X - ICSE Board

Date: 27.02.2018 **MATHS - QP + SOLUTIONS**

SECTION - A (40 Marks)

Attempt all questions from this Section

Question 1

(a) Find the value of 'x' and 'y' if : [3]

$$2\begin{bmatrix} x & 7 \\ 9 & y-5 \end{bmatrix}+\begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix}=\begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

Ans. $$2\begin{bmatrix} x & 7 \\ 9 & y-5 \end{bmatrix}+\begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix}=\begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 2x & 14 \\ 18 & 2y-10 \end{bmatrix}+\begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix}=\begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 2x+6 & 14-7 \\ 18+4 & 2y-10+5 \end{bmatrix}=\begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

Using equality of matrix

$2x+6=10$ and $2y-10+5=15$

$\therefore 2x=4$ $\quad 2y-5=15$

$\therefore x=2$ $\quad 2y=20$

$y=10$

Topic: Matrices_Subtopic:Algebra of Matrices_ Level: 1_Std. X__ICSE Board / Mathematics

(b) Sonia had a recurring deposit account in a bank and deposited Rs. 600 per month for $2^{1/2}$ years. If the rate of interest was 10% p.a., find the maturity value of this account. [3]

Ans. Recurring deposite = 600 per month

Period = $2^{1/2}$ yrs = 30 months

R.O.I = (r) = 10%

Total principal per 1 month

$$= 600\left(\frac{n(n+1)}{2}\right) = 300(30\times 31) = \text{Rs.}2,79,000$$

$$\therefore \text{ Interest} = \frac{PRT}{100} = \frac{279000\times 10\times 1}{100\times 12} = \text{Rs.}2325$$

$\therefore$ Maturity vale = 600×30+2325=Rs.20325

Topic: Banking_Subtopic: Recurring Deposits_ Level: 1_Std. X__ICSE Board / Mathematics

(c) Cards bearing numbers 2, 4, 6, 8, 10, 12, 14, 16, 18 and 20 are kept in a bag A card is drawn at random from the bag. Find the probability of getting a card which is : [4]

(i) a prime number

(ii) a number divisible by 4

(iii) a number that is a multiple of 6

(iv) an odd number

Ans. (2) (4) (6) (8) (10) (12) (14) (16) (18) (20)

$n(S) = {}^{10}C_1 = 10$

(i) A = a prime number = {2}

$$P(A) = \frac{1}{10}$$

(ii) B = Number divisible by 4 = {4, 8, 12, 16, 20}

$$P(B) = \frac{5}{10} = \frac{1}{2}$$

(iii) C = a number that is multiple of 6

= {6, 12, 18}

$$P(C) = \frac{3}{10}$$

(iv) D = an odd number = { }

$$P(D) = \frac{0}{10} = 0$$

Topic: Probability_Subtopic: Probability_ Level: 1_Std. X__ICSE Board / Mathematics

Question 2

(a) The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. Find the [3]

(i) radius of the cylinder

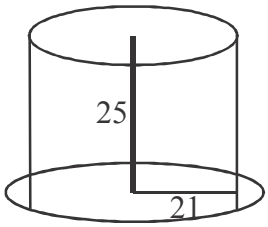
(ii) volume of cylinder $\left(\text{Use } \pi = \frac{22}{7}\right)$

Ans. (i) Given circumference $= 2\pi r$

$$132 = 2 \times \frac{22}{7} \times r$$

$$\therefore\ r = 3 \times 7 = 21 cm$$

$\therefore$ Radius = 21 cm



(ii) Volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 21 \times 21 \times 25$$

$$= 22 \times 21 \times 25 \times 3$$

$$= 34,650\ cm^3$$

Topic: Mensuration_Subtopic: Volume of cylinder_ Level: 1_Std. X__ICSE Board / Mathematics

(b) If $(k-3)$, $(2k+1)$ and $(4k+3)$ are three consecutive terms of an A.P., find the value of k. [3]

Ans. $\because\ (k-3), (2k+1), (4k+3)$ are consecutive numbers in AP.

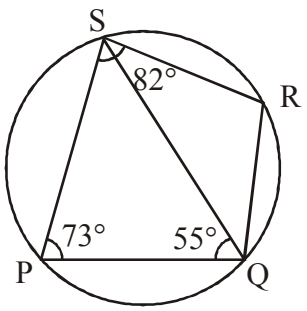
$$\therefore\ 2 \times (2k+1) = (k-3) + (4k+3)$$

$$\therefore\ 4k + 2 = k - 3 + 4k + 3$$

$$\therefore\ k = 2$$

Topic: Progression_Subtopic: A.P._ Level: 1_Std. X__ICSE Board / Mathematics

(c) $PQRS$ is a cyclic quadrilateral. Given $\angle QPS = 73°$, $\angle PQS = 55°$ and $\angle PSR = 82°$, calculate :[4]



(i) $\angle QRS$

(ii) $\angle RQS$

(iii) $\angle PRQ$

Ans. From diagram

(i) $\angle SPQ + \angle QRS = 180°$ (Opposite angles are supplimentary)

