## JAIN COLLEGE, J C Road, Bangalore <br> Mock Paper-1 December-2017 <br> II PUC - Basic Mathematics (75)

## PART A

I. Answer all the questions

1. If $\mathrm{A}=\left[\begin{array}{ll}2 & -x \\ x & -7\end{array}\right]$ find $A+A^{1}$.
2. Find ' $n$ ' if $n C_{10}=n C_{15}$.
3. Negate "If 6 is a divisor of 120 then 486 is not divisible by 6 ".
4. Find the fourth proportional of 6,12 and 18 .
5. What Income can be obtained from Rs. 8000 of $4 \%$ stock?
6. Express $\sin 5 A \cdot \operatorname{Cos} 3 A$ as sum or difference of 2 trigonometric functions.
7. Find " K " if $x^{2}+y^{2}-4 x-8 y+k=0$ represents a point circle.
8. Evaluate $\lim _{x \rightarrow 3}\left(\frac{x^{3}-27}{x-3}\right)$.
9. Find $\frac{d y}{d x}$ if $x^{2}-y^{2}=a^{2}$.
10. Evaluate $\int \frac{9}{\sin ^{2} x} d x$.

## PART B

II. Answer all the questions
11. If $\mathrm{A}=\left[\begin{array}{ll}1 & 3 \\ 1 & 0\end{array}\right]$ Prove that $A^{2}-A-3 I=0$.
12. A person has 9 friends .In how many ways can he invite one or more of them to a dinner .
13. Two dice are rolled simultaneously. Find the probability of getting a doublet of even number.
14. Write the converse and contra positive of "If $x \in(A \cup B)$ then $x \in A$ or $x \in B$ ".
15. Two numbers are in the ratio $3: 5$. If 9 is subtracted from each. The new numbers are in the ratio 12:23. Find the smaller number.
16. T.D on a bill was Rs. 100 and $B G$ was Rs. 10 . What is the face value of the bill?
17. If $\tan A=\frac{3}{4} \tan B=\frac{1}{7}$ Show that $\tan (A+B)=1$.
18. Prove that $\frac{\sin 3 A}{1+2 \cos 2 A}=\sin A$.
19. Find the focus and equation of directrix of the parabola $x^{2}+32 y=0$.
20. If $y=\sqrt{\frac{1-\cos 2 x}{1+\cos 2 x}}$ Prove That $\frac{d y}{d x}=\sec ^{2} x$.
21. Show that the function $\mathrm{f}(\mathrm{x})=\left\{\begin{array}{c}(1+3 x)^{\frac{1}{x}}, x \neq 0 \\ e^{3}, x=0\end{array}\right.$ is continous at $\mathrm{x}=0$.
22. The distance " s " feet travelled by a particle in time " t " second is given by $\mathrm{S}=t^{3}-6 t^{2}+15 t+$ 2 Find when is the accerlation is zero.
23. Evaluate $\int \frac{3 x+2}{3 x^{2}+4 x-5} d x$.

Evaluate $\int_{0}^{1} e^{2 x} d x$.
24.

