=6 \mathrm{~cm}$.
So, $\mathrm{BC}=\mathrm{BP}+\mathrm{PC}=6+6=12 \mathrm{~cm}$.
Hence the perimeter of the Triangle ABC

$$
=\mathrm{AB}+\mathrm{BC}+\mathrm{CA}=10+12+10=32 \mathrm{~cm} .
$$

Area of $\triangle \mathrm{ABC}=\frac{1}{2} \times b h$
Join $\mathrm{AP}=\mathrm{h}$ of the right triangle $\mathrm{APB}, \mathrm{BP}=6 \mathrm{~cm}$,

$$
\mathrm{AB}=10 \mathrm{~cm}
$$

$$
h=\sqrt{10^{2}-6^{2}}=\sqrt{100-36}=\sqrt{64}=8 \mathrm{~cm}
$$

Area of $\Delta \mathrm{ABC}=\frac{1}{2} \times b h=\frac{1}{2} \times 12 \times 8=48 \mathrm{~cm}^{2}$
Or area can be find out the Hero's formula .
c) Radius $=\frac{\text { A rea }}{\text { Sem i perimeter }}=\frac{48}{15}=3 \mathrm{~cm}$.

## Question.

26. Radius of a cylinder is equal to its height. If the radius is taken as $r$, volume of the cylinder is $\pi r^{2} \times r=\pi r^{3}$. Like this find the volumes of the solids, with the following measures.

| Solids | Measures | Volume |
| :--- | :--- | :--- |
| Cone | radius $=$ height $=\mathrm{r}$ |  |
| Hemisphere | radius $=\mathrm{r}$ |  |
| Sphere | radius $=\mathrm{r}$ |  |

(a) What is the ratio of the volumes of cone, hemisphere, cylinder and the sphere ?
(b) A solid metal sphere of radius 6 centimetres is melted and recast into solid cones of radius 6 centimetres and height 6 centimetres. Find the number of cones.

## Solution.

| Solids | Measures | Volume |
| :--- | :--- | :--- |
| Cone | Radius $=$ heighr $=r$ | $\frac{1}{3} \pi r^{2} h=$ |
|  |  | $\frac{1}{3} \pi \times r^{2} \times r=\frac{1}{3} \pi r^{3}$ |
| Hemisp <br> hear | Radius $=\mathrm{r}$ | $\frac{2}{3} \pi r^{3}$ |
| Sphear | Radius $=\mathrm{r}$ | $\frac{4}{3} \pi r^{3}$ |

a) The ratio of volumes of cone, hemisphere, cylinder and sphear $=\frac{1}{3} \pi r^{3}: \frac{2}{3} \pi r^{3}: \pi r^{3}: \frac{4}{3} \pi r^{3}=1: 2: 3: 4$.
b) Given radius of the sphear $=6 \mathrm{~cm}$.

Radius of cone $=6 \mathrm{~cm}$; Height of the cone $=6 \mathrm{~cm}$.
Number of cone $=\frac{\text { V olume of sphear }}{\text { V olume of cone }}$

$$
=\frac{\frac{4}{3} \pi \times 6 \times 6 \times 6}{\frac{1}{3} \pi \times 6 \times 6 \times 6}=\frac{4}{3} \times 3=4 .
$$

## Question.

27. $C$ is the centre of the circle passing through the origin. Circle cuts the $y$-axis at $A(0,4)$ and the $x$-axis at $\mathrm{B}(4,0)$.

(a) Write coordinates of C .
(b) Write the equation of the circle.
(c) $(0,0)$ is a point on the circle. There is one more point on the circle with $x$ and $y$ coordinates equal. Which is that point ?

## Solution.

Given $\mathrm{A}(0,4)$, $\mathrm{B}(4,0)$; Origin $(0,0)$
a) Coordinate of $\mathrm{C}=$ Find the mid point

$$
=\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right)=\left(\frac{0+4}{2}, \frac{4+0}{2}\right)
$$

$=\left(\frac{4}{2}, \frac{4}{2}\right)=(2,2)$.
b) Equation of the circle $=(x-a)^{2}+(y-b)^{2}=r^{2}$.

