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FOR 2019 EXAM

MAHARASHTRA HSC

SOLVED PAPER 2018

CLASS 12

MATHEMATICS & STATISTICS (COMMERCE)

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LATEST SYLLABUS

PART – I

1. MATHEMATICAL LOGIC

- Statements – Logical connectives – Statement patterns and logical equivalence – Algebra of statements.
- Venn diagram.

2. MATRICES

- Definition of matrix – Types of matrices – Algebra of matrices – Elementary transformation.
- Inverse of a matrix – Solution of system of linear equations – Inversion method – Reduction method.

3. CONTINUITY

- Continuity of a function at a point – Algebra of continuous functions – Types of discontinuity – Continuity of some standard functions.

4. DIFFERENTIATION

- Derivative of an Inverse function – Logarithmic Differentiation.
- Derivative of an Implicit function – Derivative of a parametric function-second order derivatives.

5. APPLICATIONS OF DERIVATIVE

- Increasing and decreasing functions – Applications of derivative in economic elasticity of demand – Marginal property.
- Maxima and minima.

6. INTEGRATION

- Definition of an integral – Integral of standard functions – Rules of Integration – Methods of Integration – Integration by parts.

7. DEFINITE INTEGRALS

- Definite Integrals – Properties of definite integral – Applications : Area and Volume.

...Contd.

PART – II

1. **RATIO PROPORTION AND PARTNERSHIP**
   - Ratio, Percentage, Proportion and Partnership
2. **COMMISION, BROKERAGE AND DISCOUNT**
   - Commission and Brokerage – Discount – Present worth – Sum due – True discount – Bills of exchange – Banker's discount – Banker's gain.
3. **INSURANCE AND ANNUITY**
   - Fire, Marine & accident Insurance Annuity, Various technologies of Annuity, Annuity Due, Sinking Fund.
4. **DEMOGRAPHY**
   - Definition of demography uses of vital statistics – Measurement of mortality life tables.
5. **BIVARIATE FREQUENCY DISTRIBUTION AND CORRELATION**
   - Bivariate frequency distribution : Karl Pearson's.
   - Coefficient of correlation : Rank Correlation.
6. **REGRESION ANALYSIS**
   - Equation of line of regression – Regression coefficients and their properties.
7. **RANDOM VARIABLE AND PROBABILITY DISTRIBUTION**
   - Definition and types of random variables – Probability distribution of a Discrete Random variable – Probability distribution of a Continuous random variable – Binomial Theorem – Binomial Distribution – Poisson Distribution – Normal Distribution.
8. **LINEAR INEQUATIONS AND LINEAR PROGRAMMING**
   - Inequations – Linear programming problems.
9. **ASSIGNMENT PROBLEM AND SEQUENCING**
   - Assignment problem – Sequencing.

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Solved Paper	Maharashtra HSC Exam March 2018 Set No. J-269	Mathematics & Statistics (88) (Commerce)

Time : 3 Hours **Max. Marks : 80**

General Instructions :

(i) *All questions are compulsory.*
(ii) *Figures to the right indicate full marks.*
(iii) *Graph paper is necessary for L.P.P.*
(iv) *Use of logarithmic table is allowed.*
(v) *Answer to the question in Section-I and Section-II should be written in two separate answer books.*
(v) *Questions from Section-I attempted in the answer book of Section-II and vice-versa will not be assessed/not be given any credit.*
(vi) *Answer to every question must be written on a new page.*

SECTION-I

1. Attempt any SIX of the following : **[12]**

(i) Draw Venn diagram for the truth of the following statements :

(a) All rational number are real numbers.

(b) Some rectangles are squares. **(2)**

(ii) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ using elementary transformations. **(2)**

(iii) Examine the continuity of

$f(x) = x^2 - x + 9$ for $x \le 3$

$= 4x + 3$ for $x > 3$, at $x = 3$ **(2)**

(iv) Find $\dfrac{dy}{dx}$, if $y = \cos^{-1}(\sin 5x)$ **(2)**

(v) The price P for demand D is given as $P = 183 + 120\,D - 3D^2$.
Find D for which the price is increasing. **(2)**

(vi) Evaluate : $\displaystyle\int \frac{1}{x(3 + \log x)}\,dx$ **(2)**

(vii) Find cofactors of the elements of the matrix $A = \begin{bmatrix} -1 & 2 \\ -3 & 4 \end{bmatrix}$. **(2)**

(viii) Evaluate : $\displaystyle\int \frac{1}{9x^2 + 49}\,dx$ **(2)**

2. (A) Attempt any TWO of the following : **(6) [14]**

(i) Find k, if $f(x) = \dfrac{\log(1+3x)}{5x}$ for $x \neq 0$

$= k$ for $x = 0$

is continuous at $x = 0$. **(3)**

(ii) Examine whether the following statement pattern is tautology, contradiction or contingency :
$p \vee -(p \wedge q)$ **(3)**

(iii) If $x = \cos^2\theta$ and $y = \cot\theta$ then find $\dfrac{dy}{dx}$ at $\theta = \dfrac{\pi}{4}$. **(3)**

(B) Attempt any TWO of the following : **(8)**

(i) The sum of three numbers is 6. If we multiply the third number by 3 and add it to the second number we get 11. By adding first and third numbers we get a number, which is double than the second number. Use this information and find a system of linear equations. Find these three numbers using matrices. **(4)**

(ii) Find the area of the region bounded by the parabola $y^2 = 16x$ and the line $x = 4$. **(4)**

