JAIN COLLEGE

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Bangalore - 560 098

Date: *II PUC*   *SUBJECT: Mathematics*

MOCK - II

Timings Allowed: 3 Hrs 15 Minutes   *Total Marks: 100*

PART A

I. Answer all.   **1X10=10**

1. Define an injective function.
2. Find the principle value of $Cos^{-1}(-\frac{1}{2})$
3. What is symmetric matrix?
4. Find the value of $\begin{matrix} 200 & 201 \\ 202 & 203 \end{matrix}$ without actual expansion.
5. If $2x + 3y = cosx$ $find$ $\frac{dy}{dx}$
6. Evaluate $\int x \, \overline{x} dx$
7. Find the scalar and vector components of the vector with initial points (2,1) and terminal point (5,8).
8. Find the direction cosines of the sides of the triangle whose vertices are (3,5,-4), (-1,1,2) and (-5,-5-2).
9. Define feasible solution.
10. A fair die is rolled $E = \{1,5\}$ $F = \{1,4\}$ $Find\ P\left(\frac{E}{F}\right)$.

PART-B

II. Answer any TEN questions.   **10X2=20**

11. Find gof and fog if $f(x) = 8x^2$ and $g(x) = x^{\frac{1}{3}}$
12. Prove that cos^{-1}(-x)=π-cos^{-1}x, x$\in$[-1,1]
13. Find the value of $tan^{-1}\left(\overline{3}\right) - Cot^{-1}(-\overline{3})$
14. Find the area of the triangle with vertices (1,0), (6,0), (4,3) using determinant.
15. Differentiate x^{sinx}, $x > 0$ $w.r.t$ x
16. If x= at^2 $y = 2at$ $find$ $\frac{dy}{dx}$
17. If radius of sphere is measured as 7m with error 0.02m. Find the approximate error in calculating its volume.
18. *Evaluate* $\int e^x(sinx + cosx)dx$
19. Evaluate $\int_0^{\frac{\pi}{2}} cos^2 x dx$
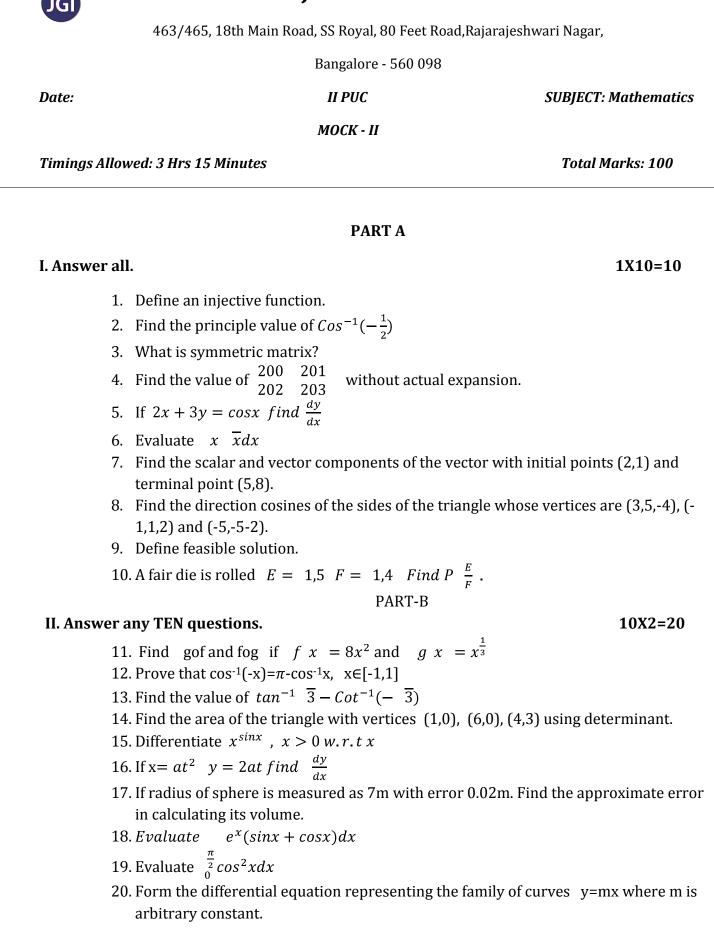20. Form the differential equation representing the family of curves y=mx where m is arbitrary constant.

