JAIN COLLEGE, Bangalore
Mock Paper - 1 January - 2017
II PUC – Mathematics (35)

Time: 3 Hours 15 Minutes **Max. Marks: 100**

PART A

I. Answer all questions **10 × 1 = 10**

1. Define an equivalence relation
2. Write the domain of $f(x)=\cos^{-1}x$
3. If a matrix has 24 elements what are the possible order it can have?
4. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ .Find the value of x.
5. Differentiate tan(2x+3) with respect to x.
6. Find the anti-derivative of $(ax+b)^2$ with respect of x.
7. Find the value of x and y so that the vectors $2\hat{i}+3\hat{j}$ and $x\hat{i}+y\hat{j}$ are equal.
8. Find the vector $\vec{PQ}$ joining the points P(2,3,0) and Q(-1,-2,-4).
9. Define Optimal solution.
10. If P(A)=0.8,P(B)=0.5, $P(B/A)=0.4$ .Find $P(A\cap B)$

PART B

II. Answer any ten **10 × 2 = 20**

11. If $f:R\to R$ is defined by $f(x)=x^2-3x+2$.Find f(f(x)).
12. Prove that $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$
13. Find the value of $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$
14. Find the equation of the line passing through (1,2) and (3,6) using the determinants.
15. If $x=at^2, y=2at$ .Find $\frac{dy}{dx}$.
16. If $y=\cos^{-1}(\sin x)$ then Prove that $\frac{dy}{dx}=-1$.
17. Find the approximate change in the volume V of a cube of side x meters caused by increasing the side by 2%.
18. Evaluate $\int\frac{\sec^2 x}{\sqrt{\tan^2 x+4}}dx$
19. Evaluate $\int_0^1\frac{x}{x^2+1}dx$
20. Find the order and degree of differential equation $\frac{d^4y}{dx^4}+\sin(y''')=0$
21. If $\vec{a}$ is a unit vector such that $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=8$.Find $|\vec{x}|$.
22. Find the area of parallelogram whose adjacent sides are the vectors $3\hat{i}+\hat{j}+4\hat{k}$ *and* $\hat{i}-\hat{j}+\hat{k}$
23. Find the distance of a point (2,5,-3) from the plane $\vec{r}.(6\hat{i}-3\hat{j}+2\hat{k})=4$.

24. Find the probability distribution of number of heads in two tosses of a coin.

PART C

III. Answer any ten **10 × 3 = 30**

25. Show that the relation R in the set Z of integers given by $R=\{(a,b):2\ divides\ a-b\}$ is an equivalence relation.

26. If $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$ .Find x.

27. By using elementary transformation, Find the inverse of the matrix $A=\begin{bmatrix}1 & -2\\ 2 & 1\end{bmatrix}$.

28. Verify the mean value theorem for $f(x)=x^2-4x-3$ in the interval [a, b] ,where $a=1\ and\ b=-1$.

29. Find two positive numbers x and y such that x + y=60 and xy^3 is maximum.

30. Find the point on the curve $y=x^3-11x+5$ at which the tangent is $y=x-11$.

31. Evaluate $\int\sin(ax+b)\cos(ax+b)dx$.

32. Evaluate $\int\frac{dx}{x+x\log x}$.

33. Find the area of the region bounded by the curve $y=x^2$ and the line y=2.

34. Find the general solution of the differential equation $\frac{dy}{dx}=e^{x+y}$

35. If $\vec{a},\vec{b}\ and\ \vec{c}$ be three vectors such that $|\vec{a}|=3,|\vec{b}|=4,|\vec{c}|=5$ and each one of being perpendicular to the sum of other two vectors .Find $|\vec{a}+\vec{b}+\vec{c}|$.

36. Prove that $[\vec{a}+\vec{b}\quad \vec{b}+\vec{c}\quad \vec{c}+\vec{a}]=2[\vec{a}\quad \vec{b}\quad \vec{c}]$.

37. Show that the shortest distance between the lines
$\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{-1}\ and\ \frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$

38. One card is drawn at random from a well shuffled deck of 52 cards. In which E: the card drawn is a king or queen, F:the card drawn is a queen or jack. Are E and F independent?

PART D

IV. Answer any six **6 × 5 = 30**

39. Prove that the function defined by $f:N\rightarrow Y$ defined by $f(x)=x^2$,where $y=\{y:y=x^2,x\in N\}$ is invertible . Also write the inverse of f(x).

40. If $A=\begin{bmatrix}0 & 6 & 7\\ -6 & 0 & 8\\ 7 & -8 & 0\end{bmatrix},B=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 2\\ 1 & 2 & 0\end{bmatrix}\ and\ C=\begin{bmatrix}2\\ -2\\ 3\end{bmatrix}$ .Calculate AC,BC and (A+B)C. Also verify that A(B+C)=AC+BC.

