

**PUBLIC EXAM MATHS FULL QUESTION PAPER ANSWER KEY-2019**

**ONE MARKS**

	<b>Type A</b>	<b>Type B</b>
1)	4	4
2)	1	1
3)	4	4
4)	1	3
5)	4	3
6)	1	2
7)	2	4
8)	2	4
9)	4	2
10)	2	1
11)	1	4
12)	3	1
13)	3	4
14)	4	2
15)	1	3
16)	4	2
17)	2	3
18)	4	4
19)	1	3
20)	3	4

**TWO MARKS**

21 )  $x+y+z=100$  ,  $1+2y+5z=30$

$$22) \quad 3\vec{i} + 2\vec{j} + 9\vec{k} = a(\vec{i} + m\vec{j} + 3\vec{k})$$

Equating component vectors  $a = 3$ ,  $am = 2$  then  $m = 2/3$

$$23) \quad \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right) = 2i/2 = i \text{ then } i^4 = 1. \text{ ie } n=4$$

24) only comet diagram

$$25) \quad dy/dx = \cos x$$

$$dy/dx = 0 \text{ then } \cos x = 0 \rightarrow x = (2n+1)\pi/2. \quad n \in \mathbb{Z}$$

26) Domain :  $(-\infty, \infty)$

Extent : vertical :  $(-\infty, \infty)$

Horizontal :  $(-\infty, \infty)$

$$27) \quad \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1 + \sqrt{\tan x}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1 + \sqrt{\tan\left(\frac{\pi}{6} + \frac{\pi}{3} - x\right)}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1 + \sqrt{\tan\left(\frac{\pi}{2} - x\right)}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1 + \sqrt{\cot x}}$$

28) closer axiom :

$$\text{Let } A = a/b \in \mathbb{Q} - 0, \quad B = -a/b \in \mathbb{Q} - 0, \text{ then } A+B = a/b - a/b = 0 \notin \mathbb{Q} - 0$$

So its not closed.

29) Limits of integral is 0 to 3.

$$F(3) = \int_0^3 3e^{-3x} dx = 3 \left[ \frac{e^{-3x}}{-3} \right]_0^3 = -(e^{-9} - 1) = -e^{-9} + 1$$

30)  $f(x)$  is not differentiable for  $x=2$  and  $x=5$ . so rolls theorem is not exists.

**THREE MARKS**

31) simply we take  $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$A+B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ SO } \rho(A) = 3, \rho(B) = 3, \rho(A+B) = 0$$

Hence  $\rho(A) + \rho(B) \neq \rho(A+B)$

32)  $\vec{a} \times \vec{b} = -\vec{i} + 2\vec{j} + 2\vec{k}$  and  $|\vec{a} \times \vec{b}| = 3$ ,  $\mu = 6$

$$\text{Required vector} = \mu \left( \pm \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} \right) = \pm (-2\vec{i} + 4\vec{j} + 2\vec{k})$$

33) let  $z = \sin\theta + i \cos\theta$  and  $\bar{z} = 1/z = \sin\theta - i \cos\theta$

$$\frac{1 + \sin\theta - i \cos\theta}{1 + \sin\theta + i \cos\theta} = \frac{1 + \bar{z}}{1 + z} = \frac{1 + \frac{1}{z}}{1 + z} = 1/z = \sin\theta - i \cos\theta$$

Hence  $\left[ \frac{1 + \sin\theta - i \cos\theta}{1 + \sin\theta + i \cos\theta} \right]^n = (\sin\theta - i \cos\theta)^n = \cos n\left(\frac{\pi}{2} - \theta\right) - i \sin\left(\frac{\pi}{2} - \theta\right)$

34) Equation of tangent  $x + yt^2 = 2ct$

A, B coordinates are  $(2ct, 0)$  and  $(2c/t, 0)$  mid point of AB =  $\left(\frac{2ct+0}{2}, \frac{0+0}{2}\right) = (ct, 0)$

35)  $f(x) = \tan^{-1}(\sin x + \cos x)$ ,  $dy/dx = \frac{\cos x - \sin x}{2 + \sin 2x} > 0$  in  $x \in (0, \pi/4)$

