JAIN COLLEGE, Bangalore
Mock Paper - 1 January - 2016
II PUC - Basic Mathematics (75)

## PART A

I. Answer any ten questions
$1 \times 10=10$

1. If $A=\left[\begin{array}{l}1 \\ 4 \\ 2\end{array}\right]$ and $B=\left[\begin{array}{lll}1 & 3 & 4\end{array}\right]$.find $A B$
2. In how many ways can 9 flowers of different colors be strung together to form a garland?
3. Write the truth value of "If $\sqrt{2}$ is irrational then $\sqrt{2}$ is a real number".
4. Find the mean proportional of $1 / 16$ and $1 / 25$.
5. Define yield
6. If $\cos A=\frac{\sqrt{3}}{2}$ find $\cos 2 A$
7. Find the other end of the diameter if one end of the diameter of the circle $x^{2}+y^{2}+4 x-6 y-12=0$ is $(-5,-1)$
8. Evaluate $\lim _{x \rightarrow 0} \frac{3^{x}-2^{x}}{x}$
9. Differentiate $x+\sqrt{x y}=x^{2}$ wrt x
10. Integrate $\frac{7^{x}-6.8^{x}}{5^{x}}$ wrt x
II. Answer any ten questions
11. If $\left[\begin{array}{ll}2 & 3 \\ 7 & 5\end{array}\right]+\left[\begin{array}{cc}2 & x-2 \\ y-1 & 5\end{array}\right]=\left[\begin{array}{cc}4 & 1 \\ 7 & 10\end{array}\right]$ find x and y .
12. In how many ways can 6 people be chosen out of 10 people if one particular person is always included.
13. The probability of occurrence of 2 events $A$ and $B$ are $1 / 4$ and $1 / 2$ respectively. The probability of their simultaneous occurrence is $7 / 50$. What is the probability that neither A nor B occurs.
14. Write the converse and contrapositive of " If 2 straight lines are parallel then they do not intersect".
15. A ratio in the lowest terms is $3: 7$. If the difference between the quantities is 24 , find the quantities.
16. Bankers discount and bankers gain on a certain bill due after sometime are Rs. 1250 and Rs. 50 respectively. Find the face value of the bill.
17. If $\tan A=5 / 6, \tan B=1 / 11$, show that $A+B=\pi / 4$
18. Prove that $\frac{\cos ^{3} A-\sin ^{3} A}{\cos A-\sin A}=1+\frac{1}{2} \sin 2 A$
19. Find the equation of the parabola given that its focus is $(0,-3)$ and directrix is $y=3$
20. Evaluate $\lim _{n \rightarrow \infty} \frac{\sum n^{3}}{n^{2} \sum n}$
21. Differentiate $(\sin x)^{\tan x}$ wrt $x$
22. If the displacement ' $s$ ' at any time ' t ' is given by $s=\sqrt{1-t}$. Show that velocity is inversely proportional to the displacement.
23. Evaluate $\int \sin ^{3} x d x$
24. Evaluate $\int_{0}^{3} \frac{x+3}{x+2} d x$

## PART C

III. Answer any ten
$3 \times 10=30$
25. Solve the equation by cramers rule
$3 x+2 y=8,4 x-3 y=5$
26. If $A=\left[\begin{array}{ccc}1 & 2 & -3 \\ 1 & -4 & 1 \\ 0 & 5 & 3\end{array}\right] B=\left[\begin{array}{ccc}4 & -2 & -3 \\ 2 & -4 & -1 \\ 0 & 1 & 3\end{array}\right] C=\left[\begin{array}{ccc}4 & 1 & 2 \\ 0 & 3 & 1 \\ -1 & -3 & 4\end{array}\right]$ verify $(\mathrm{A}+\mathrm{B}) \mathrm{C}=\mathrm{AC}+\mathrm{BC}$
27. A family of 4 brothers and 3 sisters is to be arranged for a photograph in one row. In how many ways can they be seated if
i. All the sisters sit together
ii. No 2 sisters sit together
28. Among the members of a committee, there are $75 \%$ males and $25 \%$ females. The probability that a male members becomes the president is 0.25 and probability that a female member becomes the president is 0.4 . find the probability that the person selected at random becomes the president.
29. If 10 men or 20 boys can d0 a piece of work is 30 days,how long will 30 boys and 5 men take to do the same work?
30. The bankers gain on a bill is $1 / 9^{\text {th }}$ of the bankers discount, rate of interest being $10 \%$ p.a. find the unexpired period of the bill.
31. Sukanya holds Rs. 8000 of $3 \%$ stock. She sells it at Rs. 110 and invests the proceeds in $5 \%$ stock. Therby her income increases by Rs.260. find the market price of 5\% stock.
32. When the rate of sales tax is decreased from $9 \%$ t0 7\%. For a radio ,Rahul has to pay Rs. 632 less for it . what is the listed price of the radio?
33. Find the focus, directrix, latus rectum, axis of the parabola $y^{2}=-12 x$
34. Differentiate $\log \sin x$ wrt $\sqrt{\cos x}$
35. A drop of ink spreads over a blotting papers that the circumference of the blot is $4 \pi \mathrm{~cm}$ and it changes $3 \mathrm{~cm} / \mathrm{sec}$. find the rate of increase of its radius and also find the rate of increase of its area?
36. Show that $x^{x}$ is maximum at $x=1 / e$
37. Evaluate $\int \frac{3 x+2}{2 x-5} d x$
38. Evaluate $\int \frac{5}{x(3+2 \log x)^{5}} d x$

