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Senior School Certificate Examination

March — 2014 - Compartment

Marking Scheme — Mathematics (Outside) 65/1, 65/2, 65/3

General Instructions :

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question(s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

65|1

SECTION-A

1. 11 2. $\frac{7}{2}$ 3. $-\frac{\pi}{4}$ 4. $x=4$. 5. 0 (zero)

6. $\frac{\tan^7 x}{7}+c$ 7. $\frac{\pi^2}{32}$ 8. $\frac{\pi}{6}$ 9. $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ 10. $\frac{x-\alpha}{0}=\frac{y-\beta}{0}=\frac{z-\gamma}{1}$ OR $\vec{r}=(\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k})+\lambda(\hat{k})$ — $1\times10 = 10$.

SECTION-B

11. Here $f(x)=\begin{cases}2x, & \text{if } x\geq 0\\ 0, & \text{if } x<0\end{cases}$ and $g(x)=\begin{cases}0, & \text{if } x\geq 0\\ -2x, & \text{if } x<0\end{cases}$ $\frac{1}{2}+\frac{1}{2}$

Thus for $x\geq 0$, $gof(x)=g(2x)=0$
and for $x<0$, $gof(x)=g(0)=0$ $\Rightarrow gof(x)=0\ \forall x\in R$ $1\frac{1}{2}$

and for $x\geq 0$, $fog(x)=f(0)=2(0)=0$
for $x<0$, $fog(x)=f(-2x)=-4x$ $\Rightarrow fog(x)=\begin{cases}0, & \text{if } x\geq 0\\ -4x, & \text{if } x<0\end{cases}$ $1\frac{1}{2}$.

12. Let $\cos^{-1}x=\alpha$ $\therefore$ $\cos\alpha=x$

$\therefore$ LHS $=\alpha+\cos^{-1}\left[\cos\alpha.\cos\pi/3+\frac{\sqrt{3}}{2}\sqrt{1-\cos^2\alpha}\right]$ $1\frac{1}{2}$

$=\alpha+\cos^{-1}\left[\cos\pi/3\cos\alpha+\sin\pi/3\sin\alpha\right]$ 1

$=\alpha+\cos^{-1}(\cos(\pi/3-\alpha)) = \alpha+\pi/3-\alpha=\pi/3=$ RHS $1\frac{1}{2}$

OR

$\tan^{-1}x+2\cot^{-1}x=2\pi/3$

$\Rightarrow \tan^{-1}x+2(\pi/2-\tan^{-1}x)=2\pi/3$ 1

$\Rightarrow -\tan^{-1}x=2\pi/3-\pi=-\pi/3$ or $\tan^{-1}x=\pi/3$ 1+1

$x=\tan\pi/3=\sqrt{3}$ 1.

13. $\Delta=\begin{vmatrix}b+c & a & a\\ b & c+a & b\\ c & c & a+b\end{vmatrix}=\begin{vmatrix}0 & -2c & -2b\\ b & c+a & b\\ c & c & a+b\end{vmatrix}$ (using $R_1\to R_1-(R_2+R_3)$) 1

$R_2\to cR_2$ and $R_3\to bR_3$

$=-\frac{2}{bc}\begin{vmatrix}0 & c & b\\ bc & c^2+ac & bc\\ bc & bc & ab+b^2\end{vmatrix}=-2bc\begin{vmatrix}0 & 1 & 1\\ 1 & c+a & c\\ 1 & b & a+b\end{vmatrix}$ 1

$= \underset{C_2\to C_2-C_3}{-2bc}\begin{vmatrix}0 & 0 & 1\\ 1 & a & c\\ 1 & -a & a+b\end{vmatrix}$ 1

$=-2bc(-2a) = 4abc.$ 1

14. $\lim_{x\to 0} \frac{1-\cos 4x}{8x^2} = \lim_{x\to 0} \frac{2\sin^2 2x}{8x^2}$ 1

$= \lim_{x\to 0} \left(\frac{\sin 2x}{2x}\right)^2 = 1.$ 1

f is continuous at $x=0$ $\therefore \lim_{x\to 0} f(x) = f(0) = k$ 1

$\therefore \quad 1 = k \quad$ or $k = 1$ 1

15. $y = \tan^{-1}\left(\frac{a}{x}\right) + \frac{1}{2}\left\{\log(x-a) - \log(x+a)\right\}$ 1

$\frac{dy}{dx} = \frac{1}{1+\frac{a^2}{x^2}} \cdot \left(-\frac{a}{x^2}\right) + \frac{1}{2}\left\{\frac{1}{x-a} - \frac{1}{x+a}\right\}$ 1+1

