Reg. No. : .....

Name : .....

Code No. 2053

Second Year – JUNE 2016
SAY / IMPROVEMENT

Time : 2½ Hours
Cool-off time : 15 Minutes

Part – III

MATHEMATICS (COMMERCE)

Maximum : 80 Scores

General Instructions to Candidates :

- There is a 'cool-off time' of 15 minutes in addition to the writing time of 2½ hrs.
- You are not allowed to write your answers nor to discuss anything with others during the 'cool-off time'.
- Use the 'cool-off time' to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- All questions are compulsory and only internal choice is allowed.
- When you select a question, all the sub-questions must be answered from the same question itself.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Malayalam version of the questions is also provided.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

നിർദ്ദേശങ്ങൾ :

- നിർദ്ദിഷ്ട സമയത്തിന് പുറമെ 15 മിനിറ്റ് 'കൂൾ ഓഫ് ടൈം' ഉണ്ടായിരിക്കും. ഈ സമയത്ത് ചോദ്യങ്ങൾക്ക് ഉത്തരം എഴുതാനോ, മറ്റുള്ളവരുമായി ആശയവിനിമയം നടത്താനോ പാടില്ല.
- ഉത്തരങ്ങൾ എഴുതുന്നതിന് മുമ്പ് ചോദ്യങ്ങൾ ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- എല്ലാ ചോദ്യങ്ങൾക്കും ഉത്തരം എഴുതണം.
- ഒരു ചോദ്യനമ്പർ ഉത്തരമെഴുതാൻ തെരഞ്ഞെടുത്തു കഴിഞ്ഞാൽ ഉപചോദ്യങ്ങളും അതേ ചോദ്യനമ്പരിൽ നിന്ന് തന്നെ തെരഞ്ഞെടുക്കേണ്ടതാണ്.
- കണക്ക് കൂട്ടലുകൾ, ചിത്രങ്ങൾ, ഗ്രാഫുകൾ എന്നിവ ഉത്തരപേപ്പറിൽ തന്നെ ഉണ്ടായിരിക്കണം.
- ചോദ്യങ്ങൾ മലയാളത്തിലും നൽകിയിട്ടുണ്ട്.
- ആവശ്യമുള്ള സ്ഥലത്ത് സമവാക്യങ്ങൾ കൊടുക്കണം.
- പ്രോഗ്രാമുകൾ ചെയ്യാനാകാത്ത കാൽക്കുലേറ്ററുകൾ ഒഴികെയുള്ള ഒരു ഇലക്ട്രോണിക് ഉപകരണവും പരീക്ഷാഹാളിൽ ഉപയോഗിക്കുവാൻ പാടില്ല.

1. Consider the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 7 \\ 3 & 0 & -2 \end{bmatrix}$

(i) Find A'. **(Score : 1)**

(ii) Find A + A' and A – A'. **(Scores : 2)**

(iii) Express A as the sum of a symmetric and skew symmetric matrices. **(Scores : 2)**

2. (i) If A is any square matrix of order 'n', then $|KA| =$ _____.

(a) $K|A|$ (b) $K^n|A|$

(c) KA (d) nA **(Score : 1)**

(ii) Prove that $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^2(x+a+b+c)$ **(Scores : 3)**

3. Consider the equations

$x + 2y + 2z = 4$

$2x - y + 3z = 9$

$3x - y - z = 2$

(i) Find $|A|$. **(Score : 1)**

(ii) Solve the above system of equations by matrix method. **(Scores : 4)**

4. (i) If $f : R \to R$ is given by $f(x) = 3x^2$, then $fof(x)$ is _____.

(a) $6x^2$ (b) $9x^2$

(c) $27x^4$ (d) $27x^2$ **(Score : 1)**

(ii) Let * be a binary operation on the set of rational number Q, defined by $a * b = ab + 1$ **(Scores : 2)**

(a) Check whether * is commutative. **(Scores : 2)**

(b) Check whether * is associative.

