A+ Mathematics Questions and answers English Variant

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Chapter 1. Arithmetic Series.

Bird's eye.

First term = f ., Common difference = d ., X_n = Nth term., S_n = Sum.
n = number of term.

$$X_n = f + (n-1)d \quad ; \; X_n = dn + (f-d) \quad ; \; X_n = an + b.$$

(a – common difference , a + d first term).

$$d = x_2 - x_1 \quad ; \quad x_2 = x_1 + d \;\text{ or }\; f + d \;\; ; \quad x_3 = 2f + d \quad ; \; d = \frac{x_m - x_n}{m-n} \quad ;$$

$$d = \frac{\text{difference of terms}}{\text{difference of posistion value}} \quad .$$

$$n = \frac{x_n - x_1}{d} + 1 \quad \text{or} \;\; n = \frac{\text{last term} - \text{first term}}{\text{common difference}} + 1$$

If a,b and c be the consecutive three terms of an AP

The arithmetic mean (AM) $b = \dfrac{a+c}{2}$.

$$S_n = \frac{n}{2}[2f + (n-1)d] \quad ; \qquad S_n = \frac{n}{2}[x_1 + x_2] \;\; \text{or} \;\; S_n = \frac{n}{2}[f + l]$$

$$S_n = \frac{d}{2}n + n\left(f - \frac{d}{2}\right) .$$

If $x, x+d, x+2d, x+3d, x+4d$ be the consecutive five terms of an AP ,

Then the sum of the AP = $5 \times \text{middle term}$. or $5 \times (x+2d)$.

If $x, x+d, x+2d, x+3d, x+4d, x+5d, x+6d$ be the consecutive seven terms of an AP ,

Then the sum of the AP = $7 \times \text{middle term}$ or $7 \times (x+3d)$

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Question 1.

Second and fourth terms of the following arithmetic sequence are missing. Find the numbers at these positions.

11, —, 19, —, ...

Answer:-

Given a = 11 , b = ? , c = 19

If a, b, and c be the three consecutive numbers ,

$b=\frac{a+b}{2} = \frac{11+19}{2} = \frac{30}{2}=15$

$\therefore$ AP = 11,15,19,……

d = $x_2-x_1=15-11=4$

4^{th} term = 19 + 4 = 23.

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Question 2

The sum of first n terms of an arithmetic sequence is $5n^2 + 2n$.

a) What is the sum of first two terms of this sequence?

b) Write the first two terms of this sequence.

Answer:-

Given , Sum of the term = $5n^2+2n$

First term = $5\times 1^2+2\times 1$

= 5 + 2 = 7

a) Sum of the first two terms = $5\times 2^2+2\times 2$ = 24.

Common difference (d) = 17 – 7 = 10.

b) The first two terms of this sequence = 7, 17.

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Question 3

a) Write the algebraic form of the arithmetic sequence

1, 4, 7, 10, ...

b) Is 100 a term of this sequence? Why?

c) Prove that the square of any term of this sequence is also a term of it.

Answer:-

Given Sequence = 1, 4, 7, 10,……………..

Common difference (d) = 4 – 1 = 3.

a) The algebraric form $X_n = dn + (f - d)$

$= 3n + (1 - 3) = 3n - 2.$

b) $X_n = 3n - 2$, ie., 100 = 3n – 2

$$n = \frac{102}{2} = 34.$$

Hear, n is a natural number being 100 in this sequence.

c) The square of the third term be $(3n-2)^2 = 9n^2 - 12n + 4$

The difference between first term and this will equal to the multiple of the common difference.

ie., $9x^2 - 12n + 4 - 1 = 9n^2 - 12n + 3$

$= 3(3n^2 - 4n + 1)$

Hence, the square of any term of this sequence is also a term of it.

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Question 4.

The n^{th} term of an AP is 3 – 5n . What is the $(n+1)^{th}$ term?

Answer.

Given n^{th} term of an AP = 3 – 5n , d = - 5 (the coefficient of n be the common difference)

$\therefore$ the $(n+1)^{th}$ term = 3 – 5n + - 5 = - 5n – 2

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Question 5.

How many terms are there in the AP 18, $15\frac{1}{2}$, 13,……, - 47.

Answer.

Given , a = 18 , d = $\frac{-5}{2}$

Let the given AP has n terms

ie., $X_n=-47$ We know that $n=\frac{X_n-x_1}{d}+1$

$$n=\frac{-47-18}{-\frac{5}{2}}+1 \;=\; -65\times\frac{-2}{5}+1 \;= 26 + 1 = 27.$$

∴ The given AP has 27 terms

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Questions 6

Is – 150 a term of AP 11, 8, 2, 5,……..?

