I. Answer any 10:
$10 \times 1=10$

1. If $\left[\begin{array}{l}x \\ y \\ z\end{array}\right]=\left[\begin{array}{lll}1 & 0 & -1 \\ 2 & 0 & -1 \\ 0 & 1 & -2\end{array}\right]\left[\begin{array}{l}1 \\ 1 \\ 1\end{array}\right]$, find $x, y, z$.

How many six digit numbers can be formed with the digits $2,7,6,1,9,8$.
2. Negate the following:" if 6 is a divisor of 120 then 486 is not divisible by 6 ".
3. Find the compound ratio of $3: 5$ and $4: 7$.
4. How much does maya realize by selling Rs. 30000 stock at 20 discounted?
5. Express the following as product of two trigonometric functions, $\cos 10^{\circ}-\cos 50^{\circ}$.
6. Show that the circle $x^{2}+y^{2}+4 x-3 y+4=0$ touches $x$-axis.
7. Evaluate : $\lim _{x \rightarrow-2}\left[\frac{x^{5}+32}{x+2}\right]$
8. Differentiate w.r.t $\mathrm{x}, \sqrt{\cot \sqrt{x}}$.
9. Integrate w.r.t $\mathrm{x}, \int \frac{5 \sin x}{3 \cos ^{2} x} d x$.
II. Answer any 10
$10 \times 2=20$
10. Solve for x : if $\left|\begin{array}{ll}2 & -x \\ x & -2\end{array}\right|=0$
11. If ${\underset{r}{n}}_{\underset{r}{2}}=3024,{ }_{r}^{n}=126$, find r .
12. What is the probability that a randomly chosen 2 digit positive integer is a multiple of 3 .
13. Write the inverse and contrapositive of the implication," if $x \in A \cup B$ then $x \in A$ or $x \in B$ ".
14. What must be added to each term in the ratio $5: 6$, so that it becomes $8: 9$.
15. A banker discounts a bill for certain amount having 73 days to run before it matures at $15 \%$ P.a . The discounted value of the bill is Rs.970, what is the face value of the bill?
16. Prove that: $\frac{\sin 3 \theta}{1+2 \cos 2 \theta}=\sin \theta$.
17. If $\sin A=\frac{7}{25}, \cos B=\frac{-12}{13}$, find the value of $\cos (A-B)$.
18. Find the equation of the parabola, focus $(1,0)$ and directrix is $x=-1$.
19. Evaluate: $\lim _{n \rightarrow 0}\left(\frac{n+3}{3}\right)^{\frac{2}{n}}$.
20. Differentiate w.r.t $\times$ (by product rule)

$$
\left(x^{2}-2 x+1\right)\left(e^{x}+4\right)
$$

21. The total cost of the commodity is given by $c=x^{2}-7 x+2$, where $x$ is the number of units and the price per unit is Rs5.00. find the profit function.
22. $\int\left(\frac{7^{x}-6.8^{x}}{5^{x}}\right) d x$.
23. Find the area bounded by the curve $x=2 y^{2}, y$-axis and the abscissa $y=2$ and $y=4$.