5. (i) Find the principal value of $\tan^{-1}(-\sqrt{3})$. **(Score : 1)**

(ii) Solve $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x(x>0)$. **(Scores : 3)**

6. Consider the function $f(x)=\begin{cases}\frac{x^2-x-6}{x+2} & , x\neq -2\\ -5 & , x=-2\end{cases}$

(i) What is the value of f(–2) ? **(Score : 1)**

(ii) Check whether the function f(x) is continuous at $x=-2$. **(Scores : 2)**

7. (i) If $y=\log\left(\frac{1}{x}\right)$, then show that $\frac{dy}{dx}+\frac{1}{x}=0$ **(Scores : 2)**

(ii) If $y=a\cos(\log x)+b\sin(\log x)$, then prove that

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$$ **(Scores : 3)**

(iii) Find $\frac{dy}{dx}$, if $y=xe^x+\frac{1}{x}$ **(Scores : 3)**

8. If the total revenue received from the sale of x units of a product is $R(x)=200+\frac{x^2}{5}$, then

(i) Find the Revenue when $x=20$ units. **(Score : 1)**

(ii) Find the marginal revenue function. **(Score : 1)**

(iii) Find the marginal revenue when $x=10$ units and $x=25$ units respectively. **(Scores : 2)**

OR

(i) Show that the function $f(x)=e^{2x}$ is strictly increasing on R. **(Score : 1)**

(ii) Find the maximum value of the function $f(x)=4x-\frac{1}{2}x^2, x\in\left[-2,\frac{9}{2}\right]$. **(Scores : 3)**

9. (i) Find $\int e^{\log x}\,dx$ **(Score : 1)**

(ii) Find $\int\frac{1}{x+x\log x}\,dx$ **(Scores : 2)**

(iii) Find $\int x \sin x \, dx$ **(Scores : 2)**

OR

(i) If $\int_0^a 3x^2 \, dx = 8$, then the value of a = _____.

(a) 2 (b) 4

(c) 6 (d) 8 **(Scores : 2)**

(ii) Show that $\int_0^{\pi/2} \log \tan x \, dx = 0$ **(Scores : 2)**

(iii) $\int_{-\pi/2}^{\pi/2} \sin^7 x \, dx =$ _____. **(Score : 1)**

10. Consider the curve $x = y^2$ and the line $x = 4$

(i) Sketch and shade the region bounded by the curve and the line. **(Score : 1)**

(ii) Find the area of the shaded region. **(Scores : 3)**

11. (i) Form the differential equation of family of curves $xy = c^2$. **(Score : 1)**

(ii) Consider the differential equation $\frac{dy}{dx} = (1 + x^2)(1 + y^2)$.

(a) Express the equation in the variable separable form. **(Score : 1)**

(b) Hence solve the differential equation. **(Scores : 2)**

12. The position vectors of the vertices of the ΔABC are $3\hat{i} - 4\hat{j} - 4\hat{k}$, $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} - 3\hat{j} - 5\hat{k}$ respectively.

(i) Find $\overrightarrow{AB}$ and $\overrightarrow{BC}$. **(Score : 1)**

(ii) Find the unit vector perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{BC}$. **(Scores : 3)**

(iii) Show that ΔABC is a right angled triangle. **(Scores : 2)**

13. (i) Find the angle between the pair of lines

$\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3i + 2\hat{j} + 6\hat{k})$ and

$\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$

(Scores : 3)

(ii) Find the equation of the plane passing through

(1, 1, 0), (1, 2, 1) and (–2, 2, –1)

(Scores : 2)

OR

(i) Find the shortest distance between the lines

$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

(Scores : 3)

(ii) Find the angle between the planes $\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} - 3\hat{j} + 5\hat{k}) = 3$

(Scores : 2)

14. A furniture dealer sells only tables and chairs. He has ₹ 12,000 to invest and a space to store 90 pieces. A table costs ₹ 400 and a chair ₹ 100. He can sell a table at a profit of ₹ 75 and a chair at a profit of ₹ 25. Assume that he can sell all the items. The dealer wants to get maximum profit.

(i) Write the objective function.

(Score : 1)

(ii) Write the constraints.

(Scores : 3)

15. Consider the linear programming problem

Maximize z = $5x + 3y$, Subject to

$3x + 5y \leq 15$

$5x + 2y \leq 10$

$x \geq 0, y \geq 0$

(i) Draw the graph of the lines $3x + 5y = 15$, $5x + 2y = 10$

(Scores : 2)

(ii) Solve the linear programming problem graphically.

(Scores : 2)

16. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls.

(i) Find the probability of both balls are red. **(Score : 1)**

(ii) Find the probability of first ball is black and the second ball is red. **(Scores : 2)**

(iii) Find the probability of one of them is black and the other is red. **(Scores : 2)**

17. A die is thrown 6 times. If "getting an odd number" is a success,

(i) Write the distribution. **(Score : 1)**

(ii) Find the probability of getting at least 5 successes. **(Scores : 2)**

(iii) Find the probability of getting atmost 5 successes. **(Scores : 2)**
