Jain College, Jayanagar
II PUC Mock Paper - I
Mathematics

Duration: 3.15 minutes **Max. Marks: 100**

Section – A

I. Answer all the following questions: **10 × 1 = 10**

1. An operation * defined by a*b = a^b on z^+. Determine whether * is commutative.
2. Find the value of $\tan^{-1}\left(-\sqrt{3}\right)-\sec^{-1}(-2)+\operatorname{cosec}^{-1}\left(\frac{-2}{\sqrt{3}}\right)$
3. What is the number of possible square matrices of order 3 with each entry 0 or 1.
4. If A is a square matrix with $|A|=6$ then find the values $|AA^1|$
5. If $y=e^{\frac{1}{2}\log\sin x}$. Find dy/dx.
6. Evaluate $\int\frac{\sin^6 x}{\cos^8 x}dx$
7. Find direction cosines of the vector $3\vec{i}-4\vec{j}+5\vec{k}$.
8. A line makes 60° and 45° with the positive direction of x-axis and z-axis. Then find inclination with y axis.
9. Define linear constraints.
10. A fair die is rolled consider the events E = {1,3,5} and F={2,3}. Find P(E/F)

Section – B

II. Answer any ten of the following: **10 × 2 = 20**

11. If F is a greatest integer function and g is absolute value function, the find $fog\left(-\frac{3}{2}\right)+gof\left(\frac{4}{3}\right)$
12. Write in simplest form $\cos^{-1}\left[\frac{2\log x}{1+(\log x)^2}\right]$
13. Solve $\tan^{-1}x+2\cot^{-1}x=\frac{2\pi}{3}$
14. If the area of the triangle with vertices (2, -6) and (5, 4) and (K, 4) is 35 sq. units. Find k using determinants.
15. If $y=\tan^{-1}\left[\frac{2e^{2x}}{1-e^{4x}}\right]$. Find dy/dx.
16. If $x=\sqrt[9]{9^{\sec^{-1}t}}$ and $y=\sqrt[9]{9^{\operatorname{cosec}^{-1}t}}$. Prove that dy/dx = -y/x.
17. Find the approximate value of f(x) = $2x^3+7x+1$ at x = 2.001
18. Evaluate $\int\left[\frac{1}{\log x}-\frac{1}{(\log x)^2}\right]dx$
19. Evaluate $\int\frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$
20. Find the points on the curve $x^2+y^2-2x-3=0$ at which the tangents are parallel to x-axis.
21. Find the order and degree of differential equation $\frac{d^2y}{dx^2}+\frac{dy}{dx}=\log\left[\frac{d^2y}{dx^2}\right]$
22. If $|\bar{a}|=8, |\bar{b}|=1$ and $|\bar{a}+\bar{b}|=15$ find $|\bar{a}-\bar{b}|$.
23. Find the equation of the line which passes through the point (1,2,3) and is parallel to the vector $3\bar{i}+2\bar{j}-2\bar{k}$ both in vector form and Cartesian form.

24. A die is rolled, if the outcome is an even number, what is the probability that it is a prime number.

Section – C

III. Answer any ten of the following: 10 × 3 = 30

25. Let L be the set of all lines in XY plane and R be a relation in L defined by R = (L_1,L_2): L_1 is parallel to L_2}. Prove that R is an equivalence relation.

26. If $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$. Prove that $x = \frac{\pi}{4}$

27. If A and B are invertible matrices of same order then prove that $(AB)^{-1} = B^{-1}A^{-1}$.

28. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ where $x \neq y$ then prove that $dy/dx = \frac{-1}{(1+x)^2}$

29. Verify Rolle's theorem for the function $f(x) = x^2 + 5x + 6$ in [-3, -2]

30. Find the equation of all the lines having slope = -1 that are tangents to the curve $y = \frac{1}{(x-1)}$

31. Evaluate $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}dx$

32. Evaluate $\int \frac{(2-x)}{(1-x)^2}e^x dx$

33. Find the area bounded by $y = x^2 + 2$, y = x and x = 0 and x = 3

34. Form the DE of family of circles having centre on Y-axis and radius 3 units.

35. Prove by vector method, diagonals of a parallelogram bisect each other.

36. If $|\overline{a}| = 2, |\overline{b}| = 5$ and $|\overline{a} \times \overline{b}| = 8$. Find $\overline{a} \cdot \overline{b}$.

37. Find the angle between the planes.
    4x + 8y + z - 8 = 0 and y + z – 4 = 0

38. A fair coin and unbiased die are tossed. Let A be the event "head appears on the coin" and B is the event "3 on the die". Check whether A and B are independent.

Section – D

IV. Answer any six of the following: 5 × 6 = 30

39. Let $A = R - \left\{\frac{4}{3}\right\}$ and B = R – {0}. If $f: A \to B$ and $g: B \to A$ are defined by $f(x) = \frac{1}{(3x-4)}$ and $g(x) = \frac{1+4x}{3x}$ respectively. Prove that gof = I_A and fog = I_B.

40. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ and $B = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$. Verify that $AA^T = B^TB = I$
    ($A^T \Rightarrow$ transpose of matrix A similarly B^T).

41. Solve the system of linear equations using matrix method.
    x – y + 2z = 7
    3x + 4y – 5z = -5
    2x – y – 3z = 12

42. If $y = \cos^{-1}x$. Find d^2y/dx^2 in terms of y - alone.

43. Find the integral of $\frac{1}{x^2 - a^2}$ with respect to x and evaluate $\int \frac{1}{x^2 - 8x + 5}dx$.

44. A point source of light is hung 25 feet directly above a straight horizontal path on which a boy of 6 feet in height is walking. How fast is the boy's shadow lengthening and fast the tip of the shadow is moving, when he is walking away from the light at the rate of 100 ft/min.

45. Find the area of the circle $x^2 + y^2 = a^2$ by integration method then find the area of the circle $x^2 + y^2 = 3$.

46. Solve the DE $\frac{dy}{dx} + y\sec x = \tan x,\ 0 \le x < \frac{\pi}{2}$.

47. Derive the equation of a line in a space passing through 2 given points both in vector form and Cartesian form.

48. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let 3/4 be the probability that he know the answer and 1/4 be the probability that he guesses. Assume that the student who guesses at the answer will be correct with the probability 1/4, what is the probability that the student knows the answer given that he answered correctly?

Section – E

V. Answer any one of the following: **1× 10 = 10**

49. a) Prove that $\int_{-a}^{a} f(x)dx = 2\int_{a}^{a} f(x)dx$

{when f(x) is an even function}

$\int_{-a}^{a} f(x)dx = 0$ when f(x) is an odd

and then evaluate $\int_{-\pi/2}^{\pi/2} \sin^7 x dx$

b) Prove that $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-x)(z-x)\times(xy+yz+zx)$

50. a) Maximize and minimize z = 3x + 9y subject to the constraints

$x + 3y \le 60$

$x + y \ge 10$

$x \le y,\ \ x \ge 0 \quad y \ge 0$

b) Find the relation between a and b so that $f(x) = \begin{cases} ax+1 & if\ \ x \le 3 \\ bx+3 & if\ \ x > 3 \end{cases}$ is continuous at x = 3.

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