Diameter of the circle $=$ Find the distance by distance formula

$$
\begin{aligned}
& =\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}} ; \text { Points be } \mathrm{A}(0,4), \mathrm{B}(4,0) \\
& =\sqrt{4^{2}+4^{2}}=\sqrt{16+16}=\sqrt{32}=4 \sqrt{2}
\end{aligned}
$$

Radius $=\frac{4 \sqrt{2}}{2}=2 \sqrt{2}$
Hence the equation of the circle be $(x-a)^{2}+(y-b)^{2}=r^{2}$.

$$
\begin{aligned}
& =(x-2)^{2}+(y-2)^{2}=(2 \sqrt{2})^{2} \\
& =x^{2}-4 x+4+y^{2}-4 y+4=8 \\
& =x^{2}+y^{2}-4 x-4 y+8-8=0 \\
& =x^{2}+y^{2}-4 x-4 y=0
\end{aligned}
$$

c) Put $x=y$ in the equation
$x^{2}+y^{2}-4 x-4 y=0$
ie., $x^{2}+x^{2}-4 x-4 x=0$
$2 \mathrm{x}^{2}-8 \mathrm{x}=0 ; 2 \mathrm{x}(\mathrm{x}-4)=0: 2 \mathrm{x}=0$ or $\mathrm{x}-4=0$
$x=0, x=4$.
Hence the coordinatr be $(4,4)$.

## Question.

28. The table below shows the number of children in a class, sorted according to their heights.

| Height <br> (Centimetres) | Number of Children |
| :---: | :---: |
| $130-140$ | 7 |
| $140-150$ | 9 |
| $150-160$ | 10 |
| $160-170$ | 10 |
| $170-180$ | 9 |

If the students are directed to stand in a line according to the order of their heights starting from the smallest, then
(a) The height of the child at what position is taken as the median ?
(b) What is the assumed height of the child in the $17^{\text {th }}$ position ?
(c) Find the median height.

## Solution.

| Height | Frequency | cf |
| :---: | :---: | :---: |
| $130-140$ | 7 | 7 |
| $140-150$ | 9 | 16 |
| $150-160$ | 10 | 26 |
| $160-170$ | 10 | 36 |
| $170-180$ | 9 | 45 |

Total Number $=45$
a) Position of the Child with median height $=\frac{45+1}{2}=23$.
b) The assumed height of the $17^{\text {th }}$ child $\frac{150+151}{2}=\frac{301}{2}$

$$
=150.5
$$

c) Median height

$$
l+\frac{\left(\frac{\mathrm{N}}{2}-m\right) c}{f}=150+\frac{\left(\frac{45}{2}-16\right) 10}{10}=
$$

$$
150+22.5-16=150+6.5=156.5
$$

## Question.

29. Read the following. Understand the Mathematical concepts in it and answer the questions that follow.
The remainders obtained on dividing the powers of two by 7 have an interesting property. We can understand it from the table given below.

| Number | $2^{1}$ | $2^{2}$ | $2^{3}$ | $2^{4}$ | $2^{5}$ | $2^{6}$ | $2^{7}$ | $\ldots \ldots \ldots .$. |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Remainder | 2 | 4 | 1 | 2 | 4 | 1 | 2 | $\ldots \ldots \ldots$. |

If the powers are $1,4,7, \ldots \ldots .$. . the remainder is 2
If the powers are $3,6,9, \ldots \ldots .$. . the remainder is 1
(a) What is the remainder on dividing $2^{8}$ by 7 ?
(b) Write the sequence of powers of 2 leaving remainder 1 on division by 7 .
(c) Check whether 2019 is a term of the arithmetic sequence $3,6,9, \ldots . . .$.
(d) What is the remainder on dividing $2^{2019}$ by 7 ?
(e) Write the algebraic form of the arithmetic sequence 1, 4, 7,
(f) Write the algebraic form of the sequence $2^{1}, 2^{4}, 2^{7}, \ldots . . .$. (powers of two leaving remainder 2 on division by 7).

## Solution.

a) Reminder be 4 .
b) $2^{3}, 2^{6}, 2^{9}$
c) Given sequence be $3,6,9, \ldots \ldots \ldots$.
$\mathrm{d}=6-3=3$.
2019 be the multiple of common difference 3 . yes, 2019 be a term of this sequence.
d) Reminder $=1$.
e) Given sequence $=1,4,7, \ldots ; d=4-1=3$.

Algebraic form = dn + (f-d) $=3 n+(1-3)=3 n-2$.
f) Algebraic form $=2^{3 n-2}$.
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