St. Xavier's Sr. Sec. School
Delhi-54

Pre Board Examination 2016-17 | **Set 2** | **Max. Marks : 100**

Std. 12 | **MATHEMATICS** | **Time : 3 hrs.**

12-01-2017

General instructions:

i) Questions of section A consists 1 mark each.

ii) Questions of section B consists 2 marks each.

iii) Questions of section C consists 4 marks each.

iv) Questions of section D consists 6 marks each.

SECTION – A

1. Find the angle between $\vec{a}$ and $\vec{b}$ if $\left|\vec{a}.\vec{b}\right| = \left|\vec{a} \times \vec{b}\right|$.

2. If f: R $\rightarrow$ R is given by f(x) = $\left(5 - x^5\right)^{\frac{1}{5}}$ then find fof(x).

3. Find the value of $\begin{vmatrix} \sin A & -\sin B \\ \cos A & \cos B \end{vmatrix}$, where A = 63^0 and B = 27^0.

4. State the reason why the relation R = {(a, b):a $\leq b^2$} on the set R of real numbers is not reflexive.

SECTION – B

5. For 6 trials of an experiment, let X be a binomial variate which satisfies the relation 9P(X=4)=P(X=2). Find the probability of success.

6. Find inverse of matrix A = $\begin{bmatrix} 2 & 5 \\ 1 & 4 \end{bmatrix}$ using elementary row transformation.

7. Evaluate $\int \frac{2 + \sin 2x}{1 + \cos 2x} e^x \, dx$.

8. Evaluate $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$.

9. Prove that $\vec{a} = (\vec{a}.\vec{i})\vec{i} + (\vec{a}.\vec{j})\vec{j} + (\vec{a}.\vec{k})\vec{k}$.

10. If $y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$ find dy/dx.

11. The radius of a circle is increasing at the rate of 0.7cm/sec. What is the rate of increase of it's circumference?

12. Form the differential equation of family of circles in the second quadrant and touching the coordinate axes.

SECTION – C

13. Prove that the curves $y^2 = 4ax$ and $xy = c^2$ cut at right angles if $c^4 = 32a^4$.

(OR)

Separate the interval $[0, \pi/2]$ into subinterval in which $f(x) = \sin^4 x + \cos^4 x$ is strictly increasing or strictly decreasing.

14. Evaluate $\sqrt{0.037}$ using differential approximation.

15. Evaluate : $\int \frac{\sqrt{x}}{1 + x^{3/4}} dx$.

16. Prove : $\vec{a} \cdot (\vec{b} + \vec{c}) \times (\vec{a} + 2\vec{b} + 3\vec{c}) = \vec{a}.\vec{b} \times \vec{c}$

17. Find the distance of the point (3,4,5) from the plane x + y + z = 2 measured parallel to line 2x = y = z

18. Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find expected value of x.

19. If a machine is correctly set up, it produces 90% acceptable items. If it is incorrectly set up only 40% items produced are acceptable. Past experience shows that 80% of the setups are correctly done. If after a certain set up, the machine produces 2 acceptable items, find the probability that the machine is correctly set up.

20. A trust invested some money in two type of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received Rs. 2800/- as interest. However, if trust have interchanged money in bonds, they would have got RS 100 less as interest. Using matrix method, find the amount invested by the trust. Interest received on this amount will be given to Help Age India as donation. Which value is reflected in this question?

21. Discuss the differentiability of the function $f(x) = \begin{cases} 2x-1, & x < 1/2 \\ 3-6x, & x > 1/2 \end{cases}$ at x=1/2.

(OR)

Find the value of k so that the function given by

$$f(x) = \begin{cases} k, & \text{if } x = 2 \\ \dfrac{2^{x+2} - 16}{4^x - 16}, & \text{if } x \neq 2 \end{cases}$$

22. If $x^y = e^{x-y}$ prove that $\dfrac{dy}{dx} = \dfrac{\log x}{(\log ex)^2}$.

23. Solve : $(1+y^2)\tan^{-1} x\,dx + 2y(1+x^2)dy = 0$ (OR) Solve $x\dfrac{dy}{dx} = y(\log y - \log x + 1)$.

SECTION – D

24. Using integration, find the area of the region by the following curves

$\{(x, y) : 0 \le y \le x^2 + 3,\ 0 \le y \le 2x + 3, 0 \le x \le 3\}$.

25. Let f:N→R be a function defined as $f(x) = 4x^2+12x+15$. Show that f: N→S where S is range of f, is invertible. Find the inverse of f.

(OR)

Let * be a binary operation on Q x Q by (a, b)*(c, d) = (ac, b + ad), where Q is the set of rational numbers. Determine whether * is commutative and associative. Find the identity element for * and the invertible element of Q x Q.

26. A company manufacture two type of novelty souvenirs made of plywood. Souvenirs of type A requires 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3hrs, 20 minutes available for cutting and 4 hrs available for assembling. The profit is Rs. 5/- each for type A and Rs. 6/- each for type B souvenirs. How many souvenirs of each type should the company manufactures in order to maximize the profit?

27. If a line L makes angles $\alpha, \beta, \gamma, \delta$ with the four diagonal of a cube prove that

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = 4/3.$$

28. Using properties of determinant

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

(OR)

Prove the following using properties of determinant

$$\begin{vmatrix} a+bx^2 & c+dx^2 & p+qx^2 \\ ax^2+b & cx^2+d & px^2+q \\ u & v & w \end{vmatrix} = (x^4-1)\begin{vmatrix} b & d & q \\ a & c & p \\ u & v & w \end{vmatrix}$$

29. Evaluate $\int_{-1}^{3/2} |x \sin \pi x| dx$ (OR) Evaluate using limit of sum method $\int_0^4 (x + e^{2x}) dx$.

-X-X-X-X-X-X-