## III. Answer any 10

$10 \times 3=30$
24. Prove that: $\left|\begin{array}{lll}1 & a+b & a^{2}+b^{2} \\ 1 & b+c & b^{2}+c^{2} \\ 1 & c+a & c^{2}+a^{2}\end{array}\right|=(a-b)(b-c)(c-a)$
25. If $A=\left[\begin{array}{ll}2 & 1 \\ 1 & 2\end{array}\right]$, find $A^{-1}$
26. A man has 10 relatives, 4 of them are ladies 3 are gentle men and 3 children. In how many ways can he invite 7 relatives to a dinner party so that.
i. There are exactly 2 ladies, 3 gentle men and 2 children.
ii. There are exactly 2 gentlemen and atleast 3 ladies.
27. The probability that a MBA aspirant will join IIM is $2 / 5$ and that he will join XLRI is $1 / 3$. Find the probability that
a. He will join IIM or XLRI
b. He will join neither IIM nor XLRI.
28. Four numbers are in proportion. The sum of the extremes is 54 and the sum of the mean is 36 . If the ratio of their means is $2: 1$, find the numbers.
29. The bankers gain on a bill is $1 / 5^{\text {th }}$ of the bankers discount and the rate of interest is $20 \%$ p.a, find the unexpired period of the bill.
30. A man invested equal sum of money in $4 \%, 5 \%$ and $6 \%$ stock, each stock being at par, if the total income of the man is Rs3600. Find the total investment.
31. Sanju, owner of a jeweler shop purchased a ear ring of Rs 2000 at $12 \%$ VAT and sells it at 2,300 to Radhika. If Radhika also pays $12 \%$ VAT to the shopkeeper how much did the shopkeeper deposit to the government as VAT.
32. Find the equation of the parabola given that vertex is at origin, axis is $y$-axis and passes through (1/2,2).
33. If $y=\log \left[\frac{1-\cos x}{1+\cos x}\right]$, prove that $\frac{d y}{d x}=2 \operatorname{cosec} x$
34. The sides of an equilateral triangle are increased at the rate $3 \mathrm{~cm} / \mathrm{sec}$, how fast is its area increasing when the side is 10 cm .
35. The demand function of a firm is $p=500-0.2 q$ and the total cost $c=25 q+10000$ ( $p=$ price, $\mathrm{q}=$ output). Find the output at which the profit of the firm is maximized .what is the price change
36. $\int \frac{\sin 2 x}{\left(1-\cos ^{2} x\right)^{3}} d x$
37. Integrate w.r.t $\mathrm{x}, \sec ^{2} x \sqrt{1+\tan x}$.

## IV. Answer any six

38. Find the coefficient of $\mathrm{x}^{18}$ in $\left(x^{2}-\frac{6}{x}\right)^{15}$.
39. Resolve $\frac{3 x+5}{(x+2)^{2}(x-3)}$ into partial fractions
40. Construct the truth table for $(p \rightarrow r) \Lambda(p \rightarrow q)$.
41. 8 men and 16 women can finish a job in 6 days but 12 men and 24 women can finish it in 8 days. How many days will 26 men and 20 women take to finish the same job?
42. XYZ Company supplies water tankers to government. The first water tanker takes 20000 labour hours. The government auditors suggest that there should be $90 \%$ learning effect rate. The management expects an order of 8 water tankers in the next year. What will be the labour cost the company will incur at the rate of Rs20 per hour?
43. Solve graphically: Maximize $\mathrm{z}=\mathrm{x}+\mathrm{y}$, subjected to the constraints: $2 x-y+1 \geq 0, x+y \leq 3, x \leq 2$ and $x, y \geq 0$.
44. If $\mathrm{A}+\mathrm{B}+\mathrm{C}=180^{\circ}$, prove that $\sin ^{2} \mathrm{~A}+\sin ^{2} \mathrm{~B}+\sin ^{2} \mathrm{C}=2+2 \cos \mathrm{~A} \cos \mathrm{~B} \cos \mathrm{C}$.
45. Find the equation of the circle passing through the point $(-1,2)$ and $(3,-2)$ and has its centre on $x=2 y$.
46. Differentiate 'tanx' from the first principle.
47. The marginal cost is $8+0.08 x$ and the marginal revenue is 16 . Find the total revenue, total cost and total profit. Assume that the fixed cost is nil.

## V. Answer any one

$10 \times 1=10$
48.
a. A sales person has the following records of sales for the month of January February and march 1996 for the product $A, B, C$. the person is paid a fixed rate of commission per unit but a varying rates for product $A, B$ and $C$.

| Months | Sales in Units |  |  | Commission in Rs. |
| :---: | :---: | :---: | :---: | :---: |
|  | A | B | C |  |
| January | 9 | 10 | 2 | 800 |
| February | 15 | 5 | 4 | 900 |
| March | 6 | 10 | 3 | 850 |

Find the rate of commission payable on $A, B$ and $C$ per unit sold.
b. Expand $(0.99)^{5}$ using binomial theorem up to 4 decimals.
49.
a. Prove: $\lim _{\theta \rightarrow 0} \frac{\sin \theta}{\theta}=1, \theta$ is in radians
b. A person standing on the bank of a river observe that the angle subtended by a tree on the opposite bank is $60^{\circ}$. When he returns 40 meters from the bank he finds the angle to be $30^{\circ}$, find the height of the tree and the breath of the river.