(iii) The consumption expenditure E_c of a person with the income x. is given by $E_c = 0.0006x^2 + 0.003x$. Find MPC, MPS, APC and APS when the income $x = 200$. **(4)**

3. (A) Attempt any TWO of the following : **(6) [14]**

(i) Discuss continuity of $f(x) = \dfrac{x^3 - 64}{\sqrt{x^2 + 9} - 5}$ for $x \neq 4$

$= 10$ for $x = 4$

at $x = 4$ **(3)**

(ii) Find $\dfrac{dy}{dx}$, if $e^x + e^y = e^{x-y}$ **(3)**

(iii) Using truth table show that $-(p \to -q) \equiv p \wedge q$ **(3)**

(B) Attempt any TWO of the following : **(8)**

(i) Evaluate : $\displaystyle\int \frac{\sin x}{\sqrt{\cos^2 x - 2\cos x - 3}}\,dx$ **(4)**

(ii) The total cost function of a firm is $C = x^2 + 75x + 1600$ for output x. Find the output (x) for which average cost is minimum. Is $C_A = C_M$ at this output? **(4)**

(iii) Evaluate : $\displaystyle\int_1^2 \frac{1}{(x+1)(x+3)}\,dx$ **(4)**

SECTION-II

4. Attempt any SIX of the following : **[12]**

(i) A shop valued at ₹ 2,40,000 is insured for 75% of its value. If the rate of premium is 90 paise percent, find the premium paid by the owner of the shop. **(2)**

(ii) Find the Age-Specific Death Rate (Age-SDR) for the following data :

Age groups (in years)	Population (in '000)	Number of deaths
1 - 10	11	240
10 - 20	12	150
20 - 60	9	125
60 and above	2	90

(iii) If $\Sigma d_i^2 = 25$, $n = 6$ find rank correlation coefficient where d_i, is the difference between the ranks of i^{th} values. **(2)**

(iv) The following table gives the ages of husbands and wives : **(2)**

Age of wives (in years)	Age of husbands (in years)			
	20 - 30	30 - 40	40 - 50	50 - 60
15 - 25	5	9	3	—
25 - 35	—	10	25	2
35 - 45	—	1	12	2
45 - 55	—	—	4	16
55 - 65	—	—	—	4

Find : (i) The marginal frequency distribution of the age of husbands.

(ii) The conditional frequency distribution of the age of husbands when the age of wives lies between 25 - 35. **(2)**

(v) The regression equation of Y on X is $y = \frac{2}{9}x$ and the regression equation of X on Y is $x = \frac{y}{2} + \frac{7}{6}$

Find : (i) Correlation coefficient between X and Y.

(ii) σ_y^2 if $\sigma_x^2 = 4$. **(2)**

(vi) Identify the regression equations of X on Y and Y on X from the following equations :
$2x + 3y = 6$ and $5x + 7y - 12 = 0$ **(2)**

(vii) If X has Poisson distribution with parameter $m = 1$, find $P[X \leq 1]$. (Use $e^{-1} = 0.3679$) **(2)**

(viii) Three fair coins are tossed simultaneously. If X denotes the number of heads, find the probability distribution of X.

5. **(A) Attempt any TWO of the following :**

(i) Ramesh, Vivek and Sunil started a business by investing capitals in the ratio 4 : 5 : 6. After 3 months Vivek withdrew all his capital and after 6 months Sunil withdrew all his capital from the business. At the end of the year Ramesh received ₹ 6,400 as profit. Find the profit earned by Vivek. **(3)**

(ii) Solve the following minimal assignment problem and hence find the minimum value :

	I	II	III	IV
A	2	10	9	7
B	13	2	12	2
C	3	4	6	1
D	4	15	4	9

(3)

(iii) Calculate from e^0_0, e^0_1, e^0_2 from the following data :

Age x	0	1	2
l_x	1000	900	700
T_x	—	—	11500

(3)

(B) Attempt any TWO of the following :

(i) A bill was drawn on 12th April for ₹ 3,500 and was discounted on 4th July at 5% p.a. If the banker paid ₹ 3,465 for the bill. Find period of the bill. **(4)**

(ii) Find Karl Pearson's correlation coefficient for the following data :

X	3	2	1	5	4
Y	8	4	10	2	6

(3)

(iii) Solve the following using graphical method :

Minimize : $Z = 3x + 5y$

Subject to
$2x + 3y \geq 12$
$-x + y \leq 3$
$x \leq 4, y \geq 3, x \geq 0, y \geq 0.$ **(4)**

6. **(A) Attempt any TWO of the following :** **(6) [14]**

(i) Given the following information :

Age groups (in years)	Population	Number of deaths
0 - 20	40,000	350
20 - 65	65,000	650
65 and above	15,000	x

Find X, if the $CDR = 13.4$ per thousand.

(ii) The manager of a company wants to find a measure which he can use to fix the monthly wages of persons applying for a job in the production department. As an experimental project, he collected data of 7 persons from that department referring to years of service and their monthly income :

Years of service	11	7	9	5	8	6	10
Income (₹ in thousands)	10	8	6	5	9	7	11

Find regression equation of income on the years of service. **(3)**

(iii) Solve the following inequation :
$-8 < -(3x-5) < 13$. **(3)**

(B) Attempt any TWO of the following : (8)

(i) Find the probability of guessing correctly at most three of the seven answers in a True or False objective test. **(4)**

(ii) A person bought a television set paying ₹ 20,000 in cash and promised to pay ₹ 1,000 at the end of every month for the next 2 years. If the money is worth 12% p.a. converted monthly, what is the cash price of the television set?
$[(1.01)^{-24} = 0.7884]$ **(4)**

(iii) There are four jobs to be completed. Each job must go through machines M_1, M_2, M_3 in the order $M_1 - M_2 - M_3$. Processing time in hours is given below. Determine the optimal sequence and idle time for Machine M_1.