$73° + \angle QRS = 180°$

$\angle QRS = 180° - 73° = 107°$

(ii) $\angle PSR + \angle PQR = 180°$

$82° + \angle PQR = 180°$

$\angle PQR = 180° - 82°$

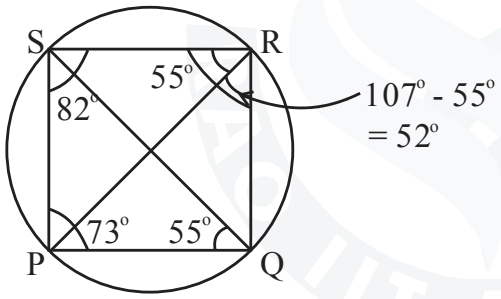
$\angle PQR = 98°$

But $\angle PQR = \angle PQS + \angle RQS$

$98° = 55° + \angle RQS$

$98° - 55° = \angle RQS = 43°$

(iii)



$\therefore \angle PRQ = 52^o$

Topic: Circles_Subtopic:Circles_ Level:2 _Std. X__ICSE Board / Mathematics

Question 3

(a) If $(x+2)$ and $(x+3)$ are factors $x^3 + ax + b$, find the values of 'a' and 'b'. [3]

Ans. $\because\ x+2$ is factor of $x^3 + ax + b$

$\therefore\ (-2)^3 + a(-2) + b = 0$

$-8 - 2a + b = 0$

$2a - b = -8$ ...(i)

$\because\ x+3$ is factor of $x^3 + ax + b$

$(-3)^3 + a(-3) + b = 0$

$-27 - 3a + b = 0$

$3a-b=-27$ ...(ii)

(ii) – (i)

$(3a-b)-(2a-b)=-27-(-8)$

$3a-b-2a+b=-27+8$

$a=-19$

Put $a=-19$ in (i)

$2(-19)-b=-8$

$-38-b=-8$

$-38+8=b=-30$

Topic: Remainder & Factor_Subtopic:Factor theory__ Level:2 _Std. X__ICSE Board / Mathematics

(b) Prove that $\sqrt{\sec^2\theta+\text{cosec}^2\theta}=\tan\theta+\cot\theta$ [3]

Ans. L.H.S

$\sqrt{\sec^2\theta+\text{cosec}^2\theta}$

$\therefore\ \sqrt{\tan^2\theta+1+\cot^2\theta+1}$

$\therefore\ \sqrt{\tan^2\theta+2+\cot^2\theta}$

$\therefore\ \sqrt{\tan^2\theta+2\tan\theta\cot\theta+\cot^2\theta}$

$\{\tan\theta\times\cot\theta=1\}$

$\therefore\sqrt{(\tan\theta+\cot\theta)^2}$

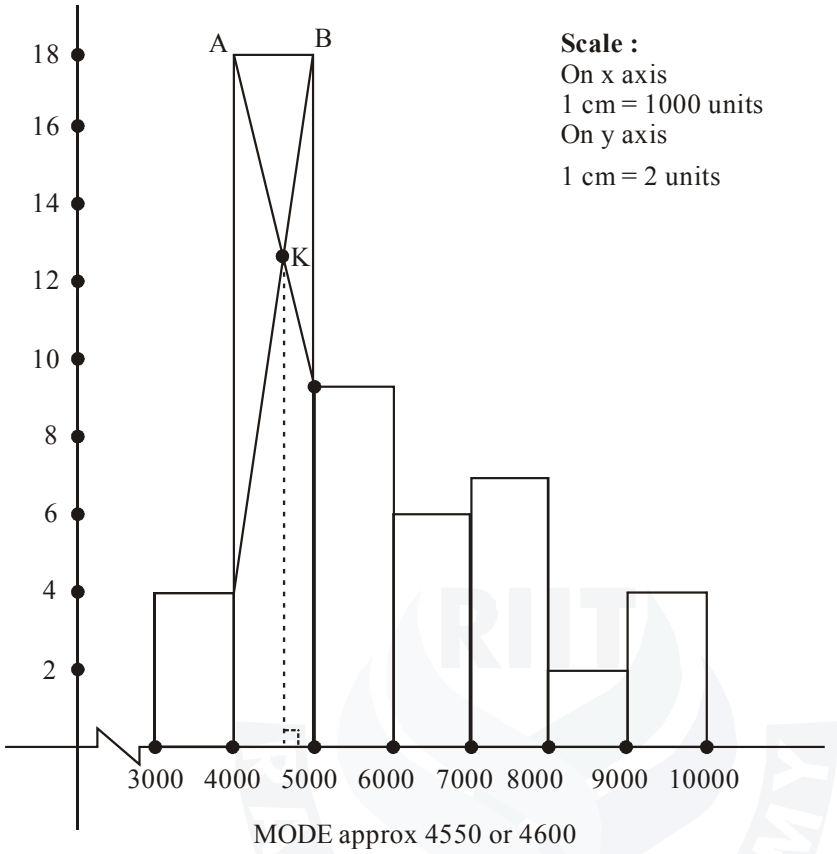
$\therefore\ \tan\theta+\cot\theta$

Topic: Trigonometric Functions_Subtopic: Identities__ Level:2_Std. X__ICSE Board / Mathematics

(c) Using graph paper draw a histogram for the given distibution showing the number of runs scored by 50 batsman. Estimate the mode of the data : [4]

Runs Scored	1000-4000	4000-5000	5000-6000	6000-7000	7000-8000	8000-9000	9000-10000
No. of batsman	4	18	9	6	7	2	4

Ans.