21. Show that the vector $2i - 3j + 4k$ and $-4i + 6j - 8k$ are collinear.

22. Find the area of the parallelogram whose adjacent sides are determined by
$$a = i - j + 3k, \quad b = 2i - 7j + k$$

23. Find the distance of the plane $3x - 3y + 4z - 6 = 0$ $from\ the\ origin.$

24. Two cards are drawn at random without replacement from a deck of 52 cards. Find the probability that both cards are red

Part-C

III. Answer any TEN questions.   10X3=30

25. Show that the relation R in the set of all integers Z defined by
$$R\ \ a, b\ \ /2\ divides\ a - b\ \ is\ an\ equivalence\ relation.$$

26. Prove that $tan^{-1}x + tan^{-1}\ \ \frac{2x}{1-x^2}\ = tan^{-1}\ \ \frac{3x-x^3}{1-3x^2}\ , \quad x\ <\frac{1}{\sqrt{3}}$

27. Using the elementary transformation find the inverse of $\begin{matrix} 1 & 3 \\ 5 & -1 \end{matrix}$

28. If $y = Sin^{-1}x$ $show\ that$ $1 - x^2\ y'' - xy' = 0$

29. If $y = tan^{-1}x\ ^2$ $S.T$ $x^2 + 1\ ^2y'' + 2x\ x^2 + 1\ y' = 1$

30. Find the equation of the tangent and normal to the circle $x^2+y^2=1$ at the point x_o, y_o

31. $Evaluate\ \ \int_0^\pi log\ 1 + sinx\ dx$

32. Express $\int_0^1 e^{2x}\ dx$ as limit of a sum

33. Find the area of the region bounded by the curve $x^2=4y$ and its latus rectum

34. In a cultural test, the bacteria count is 1, 00,000. The number is increased by 10% in 2 hours. In how many hours the count reaches 2, 00,000 if the rate of growth of the bacteria is proportional to the number present.

35. Find a unit vector perpendicular to each of $a + b\ and\ a - b$ where $a = i + j + k\ and\ b = i + 2j + 3k$

36. Show that the position vector of the point p which divides the line joining the points A and B internally in the ratio m:n is $\frac{mb+na}{m+n}$

37. Find the vector and Cartesian equation of the line that passes through the points (3,-2,-5) and (3,-2,6)

38. A fair coin is tossed 8times. Find the probability of at least six tails.

PART-D

IV Answer any SIX questions.   6X5=30

39. Consider the function f:$R_+ \rightarrow\ 4, \infty)$ $given\ by\ f\ x\ = x^2 + 4$ show that f is invertible and find the inverse of f.

40. If $A^T = \begin{matrix} 5 & 3 \\ 1 & -2 \end{matrix}$ $B = \begin{matrix} 1 & 0 \\ 1 & 2 \end{matrix}$ $verify\ \ AB\ ^T = B^T A^T$

41. Solve the following system of equations by matrix method x-y+2z=7,   3x+4y-5z=5,   2x-y+3z=2.

42. If $y = Ae^{mx} + Be^{nx}$ show that $y'' - m+n\ y' + mny = 0$.

43. Find the integral of $\dfrac{dx}{a^2+x^2}$ hence evaluate $\dfrac{dx}{x^2+9}$

44. Find the area of the region bounded by the curve $y^2=4x$ and $x^2=4y$

45. Find the particular solution of $\left(1+x^2\right)\dfrac{dy}{dx}+2xy=\dfrac{1}{1+x^2}$

46. Derive the equation of a plane passing through three non collinear points both in vector and Cartesian form

47. Find the probability of getting 5 exactly twice in 7 throws of a die.

48. Derive equation for the distance between two skew lines.

PART-E

V Answer any ONE question   **1X10=10**

49 a). Prove that $\displaystyle\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

Hence evaluate $\displaystyle\int_0^{\pi}\dfrac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}}dx$

b). Find the value of k if

$f(x) = \begin{cases} kx+1 & if\ x \le 5 \\ 3x-5 & if\ x > 5 \end{cases}$ is continuous at x=5.

50 a).A cooperative society of farmers has 50 hectare of land to grow two crops X and Y. The profit from crops X and Y per hectare are estimated as Rs.10,500 and Rs.900 respectively .To control weeds , a liquid herbicide has to be used for crops X and Y at rates of 20 liters and 10 liters per hectare. Further, no more than 800 liters of herbicide should be used in order to protect fish and wild life using a pond which collects drainage from this land. How much land should be allocated to each crop so as to maximize the total profit of the society?

b). Prove that $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = \left(x^3-1\right)^2$

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