Jain College, Jayanagar
II PUC Mock Paper - II
Mathematics

Duration: 3.15 minutes **Max.Marks: 100**

Section – A

I. Answer all the following questions: **10 × 1 = 10**

1. An operation * on z^+ defined by $a*b = 2^{ab}$. Determine whether * is associative.
2. Find the value of $\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)+\tan^{-1}(1)$
3. Define scalar matrix.
4. If $A=\begin{bmatrix}4 & 7\\ 6 & 5\end{bmatrix}$ find $|3A|$
5. If $y=a^{\frac{1}{2}\log_a \tan x}$. Find dy/dx.
6. Evaluate $\int e^{-\log x}dx$.
7. If $|\bar{a}+\bar{b}|=|\bar{a}-\bar{b}|$. Find the angle between $\bar{a}$ and $\bar{b}$.
8. If α,β,γ are the direction angles of a line where $\alpha=\beta$ and $\gamma=\frac{\pi}{4}$. Find the direction cosines of the line.
9. Define linear programming problems.
10. If E is an event of a sample space ‘S’ of an experiment then find P(S/F).

Section – B

II. Answer any ten of the following: **10 × 2 = 20**

11. If $f:R\to R$ defined by f(x) = 10x – 7 and g = f^{-1} then find g(x).
12. Write in simplest form $\sin^{-1}\left[\frac{1-e^{4x}}{1+e^{4x}}\right]$
13. Solve $\tan^{-1}(1+x)+\tan^{-1}(1-x)=\frac{\pi}{2}$.
14. Find the equation of the line passing through (1,2) and (3,6) using determinants.
15. If $y=\cos^{-1}\left[\frac{1-(\log x)^2}{1+(\log x)^2}\right]$. Find dy/dx.
16. If $x=a[\cos t+\log(\tan t/2)]$, y = a sin t. Then prove that dy/dx = tan t.
17. Find the approximate value of $(401)^{1/2}$.
18. Prove that the function f(x) = log(sinx) is strictly increasing on $\left(0,\frac{\pi}{2}\right)$ and strictly decreasing on $\left(\frac{\pi}{2},\pi\right)$.
19. Evaluate $\int \sec^3 x\tan x\,dx$
20. Evaluate $\int \frac{\sin 2x}{(a+b\cos x)^2}dx$

21. Find the order and degree of DE $y^{11}+2y^{1}+\sin y=0$

22. Find $|\vec{x}|$ if $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a})=12$ *and* $\vec{a}$ *is a unit vector*

23. Find the angle between lines
$\vec{r}=3\vec{i}+\vec{j}-2\vec{k}+\lambda(\vec{i}-\vec{j}-2\vec{k})$ and
$\vec{r}=2\vec{i}-\vec{j}+56\vec{k}+\lambda(3\vec{i}-5\vec{j}-4\vec{k})$

24. An electronic assembly consists of 2 subsystems say A and B. From the previous testing, the probability are P(A fails) = 0.2, P(B fails alone) = 0.15. P(A and B fails) = 0.15. Evaluate P(A fails/B has failed).

Section – C

III. Answer any six of the following: **5 × 6 = 30**

25. Prove that $R=\{(a-b): a,b\in z,\ \ a-b\,\textit{is a multiple of}\ 5\}$ is an equivalence relation.

26. If $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x$. Prove that $x=\frac{1}{\sqrt{3}}$.

27. For any square matrix A with real entries. Then prove that ($A + A^1$) is a symmetric and ($A - A^1$) is skew symmetric.

28. If $x^y=e^{y-x}$. Then prove that $dy/dx=\frac{2-\log x}{(1-\log x)^2}$

29. Verify Lagrange's mean value theorem for the function $f(x) = x^3 - 5x^2 - 3x$ in [1, 3].

30. Find the points on the curve $4x^2 + 9y^2 = 1$. Where the tangents are perpendicular to the line $2y + x = 0$.

31. Evaluate $\int\frac{\sqrt{16+(\log x)^2}}{x}dx$

32. Evaluate $\int\frac{(x-3)e^x}{(x-1)^3}dx$.

33. Find the area of the region lying in the first quadrant and bounded by $y = 4x^2$, $x = 0$ $y = 1$ and $y = 4$.

34. Form the differential equation representing the family of curves given by $(x - a)^2 + 2y^2 = a^2$.

35. If G is the centroid of the triangle ABC prove that $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}=0$

36. If $\vec{a}+\vec{b}+\vec{c}=0$ then prove that $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$.

37. Find the equation of the plane passing through (1, 1, 0), (1, 2, 1), (-2, 2, 1).

38. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently find the probability that: a) the problem is solved. B) Exactly one of them solves the problem.

Section – D

IV. Answer any six of the following: **5 × 6 = 30**

39. Let $A=R-\left\{\frac{7}{5}\right\}$ and $B=R-\left\{\frac{3}{5}\right\}$. If $f:A\to B$ is defined by $f(x)=\frac{3x+4}{5x-7}$ and $g:B\to A$ is defined by $g(x)=\frac{7x+4}{5x-3}$. Prove that fog = I_B and gof = I_A. Where I_A and I_B are identify functions defined on B and A respectively.

40. Find the value of x if $\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$

41. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$. Find A^{-1} using A^{-1} solve the system of equations $\begin{matrix} 2x - 3y + 5z = 11 \\ 3x + 2y - 4z = -5 \\ x + y - 2z = -3 \end{matrix}$

42. If $y = \cos(a\sin^{-1} x)$. Prove that $(1 - x^2)y_2 - xy_1 + a^2 y = 0$

43. Find the integral of $\sqrt{a^2 - x^2}$ with respect to x then evaluate $\int \sqrt{1 - 4x - x^2}\,dx$.

44. A cone has a depth of 10 cms and the base of 5 cms radius. Water is poured into it at the rate of $\frac{3}{2}$ c.c/min. Find the rate at which the level of water in the cone is rising when the depth is 4 cms.

45. Using integration find the area of the region bounded by the triangle whose vertices are (-1, 0), (1, 3) and (3, 2).

46. Find the equation of the curve passing through the point (0,2) given that the sum of the co-ordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5.

47. Derive equation of plane perpendicular to a given vector and passing through a given point both in vector and Cartesian form.

48. A card from a pack of 52 cards is lost. From the remaining cards of the pack, 2 cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.

Section – E

V. Answer any six of the following: $1 \times 10 = 10$

49. a) Prove that $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^{a} f(x)dx, & f(2a - x) = f(x) \\ 0 & \text{if } f(2a - x) = -f(x) \end{cases}$ and hence evaluate $\int_0^{2\pi} \cos^5 x\,dx$

b) Find k if $f(x) = \begin{cases} \dfrac{1 - \cos 2x}{1 - \cos x} & \text{if } x \neq 0 \\ k & \text{if } x = 0 \end{cases}$ is continuous at x = 0

50. a) One kind of cake requires 200 gm of flour and 25 gm of fat, and another kind of cake require 100gm of flour and 50 gm of fat. Find maximum number of cake that can be made from 5 kg of flour and 1 kg of fat. Using graphical method.

b) Prove that $\begin{vmatrix} a^2 + 1 & ab & ac \\ ab & b^2 + 1 & bc \\ ca & cb & c^2 + 1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$

****