Topic:Graphical Representation_Subtopic: Histogram_ Level: 2_Std. X__ICSE Board / Mathematics

Question 4

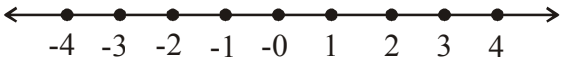
(a) Solve the following inequation, write down the solution set and represent it on the real number line:

$-2+10x \le 13x+10 < 24+10x, x \in Z$ [3]

Ans. $-2+10x \le 13x+10$ and $13x+10 < 24+10x$

$-12 \le 3x$ and $3x < 14$

$-4 \le x$ and $x < \frac{14}{3}$ $\because x \in Z$



$\therefore$ x = {-4,-3,-2,-1,0,1,2,3,4}

Topic: Inequality_Subtopic: Inequality_ Level: 2_Std. X__ICSE Board / Mathematics

(b) If the straight lines $3x-5y=7$ and $4x+ay+9=0$ are perpendicular to one another, find the value of a. [3]

Ans. Slope of $3x-5y=7$ is

$$m_1=\frac{-3}{-5}=\frac{3}{5}$$

Slope of $4x+ay+9=0$ is

$$m_2=\frac{-4}{a}$$

$\because$ lines are $\perp$

$$m_1\times m_2=-1$$

$$\frac{3}{5}\times\frac{-4}{a}=-1$$

$$\frac{-12}{5}=-a$$

$$\therefore\ a=\frac{12}{5}$$

Topic: Coordinate Geometry_Subtopic: Equation of line_ Level: 1_Std. X__ICSE Board / Mathematics

(c) Solve $x^2+7x=7$ and give your answer correct to two decimal places. [4]

Ans. $x^2+7x=7$

$$x^2+7x+\frac{49}{4}=7+\frac{49}{4}$$

$$\left(x+\frac{7}{2}\right)^2=\frac{77}{4}$$

$$x+\frac{7}{2}=\pm\sqrt{\frac{77}{4}}$$

$$x=\pm\sqrt{\frac{77}{4}}-\frac{7}{2}$$

$$x=\pm\sqrt{\frac{77}{4}}-\frac{7}{2}$$

$$x=\pm\frac{\sqrt{77}-7}{2}$$

$x=0.88$ or $x=-7.88$

Topic: Algebra_Subtopic: Qudratic Equation_ Level: 1_Std. X__ICSE Board / Mathematics

SECTION - B (40 Marks)

Attempt any four questions from this Section

Question 5

(a) The 4th term of a G.P. is 16 and the 7th terms is 128. Find the first term and common ratio of the series. [3]

Ans. Let the first term of a G.P. a and common ratio r

$a_4 = ar^3 = 16$ ...(i)

$a_7 = ar^6 = 128$ ...(ii)

$(ii) \div (i)$

$\frac{ar^6}{ar^3} = \frac{128}{16}$

$r^3 = 8$

$r = 2$

Put $r = 2$ in equation (ii)

$a(2)^3 = 16$

$a = 2$

$\therefore$ First term $(a) = 2$

Common ratio = 2

Topic: Progression_Subtopic: G.P._ Level:1 _Std. X__ICSE Board / Mathematics

(b) A man inversts Rs.22,500 in Rs.50 shares available at 10% discount. If the dividend paid by the company is 12%, calculate : [3]

(i) The number of shares purchased

(ii) The annual dividend received

(iii) The rate of return he gets on his investment. Give your answer correct to the nearest whole number.

Ans. Actual price = 50 Rs./Share

Price after discount = 50 – 10% of 50 = Rs.45

(i) Total shares bought (Purchased)

$= \frac{22500}{45} = 500$

(ii) Annual dividend received

$= 500 \times 50 \times 12\%$

$$= 500 \times 50 \times \frac{12}{100}$$

$$= 5 \times 600$$

$$= \text{Rs. } 3000$$

(iii) $$\text{ROR} = \frac{25500 - 22500}{22500} \times 100$$

$$= \frac{3000}{22500} \times 100 = 13.33\%$$

aprrox. 13%

Topic: Shares_Subtopic:Shares_ Level: 2_Std. X__ICSE Board / Mathematics

(c) Use graph paper for this question (Take 2cm = 1 unit along both x and y axis). $ABCD$ is a quadrilateral whose vertices are $A(2, 2)$, $B(2, -2)$, $C(0, -1)$ and $D(0, 1)$.

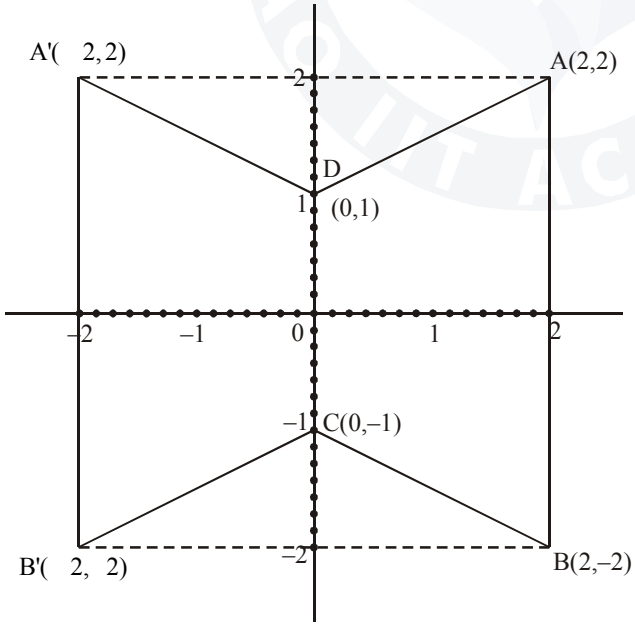
(i) Reflect quadrilateral $ABCD$ on the y-axis and name it as $A'B'CD$.