Jobs	**A**	**B**	**C**	**D**
M_1	5	8	7	3
M_2	6	7	2	5
M_3	7	8	10	9

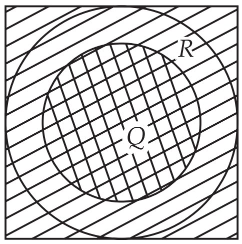
(4)

■■

Solutions

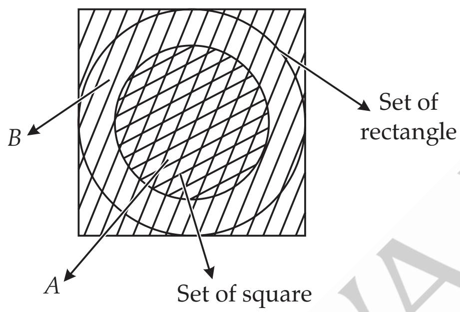
SECTION-I

1. (i) (a) We know that all rational numbers are real number *i.e.*, $Q \subset R$.



$(Q \subset R)$

(b) We know that all squares are also rectangles us *i.e.*, set of squares c set of rectangles.



(ii)

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

We have $A = IA$

$$\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} A$$

Hence $A^{-1} = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$

(iii)

$$f(x) = \begin{cases} x^2 - x + 9 & \text{for} \quad x \le 3 \\ 4x + 3 & \text{for} \quad x > 3 \end{cases}$$

We check the continuity at $x = 3$

LHL (at $x = 3$)

$$= \lim_{x \to 3^-} f(x)$$

$$= \lim_{h \to 0} f(3 - h)$$

$$= \lim_{h \to 0} [(3 - h)^2 - (3 - h) + 9]$$

$$= \lim_{h \to 0} [9 + h^2 - 6h - 3 + h + 9] = 15$$

RHL (at $x = 3$) $= \lim_{x \to 3^+} f(x) = \lim_{h \to 0} f(3 + h)$

$$= \lim_{h \to 0} [4(3 + h) + 3] = 15$$

Also $f(3) = 9 - 3 + 9 = 15$

Hence LHL $= f(3) =$ RHL

Hence $f(x)$ is continuous at $x = 3$

(iv)

$$y = \cos^{-1}(\sin 5x)$$

$$= \cos^{-1}\left\{\cos\left(\frac{\pi}{2} - 5x\right)\right\}$$

$$= \frac{\pi}{2} - 5x$$

diff. *w.r.t.* x

$$\frac{dy}{dx} = -5$$

(v) The given function is $p = 183 + 120D - 3D^2$

$$\therefore \quad \frac{dP}{dD} = 120 - 6D$$

Now $\dfrac{dP}{dD} = 0 \Rightarrow 120 - 6D = 0 \Rightarrow D = 20$

For increasing the price

$$\frac{dP}{dD} > 0$$

$$120 - 6D > 0$$

$$-6D > -120$$

$$D < \frac{120}{6}$$

$$D < 20.$$

Demand and price cannot be negative

$\therefore$ Price is increasing in the internal (0, 20).

(vi) $\int \frac{1}{x(3+\log x)} dx$

Put $\quad 3 + \log x = t$

$$\frac{1}{x} dx = dt$$

$$\therefore \quad \int \frac{dt}{t} + c = \log t + c = \log (3 + \log x) + c$$

(vii) $$A = \begin{bmatrix} -1 & 2 \\ -3 & 4 \end{bmatrix}$$

Cofactors are $\quad A_{11} = (-1)^{1+1}(4) = 4$

$$A_{12} = (-1)^{1+2}(-3) = 3$$

$$A_{21} = (-1)^{2+1}(2) = -2$$

$$A_{22} = (-1)^{2+2}(-1) = -1$$

(viii) $\int \frac{1}{9x^2+49} dx$

$$= \int \frac{1}{(3x)^2+(7)^2} dx$$

Put $\quad 3x = t$

$$dx = \frac{dt}{3}$$

$$= \frac{1}{3}\int \frac{dt}{t^2+7^2}$$

$$= \frac{1}{3}\left[\frac{1}{7}\tan^{-1}\frac{t}{7}\right] + c$$

$$= \frac{1}{21}\tan^{-1}\frac{3x}{7} + c$$

2. (A) (i) $\lim_{x\to 0}\left[\frac{\log(1+3x)}{5x}\right]$

$$= \lim_{x\to 0}\left[\frac{3x - \frac{(3x)^2}{2} + \frac{(3x)^3}{3} - \ldots\ldots}{5x}\right]$$

$$= \lim_{x\to 0}\left[\frac{3}{5} - \frac{9x}{10} + \frac{9}{5}x^2 \ldots\ldots\right]$$

$$= \frac{3}{5}$$

$\because f$ is continuous at $x = 0$

$$\therefore \quad \lim_{x\to 0} f(x) = f(0) \Rightarrow k = \frac{3}{5}$$

(ii) We make the truth table as follows

P	q	$P \wedge q$	$-(P \wedge q)$	$Pv-(P \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

So the final statement is always true. So it is a tautology.