Answer.

Given a = 11, d = 8 – 11 = - 3 . $X_n=-150$

We know $n=\frac{X_n-x_1}{d}+1$, = $n=\frac{-150-11}{-3}+1$ = $\frac{-161}{-3}+1$ = $54\frac{2}{3}$.

This is not possible because "n" is +ve integer.

Hence, - 150 is not a term of given AP.

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Question 7.

a) How many three digit numbers are divisible by 7 ?

b) How many multiples of 4 lie between 10 and 250?

Answer

a) Three digit numbers divisible by 7 are 105, 112, 119, ……….. 994.

It is an AP, a = 105 d = 7 $X_n=994$ Find n ?

We know $n=\frac{X_n-x_1}{d}+1$ = $\frac{994-105}{7}+1$ = 128.

Hence, there are 128 three digits numbers which is divisible by 7.

b) Multiples of 4 b/w 250 be 12, 16, 20, 24, 28, …………. 248.

Which is an AP . A = 12, d = 4, $X_n=248$, n = ?

We know that, $n=\frac{X_n-x_1}{d}+1$ = $f+2d=20$

So, there are 60 multiples of 4 lying b/w 10 and 250.

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Question.8.

If the sum of the 6^{th} and 20^{th} term of an AP is 84, what is the sum of the first 25 terms?

Answer.

Let “a” and “d” be the first term and common difference of the given AP respectively.

6^{th} term = 20^{th} term , sum 84.

Given condition $f+5d+f+19d=84$

ie., $2f+24d=84$

Hence the sum of the first 25 terms

$S_n=\frac{n}{2}[2f+(n-1)d]$, = $\frac{25}{2}[2f+24d]$

ie., $\frac{25}{2}[2f+24d]$ = $\frac{25}{2}\times 84=1050$.

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Question 9

In a flower decoration , flowers are arrange in ten concentric circles . If the third and eight circles have 20 and 35 flowers respectively . Find the number of flowers in the outermost circle, using the idea of AP.

Answer.

Given n = 10 , Third term = 20 ., ie., $f+2d=20 \quad \rightarrow(1)$

Eight term = 35. , ie., $f+7d=35 \quad \rightarrow (2)$

Solve equation (1) and (2) We get $f=14$ and $d=3$

The number of outermost circle = X_{10} = $f+9d=14+9\times 3=41.$

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Question 10.

See these figures made with matchsticks :

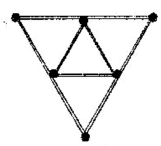

Figure 1

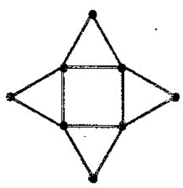

Figure 2

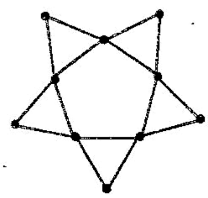

Figure 3

(a) How many sticks are needed for the next figure ?

(b) If we continue this, what is the relation between the numbers 1, 2, 3, ... and the number of matchsticks used in Figure 1, Figure 2, Figure 3 and so on ?

(c) If we write the number of matchsticks in order, what is the algebraic expression to find the n^{th} term of this sequence ?

Answer.

a) Number of sticks are needed for the first figure = 9

Number of sticks are needed for the second figure = 12

Number of sticks are needed for the third figure = 15

Common difference = 12 – 9 = 3.

∴ Number of sticks are needed for the next figure = 15 + 3 = 18.

b) Here the number of sticks will be used the 3 times of each number of 1, 2, 3, …. be added 6 will get 1,2,3..

(ie., $1\times 3+6=9$; $2\times 3+6=12$; $3\times 3+6=15$)

c) Sequence = 9, 12, 15, …….

The algebraical form of n^{th} term

$X_n = dn + (f - d)$

= 3n + (9 – 3)

= 3n + 6.

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Question 11

Consider the arithmetic sequence 12, 23, 34, ...

(a) What is the 10^{th} term of this sequence ?

(b) Is 1111 a term of this sequence ? Why ?

Answer.

Given AP = 12, 23, 34, ……………

Common difference (d) = 23 – 12 = 11.

a) The 10^{th} term = $f+9d=12+9\times 11=111$.

b) We know the algebraical form $X_n = dn + (f - d)$

= 11n + (12 – 11)

= 11n + 1

ie., 11n + 1 = 1111

11n = 1111 – 1

= 1110

n = $\frac{1110}{11}$ = 100.9090….

n is not an integer ,so 1111 is not a term in this sequence.