(ii) Write down the coordinaes of A' and B'.

(iii) Name two points which are invariant under the above reflection.

(iv) Name the polygon $A'B'CD$.

Ans.



(ii) $A' = (-2, 2)$ $B' = (-2, -2)$

(iii) C and D

(iv) $A'B'CD$ is trapezium

Topic: Coordinate Geometry_Subtopic: Reflection_ Level:2 _Std. X__ICSE Board / Mathematics

Question 6

(a) Using properties of proportion, solve for x. Given that x is positive : [3]

$$\frac{2x+\sqrt{4x^2-1}}{2x-\sqrt{4x^2-1}}=4$$

Ans. $$\frac{2x+\sqrt{4x^2-1}}{2x-\sqrt{4x^2-1}}=\frac{4}{1}$$

Applying componendo and dividendo

$$\frac{\left(2x+\sqrt{4x^2-1}\right)+\left(2x-\sqrt{4x^2-1}\right)}{\left(2x+\sqrt{4x^2-1}\right)-\left(2x-\sqrt{4x^2-1}\right)}=\frac{4+1}{4-1}$$

$$\Rightarrow \frac{4x}{2\sqrt{4x^2-1}}=\frac{5}{3}$$

Squaring on both sides, we get

$$\Rightarrow \frac{4x^2}{4x^2-1}=\frac{25}{9}$$

$$\Rightarrow 36x^2=100x^2-25$$

$$\Rightarrow 64x^2=25$$

$$\Rightarrow x^2=\frac{25}{64}$$

$$x=\pm\frac{5}{8}$$

Topic: Algebra_Subtopic: Ratio & Proportion_ Level:2 _Std. X__ICSE Board / Mathematics

(b) If $A=\begin{bmatrix}2 & 3\\ 5 & 7\end{bmatrix}$, $B=\begin{bmatrix}0 & 4\\ -1 & 7\end{bmatrix}$ and $C=\begin{bmatrix}1 & 0\\ -1 & 4\end{bmatrix}$, find $AC+B^2-10C$. [3]

Ans. $A=\begin{bmatrix}2 & 3\\ 5 & 7\end{bmatrix}$, $B=\begin{bmatrix}0 & 4\\ -1 & 7\end{bmatrix}$ and $C=\begin{bmatrix}1 & 0\\ -1 & 4\end{bmatrix}$

$$AC+B^2-10C=\begin{bmatrix}2 & 3\\ 5 & 7\end{bmatrix}\begin{bmatrix}1 & 0\\ -1 & 4\end{bmatrix}+\begin{bmatrix}0 & 4\\ -1 & 7\end{bmatrix}\begin{bmatrix}0 & 4\\ -1 & 7\end{bmatrix}-10\begin{bmatrix}1 & 0\\ -1 & 4\end{bmatrix}$$

$$AC+B^2-10C=\begin{bmatrix}2-3 & 12\\ 5-7 & 28\end{bmatrix}+\begin{bmatrix}-4 & 28\\ -7 & 45\end{bmatrix}-\begin{bmatrix}10 & 0\\ -10 & 40\end{bmatrix}$$

$$AC+B^2-10C=\begin{bmatrix}-1-4-10 & 12+28\\ -2-7+10 & 28+45-40\end{bmatrix}$$

$$AC+B^2-10C=\begin{bmatrix}-15 & 40\\ 1 & 33\end{bmatrix}$$

Topic:Algebra_Subtopic:Matrices_ Level:1 _Std. X__ICSE Board / Mathematics

(c) Prove that $(1+\cot\theta-\operatorname{cosec}\theta)(1+\tan\theta+\sec\theta)=2$ [4]

Ans. Taking LHS :

$$(1+\cot\theta-\operatorname{cosec}\theta)(1+\tan\theta+\sec\theta)$$

$$=\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)$$

$$=\frac{(\sin\theta+\cos\theta-1)}{\sin\theta}\frac{(\sin\theta+\cos\theta+1)}{\cos\theta}$$

$$=\frac{(\sin\theta+\cos\theta)^2-1^2}{\sin\theta\cdot\cos\theta}$$

$$=\frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cdot\cos\theta-1}{\sin\theta\cdot\cos\theta}$$

$$=\frac{1+2\sin\theta\cdot\cos\theta-1}{\sin\theta\cdot\cos\theta}$$

$$=\frac{2\sin\theta\cdot\cos\theta}{\sin\theta\cdot\cos\theta}$$

$$=2$$

Topic: Trigonometry_Subtopic:Trigonometrical Identities_ Level: 2_Std. X__ICSE Board / Mathematics

Question 7

(a) Find the value of k for which the following equation has equal roots. [3]

$$x^2+4kx+\left(k^2-k+2\right)=0$$

Sol. $x^2+4kx+\left(k^2-k+2\right)=0$

following equation having equal roots

$\therefore\ b^2-4ac=0$

here $a=1,\quad b=4k,\quad c=k^2-k+2$

$$(4k)^2-4(1)\left(k^2-k+2\right)=0$$

$\Rightarrow 16k^2 - 4k^2 + 4k - 8 = 0$

$\Rightarrow 12k^2 + 4k - 8 = 0$

$\Rightarrow 3k^2 + k - 2 = 0$

$\Rightarrow 3k^2 + 3k - 2k - 2 = 0$

$\Rightarrow 3k(k+1) - 2(k+1) = 0$

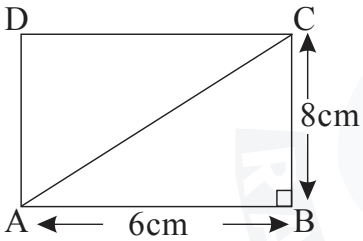
$(k+1)(3k-2) = 0$

$k = -1 \quad \text{or} \quad k = \frac{2}{3}$

Topic: Algebra_Subtopic: Quadratic Equation_ Level: 2_Std. X__ICSE Board / Mathematics