$= \frac{-a}{x^2+a^2} + \frac{a}{x^2-a^2} = \frac{2a^3}{x^4-a^4}$ 1

16. $f'(x) = \frac{12}{10}x^3 - \frac{12}{5}x^2 - 6x + \frac{36}{5}$ $\frac{1}{2}$

$= \frac{6}{5}(x-1)(x+2)(x-3)$ $1\frac{1}{2}$

$f'(x) = 0 \Rightarrow x = -2, 1, 3$ $\frac{1}{2}$

$\therefore$ Intervals are $(-\infty,-2)$, $(-2,1)$, $(1,3)$, $(3,\infty)$ $\frac{1}{2}$

$f(x)$ is st. increasing in $(-2,1) \cup (3,\infty)$ $\frac{1}{2}$

and, st. decreasing in $(-\infty,-2) \cup (1,3)$ $\frac{1}{2}$

OR.

Let s be the side of equilateral triangle

then $\frac{ds}{dt} = 2$ cm/s, $\frac{1}{2}$

and Area$(A) = \frac{\sqrt{3}s^2}{4}$ 1

$\therefore \quad \frac{dA}{dt} = \frac{2\sqrt{3}s}{4} \cdot \frac{ds}{dt}$ 1

$= \frac{2\sqrt{3}}{4} \cdot 10 \cdot 2$, (When $s = 10$cm) 1

$= 10\sqrt{3}$ cm²/s $\frac{1}{2}$

17. $I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}\,dx = \int_0^{\pi/2} \frac{\sin^2(\pi/2 - x)}{\sin(\pi/2-x) + \cos(\pi/2-x)}\,dx$ 1

$\therefore 2I = \int_0^{\pi/2} \frac{1}{\sin x + \cos x}\,dx = \frac{1}{\sqrt{2}}\int_0^{\pi/2} \frac{1}{\cos(x-\pi/4)}\,dx$ 1

$2I = \frac{1}{\sqrt{2}}\left[\log\left|\sec(x-\pi/4)+\tan(x-\pi/4)\right|\right]_0^{\pi/2}$ 1

$= \frac{1}{\sqrt{2}}\left[\log|\sqrt{2}+1| - \log|\sqrt{2}-1|\right] = \frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{2}+1}{\sqrt{2}-1}\right|$ $\frac{1}{2}$

$I = \frac{2}{2\sqrt{2}}\log|\sqrt{2}+1| = \frac{1}{\sqrt{2}}\log|\sqrt{2}+1|$ $\frac{1}{2}$

OR.

$I = \int \frac{x^2}{x^4+3x^2+2}\cdot x\,dx = \frac{1}{2}\int \frac{t\,dt}{t^2+3t+2}$ where $x^2 = t$ 1

$= \frac{1}{2}\left[\int\left(\frac{2}{t+2} - \frac{1}{t+1}\right)dt\right] = \frac{1}{2}\left\{2\log|t+2| - \log|t+1|\right\} + c$ 1+1

$= \log\left|\frac{t+2}{\sqrt{t+1}}\right| + c = \log\left|\frac{x^2+2}{\sqrt{x^2+1}}\right| + c$ 1

18. here, $\frac{dy}{dx} = \frac{y}{x} - \operatorname{cosec}\left(\frac{y}{x}\right)$ $\frac{1}{2}$

$v + x\frac{dv}{dx} = v - \operatorname{cosec} v$, where $\frac{y}{x} = v$. 1

$-\int \sin v\,dv = \int \frac{dx}{x}$ $\frac{1}{2}$

$\Rightarrow \cos v = \log|x| + c \Rightarrow \cos\left(\frac{y}{x}\right) = \log|x| + c$ 1

$x = 1, y = 0 \Rightarrow c = 1.$ $\frac{1}{2}$

$\therefore \cos\left(\frac{y}{x}\right) = 1 + \log|x|$ $\frac{1}{2}$

19. $x\frac{dy}{dx} + y = x\cos x + \sin x$

$\Rightarrow \frac{dy}{dx} + \frac{1}{x}\cdot y = \cos x + \frac{1}{x}\sin x.$ $\frac{1}{2}$