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Question 12.

The first term of an arithmetic sequence is 6 and the sum of the first 6 terms is 66.

(a) What is its 6^{th} term ?

(b) What is the common difference of the sequence ?

(c) What are the first 6 terms of this sequence ?

Answer.

First term (f) = 6. Sum of the first 6 terms = 66., n = 6.

a) $S_n = \frac{n}{2}[2a + (n-1)d]$

ie., 66 = $\frac{6}{2}[2 \times 6 + (6-1)d]$

66 = 3 [12 + 5d]

22 = 12 + 5d

5d = 22 – 12 = 10

d = 10/5 =2.

The 6^{th} term = f + 5d

= 6 + 10 = 16.

b) Common difference = 2.

c) First 6 terms of the sequence = 6, 8, 10, 12, 14, 16.

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Question13.

The first term of an arithmetic sequence is 6 and the common difference is 4.

(a) What is the algebraic form of this sequence ?

(b) What is the algebraic expression to find the sum of the first *n* terms of this sequence ?

(c) How many terms of this sequence, starting from the first, are to be added to get 510 ?

Answer{-

Given, First term (f) = 6 ; Comm0n difference = 4.

a) Algebraical form $X_n = dn + (f - d)$

= 4n + (6 – 4)

= 4n + 2.

b) The algebraic expression to find the sum of the first *n* terms

$S_n = \frac{n}{2}[2f + (n-1)d]$

$= \frac{n}{2}[2 \times 6 + (n-1)4]$

$= \frac{n}{2}[12 + 4n - 4]$

$= \frac{n}{2} \times 2[6 + 2n - 2]$

= n (2n + 4) = $2n^2 + 4n$

c) Consider the sum of the first *n* terms be 510.

ie., $2n^2 + 4n$ = 510.

$2n^2 + 4n$ - 510 = 0 Solve the equation to get *n*

$2n^2$ + 4n – 510 = 0 . Dividing through out by 2

$n^2 + 2n - 255 = 0$

(n + 17) (n – 15) = 0

n + 17 – 0 or n – 15 = 0

n = - 17 or n = 15 -ve 17 rejected

$\therefore$ n = 15.

15 terms to be added to get 510.

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Question 14.

The 25th term of an arithmatic sequence is 140 and the 27th term is 166 What is the common difference? What is the 35th term?

Answer:-

Given 25th term = 140 and 27th term is 166.

But $x_{27} - x_{25} = 2d$ ie., 2d = 166 – 140 = 26

d = $\frac{26}{2}$ = 13.

35th term = $x_{25} + 10d$

= $140 + (10 \times 13) = 140 + 130 = 270$.

OR

Given 25^{th} term = 140 and 27^{th} term is 166.
a + 24d = 140 and a + 26d = 166.
Solve this equations for a and d
Then we get d = 13 and a = - 172
$\therefore 35^{th}$ term = a + 34d
= - 172 + 34 × 13
= - 172 + 442
= 270.

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Question 15.

Sum of the first five terms of an arithmetic sequence is 45. What is the third term ?
The common difference of the sequence is 4. Write the first two terms. Write another arithmetic sequence having the sum of the first five term 45.

Answer:-

Given the sum of the first five term of an AP = 45.
Common difference = 4
Third term
Sum = middle term × number of terms
ie., $x_3 \times 5 = 45$
$\therefore \quad x_2 = \frac{45}{5} = 9$
Second term = third term – common difference
= 9 – 4 = 5
First term = Second term – common difference.
= 5 – 4 = 1.
The first two terms = 1 and 5.
If the sum of the first five term of an AP is 45, then the third term should be 9 but the common difference may be changed . In this condition we can make many AP's.
Hence The AP = 5, 7, 9, 11, 13, …..
or = 3, 6, 9, 12, 15, …….

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Question 16.

(a) Find the least and heighest three digit number which leave a remainder 1 on division by 9.

(b) How many three digit numbers are there, which leave a remainder one on division by 9 ?

(c) Find the sum of all such numbers.

Answer:-

The smallest three digit number dividing by 9, the reminder comes up 1 = 99 + 1 = 100.