(b) On a map drawn to a scale of 1 : 50,000, a rectangular plot of land ABCD has the following dimensions. AB = 6cm, BC = 8 cm and all angles are right angles. Find :
(i) the actual length of the diagonal distance AC of the plot in km.
(ii) the actual area of the plot in sq km. [3]

Sol.



Using pythagoras theory

$AC^2 = AB^2 + BC^2$

$AC^2 = 6^2 + 8^2$

$AC^2 = 100$

$AC = 10\ cm$

1) $\therefore$ Actual length

$= 10 \times 50000 = 500000\ cm$

$= \frac{500000}{1000 \times 100} = 5\ \ km$

2) Area of ABCD $= 6 \times 8$

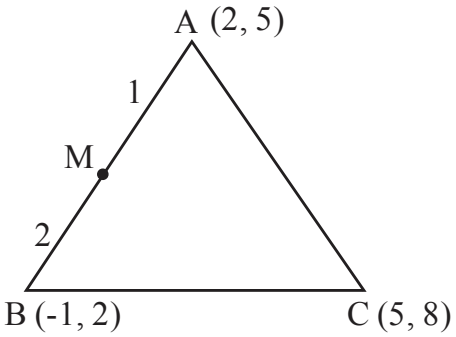
$= 48\ cm^2$

$= \frac{48 \times 50000 \times 50000}{100000 \times 100000} = \frac{48}{4} = 12\ sq.\ km$

Topic:Mensuration_Subtopic: Area_ Level: 2_Std. X__ICSE Board / Mathematics

(c) A (2, 5), B(-1, 2) and C(5, 8) are the vertices of a triangle ABC, 'M' is a point on AB such that AM : MB = 1 : 2. Find the co-ordinates of 'M'. Hence find the equation of the line passing through the points C and M. [4]

Sol. $A(2,5)$, $B(-1,2)$ and $C(5,8)$



Let the co-ordinates of M is (x, y)

$$x = \frac{2\times 2+1\times(-1)}{2+1} = \frac{4-1}{3} = 1$$

$$y = \frac{2\times 5+1\times 2}{2+1} = \frac{12}{3} = 4$$

$\therefore$ point M = (1, 4)

Equation of line passing through C (5, 8) and M (1, 4).

$$y-8 = \frac{4-8}{1-5}(x-5)$$

$$y-8 = \frac{-4}{-4}(x-5)$$

$$y-8 = 1(x-5)$$

$$y-8 = x-5$$

$$\boxed{x-y+3=0}$$

Topic: Coordinate Geometry_Subtopic: Equation of line__ Level:2 _Std. X__ICSE Board / Mathematics

Question 8

(a) Rs. 7500 were divided equally among a certain number of children. Had there been 20 less children, each woule have received Rs. 100 more. Find the original number of chlidren. [3]

Sol. Let the original number of person be x, then 7500 divided equally between x person,

each one get $= \frac{7500}{x}$

7500 divided equally between x - 20 children

each one get $75 = \frac{7500}{x-20}$

According to the question

$$\frac{7500}{x-20}=\frac{7500}{x}+\frac{100}{1}$$

$$\frac{7500}{x-20}=\frac{7500+100x}{x}$$

$$7500x=(x-20)(7500+100x)$$

$$75x=(x-20)(75+x)$$

$$75x=75x+x^2-1500-20x$$

$$x^2-20x-1500=0$$

$$x=\frac{20\pm\sqrt{400-4(-1500)}}{2}$$

$$x=\frac{20\pm\sqrt{400+6000}}{2}$$

$$x=\frac{20\pm80}{2}$$

$$x=\frac{20+80}{2} \quad \text{or} \quad x=\frac{20-80}{2}$$

$x=50$ or $x=-30$ (not possible)

$\therefore$ original number of children $=50$

Topic: Algebra_Subtopic: Quadratic Equation_ Level:1 _Std. X__ICSE Board / Mathematics

(b) If the mean of the following distribution of 24, find the value of 'a' [3]

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Number of students	7	a	8	10	5

Sol. Mean = 24

Class	Frequency (f_i)	Class mark (x_i)	$f_i \times x_i$
0 - 10	7	5	35
10 - 20	a	15	15a
20 - 30	8	15	200
30 - 40	10	35	350
40 - 50	5	45	225
Total	30 + a		810 + 15a

$$\therefore \text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = 24$$

$$\frac{810+15a}{30+a} = 24$$

$$810+15a = 720+24a$$

$$90 = 9a$$

$$\boxed{a = 10}$$

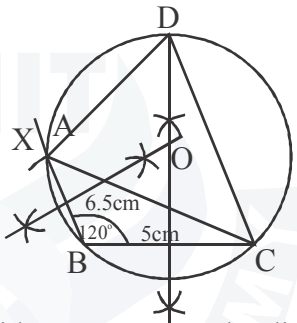
Topic: Statistics_Subtopic: Mean_ Level: 1_Std. X__ICSE Board / Mathematics

(c) Using ruler and compass only, construct a ΔABC such that BC = 5 cm and AB = 6.5 cm and $\angle ABC = 120^o$ [4]

(i) Construct a circm - circle of ΔABC

(ii) Construct a cyclic quadrilateral ABCD, such that D is equidistant from AB and BC.