$\therefore$ I.F $= e^{\int \frac{1}{x}dx} = e^{\log x} = x,$ 1

$\therefore$ Solution is $xy = \int(x\cos x + \sin x)\,dx$ $\frac{1}{2}$

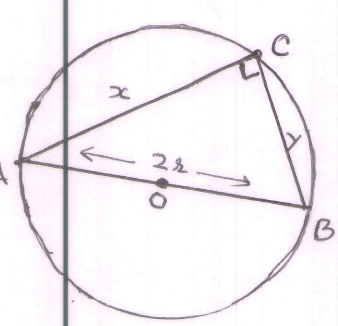
$\Rightarrow xy = x\sin x + C.$ 1

$\therefore y = \sin x + C\cdot\frac{1}{x}$

$x = \frac{\pi}{2}, y = 1 \Rightarrow 1 = 1 + C\cdot\frac{2}{\pi} \Rightarrow C = 0$ $\frac{1}{2}$

$\therefore$ Solution is $y = \sin x.$ $\frac{1}{2}$

20.	Any point on the given line is given as	
	$(2+3\lambda, -1+4\lambda, 2+2\lambda)$	1
	If this point lies on the plane, then	
	$(2+3\lambda) - (-1+4\lambda) + (2+2\lambda) = 5$	
	$\Rightarrow \lambda = 0$	1
	$\therefore$ Point of intersection is $(2, -1, 2)$	1
	Distance of $P(2,-1,2)$ from $Q(-1,-5,-10)$ is	
	$= \sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2} = \sqrt{9+16+144} = 13$ units	1
21.	Any vector perpendicular to both $\vec{\alpha}$ and $\vec{\beta} = \vec{\alpha} \times \vec{\beta}$	$\frac{1}{2}$
	$\therefore \vec{p} = \lambda(\vec{\alpha} \times \vec{\beta}) = \lambda(21\hat{i} - 21\hat{j} - 21\hat{k})$	$1\frac{1}{2}$
	$\vec{p} \cdot \vec{q} = 21 \Rightarrow \lambda(63 - 21 + 21) = 21 \Rightarrow \lambda = \frac{1}{3}$	$1\frac{1}{2}$
	$\therefore \vec{p} = 7\hat{i} - 7\hat{j} - 7\hat{k}$	$\frac{1}{2}$
	OR	
	Required vector $= \dfrac{\vec{AB} \times \vec{AC}}{\lvert \vec{AB} \times \vec{AC} \rvert}$	1
	$\vec{AB} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{AC} = 0\hat{i} + \hat{j} + 2\hat{k}$	1
	$\vec{AB} \times \vec{AC} = 3\hat{i} + 2\hat{j} - \hat{k}$	1
	$\therefore$ required unit vector $= \frac{1}{\sqrt{14}}(3\hat{i} + 2\hat{j} - \hat{k})$	1
22.	X : 14 15 16 17 18 19 20 21 ; $P(x)$: $\frac{2}{15}$ $\frac{1}{15}$ $\frac{2}{15}$ $\frac{3}{15}$ $\frac{1}{15}$ $\frac{2}{15}$ $\frac{3}{15}$ $\frac{1}{15}$	2.
	Mean of $x = \sum xP(x) =$	$\frac{1}{2}$
	$= \frac{1}{15}[28+15+32+51+18+38+60+21]$	1
	$= \frac{1}{15}[263] = 17.53$	$\frac{1}{2}$

23. 

Let the sides of rt. Δ ABC be x and y. Correct fig 1

$\therefore\ x^2+y^2=4r^2$ and

$A=$ Area of $\Delta = \frac{1}{2}\cdot x\cdot y$ $\Big\}$ $\frac{1}{2}$

Let. $S = A^2 = \frac{1}{4}x^2y^2$

$= \frac{1}{4}x^2(4r^2-x^2)$

$= \frac{1}{4}(4r^2x^2-x^4)$ 1

$\therefore \frac{ds}{dx} = \frac{1}{4}\left[8r^2x-4x^3\right]$ $\frac{1}{2}$

$\frac{ds}{dx}=0 \Rightarrow x^2=2r^2$ or $x=\sqrt{2}r$ 1

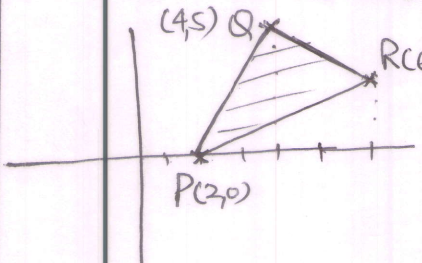
and $y^2=4r^2-2r^2=2r^2$

$\Rightarrow y=\sqrt{2}r$

i.e $x=y$ and $\frac{d^2s}{dx^2}=(2r^2-3x^2)$ $\frac{1}{2}$

$=2r^2-6r^2<0$ 1

$\Rightarrow$ Area is maximum, when Δ is isosceles. $\frac{1}{2}$

24. 