## PART C

## III. Answer all the questions

25. Show that in a determinant " if each element of a row (or column) is multiplied by a scalar and added to the corresponding elements of any other row (or column) then the value of the determinant is unaltered.
26. P.T $\left|\begin{array}{ccc}1 & 1 & 1 \\ b & c & a \\ b^{2} & c^{2} & a^{2}\end{array}\right|=(a-b)(b-c)(c-a)$.
27. A family of 4 brothers and 3 sisters is to be arranged for a photograph in one row .In how many ways can they be seated if
i) All the sisters sit together
ii) No two sisters sit together.
28. In a class of 80 students 40 take English, 25 take Hindi. Find the probability that the student take 1) $\begin{array}{llll}\text { Both English and Hindi } & 2 \text { ) only English } & 3 \text { ) only Hindi. }\end{array}$
29. If 8 men and 16 boys can do a piece of work in 6 days and 12 men and 24 boys can do the same work in 8 days. In how many days can 16 men and 20 boys do it?
30. The banker's gain on a bill is $1 / 5^{\text {th }}$ of the banker's discount and the rate of interest is $20 \%$ p.a. Find the unexpired period of the bill.
31. Sandeep invests Rs 15000 cash partly in $3 \%$ stock at 75 , and partly $6 \%$ debentures at 125 in such a way as to get a return on $4.5 \%$ for his money. How much does he invest in each.
32. Mr. Govind buys a tape recorder for Rs. 10260 including sales tax. If the list price of the tape recorder is Rs.9500.Find the rate of sales tax charged.
33. Differentiate $\sin x^{(\tan x)}$ w.r.t $x$.
34. IF $\mathrm{y}=-4$ is the equation of the directrix, axis is x -axis. find the equation of the parabola.
35. The side of a equilateral tringle increases at the rate of $\sqrt{3} \mathrm{~cm} / \mathrm{sec}$. Find the rate at which its area increases when the side is 2 mts .
36. Find the maximum and minimum value of the function $f(x)=x^{5}-5 x^{4}+5 x^{3}-1$.
37. Evaluate $\int \frac{x}{\sqrt{x+9}} d x$.
38. Evaluate $\int_{0}^{\frac{\pi}{2}} x \cos x d x$.

## PART D

## IV. Answer all the questions

39. Find the term independent of ' $x$ ' in $\left(\frac{\sqrt{x}}{2}-\frac{2}{x^{2}}\right)^{10}$.
40. Resolve $\frac{x^{2}}{(x+1)^{2}(x-5)}$ into partial fractions.
41. Find whether $p \rightarrow(\sim p \vee q)$ is a tautology or contradiction.
42. Divide Rs. 17640 into $P, Q, R$ and $S$ such that $Q$ gets $7 / 5^{\text {th }}$ of $P$, $R$ gets $5 / 8^{\text {th }}$ of $Q$ and $S$ gets $2 / 13^{\text {th }}$ of sum $Q$ and $R$.
43. A company required 1000 hours to produce first 30 engines .If the learning effect is $90 \%$. Find the total labour cost at Rs. 20 per hour to produce total of 120 engines .
44. Solve the LPP graphically

Maximize $Z=5 x+3 y$
subject to $3 x+5 y \leq 15$

$$
5 x+2 y \leq 10 \text { and } x, y \geq 0
$$

45. Show that $\frac{\cos 2 A}{\operatorname{Sec} A}+\frac{\operatorname{Sin} 2 A}{\operatorname{cosec} A}=\cos A$.
46. Find the equation of the circle passing through the points $(2,-4)(3,1)(3,-3)$.
47. If $x^{y}=e^{x-y}$ P.T $\frac{d y}{d x}=\frac{\log x}{(1+\log x)^{2}}$.
48. Find the area enclosed between the parabola $x^{2}=4 y$ and the line $x=4 y-2$.

## PART E

## V. Answer all the questions

49. a) Prove that $\lim _{x \rightarrow a} \frac{x^{n}-a^{n}}{x-a}=n a^{n-1}$ for all rationals. All three cases.
b) From the top of a cliff, the angles of depression of two boats in the same vertical planes as the observer are $30^{\circ}$ and $45^{\circ}$.If the distance between the boats 100 m ; Find the height of the cliff.
50. a) A school wants to award its students for the values of punctuality, Good behavior and Hard work with a total cash award Rs.6000.Three times the award money for hard work added to that given for punctuality amounts to Rs.11000.The award money for punctuality and hard work is double the one given for good behavior. Represent the above situation algebraically and also find the award money for each value using matrix method .
b) Find the marginal cost is $f^{1}(x)=1-2 x+12 x^{3}$ where x is the output, Find the total cost, average cost, total variable cost and average variable cost given that fixed cost is Rs.50.