The largest three digit number dividing by 9 = 999 – 8 = 991.

b) n = $\boxed{\frac{x_n - f}{d} + 1}$ = $\boxed{\frac{991 - 100}{9} + 1}$ = $\boxed{\frac{891}{9}}$ + 1 = 99 + 1 = 100.

c) Sum = $\boxed{\frac{n}{2}[f + x_n]}$ = $\boxed{\frac{100}{2}[100 + 991]}$ = 50 $\boxed{\times}$ 1091 = 54550

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Question 17

Write down the first three terms of the sequence of natural numbers leaving remainder 1 on division by 5. Check whether 510 is a term of above sequence.

Answer:-

The first natural number leaving remainder 1 on division by 5 = 1

The second natural number leaving remainder 1 on division by 5 = 6

The third natural number leaving remainder 1 on division by 5 = 11

∴ The first three terms of the sequence = 1, 6, and 11.

We know that $n = \frac{X_n - f}{d} + 1$; Here f = 1 ; d = 6 – 1 = 5. $X_n = 510$

$\therefore\ n = \frac{510 - 1}{5} + 1$ = $\frac{509}{5}$ + 1

= 102.8

Hence, *n* is not an intiger so 510 is not belonging this sequence.

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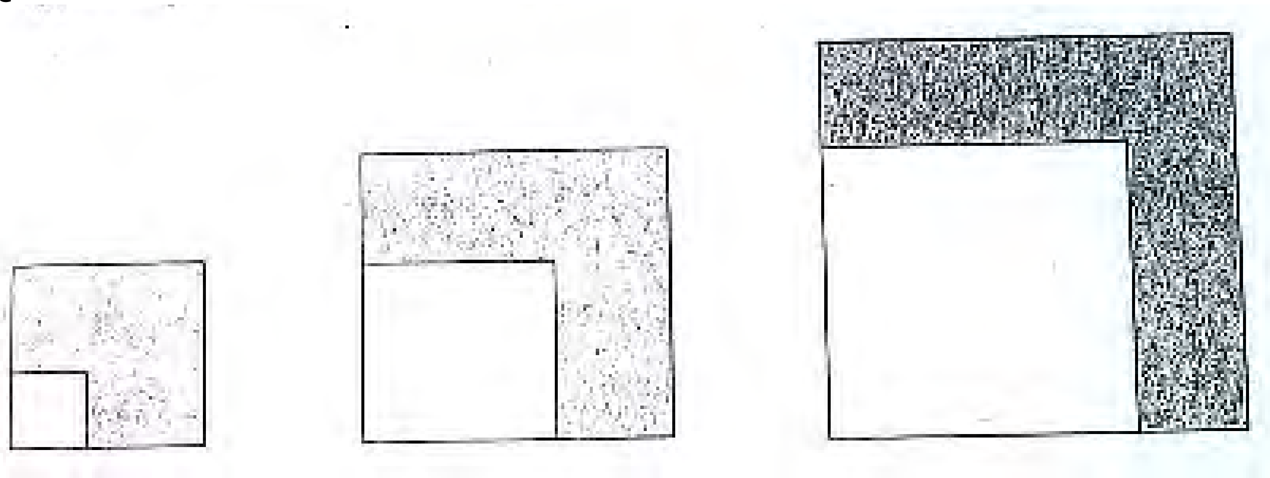
Question 18.



In these figures , the sides of the smaller squares are in the arithmetic sequence 2, 5, 8,…………. and the sides of the larger squares are in the arithmetic sequence 5, 8, 11,…………

a) Write down the algebraic form of each sequence.

b) Write down the algebraic form of the sequence of areas of the shaded portion in each figure.

Answer:-

Given sequence = 2, 5, 8,………………….

f = 2 ; d = 5 – 2 = 3.

a) Algebraical form $X_n = dn + (f - d)$

= 3n + (2 – 3)

= 3n – 1 .

The given sequence = 5, 8, 11,………… f= 5; d = 8 – 5 = 3.

lgebraical form $X_n = dn + (f - d)$

= 3n + (5 – 3)

= 3n + 2 .

b) The first figure, larger side of the square = 5cm.

The smaller square side = 2 cm.

∴ The area of the shaded region = $5^2 - 2^2$ (Area of a square = a^2)

= 25 – 4 = 21.

Similarly, the second figure area

The shaded aera = $8^2 - 5^2$

= 64 – 25 = 39

The third figure aera = $11^2 - 8^2$

= 121 – 64 = 57.

$\therefore$ The sequence of the aeras of the shaded region = 21, 39, 57,……

f = 21 ; d = 39 – 21 = 18.

Hence the algebraic form $X_n = dn + (f - d)$

$= 18n + (21 - 18)$

$= 18n + 3$.