For Correct fig. 1

Eqns. of PQ, QR and PR are:

PQ : $y=\frac{5}{2}(x-2)$

QR: $y=9-x$.

PR: $y=\frac{3}{4}(x-2)$ $\Big\}$ $1\frac{1}{2}$

Req. area $= \int_2^4 \frac{5}{2}(x-2)\,dx + \int_4^6 (9-x)\,dx - \int_2^6 \frac{3}{4}(x-2)\,dx$ $1\frac{1}{2}$

$= \frac{5}{4}(x-2)^2\Big]_2^4 - \frac{1}{2}(9-x)^2\Big]_4^6 - \frac{3}{8}(x-2)^2\Big]_2^6$ $1\frac{1}{2}$

$= 5 + 8 - 6 = 7$ sq. units $\frac{1}{2}$

25. $\int \frac{\sqrt{x^2+1}\,(\log(x^2+1)-2\log x)}{x^4}dx = \int \sqrt{1+\frac{1}{x^2}}\left(\log\left(1+\frac{1}{x^2}\right)\right)\cdot\frac{1}{x^3}dx$ 1

$1+\frac{1}{x^2}=t^2$

$\Rightarrow -\frac{2}{x^3}dx = 2t\,dt$

$= -\int t\,(2\log t)\,t\,dt = -2\int \log t\cdot t^2\,dt$ $\qquad \frac{1}{x^3}dx = -t\,dt$ 1

$= -2\log t\cdot\frac{t^3}{3} + \int 2\,\frac{1}{t}\cdot\frac{t^3}{3}\,dt$ 1

1

$= -\frac{2}{3}\log t\cdot t^3 + \frac{2}{9}t^3 + c$ 1

1

$= \frac{1}{3}\left(1+\frac{1}{x^2}\right)^{\frac{3}{2}}\left(-\log\left(1+\frac{1}{x^2}\right)+\frac{2}{3}\right)+c$

OR

$I = \int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\,dx \quad = \frac{2}{\pi}\int \left[\sin^{-1}\sqrt{x} - \left(\pi/2 - \sin^{-1}\sqrt{x}\right)\right] dx$

$= \frac{2}{\pi}\int 2\sin^{-1}\sqrt{x}\,dx \quad - \int 1\,dx$ 1

$= \frac{4}{\pi}\left[\sin^{-1}\sqrt{x}\cdot x - \int \frac{1}{\sqrt{1-x}}\,\frac{1}{2\sqrt{x}}\cdot x\,dx\right] - x + C$ 1

$= \frac{1}{\pi}\left[4x\sin^{-1}\sqrt{x} - 2\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx\right] - x + C$ $\frac{1}{2}$

$x = \sin^2\theta \Rightarrow dx = 2\sin\theta\cos\theta\,d\theta$

$= \frac{1}{\pi}\left[4x\sin^{-1}\sqrt{x} - 2\int \frac{\sin\theta}{\cos\theta}\cdot 2\sin\theta\cos\theta\,d\theta\right] - x + C$ $\frac{1}{2}$

$= \frac{1}{\pi}\left[4x\sin^{-1}\sqrt{x} - 2\int (1-\cos 2\theta)\,d\theta\right] - x + C$ $\frac{1}{2}$

$= \frac{1}{\pi}\left[4x\sin^{-1}\sqrt{x} - 2\left(\theta - \frac{\sin 2\theta}{2}\right)\right] - x + C$ 1

$= \frac{1}{\pi}\left[4x\sin^{-1}\sqrt{x} - 2\sin^{-1}\sqrt{x} + 2\sqrt{x}\sqrt{1-x}\right] - x + C$ 1

$= \frac{\sin^{-1}\sqrt{x}}{\pi}\left(2(2x-1)\right) + \frac{2}{\pi}\sqrt{x-x^2} - x + C$ $\frac{1}{2}$

26. Equation of plane passing through the line of intersection of the planes $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k}) = 1$ and $\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k}) + 4 = 0$ is

$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k}) - 1 + \lambda\left(\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k}) + 4\right) = 0$ $1\frac{1}{2}$

$\Rightarrow \vec{r}\cdot\left((1+2\lambda)\hat{i} + (1+3\lambda)\hat{j} + (1-\lambda)\hat{k}\right) = (1-4\lambda) \quad \text{---}(i)$ $1\frac{1}{2}$