(iii) $x = \cos^2\theta$ and $y = \cot\theta$

$$\frac{dx}{d\theta} = \frac{d}{d\theta}(\cos^2\theta)$$

$$\frac{dx}{d\theta} = -2\cos\theta\sin\theta$$

$$\frac{dy}{d\theta} = -\text{cosec}^2\theta$$

$$\frac{dy}{dx} = \frac{dy}{d\theta}\bigg/\frac{dx}{d\theta}$$

$$= \frac{-\text{cosec}^2\theta}{-2\cos\theta\,\sin\theta}$$

$$= \frac{1}{2\sin^3\theta\,\cos\theta}$$

$$= \left(\frac{1}{2\sin^3\theta\,\cos\theta}\right)\theta = \frac{\pi}{4}$$

$$\left(\frac{dy}{dx}\right)_\theta = \frac{\pi}{4}$$

$$= \frac{1}{2\left(\frac{1}{\sqrt{2}}\right)^3\frac{1}{\sqrt{2}}}$$

$$= \frac{1}{2\frac{1}{4}} = 2$$

$$\left(\frac{dy}{d\theta}\right)_{\theta=\frac{\pi}{4}} = 2$$

$$\theta = \frac{\pi}{4}$$

(B) **(i)** Let the three numbers are x, y and z according to condition

$$x + y + z = 6 \qquad ...(i)$$

$$3z + y = 11 \qquad ...(ii)$$

$$x + z = 2y \qquad ...(iii)$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$R_2 : R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ 0 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 11 \end{bmatrix}$$

$R_2 : R_2 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 0 \\ 0 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -6 \\ 11 \end{bmatrix}$$

$R_3 : 3R_3 + R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 0 \\ 0 & 0 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -6 \\ 27 \end{bmatrix}$$

$R_3 : \frac{R_3}{9}$, $R_2 : \frac{R_2}{-3}$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix}$$

$R_1 : R_1 - (R_2 + R_3)$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$I\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

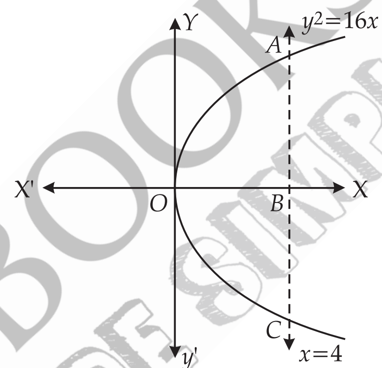
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$x = 1$, $y = 2$ and $z = 3$

(ii) The region bounded by the parabola $y^2 = 16x$ and the line $x = 4$ is the area $OACO$

The area $OACO$ is symmetrical about x-axis

Area of $OACO$ = 2(Area of OAB)



$$\text{Area of } OACO = 2\left[\int_0^4 y\,dx\right]$$

$$= 2\int_0^4 4\sqrt{x}\,dx$$

$$= 8\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^4$$

$$= \frac{16}{3}\left((4)^{\frac{3}{2}}\right)$$

$$= \frac{16}{3}(8) = \frac{128}{3}$$

Therefore, the required area is $\frac{128}{3}$ sq. units.

(iii) The expenditure E_C of a person with income x is given by

$$E_C = [0.0006]x^2 + [0.003]x$$

So, marginal propensity to consume (MPC) = $\frac{d\,E_C}{dI}$

$$\therefore \quad MPC = \frac{d\,E_C}{dx} = 2\,(0.0006)\,x + (0.003)$$

$$= 0.0012\,x + 0.003$$

$\therefore$ MPC at $x = 200$ is

$$= 0.0012\,(200) + 0.003$$
$$= 0.24 + 0.003$$
$$= 0.243$$

As $MPC + MPS = 1$

$$MPS = 1 - MPC$$
$$= 1 - 0.0243$$
$$= 0.757$$

Now $APC = \dfrac{E_C}{x}$

$$= 0.0006\,x + 0.003$$

At $x = 200$

$$APC = 0.0006 \times 200 + 0.003$$
$$= 0.12 + 0.003$$
$$= 0.123$$

As $APS = 1 - APC$

$$= 1 - 0.123$$
$$= 0.877$$

3. (i) $\lim\limits_{x\to 4} \dfrac{x^3 - 64}{\sqrt{x^2+9}-5}$ $\left(\dfrac{0}{0}\right)$Form

$$\lim_{x\to 4} \frac{x^3-64}{\sqrt{x^2+9}-5} \quad \left(\frac{0}{0}\right) \text{ form}$$

$$\lim_{x\to 4} \frac{x^3-64}{\sqrt{x^2+9}-5} \times \frac{\sqrt{x^2+9}+5}{\sqrt{x^2+9}+5}$$

$$\lim_{x\to 4} \frac{\left(x^3-64\right)\left(\sqrt{x^2+9}+5\right)}{\left(\sqrt{x^2+9}\right)^2-(5)^2}$$

$$\lim_{x\to 4} \frac{(x-4)\left(x^2+4x+16\right)\left(\sqrt{x^2+9}+5\right)}{x^2+9-25}$$

$$[a^3 - b^3 = (a-b)\,(a^2 + ab + b^2)]$$

$$\lim_{x\to 4} \frac{(x-4)(x^2+4x+16)(\sqrt{x^2+9}+5)}{x^2-16}$$

$$\lim_{x\to 4} \frac{(x-4)\left(x^2+4x+16\right)\left(\sqrt{x^2+9}+5\right)}{(x-4)(x+4)}$$

$$\lim_{x\to 4} \frac{(x+4x+16)\left(\sqrt{x^2+9}+5\right)}{(x+4)} \qquad [x \neq -4]$$

$$\frac{\left\{(4)^2+4(4)+16\right\}\left(\sqrt{4^2+9}+5\right)}{(4+4)}$$

$$\frac{(16+16+16)(5+5)}{8}$$

$$\underset{\lim x\to 4}{f(x)} = 60$$

$$\lim_{x\to 4} f(x) \neq f(4)$$

$\therefore f(x)$ is not continuous at $x = 4$.