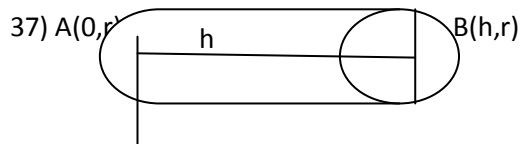
So  $f(x)$  is st. increasing in  $(0, \pi/4)$

36)  $f(tx, ty) = \frac{1}{\sqrt{(tx)^2 + (ty)^2}} = \frac{1}{t\sqrt{x^2 + y^2}} = t^{-1} f(x, y)$

So  $f$  is Homogeneous fn. of deg -1

By eulers thm

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = -1$$



Equation of the line AB is  $\frac{y-r}{r-r} = \frac{x-0}{h-0}$

$$\frac{y-r}{0} = \frac{x}{h} \rightarrow y-r=0 \text{ ie } y=r$$

Volume of the cylinder  $= \pi \int_0^h y^2 dx$

$$= \pi \int_0^h r^2 dx$$

$$= \pi r^2 [x]_0^h = \pi r^2 h \text{ cu.unit}$$

38)

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

39)  $n=120$

$$P=1/2, q=2/3$$

$$\text{Mean} = np = 120 \times 1/3 = 40$$

$$\text{Variance} = npq = 40 \times 2/3 = 40/3$$

40)  $yx^3 dx = -e^{-x} dy$

$$\frac{x^3 dx}{e^{-x}} = -\frac{dy}{y}$$

$\int e^x x^3 dx = -\int \frac{dy}{y}$  using bernoullies integral formula

$$e^x (x^3 - 3x^2 + 6x - 6) + \log y = c$$

**FIVE MARKS**

$$41) a) [A,B] = \begin{bmatrix} 1 & 1 & 3 & 0 \\ 4 & 3 & \mu & 0 \\ 2 & 1 & 2 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & \mu - 12 & 0 \\ 0 & 0 & 8 - \mu & 0 \end{bmatrix}$$

Case (i) if  $\mu \neq 8$

$$\rho(A) = \rho(A, B) = 3$$

So it has trivial solution,  $(x, y, z) = (0, 0, 0)$

Case (ii) if  $\mu = 8$

$$\rho(A) = \rho(A, B) = 2 < 3$$

So it has non trivial solution,  $(x, y, z) = (c, -4c, c)$ , where  $c \neq 0$

b) diagram and correct proof

42) a) Cartesian equation

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l & m & n \end{vmatrix} = 0$$

ie  $8x - 10y - 7z + 11 = 0$

$$\text{b) } x^6(x^5 - 1) + 1(x^5 - 1) = 0$$

$$(x^5 - 1)(x^6 + 1) = 0 \rightarrow x^6 + 1 = 0 \text{ and } x^5 - 1 = 0 \text{ ie } x^6 = -1 \text{ and } x^5 = 1$$

Case(i)  $x^6 = -1$

$$x = (-1)^{\frac{1}{6}}$$

$$= (\text{cis}\pi)^{\frac{1}{6}}$$

$$= \text{cis}\pi\left(\frac{2k+1}{6}\right), k=0,1,2,3,4,5$$

Case(ii)  $x^5 = 1$

$$x = 1^{\frac{1}{5}}$$

$$= (\text{cis}0)^{\frac{1}{5}}$$

$$= (\text{cis}2k\pi)^{\frac{1}{5}}$$

$$= \text{cis}\frac{2k\pi}{5}, k=0,1,2,3,4$$

43)a

w.k.t focal property of ellipse  $F_1P + F_2P = 2a$

$$2a=9 \quad \text{and foci}(\pm ae, 0)=(\pm 3, 0) \quad \text{so } ae=3$$

$$a=9/2, b^2 = a^2 - (ae)^2, b^2 = 45/4$$

hence the required ellipse equation is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ ie, } \frac{x^2}{\frac{81}{4}} + \frac{y^2}{\frac{45}{4}} = 1$$