If Plane (i) is parallel to x- axis then

$(1+2\lambda) = 0 \Rightarrow \lambda = -\frac{1}{2}$

$\therefore$ Eqn. of plane is $1\frac{1}{2}$

$\vec{r}\cdot\left(-\frac{1}{2}\hat{j} + \frac{3}{2}\hat{k}\right) = 3$ or $\vec{r}\cdot(-\hat{j}+3\hat{k}) = 6$ $1\frac{1}{2}$

27. let E_1 : urn has 2 white balls
E_2 : urn has 3 white balls
E_3 : urn has 4 white balls 1

A : 2 balls drawn are white

$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$ 1

$P(A/E_1) = \frac{{}^2C_2}{{}^4C_2} = \frac{1}{6}$, $P(A/E_2) = \frac{{}^3C_2}{{}^4C_2} = \frac{1}{2}$, $P(A/E_3) = \frac{{}^4C_2}{{}^4C_2} = 1.$ $1\frac{1}{2}$

$\therefore P(E_3/A) = \frac{P(E_3)\cdot P(A/E_3)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)} = \frac{\frac{1}{3}\cdot 1}{\frac{1}{3}\cdot\frac{1}{6} + \frac{1}{3}\cdot\frac{1}{2} + \frac{1}{3}\cdot 1}$ $\frac{1}{2}+1$

$= \frac{6}{10} = 0.6$ 1

OR

X	S. Rs. 5/-	F.S. Rs. 4/-	F.F.S. Rs. 3/-	FFF Rs(−3)	
$P(x)$:	$1/6$	$5/6 \cdot 1/6$	$5/6 \cdot 5/6 \cdot 1/6$	$5/6 \cdot 5/6 \cdot 5/6$	$1\frac{1}{2}$
$x \cdot P(x)$:	$5/6$	$20/36$	$75/216$	$\frac{-375}{216}$	$1\frac{1}{2}$

$1\frac{1}{2}$

$\sum x \cdot P(x) = \frac{1}{216}(180 + 120 + 75 - 375) = 0$

$\therefore$ Expected value = Rs 0. $1\frac{1}{2}$

28. We have

$$\left.\begin{aligned} 2x + 3y + 4z &= 4600 \\ 3x + 2y + 3z &= 4100 \\ x + y + z &= 1500 \end{aligned}\right\}$$ $1\frac{1}{2}$

$\begin{pmatrix} 2 & 3 & 4 \\ 3 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4600 \\ 4100 \\ 1500 \end{pmatrix}$ or $A \cdot X = B$ $\frac{1}{2}$

$|A| = 2(-1) - 3(0) + 4(1) = 2 \neq 0 \therefore X = A^{-1}B$ $\frac{1}{2}$

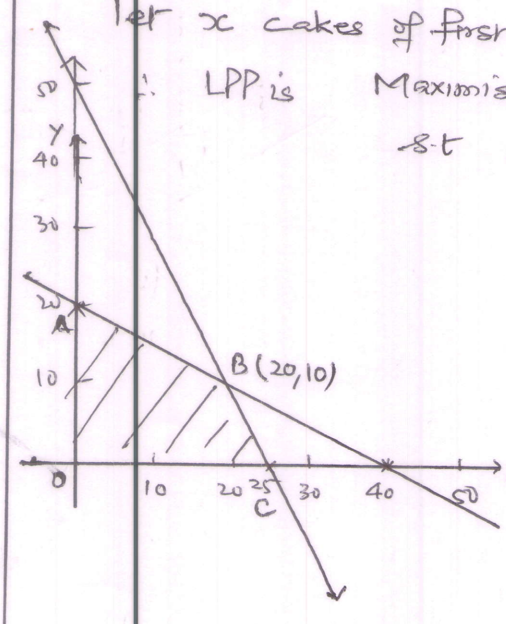
Cofactor are: $A_{11} = -1$, $A_{12} = 0$, $A_{13} = 1$
$A_{21} = 1$, $A_{22} = -2$, $A_{23} = 1$
$A_{31} = 1$, $A_{32} = 6$, $A_{33} = -5$ $1\frac{1}{2}$

$\Rightarrow \frac{1}{2}\begin{pmatrix} -1 & 1 & 1 \\ 0 & -2 & 6 \\ 1 & 1 & -5 \end{pmatrix}\begin{pmatrix} 4600 \\ 4100 \\ 1500 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$\Rightarrow x = 500, \quad y = 400, \quad z = 600$ 1

For writing one more value ——— 1

29. Let x cakes of first type and y cakes of 2nd type



$\therefore$ LPP is Maximise: $Z = x + y$ 1

s.t
$$\left.\begin{aligned} 200x + 100y &\le 5000 \\ 25x + 50y &\le 1000 \\ x \ge 0 \quad y &\ge 0 \end{aligned}\right\}$$ ——— 2

Correct graph: 2

vertices are $A(0,20)$ $B(20,10)$ $C(25,0)$.

