SRI BHAGAWAN MAHAVEER JAIN COLLEGE Vishweshwarapuram, Bangalore. **Mock Exam 2 - Feb.2016**		**Course:**	II PUC
		Subject:	**Mathematics**
		Max. Marks:	100
		Duration:	3:15 Hrs.

Instructions :

(i) The question paper has five parts namely A, B, C, D and E. Answer all the parts.

(ii) Use the graph sheet for the question on linear programming in PART E.

PART – A

I Answer ALL the questions: 10 x1=10

1. Let * be a binary operation defined on set of rational numbers by $a*b=\frac{ab}{4}$, find the identity element.
2. Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$.
3. Define scalar matrix.
4. Find the minor of an element 6 in the determinant $\Delta=\begin{vmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{vmatrix}$.
5. If $y=e^{\cos x}$, find $\frac{dy}{dx}$.
6. Find the anti-derivative of $(ax+b)^2$ with respect to x.
7. Define co-initial vectors.
8. Find the equation of the plane having intercept 3 on the y-axis and parallel to ZOX plane.
9. Define optimal solution.
10. If $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$ and $p(A\cap B)=\frac{1}{6}$, show that A and B are independent events.

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PART – B

II Answer any TEN questions: 10 x 2 = 20

11. Show that function $f:R\to R$ defined as $f(x)=x^2$ is neither one-one nor onto.
12. Prove that $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$, $\forall x\in R$.
13. Write $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$, $0<x<\pi$ in simplest form.
14. Prove that, if each element of a row of 3×3 determinant is multiplied by a constant k, then the value of determinant is multiplied by k.
15. If $y=\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ find $\frac{dy}{dx}$ where $|x|<a$.
16. Find $\frac{dy}{dx}$, if $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$.
17. Find the local maximum value of the function $f(x)=x^3-3x$.
18. Evaluate $\int\frac{\sin\left(\tan^{-1}x\right)}{1+x^2}dx$
19. Evaluate $\int\sin 4x\sin 8x\,dx$.
20. Form the differential equation representing the family of curves $y=mx$ where m is arbitrary constant.

21. Show that the vectors $\hat{i}+\hat{j}+\hat{k}$ is equally inclined to the axis OX,OY,OZ respectively.
22. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}+\hat{k}$ find the unit vector parallel to the vector $2\vec{a}-\vec{b}+3\vec{c}$.
23. Find the angle between the lines whose direction ratios are a,b,c and b – c , c – a , a – b .
24. A die is thrown. If E is the event " the number appearing is a multiple of 3" and F is the event " The number appearing is even" then find whether E and F are independent?

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PART- C

III Answer any TEN questions: 10 x 3 =30

25. If * is a binary operation defined on $A=N\times N$, by $(a,b)*(c,d)=(a+c,b+d)$. Prove that * is both commutative and associate. Find the identity if it exist.
26. Prove that $\sin^{-1}\left(\frac{12}{13}\right)+\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{63}{16}\right)=\pi$.
27. If $x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$, find the values of x and y .
28. If $x=\sqrt{a^{\sin^{-1}t}}$ and $y=\sqrt{a^{\cos^{-1}t}}$ show that $\frac{dy}{dx}=-\frac{y}{x}$.
29. Find the absolute maximum and absolute minimum value of the function $f(x)=\sin x+\cos x$, $x\in[0,\pi]$.
30. Prove that the function f given by $f(x)=\log(\cos x)$ is strictly decreasing on $\left(0,\frac{\pi}{2}\right)$ and strictly increasing on $\left(\frac{\pi}{2},\pi\right)$.
31. Evaluate $\int\frac{x+3}{\sqrt{5-4x+x^2}}dx$.
32. Find the anti-derivative of $\frac{df(x)}{dx}=4x^3-\frac{3}{x^4}$ such that $f(2)=0$.
33. Find the area of the region bounded by the curve $y=x^2$ and the line $y=2$.
34. In a bank, principal increases continuously at the rate of 5% per year. Find the principal in terms of time t.
35. Find a unit vector perpendicular to each of the vector $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ where $\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$.
36. If $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+5\hat{j}$, $3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$ are the position vector of the points A, B, C and D respectively. Then find the angle between $\vec{AB}$ and $\vec{CD}$ deduce that $\vec{AB}$ and $\vec{CD}$ are collinear.
37. Find the vector equation of the plane passing through the points $R(2,5,-3)$, $S(-2,-3,5)$ and $T(5,3,-3)$.
38. A couple has two children (i) Find the probability that both children are males, if it is known that at least one of the children is male. (ii) Find the probability that both children are females, if it is known that the elder child is female.

PART-D

IV Answer any SIX questions: 6 x 5 = 30

39. Let $f:R\rightarrow R$ be defined as $f(x)=10x+7$. Find the function $g:R\rightarrow R$ such that $g\circ f=f\circ g=I_R$.

40. If $A=\begin{bmatrix}0 & 6 & 7\\ -6 & 0 & 8\\ 7 & -8 & 0\end{bmatrix}$, $B=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 2\\ 1 & 2 & 0\end{bmatrix}$ and $C=\begin{bmatrix}2\\ -2\\ 3\end{bmatrix}$ calculate AB, AC and $A(B+C)$, verify that $A(B+C)=AB+AC$.

41. If $A=\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$, find A^{-1}, using A^{-1} solve the system of equations $2x-3y+5z=11$, $3x+2y-4z=-5$ and $x+y-2z=-3$.

42. If $y=\left(\tan^{-1}x\right)^2$ show that $\left(x^2+1\right)^2 y_2+2x\left(x^2+1\right)y_1=2$.

43. A ladder 5m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4m away from the wall?

44. Find $\int\sqrt{x^2+a^2}\,dx$ and hence evaluate $\int\sqrt{x^2+2x+5}\,dx$.

45. Find the area of the circle $4x^2+4y^2=9$ which is interior to the parabola $x^2=4y$.

46. Find the general solution of the equation $\left(1+x^2\right)dy+2xydx=\cot x dx$.

47. Derive the equation of a plane perpendicular to a given vector and passing through a given point both in vector and Cartesian form.

48. An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear a mark 'Y'. A ball is drawn at random from the urn. It's mark is noted down and it is replaced. If 6 balls are drawn in this way find the probability that (i) all will bear X mark (ii) not more than 2 will bear Y mark (iii) at least one ball will bear Y mark.

PART – E

V Answer any ONE questions: 1 x 10 = 10

49. (a) Prove that $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$ and hence evaluate $\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}}$.

(b) Prove that $\begin{vmatrix}x & x^2 & yz\\ y & y^2 & zx\\ z & z^2 & xy\end{vmatrix}=(x-y)(y-z)(z-x)(xy+yz+zx)$.

50. (a) A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods F1 and F2 are available. Food F1 costs Rs 4 per unit food and F2 costs Rs 6 per unit. One unit of food F1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem. Find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.

(b) Find the value of a and b such that the function defined by $f(x)=\begin{cases}5 & if \quad x\le 2\\ ax+b & if \quad 2<x<10\\ 21 & if \quad x\ge 10\end{cases}$ is a continuous function.

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