## PART A

## I. Answer all the questions

1. Evaluate $\left|\begin{array}{ll}41 & 42 \\ 43 & 44\end{array}\right|$
2. In how many ways can 9 people be arranged in a circle if 3 particular people are always together?
3. Negate: $x \in A$ and $x \in A \cup B$.
4. Find the third proportional to 3,12 .
5. What is the yield obtained when Rs. 5000, 3\% stock is purchased at Rs. 125.
6. Find the value of $\frac{2 \tan 15^{\circ}}{1-\tan ^{2} 15^{\circ}}$.
7. Find k if the length of the latus rectum of $\mathrm{y}^{2}=8 \mathrm{kx}$ is 4 .
8. Evaluate: $\operatorname{Lt}_{x \rightarrow 0} \frac{\sin 4 x}{\sin 2 x}$.
9. If $y=x^{2}+e^{x}+e^{2}$ find $\frac{d y}{d x}$.
10. Evaluate: $\int 7^{x}+\log 7 d x$.
II. Answer all the questions
11. Solve using Cramer's rule
a. $5 x+2 y=4$
b. $7 x+3 y=5$
12. If a convex polygon has 170 diagonals. Find the number of sides of the polygon.
13. If $P(A)=\frac{1}{4} P(B)=\frac{2}{5} \quad P(A \cup B)=\frac{1}{2}$ find $P(A \mid B)$
14. Write the converse and inverse of the statement "If Maths exam is easy then all exams are easy".
15. Two numbers are in the ratio $3: 4$. If the sum of their squares in 900 find the two numbers.
16. A banker pays Rs. 4520 on a bill of Rs. 5000,146 days before the legally due date. Find the rate of discount charged by the banker.
17. A shopkeeper purchases an article for Rs. 2500 and sells it to a customer for Rs. 3000 . If the VAT rate is $8 \%$ find the VAT paid by the shopkeeper and the total amount paid by the customer.
18. Prove that $\sin \left(45^{\circ}+A\right)+\cos \left(45^{\circ}+A\right)=\sqrt{2} \cos A$.
19. Find the equation of the parabola given that focus is $(3,0)$ directrix is $x=-3$.
20. Discuss the continuity of $f(x)=\left\{\begin{array}{cl}3 x^{2}+1 & \text { if } x<1 \\ 4 & \text { if } x=1 \\ 2 x+2 & \text { if } x>1\end{array}\right.$ at $x=1$.
21. If $x=\frac{1-t^{2}}{1+t^{2}} \quad y=\frac{2 t}{1+t^{2}}$ find $\frac{d y}{d x}$.
22. A particle shot vertically upward rises ' $s$ ' feet in ' $t$ ' sec where $s=40 t-16 t^{2}$. Find the greatest height attained by the particle.
23. Evaluate: $\int \sin ^{3} x \cos x d x$.
24. Evaluate: $\int_{0}^{\pi / 4} \sin 2 x$.
III. Answer all the questions
25. Find $A=\left[\begin{array}{ccc}2 & -1 & 3 \\ 1 & 3 & -4 \\ 4 & -2 & 0\end{array}\right] \quad B=\left[\begin{array}{ccc}1 & 2 & -3 \\ -1 & 4 & 3 \\ 5 & -1 & 1\end{array}\right]$ Find $A B$ and $B A$. Verify $A B \neq B A$.
26. Show that $\left|\begin{array}{ccc}a-b-c & 2 a & 2 a \\ 2 b & b-c-a & 2 b \\ 2 c & 2 c & c-a-b\end{array}\right|=(a+b+c)^{3}$
27. In how many ways can 9 Maths, 12 Statistics and 15 Accountancy books be arranged in a shelf if
i. All Maths books are together
ii. Books of the same subject are together
iii. No two Accountancy books are together.
28. A couple appears in an interview for two vacancies in the same post. The probability of husband's selection is $\frac{1}{7}$ and the probability of wife's selection is $\frac{1}{5}$. What is the probability that
i. both of them are selected.
ii. only one of them will be selected .
iii. none of them will be selected.
29. 5 men each working 9 hours a day can finish a work in 30 days. How many men are required to finish eight times the work in 25 days each working 8 hours a day.
30. The difference between banker's discount and true discount on a bill due after 6 months at 4\% interest p.a. is Rs. 20. Find the true discount, banker's discount and face value of the bill.
31. Ramesh has invested Rs. 4300 partly in $4.5 \%$ at Rs. 75 and partly in $5 \%$ stock at Rs. 95 . If the total income from both is Rs. 250, find the investment in both the types of stock.
32. A shop keeper announces a discount of $10 \%$ on a washing machine set. The marked price of the washing machine is Rs. 12000. How much will a customer have to pay for buying the washing machine set if the rate of sales tax is $8 \%$.
33. Find the focus, equation of directrix, end points for latus rectum of the parabola $x^{2}=32 y$.
34. If $y^{x}=x^{y}$ show that $\frac{d y}{d x}=\frac{y(y-x \log y)}{x(x-y \log x)}$
35. A ladder 17 feet long leans against a smooth vertical wall. If the lower end is moving at the rate of $2 \mathrm{ft} / \mathrm{min}$. Find the rate at which the upper end is moving when the lower end is 8 feet from the wall.
36. Let the demand function of an article be $p=75-2 x$ and the cost function be