$\therefore Z$ is max. at $(20,10)$

i.e. 20 cakes of first type and 10 cakes of 2nd type } 1

SECTION-A

1. $\sqrt{3}$ 2. $\frac{\pi}{2}$ 3. 11 4. $\frac{x-\alpha}{0}=\frac{y-\beta}{0}=\frac{z-\gamma}{1}$ or $\vec{r}=(\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k})+\lambda(\hat{k})$
5. $\frac{7}{2}$ 6. $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ 7. $x=4$. 8. $\frac{\pi}{6}$ 9. 0 (zero) 10. $\frac{\tan^7 x}{7}+c$. $1\times10=10$

SECTION-B

11 Given differential equation can be written as

$$\frac{dy}{dx}=\frac{y}{x}-\sin^2\left(\frac{y}{x}\right) \qquad \tfrac{1}{2}$$

$\frac{y}{x}=v \Rightarrow v+x\frac{dv}{dx}=v-\sin^2 v$ $\frac{1}{2}$

$\Rightarrow -\operatorname{cosec}^2 v\, dv=\frac{dx}{x}$ $\frac{1}{2}$

$\Rightarrow \cot v=\log|x|+c \quad \therefore \cot y/x=\log|x|+c$ $1+\frac{1}{2}$

$x=1,\ y=\frac{\pi}{4} \Rightarrow c=1.$ $\frac{1}{2}$

$\Rightarrow$ solution is $\cot\frac{y}{x}=1+\log|x|$ $\frac{1}{2}$

12. Getting $\sin^{-1}\left(\frac{8}{17}\right)=\tan^{-1}\frac{8}{15}$, and $\sin^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\frac{3}{4}$ 1

$\therefore$ LHS $=\tan^{-1}\frac{8}{15}+\tan^{-1}\frac{3}{4}=\tan^{-1}\left(\frac{\frac{8}{15}+\frac{3}{4}}{1-\frac{8}{15}\cdot\frac{3}{4}}\right)$ 1

$=\tan^{-1}\left(\frac{77}{36}\right)$ 1

$=\cos^{-1}\left(\frac{36}{85}\right)$ 1

OR

$\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x \Rightarrow \frac{\pi}{4}-\tan^{-1}x=\frac{1}{2}\tan^{-1}x$ 2

$\frac{3}{2}\tan^{-1}x=\frac{\pi}{4}$ or $\tan^{-1}x=\frac{\pi}{6}$ $\frac{1}{2}+\frac{1}{2}$

$\Rightarrow x=\frac{1}{\sqrt{3}}$ 1

13. Same as Q13 in 65/1
14. Same as Q19 in 65/1
15. Same as Q16 in 65/1
16. Same as Q11 in 65/1

17. Put $x=\sin\alpha$ and $\sqrt{x}=\sin\beta$ 1

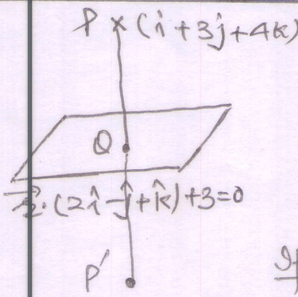
$y=\sin^{-1}(\sin\alpha\cos\beta-\cos\alpha\sin\beta)$

$=\sin^{-1}(\sin(\alpha-\beta))=\alpha-\beta$ 1

$y=\sin^{-1}x-\sin^{-1}\sqrt{x}$ $\frac{1}{2}$

$\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-x}}\cdot\frac{1}{2\sqrt{x}}$ $1\frac{1}{2}$

18.