Sol. Step of construction :



(i) Draw BC = 5 cm

(ii) At B, draw $\angle XBC = 120^o$

(iii) From BX, cut off AB = 6.5 cm

(iv) Join AC to get ΔABC

(v) Draw the perpendicular bisector of BC and AB. These bisectors meet at O. With O as centre and radius equal to OA, draw a circle, which passes through A, B and C. This is the required circumcircle of ΔABC

(vi) Produce the perpendicular bisector of BC so that it meets the circle at D. Join CD and AD to get the required cyclic quadrilateral ABCD.

Topic: Geometry_Subtopic: Construction_ Level:1 _Std. X__ICSE Board / Mathematics

Question 9

(a) Priyanka has a recurring deposit account of Rs. 1000 per month at 10% per annum. If she gets Rs. 5550 as interest at the time of maturity, find dthe total time for which the account was held. [3]

Sol. Amount of recurring deposit per month = Rs. 1000

Rate of interest = 10 % p.a.

let period = n months

Amount of interest = 5550 .....(1)

$$\text{Total principal for one month} = \frac{1000 \times n(n+1)}{2}$$

$$\text{Interest} = \frac{1000n(n+1)}{2} \times \frac{10}{100} \times \frac{1}{12}$$

$$= \frac{25}{6} n(n+1) \quad .....(2)$$

From (1) nd (2) , we get

$$\frac{25}{6} n(n+1) = 5550$$

$25n^2 + 25n = 33300$

$25n^2 + 25n - 33300 = 0$

$n^2 + n - 1332 = 0$

$n^2 + 37n - 36n - 1332 = 0$

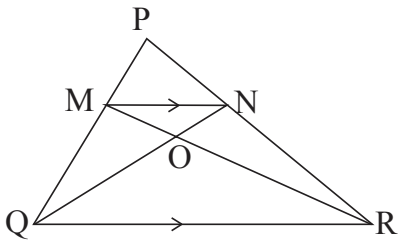
$n(n+37) - 36(n+37) = 0$

$(n-36)(n+37) = 0$

$\boxed{n = 36}$

Topic: Commercial Arithmetic_Subtopic: Banking__ Level:2 _Std. X__ICSE Board / Mathematics

(b) In ΔPQR, MN in parallel to QR and $\frac{PM}{MQ} = \frac{2}{3}$



(i) Find $\frac{MN}{QR}$

(ii) Prove that ΔOMN and ΔORQ are similar.

(iii) Find. Area of ΔOMN : Area of ΔORQ [3]

Sol. In $\Delta PQR, MN \parallel QR$ is such a way that $PM : MQ = 2:3$

(i) In ΔPQR, $MN \parallel QR$

$\therefore \frac{PM}{MQ} = \frac{PN}{NR} = \frac{2}{3} \Rightarrow \frac{MQ}{PM} = \frac{3}{2}$

Adding 1 on both sides,

$1 + \frac{MQ}{PM} = \frac{3}{2} + 1$

$\Rightarrow \frac{PM + MQ}{PM} = \frac{3+2}{2}$

$\frac{PQ}{PM} = \frac{5}{2} \Rightarrow \frac{PM}{PQ} = \frac{2}{5}$

Now in ΔPMN and ΔPQR,

$\angle PMN = \angle PQR$ (corresponding angles)

$\angle P = \angle P$ (Common)

$\therefore \Delta PMN \sim \Delta PQR$ (AA postulates)

$\therefore \frac{PM}{PQ} = \frac{MN}{QR} = \frac{PN}{NR}$

But $\frac{PM}{PQ}=\frac{2}{5}$

$\therefore \frac{MN}{QR}=\frac{2}{5}$

(ii) In ΔOMN and ΔORQ

(a) $\angle MON=\angle QOR$ (Vertically opposite angles)

Since MN | | QR,

(b) $\angle MNO=\angle OQR$ (Alternate angles)

(c) $\angle NMO=\angle ORQ$ (Alternate angles)

By AAA postulates,

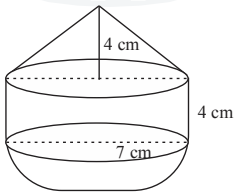
$\Delta OMN \sim \Delta ORQ$

(iii) $\frac{Ar(\Delta OMN)}{Ar(\Delta ORQ)}=\frac{MN^2}{QR^2}$

$\frac{Ar(\Delta OMN)}{Ar(\Delta ORQ)}=\frac{4}{25}$

Topic: Geometry_Subtopic: Similarity_ Level:2 _Std. X__ICSE Board / Mathematics

(c) The following figure represents a solid consisting of a right circular cylinder with a hemisphere at one end and a cone at the other. This common radius is 7 cm. The height of the cylinder and cone are each of 4 cm. Find the volume of the solid. [4]



Sol. Volume = Volume of cone + Volume of cylinder + Volume of hemishpere

$=\frac{1}{3}\pi r^2 h+\pi r^2 H+\frac{2}{3}\pi r^3$ $=\frac{1}{3}\pi r^2(h+3H+2r)$