(ii) $e^x + e^y = e^{x-y}$

diff. *w.r.t.* x

$$\frac{de^x}{dx} + \frac{de^y}{dx} = \frac{de^{x-y}}{dx}$$

$$e^x + e^y \frac{dy}{dx} = e^{x-y}\left(1 - \frac{dy}{dx}\right)$$

$$(e^y + e^{x-y})\frac{dy}{dx} = e^{x-y} - e^x$$

$$\frac{dy}{dx} = \left(\frac{e^{x-y}-e^x}{e^{x-y}+e^y}\right) = \frac{e^x+e^y-e^x}{e^x+e^y+e^y}$$

$$= \left(\frac{e^x}{2e^y}\right) = \frac{1}{2}\left[e^{x-y}\right]$$

$$\frac{dy}{dx} = \frac{1}{2}\left[e^x + e^y\right]$$

(iii) $-(P \to -q) \cong P \wedge q$

Truth table

P (1)	q (2)	$-q$ (3)	$P \to -q$ (4)	$-(P \to -q)$ (5)	$P \wedge q$ (6)
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	T	F	F

From the truth table, we get 5th and 6th columns are identical

$\therefore -(P \to -q) \cong P \wedge q$

(B) (i) $\int \frac{\sin x}{\sqrt{\cos^2 x - 2\cos x - 3}} dx$

Put $\cos x = t$

$-\sin x\, dx = dt$

$\sin x\, dx = -dt$

$$= -\int \frac{dt}{\sqrt{t^2 - 2t - 3}}$$

$$= -\int \frac{dt}{\sqrt{(t-1)^2 - 4}}$$

(Completing the square)

$$= -\int \frac{dt}{\sqrt{(t-1)^2 - (2)^2}}$$

Put $t - 1 = \theta$

$dt = d\theta$

$$= -\int \frac{d\theta}{\sqrt{\theta^2 - (2)^2}}$$

$$= -\log\left|\theta + \sqrt{\theta^2 - (2)^2}\right| + C$$

$$= -\log\left|t - 1 + \sqrt{(t-1)^2 - 4}\right| + C$$

$$= -\log\left|t - 1 + \sqrt{t^2 - 2t - 3}\right| + C$$

$$= -\log\left|\cos x - 1 + \sqrt{\cos^2 x - 2\cos x - 3}\right| + C$$

(ii) Given cost function

$$C(x) = x^2 + 75x + 1600$$

Average $\bar{C}(x) = \frac{C(x)}{x}$

$$= \frac{x^2 + 75x + 1600}{x}$$

$$= x + 75 + \frac{1600}{x}$$

Now $\bar{C}'(x) = \frac{d\bar{C}(x)}{dx} = 1 - \frac{1600}{x^2}$

For minimum average cost $\bar{C}'(x) = 0$

i.e., $1 - \frac{1600}{x^2} = 0 \quad \Rightarrow x^2 = 1600$

$\Rightarrow x = 40$

$\therefore \quad \bar{C}''(x) = \frac{d^2C(x)}{dx^2}$

$= \frac{3200}{x^3} > 0$ [For x= 40]

$\therefore$ it is minimum

$\therefore$ Minimum average cost $= \bar{C}(x) = 40 + 75 + \frac{1600}{40} = 155$

$\therefore \quad C_A = 155$

Now we find marginal cost *i.e.,*

$$C_m = \frac{dC}{dx}$$

$$C_m = \frac{d}{dx}(x^2 + 75x + 1600)$$

$= 2x + 75$ ...(1)

$\therefore$ put $x = 40$ in eq (1)

$C_m = 2 \times 40 + 75$

$= 80 + 75 = 155$

$\therefore \quad C_A = C_m$ for $x = 40$

(iii) $\int_1^2 \frac{1}{(x+1)(x+3)} dx$

$$= \frac{1}{2}\int_1^2 \left(\frac{1}{(x+1)} - \frac{1}{x+3}\right) dx$$

$$= \frac{1}{2}\left[\{\log(x+1)\}_1^2 - \{\log(x+3)\}_1^2\right]$$

$$= \frac{1}{2}[(\log 3 - \log 2) - (\log 5 - \log 4)]$$

$$= \frac{1}{2}[(\log 3 + \log 4) - (\log 2 + \log 5)]$$

$$= \frac{1}{2}[\log 12 - \log 10]$$

$$= \frac{1}{2}\log\frac{12}{10}$$

$$\therefore \int_1^2 \frac{1}{(x+1)(x+3)} dx = \frac{1}{2}\log\frac{6}{5}.$$

SECTION-II

4. (i) Given, property value = ₹ 2,40,000

Rate of premium = 90 paise% = ₹ 0.9 %

Since, the shop is insured for 75% of its value

$\therefore$ Policy value = 75% of property value

$\therefore$ Policy value $= \frac{75}{100} \times 240000$

= ₹ 180000

Now, amount of premium = 0.9% of policy value

$= \frac{0.9}{100} \times 180000$

= ₹ 1620

(ii) We present the computation in the following table

Age group	**Population $n^P x$**	**No of Deaths $n^D x$**	**Age – SDR per thousand $= \frac{n^D x}{n^P x} \times 1000$**
0 – 10	11000	240	21.81
10 – 20	12000	150	12.50
20 – 60	9000	125	13.88
60 and above	2000	90	45.00