Equation of line PQ is

$\vec{r} = (\hat{i}+3\hat{j}+4\hat{k}) + \lambda(2\hat{i}-\hat{j}+\hat{k})$

$= (1+2\lambda)\hat{i} + (3-\lambda)\hat{j} + (4+\lambda)\hat{k}$ 1

If these are coordinates of point Q, then

$2(1+2\lambda) - 1(3-\lambda) + 1(4+\lambda) + 3 = 0$ 1

$6\lambda = -3-3 \Rightarrow \lambda = -1$

$\therefore$ Coords. of Q are $(-1, 4, 3)$ 1

let coords of P' be (x, y, z) then

$\frac{x+1}{2} = -1,\ \frac{y+3}{2} = 4,\ \frac{z+4}{2} = 3 \Rightarrow (x,y,z) = (-3, 5, 2)$ 1

19. Same as Q22 in 65/1

20. Same as Q21 in 65/1

21. Same as Q17 in 65/1

22. Same as Q14 in 65/1

SECTION-C

23. Same as Q27 in 65/1

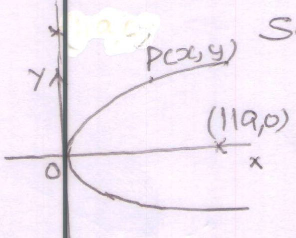
24. Same as Q28 in 65/1

25. Same as Q29 in 65/1

26. Same as Q25 in 65/1

27. Same as Q26 in 65/1

28.



let P(x,y) be the nearest point $1\frac{1}{2}$

$\therefore D = \sqrt{(x-11a)^2 + y^2}$ $\frac{1}{2}$

or $S = (x-11a)^2 + y^2 = (x-11a)^2 + 4ax$ 1

$\frac{ds}{dx} = 2(x-11a) + 4a$ 1

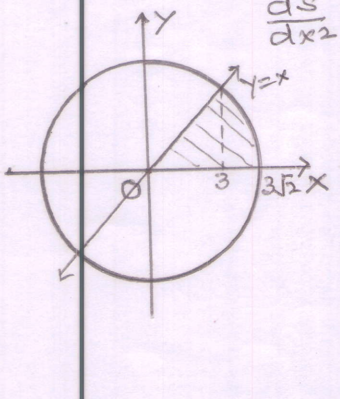
$\frac{ds}{dx} = 0 \Rightarrow x = 9a$ 1

$\therefore y = \pm 6a$

$\frac{d^2s}{dx^2} = 2 > 0 \Rightarrow$ for Minimum distance $\frac{1}{2}+\frac{1}{2}$

coords are $P(9a, \pm 6a)$

29.



Point of intersection of circle and line is (3,3) 1

correct figure 1

$\therefore$ Area $= \int_0^3 x\,dx + \int_3^{3\sqrt{2}} \sqrt{18-x^2}\,dx$ $\frac{1}{2}+1$

$= \left[\frac{x^2}{2}\right]_0^3 + \left[\frac{x}{2}\sqrt{18-x^2} + 9\sin^{-1}\frac{x}{3\sqrt{2}}\right]_3^{3\sqrt{2}}$ $\frac{1}{2}+1$

$= \frac{9}{2} + \frac{9\pi}{2} - \frac{9}{2} - \frac{9\pi}{4}$

$= \frac{9\pi}{4}$ sq. units 1

(10)

65/3

SECTION-A

1. $gof: \{(1,3),(3,1),(4,3)\}$ 2. $\frac{x-\alpha}{0}=\frac{y-\beta}{0}=\frac{z-\gamma}{1}$ or $\vec{r}=(\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k})+\lambda(\hat{k})$

3. $\frac{7}{2}$ 4. $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ 5. $-\frac{\pi}{4}$ 6. 1 7. $\frac{\pi}{6}$ 8. 4.

9. $\frac{\tan^7 x}{7}+C$ 10. 0 (zero).

SECTION-B

11. $$\Delta_1=\begin{vmatrix} A & B & C \\ x & y & z \\ zy & zx & xy \end{vmatrix}=\begin{vmatrix} A & x & zy \\ B & y & zx \\ C & z & xy \end{vmatrix}$$ 1

$R_1\to xR_1$
$R_2\to yR_2$
$R_3\to zR_3$

$$=\frac{1}{xyz}\begin{vmatrix} Ax & x^2 & xyz \\ By & y^2 & xyz \\ Cz & z^2 & xyz \end{vmatrix}$$ $1\frac{1}{2}$

$$=\frac{\cancel{xyz}}{\cancel{xyz}}\begin{vmatrix} Ax & x^2 & 1 \\ By & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix}=\Delta$$ $1\frac{1}{2}$

12. Same as QNo. 14 in 65/1

13. Same as QNo 16 in 65/1

14. $$I=\int_{\pi/6}^{\pi/3}\frac{\sin x+\cos x}{\sqrt{1-(\sin x-\cos x)^2}}\,dx$$ $\frac{1}{2}$