## JAIN COLLEGE, J C Road Bangalore

## PART - A

I. ANSWER ALL THE QUESTIONS

1. Find x , if $\left[\begin{array}{ll}3 & x \\ 4 & 7\end{array}\right]$ is symmetric.
2. Find $r$, if $15 c_{r+3}=15 c_{2 r-3}$
3. Negate the proposition $p \rightarrow(q \wedge r)$
4. If $a: b=2: 3$ and $b: c=5: 7$, find $a: c$
5. Find the index of learning for $80 \%$ learning effect.
6. Find the value of $3 \sin 10^{\circ}-4 \sin ^{3} 10^{0}$
7. Find the unit concentric circle with the circle $x^{2}+y^{2}-8 x+4 y=8$
8. Evaluate $\lim _{x \rightarrow 0} \frac{e^{-3 x}-1}{x}$
9. Differentiate $\mathrm{y}=\tan (\log (\sin \mathrm{x}))$ wrt x
10. Evaluate $\int \frac{x^{4}+3 x^{2}-5 x}{x^{2}} d x$

## PART - B

II. ANSWER ANY TEN QUESTIONS
$10 \times 2=20$
11. Prove that " If in a determinant of the elements of any row (or column) are multiplied by the same scalar say $k$, then the value of the new determinant is $k$ times the given determinant"
12. In how many ways can 6 boys and 6 girls be arranged in a circle so that no 2 boys are together?
13. A card is drawn from a pack of 52 playing card. What is the probability that the card is king given that the card is red?
14. Write the converse and contrapositive of ," if I work hard then I get the grade".
15. Three numbers are in the ratio $2: 3: 4$. If the sum of their squares is 1856 ,find the numbers.
16. A banker pays Rs. 2380 on a bill of Rs. 2500,73 days before the legal due date. Find the rate of discount charged by the banker.
17. Prove that $\cos \left(120^{\circ}+A\right)+\cos \left(120^{\circ}-A\right)+\cos A=0$
18. Solve $\frac{\cos 75^{\circ}+\cos 15^{\circ}}{\sin 75^{\circ}-\sin 15^{\circ}}$
19. Find the equation of the parabola given that its vertex is $(0,0)$, axis is $y$-axis and passes through ( $-1,-3$ ).
20. If the function $f(x)=\left\{\begin{array}{cc}\frac{e^{2 x}-1}{x} & x \neq 0 \\ a & x=0\end{array}\right.$ is continuous at $x=0$. Find a.
21. Differentiate $\sin ^{3} x$ wrt $\cos ^{3} x$
22. If the total cost function $C=9 q-3 q^{2}+\frac{q^{3}}{3}$ find the level of output at which average cost is minimized.
23. Evaluate $\int\left(4 x^{2}-2 x+7\right)^{\frac{3}{2}}(4 x-1) d x$
24. Evaluate $\int_{0}^{1} \frac{e^{x}+1}{e^{x}} d x$