41. Solve the system of equations by matrix method $x+y+z=6, y+3z=11\ and\ x-2y+z=0$

42. If $y=\sin^{-1}x$,Show that $(1-x^2)y_2-xy_1=0$.

43. A particle moving along the curve $6y=x^3+2$ .Find the points on the curve at which the y-coordinate is changing 8times as fast as the x-coordinate.

44. Find the integral of $\sqrt{x^2-a^2}$ with respect to x and hence evaluate $\int\sqrt{x^2+4x+1}\ dx$

45. Find the area of the region in the first quadrant enclosed by x-axis ,the line $x=\sqrt{3}y$ and the circle $x^2+y^2=4$.

46. Solve the differential equation $ydx-(x+2y^2)dy=0$.

47. Derive the equation of the line in the space passing through a point and parallel to a vector both in vector and Cartesian form.

48. If a fair coin is tossed 8 times. Find the probability of
1) at least five heads and
2) at most five heads.

PART E

V. Answer any one **10 × 1 = 10**

49. a) Minimize and Maximize $z=x+2y$

subject to constraints $x+2y\geq 100,\ 2x-y\leq 0,\ 2x+y\leq 200,\ x,y\geq 0$

b) Prove that $\begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix}=(x-y)(y-z)(z-x)(xy+yz+zx)$

50. a) Prove that $\int_a^b f(x)dx=\int_a^c f(x)dx+\int_c^b f(x)dx$ and hence evaluate $\int_0^4 |x-1|dx$.

b) Find the value of k so that the function given by

$f(x)=\begin{cases} kx^2 & if \quad x\leq 2 \\ 3 & if \quad x>2 \end{cases}$ is continuous at x=2.

JAIN COLLEGE, Bangalore
Mock Paper - 2 January - 2017
II PUC – Mathematics (35)

Time: 3 Hours 15 Minutes **Max. Marks: 100**

PART A

I. Answer all **10 × 1 = 10**

1. Let * be a binary operation defined on set of rational numbers, by a*b=$\frac{ab}{4}$. Find the identity element.
2. Find the principle value of $\cos^{-1}(\frac{1}{2})$.
3. What is the number of the possible square matrices of order 3 with each entry 0 or 1?
4. If A is a square matrix with IAI=6, find the values of $|AA^{I}|$.
5. Check the continuity of the function given by f(x)=2x+3 at x=1
6. Find the anti-derivative of cosec x (cosec x +cot x).
7. Define co- initial vectors.
8. Show that the planes 2x-y+4z=5 and 5x-2.5y+10z=6 are parallel.
9. Define optimal solution.
10. If P(A)=$\frac{3}{5} and P(B) = \frac{1}{5}$ find P(A∩B) where A and B are independent events.

PART B

II. Answer any 10 **10 × 1 = 10**

11. If f:R→ R be given by $f(x) = (3 - x^3)^{\frac{1}{3}}$ find $f \circ f(x)$
12. Show that $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}, \forall x \in R$.
13. Write $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$,$0< x < \pi$ *in the simplest form*
14. Prove that " if any two rows(or columns) of a determinant are interchanged, then sign of determinant changes".
15. if $y=\log_7(\log x)$ prove that $\frac{dy}{dx} = \frac{1}{x \log x \log 7}$.
16. Find the local maximum value of the function $g(x)=x^3 -3x$.
17. Find the approximate change in the surface of a cube of side x meters caused by increasing the side by 1%
18. Evaluate :$\int \frac{\sec^2 x}{\sqrt{\tan^2 x+4}}\, dx$
19. Evaluate $\int_0^{\frac{\pi}{2}} (2\sec^2 x + x^3 + 2)\, dx$
20. Find the order and degree of the differential equation $(\frac{ds}{dt})^4 + 3s\left(\frac{d^2 s}{dt^2}\right) = 0$
21. If $\underset{a}{\rightarrow} = 5i - j - 3k$ and $\underset{b}{\rightarrow} = i + 3j - 5k$ then show that the vectors $\underset{a}{\rightarrow} + \underset{b}{\rightarrow}$ *and* $\underset{a}{\rightarrow} - \underset{b}{\rightarrow}$ are perpendicular to each other.
22. Find the direction cosines of the line passing through two [points (-2,4,-5) and (1,2,3)
23. If a plane has the intercepts a, b , c and is at a distance of 'p' units from the origin , Prove that $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$.
24. Three cards are drawn successively, without replacement from a pack of 52 well shuffled cars. What is the probability that first two cars are kings and the third card drawn is an ace?.