b)  $x=2r\cos\theta, y=2r\sin\theta$

area of rectangle =  $2x \cdot 2y$

$$A(\theta) = 2r\cos\theta \cdot 2r\sin\theta$$

$$\frac{d}{d\theta} A(\theta) = 4r^2 \cos 2\theta$$

$$\frac{d}{d\theta} A(\theta) = 0 \text{ then } \theta = \pi/4$$

$$A''(\theta) = -8r^2 \sin 2\theta < 0 \text{ for } \theta = \pi/4$$

So A is maximum for  $\theta = \pi/4$

$$x=y=\sqrt{2}r,$$

Required area =  $x \cdot y$

$$= \sqrt{2}r \cdot \sqrt{2}r$$

$$= 2r^2$$

44) a

$$x = 100t - \frac{25}{2}t^2$$

(i) Initial velocity, so  $t=0$

$$v = dx/dt = 100 - (25/2)2t$$

$$v = 100 - 0 = 100$$

ii) time taken. maximum height so  $v=0$

$$\text{ie } 100 - 25t = 0$$

$$t = 4 \text{ sec}$$

iii) maximum height reached when  $t=4$  sec

$$x = 100(4) - (25/2)16$$

$$x = 200 \text{ mts}$$

iv) velocity

if  $t = 4 + 4 = 8$  sec

$$v = 100 - 25(8) = -100 \text{ m/sec}$$

b) equation

$$\frac{(y+1)^2}{16} - \frac{(x-1)^2}{9} = 1, a^2=16, b^2 = 9$$

$$e=5/4$$

centre:(1,-1)

foci:(1,4), (1,-6)

vertices : (1,3), (1,-4)

45)a)  $P(z_1 < z < z_2) = 70\%$

ie  $P(z_1 < z < z_2) = .70$  ie  $P(z_1 < z < 0) = .35$  and  $P(0 < z < z_2) = .35$

$z_1$  and  $z_2$  lies left and right side of the normal curve

So  $z_1 = -1.04$  and  $z_2 = 1.04$

$$Z = \frac{x - \mu}{\sigma} = \frac{x - 34}{16} = -1.04$$

$$X = 17.36 \text{ and } z = \frac{x - 34}{16} = 1.04 \rightarrow x = 50.64$$

70% student score b/w 17.36 to 50.64

b)  $Y = \sin x, y = \cos x$

Then  $\sin x = \cos x \rightarrow x = \frac{\pi}{4} \in (0, \pi)$

$$\text{Required area} = \int_0^{\pi/4} (\cos x - \sin x) dx + \int_{\pi/4}^{\pi} (\sin x - \cos x) dx$$

$$= 2\sqrt{2} \text{ sq.unit}$$

46) a)

$$\frac{dw}{dt} = \frac{\partial w}{\partial x} \frac{dx}{dt} + \frac{\partial w}{\partial y} \frac{dy}{dt} + \frac{\partial w}{\partial z} \frac{dz}{dt}$$

$$= -\sin t + 2\cos t + 2t \dots \dots (1)$$

$$w = \cos t + 2\sin t + t^2, \frac{dw}{dt} = -\sin t + 2\cos t + 2t \dots \dots (2)$$

hence proved

b)  $dT/dt = k(T-s)$ ,  $s=15$

$$T = s + ce^{kt} \dots\dots\dots(1)$$

$t=0$  then  $T=100$

(1)  $\Rightarrow c = 85$

$t = 5$ ,  $T=60$  then  $e^{5k} = \frac{45}{85}$

$t=10$ , then  $T = ?$

$$T = 15 + 85 e^{10k} = 38.82$$

47) a

- (i) Identity element of group is unique
- (ii) Inverse of each element of group is unique
- (iii) Reversal law
- (iv) Cancellation law
- (v)  $[a^{-1}]^{-1} = a$

b) C.F =  $Ae^{2x} + Be^{-\frac{2}{5}x}$

P.I1 =  $(-5/12) xe^{-\frac{2}{5}x}$

P.I2 =  $(-2/7) e^x$

P.I3 =  $-3/4$

G.S  $y = C.F + P.I1 + P.I2 + P.I3$

$$= Ae^{2x} + Be^{-\frac{2}{5}x} + (-5/12) xe^{-\frac{2}{5}x} + (-2/7) e^x - 3/4$$

Answer key prepared by: E.MADHESWARAN M.SC, B.Ed PGT in MATHS

Salem D.T: mob -8807120509