## PART - C

III. ANSWER ANY TEN QUESTIONS
$10 \times 3=30$
25. If $A=\left[\begin{array}{ccc}2 & -1 & 3 \\ 1 & 3 & -4 \\ 4 & -2 & 0\end{array}\right]$ and $B=\left[\begin{array}{ccc}1 & 2 & -3 \\ -1 & 4 & 3 \\ 5 & -1 & 1\end{array}\right]$ then find $A B$ and $B A$. Check whether $A B \neq B A$
26. Prove that $\left|\begin{array}{ccc}a^{2} & b c & a c+c^{2} \\ a^{2}+a b & b^{2} & a c \\ a b & b^{2}+b c & c^{2}\end{array}\right|=4 a^{2} b^{2} c^{2}$
27. A team of 8 players has to be selected from 14 players. In how many ways the selection can be made if
a. 2 particular players are always included
b. 2 particular players are always excluded
28. Among the members of a committee, there are $75 \%$ males and $25 \%$ females. The probability that a male member becomes the president is 0.25 and probability that a female member becomes the president is 0.4 . find the probability that a person selected at random becomes the president
29. Rajeev planned his journey to Mumbai as follows. He will travel $5 / 9^{\text {th }}$ of the total distance by an aeroplane, $3 / 4^{\text {th }}$ of the remainder by train and the remaining distance of 200 km by a car. What is the total distance to Mumbai?
30. A bill of Rs. 1460 was drawn on $1^{\text {st }}$ April for 6 months after date and was discounted at 5\%p.a for Rs1451. On what date was the bill discounted?
31. Mr.Sandeep invests Rs15000 cash partly in $3 \%$ stock at 75 and partly in $6 \%$ debenture at 125 in such a way as to get a return on $4.5 \%$ for his money. How much does he invest his money in each?
32. A shopkeeper buys a mobile set at a discount rate of $20 \%$ from the wholesaler,the printed price of the mobile set being Rs1600 and the rate of sale tax is $6 \%$. The shopkeeper sells it to the buyer at the printed price and charges tax at the rate. Find a. the price at which mobile set can be bought
b. the VAT paid by the shopkeeper.
33. Find the equation of the parabola whose focus is $(-1,-1)$ and directrix $x+y+1=0$
34. Find dy/dx if, $x=\frac{1-t^{2}}{1+t^{2}}, \quad y=\frac{2 t}{1+t^{2}}$
35. The volume of a sphere is increasing at the rate $4 \pi c . c / \mathrm{sec}$. Find the rate at which the area of its surface increases when its radius is 10 cm .
36. Find the maxima and minima of the function $f(x)=9 x^{2}+12 x+2$
37. Evaluate $\int \frac{2 x+5}{3 x+4} d x$
38. Evaluate $\int x^{2} \cos x d x$

## PART - D

## IV. ANSWER ANY SIX QUESTIONS

39. Solve by Matrix Method $x-y+z=2,2 x-y=0,2 y-z=1$
40. Find the term independent of x in $\left(\sqrt{x}-\frac{2}{x^{2}}\right)^{20}$
41. Resolve $\frac{2 x^{3}+x^{2}-x-3}{x(x-1)(2 x+3)}$ into partial fractions.
42. Verify $(p \leftrightarrow q)$ and $(\sim p \vee q) \wedge(\sim q \vee p)$ is logically equivalent or not.
43. If 2 men and 4 women can do a work in 33 days and 3 men and 5 women can do the same work in 24 days. How long shall 5 men and 2 women take to do the same work?
44. The production manager of a company obtained the following equation for the learning effect $y=1356 x^{-0.3219}$. This function is based on the company's experience for assembling the first 50 units of the product. Find the labour hours required to assemble 100 units.
45. Solve the following LPP graphically

Maximize $\mathrm{Z}=-x+2 y$
Subject to constraints $x+y \geq 5, x+2 y \geq 6, x \geq 3, x \geq 0, y \geq 0$
46. If $A+B+C=\pi$ Prove that $\tan (\mathrm{A} / 2) \tan (\mathrm{B} / 2)+\tan (\mathrm{B} / 2) \tan (\mathrm{C} / 2)+\tan (\mathrm{C} / 2) \tan (\mathrm{A} / 2)=1$
47. If $y=\left(x+\sqrt{x^{2}+1}\right)^{m}$ show that $\left(x^{2}+1\right) y_{2}+x y_{1}-m^{2} y=0$
48. Find the area bounded by the parabola $y^{2}=4 x$ and the line $y=2 x-4$

## PART - E

V. ANSWER ANY ONE QUESTIONS
a. Evaluate $\lim _{x \rightarrow a} \frac{x^{n}-a^{n}}{x-a}=n a^{n-1} \quad$ for all rationals
b. Expand $(0.96)^{3}$ using binomial theorem upto 5 decimal.
50.
a. Show that the points $(2,-4),(0,0),(3,-1)$ and $(3,-3)$ are concyclic
b. The angles of elevation of the top of an unfinished tower at a point distance 120mt from its base in $45^{\circ}$. How much higher must the tower be raised so that the angle of elevation at the same point may be $60^{\circ}$ ?

