MATHEMATICS
CLASS – X

Max.Marks:100   **Time: 3 Hrs**

General Instructions:
1. The question paper consists of 25 questions divided into three sections A, B and C.
2. Section A contains 10 questions of 3 marks each. Section B contains 10 questions of 4 marks each.
   Section C contains 5 questions of 6 marks each.
3. There is no overall choice. However, an internal Choice is given in some questions.

SECTION A

1. Solve for x, y :
$$(a - b)x + (a + b)y = a^2 - 2ab - b^2$$
$$(a + b)(x + y) = a^2 + b^2$$

2. Find G.C.D and L.C.M of the following polynomials:
$$24(x^3 + 9x^2 + 20x), \; 28(x^4 + x^3 - 12x^2).$$

3. Simplify:
$$\left[\frac{x^2 + y^2}{x^2 - y^2} - \frac{x^2 - y^2}{x^2 + y^2} \right] \div \left[\frac{x + y}{x - y} - \frac{x - y}{x + y} \right]$$

4. Using quadratic formula, solve the following for y:
$$3y^2 + (6 + 4a)y + 8a = 0$$

5. The 8th term of an Arithmetic Progression is 37 and its 12th term is 57. Find the A.P

6. Find the A.P whose 10th term is -8 and 20th term is 12. Also find the sum of first 10 terms

OR

7. The 5th term of an A.P is 1 and its 31st term is –77. Which term of the A.P is -17.
An article is sold for Rs.2000 cash or for Rs.1000 cashdown payment together with Rs.1040 to be paid after 3 months. Find the rate of interest charged under the instalment plan.

8. The cost of a motorcycle is Rs.27000. A customer agrees to make a down payment of Rs.7140, followed by 3 equal annual instalments. If the rate of

interest be 10% per annum, compounded annually, what is the value of each instalment?

9. In the given figure, ABE, ACF are tangents to the circle with centre O. $\angle EBD = 60^0$ and $\angle DCF = 60^0$. Find $\angle BAC$ and $\angle BDC$.
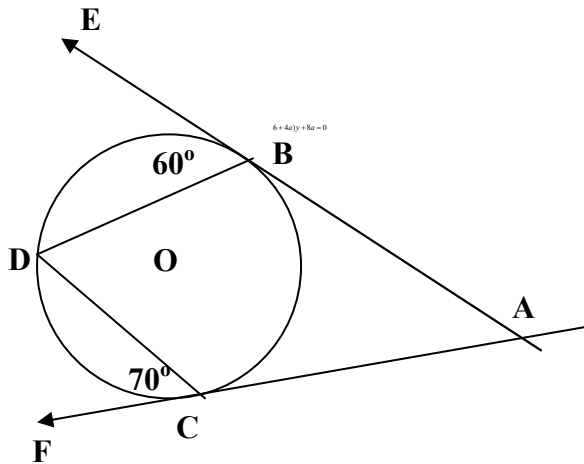
10. The diagonal BD of a parallogram $ABCD$ intersects the line segment AE at the point F, where E is any point on side BC. Prove that $DF \times EF = FB \times FA$

SECTION-B

11. Find graphically the co-ordinates of the triangle formed by the lines $y = x, y = 2x, x + y = 6$.

12. Find the point on y-axis which is equidistant from (-5,-2) and (3,2).

13. Find the co-ordinates of centroid of a triangle whose vertices are (x_1, y_1), (x_2, y_2), (x_3, y_3).

14. A card is drawn at randomfrom a pack of 52 cards. Find the probability that the card drawn is
(i) a black king
(ii) spade or an ace
(iii) neither a heart nor a king
(iv) a jack,queen or a king.

15. Prove that $\dfrac{\cos A}{1 - \tan A} - \dfrac{\sin^2 A}{\cos A - \sin A} = \sin A + \cos A$

OR

Find the value of:

$$\frac{\tan 20^o}{\cot 70^o} + \frac{\cot 50^o}{\tan 40^o} + \frac{\sin^2 20^o + \sin^2 70^o}{\sin\theta\cos(90^o - \theta) + \cos\theta\sin(90^o - \theta)}$$

16. Find the value of p, if the mean of the following data is 20.

X	15	17	19	$20 + p$	23
F	2	3	4	$5p$	6

17. A well of diameter 3m is dug 14m deep. The earth taken out of it has been spread evenly all around it to a width of 4m to form an embankment. Find the height of the embankment.

18. Solve : $\dfrac{x-1}{x-2} + \dfrac{x-3}{x-4} = \dfrac{10}{3}$, $(x \neq 2,4)$

OR

Two pipes running together can fill a tank in 6 minutes. If one pipe takes 5 minutes more than the other to fill the tank, find the time in which each pipe would fill the tank.