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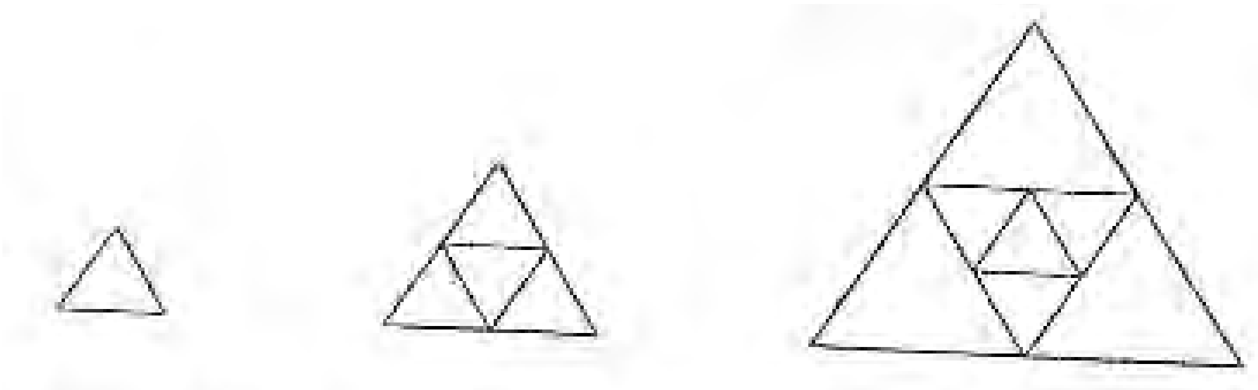
Question 19.



The first figure above is an equilateral triangle of side 2cm. The second figure is obtained by drawing lines passing through the vertices and parallel to the sides of the triangle in the first figure. The third figure is got by drawing lines passing through the vertices and parallel to the sides of the triangle in the second figure.

a) Write the sequence of perimeters of biggest triangle in each figure obtained by continuing this process.

b) Write the sequence of areas of biggest triangle in each figure.

c) Write algebraic forms of both of the above sequences.

Answer:-

In the first figure , one side of the biggest triangle = 2cm

$\therefore$ Perimeter = 3a = $3 \times 2 = 6$ cm

In the second figure , one side of the biggest triangle = $2 \times 2 = 4$ cm

$\therefore$ Perimeter = 3a = $3 \times 4 = 12$ cm

In the third figure , one side of the biggest triangle ¿$2 \times 4 = 8$ cm

$\therefore$ Perimeter = 3a = $3 \times 8 = 24$ cm

Hence the sequence of perimeter = 6, 12, 24,……………. (is a geometric sequence , GP)

b) Area of an equilateral triangle = $\dfrac{\sqrt{3}a^2}{4}$

In the figure , area of the first biggest triangle = $\frac{\sqrt{3}a^2}{4}$ (formula)

$$= \frac{\sqrt{3}2^2}{4} = \sqrt{3}\ \text{cm}^2 .$$

In the figure , area of the second biggest triangle = $\sqrt{3}\frac{(4)^2}{4} = 4\sqrt{3}$ cm^2 .

In the figure , area of the third biggest triangle = $\frac{\sqrt{3}\times(8)^2}{4}$ = $16\sqrt{3}$ cm^2 .

Hence the sequence of the area of the biggest triangle

= $\sqrt{3}, 4\sqrt{3}, 16\sqrt{3}$,………

c)Here the sequence of the perimeters of the biggest triangles

= 6, 12, 24,….. which is a GP. (This is beyond our study)

f = 6; common ratio (r) = $\frac{12}{6}$ = 2.

Algebraic form = $X_n = ar^{n-1}$ = $6 \times 2^{n-1}$ = $3 \times 2 \times 2^{n-1}$ = 3×2^n

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Question 19.

Consider the arithmetic sequence 135, 141, 147, …….. can the sum of any 25 consecutive terms of the sequence be 2016? Justify your answer.

Answer:-

Given arithmetic sequence = 135, 141, 147,………………………

f = 135 ; d = 141 – 135 = 6. n = 25 ; sum = 2016

Algebraic form sum

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$2016 = \frac{25}{2}[2 \times 135 + (25-1)6]$$

2016 $\neq$ 1550 .

Here the sum is not equal. So, 2016 is not in the sequence .

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Question 20.

The product of two consecutive terms of the arithmetic sequence 5, 8,11,…………. is 598. Find the position of the terms multiplied.