$=\frac{1}{3}\times\frac{22}{7}\times 7\times 7(4+4\times 3+2\times 7)$

$=\frac{1}{3}\times 22\times 7\times 30$

$=22\times 7\times 10$

$=1540\ cm^3$

Topic: Mensuration_Subtopic: Cylinder_ Level: 2_Std. X__ICSE Board / Mathematics

Question 10

(a) Use Remainder theorem to factorize the following polynomial : [3]

$2x^3+3x^2-9x-10$

Sol. $p(x)=2x^3+3x^2-9x-10$

$p(-1)=2(-1)+3(1)-9(-1)-10=0$

$\therefore$ $x+1$ is a factor of $p(x)$

Now, dividing $p(x)$ by $x+1$, we get

$$
\begin{array}{r|l}
x+1 & 2x^2+x-10 \\
\hline
 & 2x^3+3x^2-9x-10 \\
 & 2x^3+2x^2 \\
 & (-)\quad(-) \\
\hline
 & x^2-9x-10 \\
 & x^2+x \\
 & (-)\quad(-) \\
\hline
 & -10x-10 \\
 & -10x-10 \\
 & (+)\quad(+) \\
\hline
 & 0
\end{array}
$$

$\therefore$ $2x^3+3x^2-9x-10=(x+1)(2x^2+x-10)$

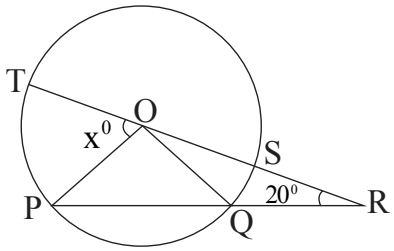
$=(x+1)[2x^2+5x-4x-10]$

$=(x+1)[x(2x+5)-1(2x+5)]$

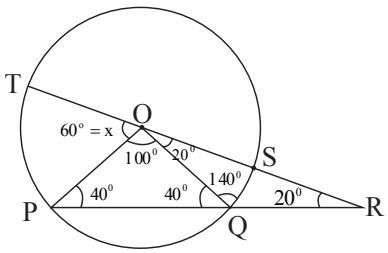
$=(x+1)(x-2)(2x+5)$

Topic: Algebra_Subtopic:Remainder & Factor theorem_ Level:1 _Std. X__ICSE Board / Mathematics

(b) In the figure given below 'O' is the center of the circle. If QR = OP and $\angle ORP = 20^o$. Find the value of 'x' giving reasons. [3]



Sol.



$\therefore QR = OP$

$\therefore QR = OP = OQ$

Hence ΔOQR is isoscleles Δ

$\angle QRO = 180 - 20 - 20$

$= 140$

$\therefore \angle OQP = 40^o$ linear pair of $\angle OQR$

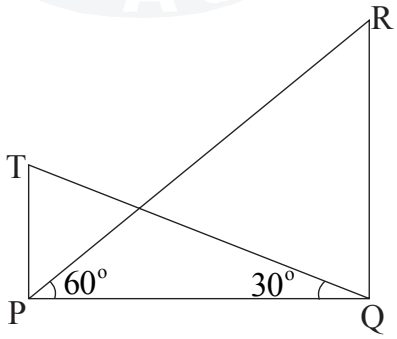
$\therefore \angle OPQ = 40^o$ as ΔOPQ is isosceles

$\therefore \angle POQ = 180^o - 40^o - 40^o = 100^o$

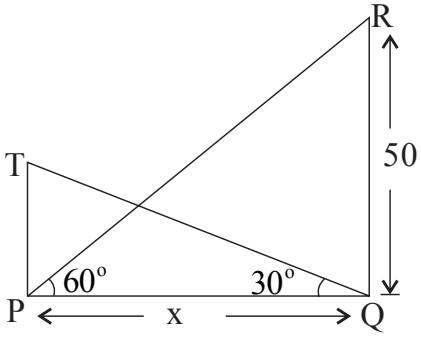
$\therefore \angle POT = x = 180^o - 100^o - 20^o = 60^o$

Topic: Geometry_Subtopic:Circle_ Level: 2_Std. X__ICSE Board / Mathematics

(c) The angle of elevation from a point P of the top of a tower QR, 50 m high is 60^o and that of the tower PT freom a point Q is 30^o. Find the height of the tower PT, correct to the nearest metre. [4]



Sol.



Let the height of the tower PT is h. and PQ is x

In ΔPQT

$$\tan 30^o = \frac{PT}{PQ}$$

$$\frac{1}{\sqrt{3}} = \frac{H}{x}$$

$$x = \sqrt{3}\, h \qquad \text{.....(1)}$$

In ΔPQR

$$\tan 60^o = \frac{50}{x}$$

$$\sqrt{3}\, x = 50 \qquad \text{.....(2)}$$

$$\Rightarrow \sqrt{3}\left(\sqrt{3}\, h\right) = 50$$

$$3h = 50$$

$$h = \frac{50}{3}$$

$$\therefore h = \frac{50}{3}$$

Topic: Trigonometry_Subtopic: Heights & Distances_ Level: 2_Std. X__ICSE Board / Mathematics

Question 11

(a) The 4th term of an A. P. is 22 and 15th term is 66. Find the first term and the common difference. Hence find the sum of the series to 8 terms. [4]

Sol. Let the first term at a A.P. is a and common difference is d.

$$a_4 = a + 3d = 22 \qquad \text{.....(1)}$$

$$a_{15} = a + 14d = 66 \qquad \text{.....(2)}$$

$$- \quad - \quad -$$

$$-11d = -44$$

$$a = 4$$

put d = 4 in equation (1)

$$a + 3 \times 4 = 22$$

$$a + 12 = 22$$

$$a = 10$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_8 = \frac{8}{2}\left[20 + 7 \times 4\right]$$

$$S_8 = 4\left[20 + 8 \times 4\right]$$

$S_8 = 4[48] = 192$

Topic: Algebra_Subtopic: A.P._ Level: 1_Std. X__ICSE Board / Mathematics

(b) Use graph paper for htis questin.