## PART D

IV. Answer any six
39. Find the coefficient of $x^{18}$ in $\left(x^{2}+\frac{3 a}{x}\right)^{15}$
40. Resolve into partial fraction $\frac{2 x^{2}+3 x+2}{x^{2}-x-2}$
41. Show that $[(p \rightarrow q) \wedge(q \rightarrow r)] \rightarrow(p \rightarrow r)$ is a tautology
42. Walking 4 kmph a student reaches his college 5 min late and if he walks at 5 kmph , he reache in $21 / 2$ min early. What is the distance from his house to the college?
43. An aircraft manufacturer supplies aircraft engines to different airlines.; they have just completed an initial order for 30 engines involving a total of 6000 direct labour hours at Rs20 per hour. They have been asked to bid for a prospective contract of 90 engines. It is expected that there will be $80 \%$ learning effect. Estimate the labour cost for the new order.
44. Solve LPP graphically

Minimize $\mathrm{Z}=\mathrm{x}-7 \mathrm{y}+190$
Subject to $x+y \leq 8, y \leq 5, x \leq 5, x+y \geq 4$

$$
x, y \geq 0
$$

45. Show that $\cos ^{2} \theta+\cos ^{2}\left(60^{\circ}-\theta\right)=3 / 2$
46. find the equation of the circle passing through the points $(1,-4)$ and $(5,2)$ and has its centre on the line $x-2 y+9=0$
47. if $y^{x}+x^{y}=a^{b}$ show that $\frac{d y}{d x}=\frac{-\left[y^{x} \log y+y x^{y-1}\right]}{x y^{x-1}+x^{y} \log x}$
48. the marginal cost=8+0.08x and the marginal revenue $=16$. Find the total revenue,total cost and total profit. Assume the fixed cost is nil.

## PART E

V. Answer any one question $10 \times 1=10$
49.
a) A sales person Samanth has the following record of sales for the month of January, Febraury and March 1996 for 3 products $\mathrm{A}, \mathrm{B}$ and C . he is paid a commission at fixed rate per unit but at varying rates for products $\mathrm{A}, \mathrm{B}$ and C . .

| Months | Sales in units |  |  | commission |
| :--- | :---: | :---: | :---: | :---: |
|  | A | B | C | In Rs |
| January | 9 | 10 | 2 | 800 |
| Febraury | 15 | 5 | 4 | 900 |
| March | 6 | 10 | 3 | 850 |

Find the rate of commission payable on $\mathrm{A}, \mathrm{B}$ and C per unit sold.
b) A person is at the top of a tower 75 ft high from there he observes a vertical pole and finds the angle of depression of the top and the bottom of the pole which are $30^{\circ}$ and $60^{\circ}$ respectively. Find the height of the pole.
50.
a) Prove that $\lim _{x \rightarrow 0} \frac{\sin x}{x}=1 \mathrm{x}$ is an radian and hence deduce that $\lim _{x \rightarrow 0} \frac{\tan x}{x}=1$
b) A producer named Samarth has 30 and 17 units of labour and capital respectively which he can use to produce 2 types of goods A and B. to produce one unit of A ,2units of labour and 3units of capital are required. Similarly 3units of labour and 1unit of capital is required to produce one unit of B . if A and B are priced at Rs. 100 and Rs .120 per unit respectively,how should the producer use his resources to maximize the total revenue. Formulate the LPP.