Answer:-

Given arithmetic sequence = 5, 8, 11,……….

f = 5 ; d = 8 – 5 = 3.

Let the next to terms be x and $x + 3$.
Given condition x(x + 3) = 598.

ie., $x^2 + 3x - 598 = 0$. Solve the equation

(x – 23 (x + 26) = 0

x – 23 = 0 or x + 26 = 0

ie., x = 23 or x = - 26 .

In this sequence all the numbers be positive , so -ve 26 be rejected
Hence x = 23. The next term 3 + 3 = 26.
Algebraic form be $X_n = dn + (f - d)$

$= 3n + (5 - 3)$

$= 3n + 2$

ie., 3n + 2 = 23 ; 3n = 23 – 2 ; n = $\frac{21}{3}$ = 7.

Next term 3n + 2 = 26 ; 3n = 26 – 2; n = $\frac{24}{3}$ = 8 .

Hence the position of the term be 7 and 8.
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Question 21.

a) Find the sum of the natural numbers from 1 to 15.
b) The sum of 15 terms of an arithmetic sequence with common difference 6 is 780 . Write the algebraic expression of the sequence .
c) Write the algebraic expression for the sum of the sequence.

Answer:-

a)The sum of the first natural numbers be $\frac{n(n+1)}{2}$; (formula) n = 15

ie., sum = $\frac{15(15+1)}{2}$ = 120.

b) Given, d = 6 ; sum = 780. n = 15 . f = ? *f* can be find out by the sum formula

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

ie., $780 = \frac{15}{2}[2a + (15-1)6]$

$$750=\frac{15}{2}[2a+84]$$

$2a+84=\frac{780\times 2}{15}$; 2a + 84 = 104 ; 2a = 104 – 84

2a = 20 ; a = 20/2 = 10.

Algebraic expression for the sum of the sequence.

= dn + (f – d)

= 6n + *10 – 6)

= 6n + 4 .

c) The algebraic expression for the sum of the sequence.

$$\frac{d}{2}n^2+\left(f-\frac{d}{2}\right)n = \frac{6}{2}n^2+\left(10-\frac{6}{2}\right)n$$

$$= 6n^2 + 7n .$$

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Question .22.

How many integers are tere in between 100 an 300 which are exactly divisible by 11?.

Answer:-

The multiple of 11 form an arithmetic sequence

The first number after 100 which is divisible by 11 is 110 and the last number before 100 is 297.

f = 110; d = 11; $X_n=297$

$n=\frac{s_n-f}{d}+1$; $n=\frac{297-110}{11}+1$ = 18.

Hence there are 18 numbers between 100 and 300 which are divisibl by 11.

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Question 23.

In an arithmetic sequence nth term be 6n – 5 . Find the sum of the first 20 terms.

Answer{-

Given nth term = 6n – 5.

First term, put n = 1 in the nth term
ie., $6 \times 1 - 5$ = 6 – 5 = 1. d = 6 (the coefficient of *n* all ways *d* in the case of AP)
Put n = 20 to get last term

ie., $S_n = 6 \times 20 - 5 = 115$
Hence sum 20 terms = $\frac{n}{2}[s_n + f]$ = $\frac{20}{2}[115+1]$ - 1160.
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Question 24

Find three numbers in arithmetic sequence such that their sum is 30 nd product is 840.

Answer:-

Given , sum of three numbers = 30 ; Product = 840
Let consider the three numbers are f – d , f, f+d.
ie., f – d + f + f+d = 30
3f = 30 ; f = 30/3 = 10
All so product , (f – d)(f)(f+d) = 840.

$$f(f^2 - d^2) = 840$$
$$10(100 - d^2) = 840$$
$$100 - d^2 = 840/10$$
$$-d^2 = 84 - 100$$
$$-d^2 = -16.$$
$$d = 4 .$$

Hence the numbers are f – d = 10 – 4 = 6 , f = 10 ,
f + d = 10 + 4 = 14 .
ie., 6, 10, 14.
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Question 25.

The angles of a right angled triangle are in arithmetic sequence . Find the angle by forming suitable equation.

Answer:-

Let consider the angles be f – d, f , f + d.
ie., f – d +f + f + d = 180 (sum of the three angle be 180)
3f = 180 ; f = 180/3 ; f = 60.

We know that f + d = 90 (largest angle of right angle be 90.)
ie., 60 + d = 90 ; d = 90 – 60 = 30.
The angles are 30, 60, 90 .
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The end of the chapter – 1 ,Arithmetic sequence.
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