19. Construct $\triangle ABC$ in which $BC = 7cm$, $\angle A = 70^0$ and foot of the perpendicular D on BC from A is $4.5cm$ away from B. How many such triangles are possible?

20. The pie chart alongside shows the monthly expenditure of a family on food,clothing, rent, miscellaneous expences and savings. Read the pie chart carefully and answer the following:
(i) What is the central angle for savings?
(ii) What is the ratio of expenditure on food to that of rent?
(iii) If family spends Rs.825 on clothing, what is the total monthly income?
(iv) What percent of the total income does the family save?

Pie chart with sectors: Savings, Rent 90°, Misc. 72°, Food 108°, Clothing 36°
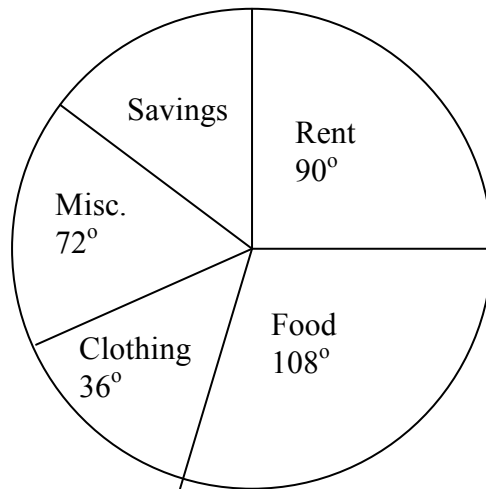
SECTION-C

21.(i) Prove that the sum of either pair of the opposite angles of a cyclic quadrilateral is 180⁰.

(ii) *ABCD* is a cyclic trapezium with *AD // BC* . If $\angle B = 70^\circ$, find the other three angles of the trapezium.

22. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.
Using the above result, prove that the area of equilateral triangle described on the side of a square is half the area of the equilateral triangle described on its diagonal.

23 .A pole 5m high is fixed on the top of a tower. The angle of elevation of the top of the pole observed from a point 'A' on the ground is 60⁰ and the angle of depression of the point 'A' from the top of the tower is 45⁰. Find the height of the tower.

24. A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface area of the remaining portion is 8/9 of the curved surface of the whole cone, find the ratio of the line-segments into which the cone's altitude is divided by the plane.

OR

The interior of a building is in the form of a cylinder of base radius 12m and height 3.5m, surmounted by a cone of equal base and slant height 12.5m. Find the internal curved surface area and the capacity of the building.

25. Shahid income is Rs.60, 000 (excluding   HRA). He pays a premium of Rs.30, 000 annually towards L.I.C, and contributes Rs.5, 000 per month towards P.F. He purchases NSC for Rs.20, 000 and contributes Rs.10, 000 towards P.M.s National Relief fund. He also donates Rs.8, 000 to a school where he studied, earning deduction 100% relief and 50% of the amount donated respectively. Find the total income tax to be paid by him for that year. If Rs.8000 is deducted from his salary every month towards income tax, find the tax to be paid by him in the last month of the year.

For calculation purpose use the following instructions:

Deduction u/s.80c: An individual is allowed a deduction up to Rs.1, 00,000 from gross salary on account of savings for contributions in PF, PPF, LIC, NSC, Infrastructure bonds etc.

 Income tax rates for individuals:

Taxable income up to 1,00,000 -	Nil
1,00,000 to 1,50,000 -	10% of the amount exceeding 1,00,000
1,50,001 to 2,50,000 -	Rs.5,000 + 20% of the amount exceeding 1,50,000
2,50,001 & above -	Rs.25,000 + 30% of the amount exceeding 2,50,000
Education Cess:	2% of the total tax payable.

6. .   Prove any four vertices of a regular pentagon are concyclic.

7. Find mean by step deviation method

Class Interval	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	235	340	340	360	420	220

8. State and prove alternate segment theorem.

9. Prove: $\cot^2\theta\left(\dfrac{\sec\theta-1}{1+\sin\theta}\right) + \sec^2\theta\left(\dfrac{\sin\theta-1}{1+\sec\theta}\right) = 0$

10. A car is available for Rs. 4,02,200 cash or Rs. 1,50,000 cash down payment and three equal half
yearly instalments. If the interest is charged at 10% per annum compounded half yearly, find the
value of each instalment.

SECTION – B

11. Solve the following system of linear equations graphically: 3x – 5y = 19; 3y – 7x +1 =0. Does the point
(4,9) lie on any of these lines? Write its equation.

12. Find the ratio in which the segment joining the points (5,6) and (2, -3) is divided by x-axis. (and also)

By distance formula, prove that (2,5), (-1,2) and (4,7) are collinear.