PART C

III. Answer any 10 **10 × 3 = 30**

25. Let f: X→ Y and $g{:}Y \rightarrow Z$ be two invertible functions . Then show that (gof) is also invertible with $(gof)^{-1}=f^{-1}og^{-1}$.
26. Show that $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{4}{3}=\frac{\pi}{2}$.
27. If A and B are symmetric matrices of same order, then show that AB is symmetric if and only if A and B commute , that is AB=BA.
28. if $x=a\cos^3\theta$ and $y=a\sin^3\theta$, Prove that $\frac{dy}{dx}=-\sqrt[3]{\frac{y}{x}}$
29. Verify Rolle's theorem for the function $f(x)= x^2+2x-8$, $x\epsilon\ [-4,2]$.
30. Find the least value of 'a' such that the function f given by $f(x)=x^2+ax+1$ is strictly increasing on (1,2).
31. Find $\int\frac{(x^2+1).e^x}{(x+1)^2}dx$
32. Evaluate $\int\frac{1}{x(x^n+1)}dx$
33. Find the area bounded by the curve $x^2=4y$ and the line $x=4y-2$.
34. Find the particular solution of the differential equation $x(x^2-y)\frac{dy}{dx}=1, y=0\ when\ x=2$.
35. The two adjacent sides of a parallelogram are $\hat{\imath}-2\hat{\jmath}+3\hat{k}\ and\ 2\hat{\imath}-4\hat{\jmath}+5\hat{k}$. Find the unit vector parallel to its diagonal. Also find its area.
36. Show that the vectors $2\hat{\imath}-\hat{\jmath}+\hat{k}, \hat{\imath}-3\hat{\jmath}-5\hat{k}\ and\ 3\hat{\imath}-4\hat{\jmath}-4\hat{k}$ form the vertices of a right angled triangle.
37. Find the coordinates of the point where the line through (5,1,6) and (3,4,1) crosses the yz-plane.
38. One card is drawn at random from a well shuffled deck of 52 cards. In which E:the card drawn is a king or queen, F: the card drawn is a queen or jack. Are E and F independent?.

PART D

IV. Answer any six **6 × 5 = 30**

39. 39. Let f:N→R be defined by $f(x)=4x^2+12x+15$. Show that f:N→S where S is the range of function f,is invertible. Also find the inverse of f.
40. Solve the system of linear equations by matrix method: 2x-3y+5z=11, 3x+2y-4z=-5, x+y-2z=-3.
41. If $A=\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}, B=\begin{bmatrix}2 & 0\\ 1 & 3\end{bmatrix}\ and\ C=\begin{bmatrix}1 & 1\\ 2 & 3\end{bmatrix}$. Calculate AB, AC, and A(B+C). Verify that AB+AC=A(B+C).
42. If $y=Ae^{mx}+Be^{nx}$, show that $\frac{d^2y}{dx^2}-(m+n)\frac{dy}{dx}+mn\ y=0$.
43. A particle moving along the curve $6y=x^3+2$. Find the points on the curve at which the y- coordinate is changing 8 times as fast as the x-coordinate.
44. Find the integration of $\sqrt{a^2+x^2}$, w.r.t. x and hence evaluate $\int\sqrt{x^2+2x+5}\,dx$.
45. Using integration find the area of the region bounded by the triangle whose vertices are (-1,0),(1,3) and (3,2).
46. Find the general solution of the differential equation $x\frac{dy}{dx}+2y=x^2\log x$.
47. Derive the formula to find the shortest distance between two skew lines in vector and Cartesian form.
48. The probability that a student is not a swimmer is $\frac{1}{5}$. Find the probability that out of 5 students ,(i) at least four are swimmers and (ii) at most three are swimmers.

PART –E

V. Answer any ONE question **1 × 10 =10**

49. (a) Prove that $\int_0^a f(x)dx = \int_o^a f(a-x)dx$ *and hence evaluate* $\int_0^{\frac{\pi}{4}} \frac{sinx \cos x}{cos^4x+sin^4x} dx$

(b) find the relationship between 'a' and 'b' so that the function f defined by a

$f(x) = \begin{cases} ax+1 & if\ x \le 3 \\ bx+3 & if\ x > 3 \end{cases}$ is continuous at x=3.

50. a) a manufacturer produces nuts and bolts it takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts.It takes 3 hours on machine A and 1hour on machine B to produce a package of bolts. He earns a profit of Rs.17.50 per package on nuts and Rs.7 per package on bolts. How many packages of each should be produced each day so as to maximize its profit if he operates his machine for at the most 12 hours a day.

b) Prove that $\begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix} = (1-x^2)\begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$