A survey regarding height (in cm) of 60 boys belonging to Class 10 of a school was conducted. The following data was recorded : [6]

Height in cm	135-140	140-145	145-150	150-155	155-160	160-165	165-170
No. of boys	4	8	20	14	7	6	1

Taking 2cm = height of 10 cm along one axis and 2 cm = 10 boys along the other axis draw an ogive of the above distribution. Use the graph to estimate the following :

(i) the medium

(ii) lower Quarile

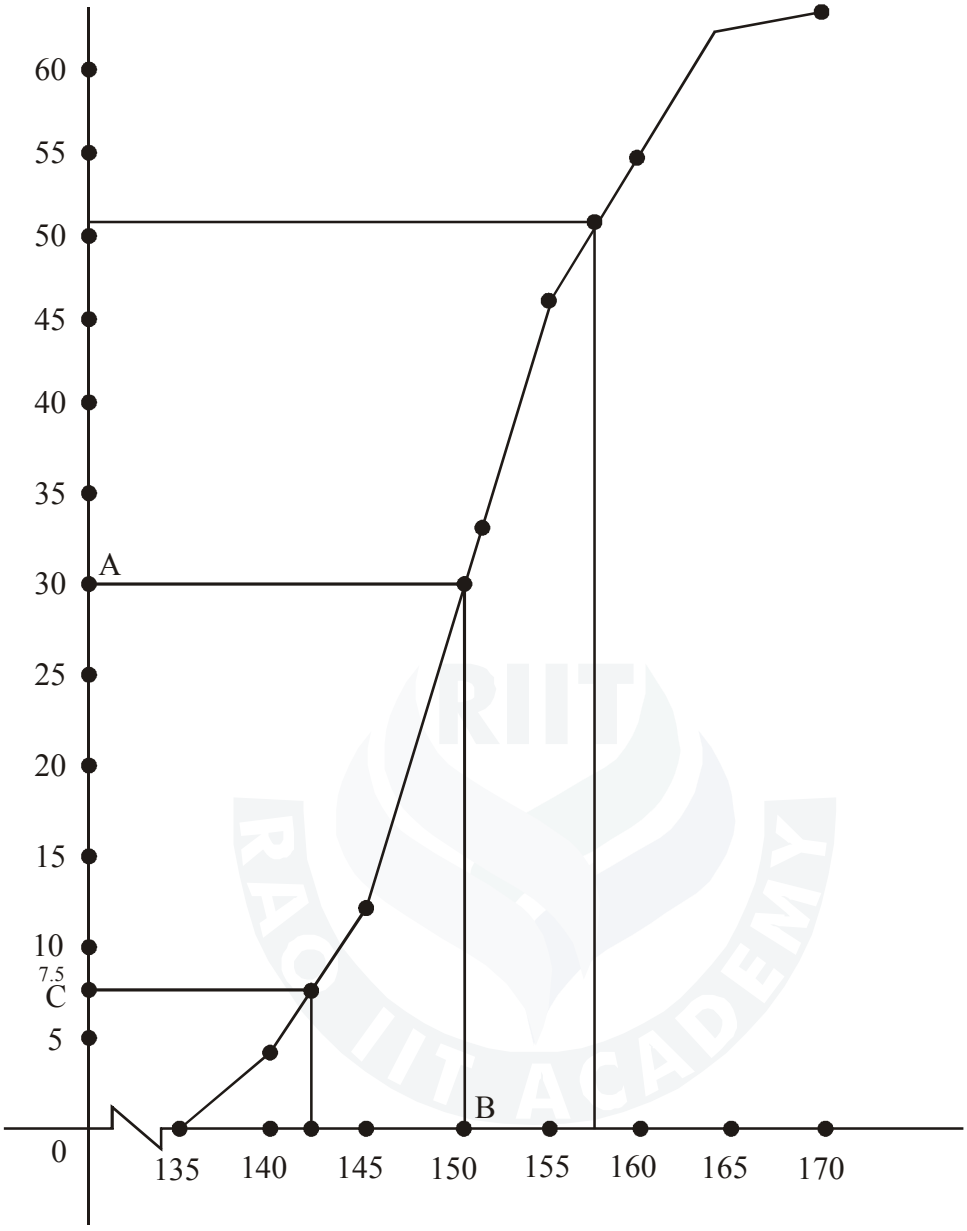
(iii) if above 158 cm is considered as the tall boys of the class. Find the number of boys in the class who are tall.

Topic: Statistics_Subtopic: Median & Quartiles_ Level:2_Std. X__ICSE Board / Mathematics

Sol.

Height (in cm)	No.of boys	$C.f.$
135-140	4	4
140-145	8	12
145-150	20	32
150-155	14	46
155-160	7	53
160-165	6	59
165-170	1	60

$\text{Median} = \frac{60}{2} = 30^{th}\text{ item}$



(i) Meadian = 150.5

(ii) Lower quartile = 142.5

(iii) 10 boys are above 158 cm