## PART A

I. Answer any ten questions
$1 \times 10=10$

1. Solve for $\mathrm{x}:\left|\begin{array}{cc}x & 3 \\ 12 & x\end{array}\right|=0$

2. Negate the following : $(p \rightarrow(q \wedge r))$
3. Find the compound ratio of the ratios 1:2,2:3,3:4.
4. Define learning curve?
5. Find the value of $3 \sin 10^{\circ}-4 \sin ^{3} 10^{0}$
6. Find the equation of the circle centre is at $(-1,-2)$ and diameter $\mathrm{d}=25 \mathrm{cms}$
7. Evaluate: $\lim _{x \rightarrow 0} \frac{\cos ^{2} x}{1-\sin x}$
8. Differentiate $x^{e}+e^{x}+e^{e}$ wrt x
9. Integrate $\int \frac{8}{\cos e c x} d x$ wrt x
II. Answer any ten questions
10. If $A=\left[\begin{array}{cc}1 & -3 \\ -4 & -1\end{array}\right]$ and $B=\left[\begin{array}{cc}3 & 4 \\ -5 & 1\end{array}\right]$ find C , if $2 \mathrm{C}=\mathrm{A}+\mathrm{B}$
11. How many 6 digit numbers can be formed from $1,2,3,4,5,6$ (no digit are repeated) which is divisible by 5
12. A bag has 15 tickets numbered from 1 to 15 , two tickets are drawn at random from the bag find the probability that both the numbers are prime?
13. Write the converse and contrapositive of the following statement: if $x \in(A \cup B)$ then $x \in A$ or $x \in B$
14. What must be subtracted from the ratio 7:4, so that it becomes 5:2.
15. The bankers gain on a bill is $1 / 5^{\text {th }}$ of the bankers discount and the rate of interest is $20 \%$ p.a, find the unexpired period of the bill.
16. Prove that $\tan C=3$, if $\tan A=1$ and $\tan B=2$ and also given that $A+B+C=180^{\circ}$
17. Prove that $\cos ^{4} x-\sin ^{4} x=\cos 2 x$
18. Write the equation of axis, directrix and tangent at the vertex for the parabola $X^{2}+16 y=0$
19. Evaluate the limit: $\lim _{x \rightarrow 0}\left(\frac{e^{x}-e^{-x}}{x}\right)$
20. If $\mathrm{y}=\mathrm{x}+\tan \mathrm{x}$ show that $\cos ^{2} \mathrm{x} \frac{d y}{d x}=2-\sin ^{2} \mathrm{x}$.
21. The total cost of the commodity is given by $\mathrm{c}=\mathrm{x}^{2}-7 \mathrm{x}+2$, where x is the number of units produced. If price per unit is Rs. 5 find the profit function.
22. Integrate wrt $x \cos ^{3} x$
23. Find the area bounded by the curve $3 x^{2}=4 y, y$ axis $y=1$ and $y=2$.
III. Answer any ten questions
24. If $A=\left[\begin{array}{ll}3 & 7 \\ 2 & 5\end{array}\right]$ and $B=\left[\begin{array}{ll}6 & 8 \\ 7 & 9\end{array}\right]$ verify $(A B)^{-1}=B^{-1} A^{-1}$
25. Solve by cramer's rule : $2 x+y=1, x-3 y=4$
26. Find the number of permutation of the letters of the word "Engineering". How many of these
a. begin with GIN and end with GRIN
b. all vowels are together
27. What is the probability that a card drawn from a pack of 52 cards is red or a queen?
28. A mixture contains milk and water in the ratio $6: 1$ on adding 5 liters of water the ratio of the milk and water becomes 7:2, find the quantity of milk in original mixture.
29. A bill for Rs. 2920 was drawn on sep $11^{\text {th }}$ for 3 months after due and was discounted at $16 \%$ p.a for Rs.2875.20, on what date was the bill discounted?
30. Sukanya holds Rs. 8000 of $3 \%$ stock she sells it at Rs. 110 and invest the proceeds in $5 \%$ stock thereby her income increased by Rs. 260 find the market price of 5\% stock.
31. Sharath goes to a departmental stores and purchase the following articles
a. A rain coat for Rs. 300 S.T @ 10\%