4. **(iii)** Here given $\Sigma di^2 = 25$

$n = 6$

The rank correlation coefficient is given by

$$r = 1 - \frac{6\Sigma di^2}{n(n^2-1)}$$

$$= 1 - \frac{6 \times 25}{6(36-1)}$$

$$= 1 - \frac{150}{6 \times 35}$$

$$= 1 - \frac{5}{7}$$

$$r = \frac{2}{7}$$

(iv)

Age of wives (in years)	**Age of husbands (in years)**			
	20 – 30	**30 – 40**	**40 – 50**	**50 – 60**
15 – 25	5	9	3	—
25 – 35	—	10	25	2
35 – 45	—	1	12	2
45 – 55	—	—	4	16
55 – 65	—	—	—	4
Total	**5**	**20**	**44**	**24**

Now, marginal distribution of age of husbands.

Age (years)	20 – 30	30 – 40	40 – 50	50 – 60	Total
Number	5	20	44	24	93

Now marginal distribution of husband, when the age of wives lies between 25 – 35

Age (years)	20 – 30	30 – 40	40 – 50	50 – 60	Total
Number	—	10	25	2	37

(v) The repression eqn. of y on x is $y = \frac{2}{9}x$

Comparing with $y - \bar{y} = b_{yx}(x - \bar{x})$

Here $b_{yx} = \frac{2}{9}$

Now the regression eqn. of x on y is $x = \frac{y}{2} + \frac{7}{6}$

Comparing with $x - \bar{x} = b_{xy}(y - \bar{y})$

Here $b_{xy} = \frac{1}{2}$

(a) we have, correlation coefficient between x and y is

$$r = \sqrt{b_{yx} . b_{xy}}$$

$$= \sqrt{\frac{2}{9} \times \frac{1}{2}} = \sqrt{\frac{1}{9}} \pm \frac{1}{3}$$

$\therefore \quad r = \frac{1}{3}$ ($\because b_{yx}$ and b_{xy} are positve)

(b) $\because \quad \sigma_x^2 = 4 \Rightarrow \sigma_x = 2$

We have $\quad b_{yx} = r.\frac{\sigma_y}{\sigma_x}$

$\therefore \quad \frac{2}{9} = \frac{1}{3}.\frac{\sigma_y}{2}$

$\Rightarrow \quad \sigma_y = \frac{12}{9}$

$\Rightarrow \quad \sigma_y = \frac{4}{3}$

(vi) Let the regression eqn. of y on x is

$$2x + 3y = 6$$

$\Rightarrow \quad y = \frac{-2}{3}x - 2$

Here $\quad b_{yx} = \frac{-2}{3} \quad \ldots$(i)

The regression eqn. of x on y is $5x + 7y - 12 = 0$

$\Rightarrow \quad x = \frac{-7y}{5} + \frac{12}{5}$

here $\quad b_{xy} = \frac{-7}{5} \quad \ldots$(ii)

from eqn. (i) and (ii)

$$b_{yx} \times b_{xy} = \frac{-2}{3} \times \frac{-7}{5}$$

$$= \frac{14}{15} < 1$$

Hence regression eqn. of y on x is $2x + 3y = 6$

and regression eqn. of x on y is $5x + 7y - 12 = 0$

(vii) We have, the Poisson distribution is given by

$$P(r) = \frac{e^{-m}.m^r}{r!}$$

Here $\quad m = 1$ (given)

Then $\quad P(X \leq 1) = P(0) + P(1)$

$$= \frac{e^{-m}.(1)^0}{0!} + \frac{e^{-1}(1)^1}{1!}$$

$$= e^{-1} + e^{-1}$$

$$= 2e^{-1}$$

$$= 2(0.3679)$$

$e^{-1} = 0.3679$ (given)

$$= 0.7358$$

(viii) probability of getting head

$$P = \frac{1}{2}$$

$$q = 1 - \frac{1}{2} = \frac{1}{2}$$

The probability distribution is as follows

x **(No. of heads)**	**0**	**1**	**2**	**3**
$P(x)$	${}^3C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^3 = \frac{1}{8}$	${}^3C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^2 = \frac{3}{8}$	${}^3C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) = \frac{3}{8}$	${}^3C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^0 = \frac{1}{8}$

5. (A) (i) Since Ratio of their capital is 4 : 5 : 6

$\therefore$ Let Ramesh capital be $4x$, Vivek capital be $5x$ and Sunil capital be $6x$

$\therefore$ Ramesh invested ₹ $4x$ for 12 month

Vivek invested ₹ $5x$ for 3 month

Sunil invested ₹ $6x$ for 6 month

$\therefore$ Profit is distributed in the ratio

i.e., $4x \times 12 : 5x \times 3 : 6x \times 6$

$16x : 5x : 12x$

$16 : 5 : 12$

Also $16 + 5 + 12 = 33$

Now given that Ramesh profit is ₹ 6400

$\therefore$ Ramesh share in the profit

$$= \frac{16}{33} \times \text{Total profit}$$

$$6400 = \frac{16}{33} \times \text{Total profit}$$

$$\text{Total profit} = ₹\ 13{,}200$$

Now Vivek share in the profit $= \frac{5}{33} \times 13200$

$= ₹\ 2000$

and Sunil share in the profit $= \frac{12}{33} \times 13200$

$= ₹\ 4800$

(ii)

