Question Paper - Outside Delhi (2012)

General Instructions:

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into three Sections A, B and C, Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

SECTION-A

Questions numbers 1 to 10 carry 1 mark each.

Q1. The binary operation * : R × R → R is defined as $a * b = 2a + b$. Find (2 * 3) * 4.

Q2. Find the principal value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$.

Q3. Find the value of $x + y$ from the following equation :

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

Q4. If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then find $\mathbf{A}^T - B^T$.

Q5. Let A be a square matrix of order 3 × 3. Write the value of |2A|, where |A| = 4.

Q6. Evaluate : $\int_0^2 \sqrt{4 - x^2}\, dx$

Q7. Given $\int e^x (\tan x + 1) \sec x\, dx = e^x f(x) + c$.

Write $f(x)$ satisfying the above.

Q8. Write the value of $(\hat{i} \times \hat{j}).\hat{k} + \hat{i}.\hat{j}$.

Q9. Find the scalar components of the vector $\overrightarrow{AB}$ with initial point A(2, 1) and terminal point B (–5, 7).

Q10. Find the distance of the plane $3x - 4y + 12z = 3$ from the origin.

SECTION-B

Questions numbers 11 to 22 carry 4 mark each.

Q11. Prove the following :

$$\cos\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)=\frac{6}{5\sqrt{13}}$$

Q12. Using properties of determinants, show that

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$$

Q13. Show that $f : \mathrm{N} \to \mathrm{N}$, given by

$$f(x) = \begin{cases} x+1, & \text{if } x \text{ is odd} \\ x-1, & \text{if } x \text{ is even} \end{cases}$$

is both one-one and onto.

OR

Consider the binary operations $*$: R × R → R and o : R × R → R defined as $a * b = |a - b|$ and $a \, o \, b = a$ for all $a, b \in$ R. Show that '*' is commutative but not associative, 'o' is associative but not commutative.

Q14. If $x = \sqrt{a^{\sin^{-1}t}}, y = \sqrt{a^{\cos^{-1}t}}$, show that $\dfrac{dy}{dx} = -\dfrac{y}{x}$.

OR

Differentiate $\tan^{-1}\left[\dfrac{\sqrt{1+x^2}-1}{x}\right]$ with respect to x.

Q15. If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$, $0 < t < \dfrac{\pi}{2}$, find $\dfrac{d^2x}{dt^2}, \dfrac{d^2y}{dt^2}$ and $\dfrac{d^2y}{dx^2}$.

Q16. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?

Q17. Evaluate : $\displaystyle\int_{-1}^{2} |x^3 - x| \, dx$

OR

Evaluate : $\displaystyle\int_{0}^{\pi} \frac{x\sin x}{1+\cos^2 x} dx$

Q18. Form the differential equation of the family of circles in the second quadrant and touching the coordinate axes.

OR

Find the particular solution of the differential equation

$x(x^2-1)\dfrac{dy}{dx}=1;\ \ y=0$ when $x = 2$.

Q19. Solve the following differential equation :

$((1+x^2)dy + 2xy\,dx = \cot x\,dx; x \neq 0$

Q20. Let $\vec{a} = \hat{i}+4\hat{j}+2\hat{k},\ \vec{b} = 3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c} = 2\hat{i}-\hat{j}+4\hat{k}$. Find a vector $\vec{p}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{p}\,.\,\vec{c}=18$.

Q21. Find the coordinates of the point where the line through the points A(3, 4, 1) and B(5, 1, 6) crosses the XY-plane.

Q22. Two cards are drawn simultaneously (without replacement) from a well-shuffled pack of 52 cards. Find the mean and variance of the number of red cards.

SECTION-C

Questions numbers 23 to 29 carry 6 mark each.

Q23. Using matrices, solve the following system of equations :

$2x+3y+3z=5$, $x-2y+z=-4$, $3x-y-2z=3$.

Q24. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

OR

An open box with a square base is to be made out of a given quantity of cardboard of area c^2 square units. Show that the maximum volume of the box is $\dfrac{c^3}{6\sqrt{3}}$ cubic units.

Q25. Evaluate : $\displaystyle\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$

OR

Evaluate : $\displaystyle\int \frac{x^2+1}{(x-1)^2(x+3)}dx$

Q26. Find the area of the region $\{(x, y) : x^2 + y^2 \leq 4,\ x + y \geq 2\}$.

Q27. If the lines $\dfrac{x-1}{-3}=\dfrac{y-2}{-2k}=\dfrac{z-3}{2}$ and $\dfrac{x-1}{k}=\dfrac{y-2}{1}=\dfrac{z-3}{5}$ are perpendicular, find the value of k and hence find the equation of plane containing these lines.

Q28. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin 3 times and notes the number of heads. If she gets 1, 2, 3 or 4 she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die?

Q29. A dietician wishes to mix two types of foods in such a way that the vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C while Food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs ` 5 per kg to purchase Food I and ` 7 per kg to purchase Food II. Determine the minimum cost of such a mixture. Formulate the above as a LPP and solve it graphically.