Question Paper-Foreign (2013)

General Instructions:

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION-A

Questions numbers 1 to 10 carry 1 mark each.

Q1. Write the principal value of $\tan^{-1}\left(\tan\frac{9\pi}{8}\right)$.

Q2. Write the value of $\sin\left(2\sin^{-1}\frac{3}{5}\right)$.

Q3. If A is a 3 × 3 matrix, whose elements are given by $a_{ij} = \frac{1}{3}|-3i + j|$, then write the value of a_{23}.

Q4. If A is a square matrix and | A | = 2, then write the value of |AA′ | where A′ is the transpose of matrix A.

Q5. If $A = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$, then write A^{-1}.

Q6. Write the differential equation formed from the equation $y = mx + c$, where m and c are arbitrary constants.

Q7. If $\vec{a}$ is a unit vector and $(\vec{x}-\vec{a}).(\vec{x}+\vec{a}) = 24$, then write the value of $|\vec{x}|$.

Q8. For any three vectors $\vec{a}, \vec{b}$ and $\vec{c}$, write the value of the following :

$\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b})$

Q9. Write the cartesian equation of a plane, bisecting the line segment joining the points A(2, 3, 5) and B(4, 5, 7) at right angles.

Q10. If $C = 0.003x^3 + 0.02x^2 + 6x + 250$ gives the amount of carbon pollution in air in an area on the entry of x number of vehicles, then find the marginal carbon pollution in the air, when 3 vehicles have entered in the area and write which value does the question indicate.

SECTION B

Question numbers 11 to 22 carry 4 marks each.

Q11. Prove that the relation R in the set A = {5, 6, 7, 8, 9} given by R = {(a, b) : | a – b | is divisible by 2}, is an equivalence relation. Find all elements related to the element 6.

Q12. If $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$, then find the value of x.

OR

If $\cot^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$, then prove that $\sin y = \tan^2\left(\frac{x}{2}\right)$.

Q13. Using properties of determinants, prove the following

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

Q14. Differentiate the following with respect to x :

$x^{\sin x} + (\sin x)^{\cos x}$

Q15. If $y = \sin(\log x)$, then prove that

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

Q16. Show that the function $f(x) = 2x - |x|$ is continuous but not differentiable at $x = 0$.

OR

Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}x$, when $x \neq 0$.

Q17. Evaluate : $\int \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$

OR

Evaluate : $\int x^2 \log(1 + x) dx$

Q18. Evaluate : $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$

Q19. The magnitude of the vector product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to $\sqrt{2}$. Find the value of λ.

Q20. Evaluate : $\int \frac{x^2 + 1}{x^2 - 5x + 6} dx$

Q21. Find the shortest distance between the following lines :

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}; \frac{3-x}{-1} = \frac{y-5}{-2} = \frac{z-7}{1}.$$

OR

Find the equation of the plane through the points (2, 1, – 1) and (– 1, 3, 4) and perpendicular to the plane $x - 2y + 4z = 10$.

Q22. In a group of 50 scouts in a camp, 30 are well trained in first aid techniques while the remaining are well trained in hospitality but not in first aid. Two scouts are selected at random from the group. Find the probability distribution of number of selected scouts who are well trained in first aid. Find the mean of the distribution also.

Write one more value which is expected from a well trained scout.

SECTION C

Question numbers 23 to 29 carry 6 marks each.

Q23. 10 students were selected from a school on the basis of values for giving awards and were divided into three groups. The first group comprises hard workers, the second group has honest and law abiding students and the third group contains vigilant and obedient students. Double the number of students of the first group added to the number in the second group gives 13, while the combined strength of first and second group is four times that of the third group. Using matrix method, find the number of students in each group. Apart from the values, hard work, honesty and respect for law, vigilance and obedience, suggest one more value, which in your opinion, the school should consider for awards.

Q24. Prove that the volume of the largest cone, that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

OR

Show that the normal at any point θ to the curve $x = a\cos\theta + a\theta\sin\theta$, $y = a\sin\theta - a\theta\cos\theta$ is at a constant distance from the origin.

Q25. Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$.

Q26. Solve the differential equation $x^2\frac{dy}{dx} - xy = 1 + \cos\left(\frac{y}{x}\right), x \neq 0$. Find the particular solution of,this differential equation, given that when $x = 1, y = \frac{\pi}{2}$.

Q27. Find the image of the point having position vector $\hat{i} + 3\hat{j} + 4\hat{k}$ in the plane $\vec{r}.(2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$.

OR

Find the equation of a plane which is at a distance of $3\sqrt{3}$ units from origin and the normal to which is equally inclined to the coordinate axes.

Q28. An aeroplane can carry a maximum of 200 passengers. A profit of ₹ 500 is made on each executive class ticket out of which 20% will go to the welfare fund of the employees. Similarly a profit of ₹ 400 is made on each economy ticket out of which 25% will go for the improvement of facilities provided to economy class passengers. In both cases, the remaining profit goes to the airline's fund. The airline reserves at least 20 seats for executive class. However at least four times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximise the net profit of the airline. Make the above as an LPP and solve graphically.

Do you think, more passengers would prefer to travel by such an airline than by others ?

Q29. Often it is taken that a truthful person commands more respect in the society. A man is known to speak the truth 4 out of 5 times. He throws a die and reports that it is actually a six. Find the probability that it is actually a six. Do you also agree that the value truthfulness leads to more respect in the society?