	I	II	III	IV
A	2	10	9	7
B	13	2	12	2
C	3	4	6	1
D	4	15	4	9

Subtracting minimum element from each row, we have

	I	II	III	IV
A	0	8	7	5
B	11	0	10	0
C	2	3	5	0
D	0	11	0	5

Subtracting minimum element from each Column, we have

	I	II	III	IV
A	$\boxed{0}$	8	7	5
B	11	$\boxed{0}$	10	$\cancel{0}$
C	2	3	5	$\boxed{0}$
D	$\cancel{0}$	11	$\boxed{0}$	5

Hence $A \to$ I, B $\to$ II, C $\to$ IV, D $\to$ III

Minimum value is 2 + 2 + 1 + 4 = 9

(iii) $l_0 = 1000, l_1 = 880, l_2 = 876$

$$T_2 = 3323$$

$$L_0 = \frac{l_0 + l_1}{2} = \frac{1000+880}{2}$$

$$= \frac{1880}{2} = 940$$

$$L_1 = \frac{l_1 + l_2}{2} = \frac{880+876}{2}$$

$$= \frac{1756}{2} = 878$$

$$T_1 = L_1 + T_2$$
$$= 828 + 3323$$
$$= 4201$$

$$T_0 = L_0 + T_1$$
$$= 940 + 4201$$
$$= 5141$$

$$e_0^0 = \frac{T_0}{l_0} = \frac{5141}{1000} = 5.141$$

$$e_1^0 = \frac{T_1}{l_1} = \frac{4201}{880} = 4.7738$$

$$e_2^0 = \frac{T_2}{l_2} = \frac{3323}{876} = 3.7933$$

(B) (i) S.D. = 3500, C.V. = 3465

$$r = 5\%$$

Now B.D. = S.D. – C.V.

= 3500 – 3465

= 35

Also $\text{B.D.} = \frac{\text{S.D} \times n \times r}{100}$

$$35 = \frac{3500 \times n \times 5}{100}$$

$$n = \frac{1}{5}\text{year} = \frac{365}{5} = 73 \text{ days}$$

The period for which the discount is deducted is 73 days which is counted from the date of discounting *i.e.*, 4th July

July	August	September	Total
27	31	15	73

$\therefore$ The legal due date is 15 September. Hence the period of the bill is from 12th April to 15 September *i.e.*, 5 month.

(ii)

x	y	x^2	y^2	xy
3	8	9	64	24
2	4	4	16	8
1	10	1	100	10
5	2	25	4	10
4	6	16	36	24
15	**30**	**55**	**220**	**76**

Here $n = 5, \Sigma x = 15, \Sigma y = 30, \Sigma x^2 = 55, \Sigma y^2 = 220, \Sigma xy = 76$

Karl pearson coefficient of correlation between x and y is

$$r(x, y) = \frac{n\,\Sigma xy - \Sigma x\,\Sigma y}{\sqrt{n\,\Sigma x^2 - (\Sigma x)^2}\ \sqrt{n\,\Sigma y^2 - (\Sigma y)^2}}$$

$$= \frac{5 \times 76 - 15 \times 30}{\sqrt{5 \times 55 - (15)^2}\ \sqrt{5 \times 220 - (30)^2}}$$

$$= \frac{380 - 450}{\sqrt{275 - 225}\ \sqrt{1100 - 900}}$$

$$= \frac{-70}{\sqrt{50}\ \sqrt{200}}$$

$$= \frac{-70}{100} = -0.7$$

(iii) Min $Z = 3x + 5y$

S.t. $2x + 3y \geq 12$ ...(i)

$-x + y \leq 3$ ...(ii)

$x \leq 4, y \geq 3, x \geq 0, y \geq 0$

Taking eqn (i)

$$2x + 3y = 12$$

Putting $x = 0$, $y = 4$ Let the point is (0, 4)

Now putting $y = 0$, $x = 6$ Let the point is (6, 0)

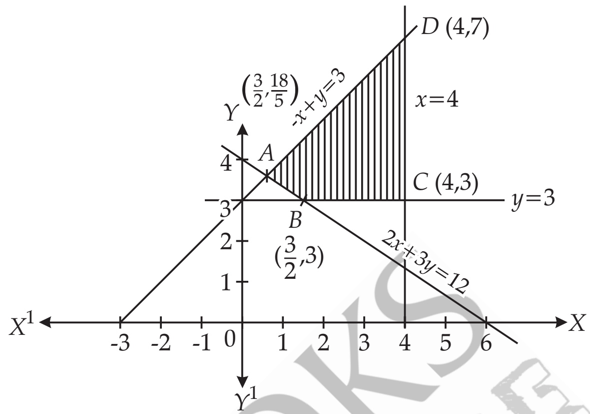
Now taking eqn (ii)

$$-x + y = 3$$

Putting $x = 0$, $y = 3$ (0, 3)

Putting $y = 0$, $x = -3$ (– 3, 0)

The graph is as follows



$ABCDA$ be the feasible region bounded by these lines Now we find the coordinates of A, B, C and D for A, Solving the eqns.

$$2x + 3y = 12 \text{ and } -x + y = 3$$

We get $x = \frac{+3}{5}$ and $y = \frac{18}{5}$

coordinate of $A\left(\frac{+3}{5}, \frac{18}{5}\right)$

Now $Z = 3 \times \left(\frac{+3}{5}\right) + 5 \times \frac{18}{5}$

$$= \frac{+9}{5} + \frac{90}{5} = \frac{99}{5}$$

For B, Solving the eqns

$$2x + 3y = 12 \text{ and } y = 3$$

We get $x = \frac{3}{2}, y = 3$

$\therefore$ Coordinate of $B\left(\frac{3}{2}, 3\right)$

Now $Z = 3 \times \frac{3}{2} + 5 \times 3$

$$= \frac{9}{2} + 15 = \frac{39}{2}$$

For C. Solving the eqn $x = 4$ and $y = 3$

$\therefore$ Coordinate of C (4, 3)