13. Simplify: $\dfrac{1}{x+a} + \dfrac{1}{x+b} + \dfrac{1}{x+c} + \dfrac{ax}{x^3+ax^2} + \dfrac{bx}{x^3+bx^2} + \dfrac{cx}{x^3+cx^2}$

14. If PAB is a secant to a circle, intersecting the circle at A and B and PT is a tangent segment, Prove that PA × PB=PT^2.
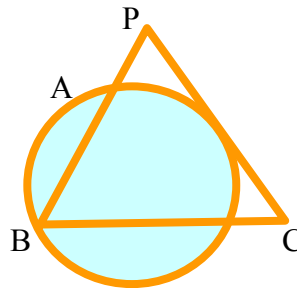
15. Construct a ΔABC in which AB = 5 cm. ∠B = 60° and altitude CD = 3 cm. Construct a ΔAQR similar to Δ ABC such that each side of Δ AQR is 1.5 times that of the corresponding side of ΔABC.

16. Show that a-b, a and a+b form consecutive terms of an A.P. (and)

The first term of an AP is 5 and its 100th term is -292. Find the 50th term of this AP.

17. The number of students in various classes in a hobby school are as the following table :-

Hobby	Computers	Painting	Dancing	Reading	Playing
Students	180	150	27	75	108

Represent the data by a pie chart.

17. Evaluate:

$\dfrac{\sin\theta.\cos\theta.\cos(90-\theta)}{\cos(90-\theta)} + \dfrac{\cos\theta.\sin\theta.\cos(90-\theta)}{\sin(90-\theta)} + \dfrac{\sin^2 27 + \sin^2 63}{\cos^2 40 + \cos^2 50}$ (and)

Prove the identity: $\dfrac{\cot\theta + \cos ec\theta - 1}{\cot\theta - \cos ec\theta + 1} = \dfrac{1+\cos\theta}{\sin\theta}$

18 State and prove Basic Proportionality Theorem.

Using above result, prove that line drawn parallel to parallel sides of a trapezium divides the non –parallelsides proportionally.

19. Water in a canal, 30 dm wide and 12 dm deep is flowing with a velocity of 10 km/h. How much area will it irrigate in 30 minutes, if 4 cm of standing water is required for irrigation?

20. At the foot of a mountain, the elevation of its summit is 45 . After ascending 1000 m towards the mountain up a slope of 30 inclination the elevation is found to be 60. Find the height of the mountain.

SECTION – C

21. At the foot of a mountain, the elevation of its summit is 45 . After ascending 1000 m towards the mountain up a slope of 30 inclination the elevation is found to be 60. Find the height of the mountain
OR   The angle of elevation A of the top of a light house, as seen by a person on the ground,
is such that tanA = 5/12. When the person moves a distance of 240 m towards the light house, the angle   of elevation becomes B. Such that tan B = 3/4, find the height of the light house.

22. The percentage of various categories of workers in a state is given in the following table. Present the information in the form of pie chart

Cultivators	Agricultural	Industrial	Commercial	Others
49	25	12.5	10	12.5

23. If a line is drawn parallel to one side of a triangle, then the other two sides are divided in the
same ratio - Prove.
Using the above result, prove the following.
In the following figure, ABCD is a parallelogram,
P is a point on BC and DP when produced meets
AB produced at L. Prove that DP/DL = DC/BL

24.   Prove that the sum of either pair of the opposite angles of a cyclic quadrilateral is 180.
using the above result, solve the following :
In fig. BD = DC and angle DBC = 25
find the measure of angle BAC.

25.   Annual income from salary of Shyam is Rs.2,40,000.
He contributes Rs. 2,000 per month to provident
fund, pay annual LIC premium of Rs. 5,000 invests
Rs. 15,000 in NSC's and donates Rs.5,000 to PM's National Relief Fund
carrying 100% relief. Calculate the income tax, he has to pay for the year.

Standard Deduction : 1/3rd of the total annual gross salary subject to maximum of Rs. 30,000 if income is les than Rs. Rs.1.5 lakh and Rs. 25,000 if income is from Rs..
1.5 lakh to Rs 3 lakh

Rate of income tax :

a) Upto Rs. 50,000	No tax
b) From Rs. 50,001 to Rs. 60,000	10% of the amount exceeding Rs. 50,000
c) From 60,001 to Rs. 1,50,000	Rs. 1000 + 20% of the amount exceeding Rs. 60,000
d) from Rs. 1,50,001 and above	Rs. 19,000 + 30% of the amount exceeding Rs. 1,50,000

REbate :   20% of the amount of saving subject to maximum Rs. 14,000 if taxable income is upto Rs. 1,50,000.

15% of the amount of saving subject to a maximum of Rs. 10,500 if taxable income is above Rs. 1,50,000.

Surcharge : 5% of the total tax payable. (After rebate)