b. A pair of shoes for Rs. 460 S.T @ 9\%
c. Food article for 450 S.T @ 5\%
d. Cloth for Rs. 800 S.T @ $1 \%$. Calculate the total amount of the bill?
32. Find the equation of the parabola given vertex at origin and passing through the point (2,-3) and which is symmetric about y axis.
33. If $x=e^{\log \cos 4 \theta}, y=e^{\log \sin 4 \theta}$ show that $\frac{d y}{d x}=\frac{-x}{y}$.
34. A ladder of 15 ft long leans against the wall , if the top slides downwards at the rate of $2 \mathrm{ft} / \mathrm{sec}$. find how fast is the lower end moving when the lower end is at a distance 12 ft from the wall?
35. The demand function of a firm is $p=500-0.2 q$ and the total cost fun $C=25 q+10000 p$ is the price and q is the quantity at which the profit of the firm is maximized what is the price charged
36. Integrate $\int \sin 2 x \sqrt{1+\sin ^{2} x} d x$ wrt x
37. Integrate by parts $\int \log x(1+x) d x$
IV. Answer any six $5 \times 6=30$
38. If $A=\left[\begin{array}{ccc}1 & -4 & -2 \\ -2 & -5 & 4 \\ 1 & 2 & 1\end{array}\right]$ verify $A \cdot \operatorname{adj} A=\operatorname{adj} A \cdot A=|A| \mid$
39. The second ,third and fourth term in the binomial expression $(x+a)^{n}$ are 240,720 and 1080 respectively find $\mathrm{x}, \mathrm{a}$ and n .
40. Resolve into partial fractions : $\frac{x^{2}+1}{(x+1)(x-2)^{2}}$
41. Examine whether the given equation is logically equivalent $p \wedge q$ and $\sim(p \rightarrow \sim q)$
42. The monthly income of $A$ and $B$ are in the ratio $9: 7$ and those of $B: C$ is in the ratio $3: 2$. If $10 \%$ of $A$ ' $s$ income and $15 \%$ of $C$ 's income differ by Rs.18. find the income of $A, B$ and $C$
43. The production manager of a company obtained the following equation for learning effect $\mathrm{Y}=1400 \mathrm{X}$ ${ }^{0.3}$ this function is based on the company's experience for assembling the first 50 units of the
product. The company was asked to bid a new order of 100 additional units and the labour cost for producing an additional 100 units is at the rate of Rs.20/hr.
44. Archana, a dietician wishes to mix two types of food F1 and F2 in such a way that the vitamin contents of the mixture contains atleast 6 units of vitamin $A$ and 8 units of vitamin $B$. food $F 1$ contains 2 Kg of vitamin A and 3 Kg of vitamin B while food F 2 contains 3 Kg of vitamin A and 4 Kg of vitamin B. food F1 cost Rs. $50 / \mathrm{Kg}$ anf F2 cost Rs. $75 / \mathrm{Kg}$ (solve graphically to minimize the cost of the mixture)
45. Prove that: $\frac{\cot A}{\cot A-\cos 3 A}+\frac{\tan A}{\tan A-\tan 3 A}=1$
46. If $x \sqrt{1+y}+y \sqrt{1+x}=0$ where $x \neq y$ showthat $\frac{d y}{d x}=\frac{-1}{(1+x)^{2}}$
47. Evaluate $\int_{2}^{3} \frac{1}{(x+1)(x+2)} d x$
V. Answer any one question $10 \times 01=10$
48. 

a. Show that the points are concylic $(2,-4)(3,-1)(3,-3)$ and $(0,0)$.
b. The angle of depression of two boats as observed from the mast of the ship 50 mts are $45^{\circ}$ and $30^{\circ}$ respectively what is the distance between the boats if they are on the same side of the mast head in line with it.
50.
a. If n is any rational integer and a is a non zero number, prove that $\lim _{x \rightarrow a}\left(\frac{x^{n}-a^{n}}{x-a}\right)=n a^{n-1}$ \{all 3 cases\}
b. Simplify $(3+\sqrt{2})^{4}+\left(3-\sqrt{2}^{4}\right.$ using binomial theorem.