Now $Z = 3 \times 4 + 5 \times 3$

$$= 12 + 15 = 27$$

For D, Solving the eqn

$$-x + y = 3 \text{ and } x = 4$$

We get $x = 4, y = 7$

Now $Z = 3 \times 4 + 5 \times 7$

$= 12 + 35 = 47$

Min $Z = \frac{39}{2}$, for $x = \frac{3}{2}, y = 3$

6. **(A)** (i) Given $CDR = 13.4$

Total population $(\Sigma Pi) = 40000 + 65000 + 15000$

$= 120000$

No of Deaths $(\Sigma Di) = 350 + 650 + x$

$= 1000 + x$

$$CDR = \frac{\Sigma Di}{\Sigma Pi} \times 1000$$

$$\Rightarrow \quad 13.4 = \frac{1000 + x}{120000} \times 1000$$

$$\Rightarrow \quad 13.4 = \frac{1000 + x}{120}$$

$$\Rightarrow \quad x = 1608 - 1000$$

$$\therefore \quad x = 608$$

(ii) For the given data we have the following table

Year of service x	income y	xy	x^2	y^2
11	10	110	121	100
7	8	56	49	64
9	6	54	81	36
5	5	25	25	25
8	9	72	64	81
6	7	42	36	49
10	11	110	100	121
56	**56**	**469**	**476**	**496**

Here $n = 7$, $\Sigma x = 56$, $\Sigma y = 56$, $\Sigma xy = 469$, $\Sigma x^2 = 476$, $\Sigma y^2 = 496$

Now $$\bar{x} = \frac{\Sigma x}{n} = \frac{56}{7} = 8$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{56}{7} = 8$$

The regression of y on x is given by

$$b_{yx} = \frac{n\,\Sigma xy - \Sigma x\,\Sigma y}{n\,\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{7 \times 469 - 56 \times 56}{7 \times 476 - (56)^2}$$

$$= \frac{3283 - 3136}{3332 - 3136}$$

$$= \frac{147}{196} = 0.75$$

Hence the regression line of y on x is given by

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - 8 = 0.75\,(x - 8)$$

$$y - 8 = 0.75\,x - 6.00$$

$$y = 0.75\,x + 2$$

or $$y = \frac{3}{4}x + 2$$

$$\Rightarrow \quad 4y = 3x + 8$$

(iii) The given inequality is

$$-8 < -(3x - 5) < 13$$

$$= -8 < -3x + 5 < 13$$

$$= -8 - 5 < -3x + 5 - 5 < 13 - 5$$

$$= -13 < -3x < 8$$

Multiplying by -1, we have

$$13 > 3x > -8$$

$$= \frac{13}{3} > x > \frac{-8}{3}$$

reversing the order of inequality

$$\frac{-8}{3} < x < \frac{13}{3}$$

$\therefore$ The solution set is $x \in \left(\frac{-8}{3}, \frac{13}{3}\right)$

(B) (i) For true, false question

Let the prob. of true $P = \frac{1}{2}$ and $q = 1 - P = 1 - \frac{1}{2} = \frac{1}{2}$

Now we have to find

$$P(X \le 3) = P(0) + P(1) + P(2) + P(3)$$

$$= {}^7C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^7 + {}^7C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^6$$

$$+{}^7C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^5+{}^7C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^4$$

$$=\left(\frac{1}{2}\right)^7+7.\left(\frac{1}{2}\right)^7+21\left(\frac{1}{2}\right)^7+35.\left(\frac{1}{2}\right)^7$$

$$=64\times\left(\frac{1}{2}\right)^7$$

$$=64\times\frac{1}{128}$$

$$=\frac{1}{2}$$

$\therefore$ Prob. of guessing at most three question correctly $=\frac{1}{2}$

(ii) C = ₹ 1000, n = 2 years = 24 months

r = 12% per annum

= 1% per month

$$i=\frac{r}{100}=\frac{1}{100}=0.01$$

$$\text{Present value } P=\frac{C}{i}[1-(1+i)^{-n}]$$

$$=\frac{1000}{0.01}[1-(1+0.01)^{-24}]$$

$$=\frac{1000}{0.01}[1-0.7884]$$

$$=\frac{1000}{0.01}\times 0.2116$$

= ₹ 21160

Cash price of television = 20000 + 21160

= ₹ 41160

(iii) The given problem is of n jobs and three machines. We change the problem in of n jobs and two machines.

For this either Min $M_1 \le M_3 \times M_2$ or min $M_3 \le$ max M_2

Here min $M_3 = 7 = M_1 \times M_2$

Hence we write $M_1 + M_2$ = G and $M_2 + M_3 = H$ the problem will be as follows

Jobs	**A**	**B**	**C**	**D**
$G = M_1 + M_2$	11	15	9	8
$H = M_2 + M_3$	13	15	12	14

The sequence as follows

Now	D	C	A	B

Job	M_1		M_2		M_3	
	Time in	**Time out**	**Time in**	**Time out**	**Time in**	**Time out**
D	0	3	3	8	8	17
C	3	10	10	12	17	27
A	10	15	15	21	27	34
B	15	23	23	30	34	42

The minimum elapsed time = 42 hrs.

Ideal time for M_1 = 42 – 23 = 19 hrs.