$$
\begin{aligned}
& c(x)=350+12 x+\frac{x^{2}}{4} \text {. Find the number of units and the price at which the total profit is } \\
& \text { maximum } \quad(p=\text { price, } x=\text { output }) .
\end{aligned}
$$

37. Evaluate: $\int x^{2} e^{x} d x$.
38. Evaluate : $\int \frac{1}{x(x+1)(x+2)} d x$.

## IV. Answer all the questions

39. Solve using matrix method
a. $x-y-2 z=3$
b. $2 x+y+z=5$
c. $4 x-y-2 z=1$
40. Find the term independent of $x$ in the expansion of $\left(\frac{3}{2} x^{2}-\frac{1}{3 x}\right)^{6}$.
41. Resolve into partial fractions $\frac{3 x+4}{(x+1)^{2}(x-1)}$.
42. Verify whether $(p \rightarrow q) \leftrightarrow(\sim q \rightarrow \sim p)$ is a tautology or not.
43. A can do a piece of work in 20 days $B$ in 30 days and $C$ in 60 days. All of them began to work together. However A left the job after 6 days and $B$ quit work 6 days before the completion of work. How many days did the work last.
44. A company requires 600 has to produce first 20 units of a product. If the learning effect is $90 \%$ find the total cost of manufacturing next 140 units if labour cost is Rs. 35 per hour.
45. Solve the following LPP graphically.
a. Maximize $z=10 x+9 y$
b. Subject to

$$
\begin{aligned}
& x+y \leq 50 \\
& 2 x+y \leq 80 \\
& x, y \geq 0
\end{aligned}
$$

46. Prove that $\sin 3 A=3 \sin A-4 \sin ^{3} A$. If $\sin A=\frac{3}{5}$ find $\sin 3 A$.
47. If $x^{2}+2 x y+3 y^{2}=1$ show that $y_{2}=\frac{-2}{(x+3 y)^{3}}$.
48. Find the area enclosed between the parabola $y^{2}=x$ and $x^{2}=y$.

## V. Answer all the questions

49. a) Prove that $\operatorname{Lt}_{x \rightarrow a} \frac{x^{n}-a^{n}}{x-a}=n . a^{n-1}$ for all $n$
b) Expand (0.9) ${ }^{5}$ upto 4 decimal places using binomial theorem.
50. a) Show that the points $(2,0)(-1,3),(-2,0)(1,-1)$ are concyclic.
b) The angle of elevation of the summit of a hill from the top and the bottom of a tower are $30^{\circ}$ and $60^{\circ}$ respectively. If the height of the tower is $h$,show that the height of the hill is $3 h / 2$.