Let $\sin x-\cos x=t$ $\frac{1}{2}$

$\therefore$ $(\cos x+\sin x)\,dx=dt$

$$=\int_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}}\frac{dt}{\sqrt{1-t^2}}=\left[\sin^{-1}(t)\right]_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}}$$ $\frac{1}{2}+1$

$$=\sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)-\sin^{-1}\left(\frac{1-\sqrt{3}}{2}\right)=2\sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$$ $1+\frac{1}{2}$

OR

$$I=\int\frac{x\cos^{-1}x}{\sqrt{1-x^2}}\,dx$$ Put $x=\cos\theta$, $dx=-\sin\theta\,d\theta$ $\frac{1}{2}$

$$=-\int\theta\cos\theta\,d\theta$$ $\frac{1}{2}$

$$=-\left[\theta\sin\theta-\int\sin\theta\,d\theta\right]$$ $\frac{1}{2}$

$$=-\theta\sin\theta-\cos\theta+C$$ 1

$$=-\sqrt{1-x^2}\cos^{-1}x-x+C$$ $1\frac{1}{2}$

65|3

15. Same as Q. NO 11 in 65|1

16. Same as Q NO 12 in 65|1

17. Given equation can be written as

$$\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = \left(\frac{y}{x}\right)\cos\left(\frac{y}{x}\right) + 1$$

$$\frac{dy}{dx} = \frac{1+(y/x)\cos\left(\frac{y}{x}\right)}{\cos\left(\frac{y}{x}\right)}$$ 1.

$$\frac{y}{x} = v$$

$$\therefore \quad v + x\frac{dv}{dx} = \frac{1+v\cos v}{\cos v}$$ $\frac{1}{2}$.

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{1+v\cos v}{\cos v} - v = \frac{1}{\cos v}$$ 1

$$\int \cos v \cdot dv = \int \frac{dx}{x}$$ 1

$$\Rightarrow \quad \sin v = \log|x| + c$$ $\frac{1}{2}$

$$\text{or} \quad \sin\left(\frac{y}{x}\right) = \log|x| + c$$

18. Same as Q22 in 65|1

19. Same as Q19 in 65|1

20. Same as Q21 in 65|1

21. Same as Q20 in 65|1

22. Let $(\tan^{-1}x)^y = u$ and $(y)^{\cot x} = v$

$\log u = y \log \tan^{-1}x$ and $\log v = \cot x \cdot \log y$. ------ $\frac{1}{2}+\frac{1}{2}$

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{y}{\tan^{-1}x}\cdot\frac{1}{1+x^2} + \log\tan^{-1}x\cdot\frac{dy}{dx} \;\Bigg|\; \frac{1}{v}\frac{dv}{dx} = \frac{\cot x}{y}\frac{dy}{dx} - \operatorname{cosec}^2 x\cdot\log y$$ $1+\frac{1}{2}$

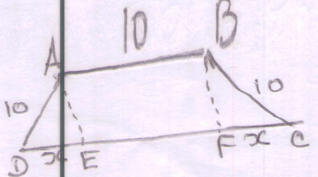
$$\frac{du}{dx} = (\tan^{-1}x)^y\left[\frac{y}{(\tan^{-1}x)(1+x^2)} + \log\tan^{-1}x\frac{dy}{dx}\right] \text{ and } \frac{dv}{dx} = y^{\cot x}\left[\frac{\cot x}{y}\frac{dy}{dx} - \operatorname{cosec}^2 x\log y\right]$$ $\frac{1}{2}$

$$u + v = 1 \Rightarrow \frac{du}{dx} + \frac{dv}{dx} = 0$$

$$\therefore \frac{y\cdot(\tan^{-1}x)^{y-1}}{(1+x^2)} + (\tan^{-1}x)^y\cdot\log\tan^{-1}x\frac{dy}{dx} + \cot x\cdot y^{\cot x-1}\cdot\frac{dy}{dx} - y^{\cot x}\cdot\operatorname{cosec}^2 x\cdot\log y = 0$$ $\frac{1}{2}$

$$\Rightarrow \frac{dy}{dx} = \frac{y^{\cot x}\cdot\operatorname{cosec}^2 x\cdot\log y - \frac{y(\tan^{-1}x)^{y-1}}{(1+x^2)}}{(\tan^{-1}x)^y\cdot\log\tan^{-1}x + \cot x\cdot y^{\cot x-1}}$$ $\frac{1}{2}$

figure . . . $\frac{1}{2}$

23. 

let $DE = CF = x$. $\frac{1}{2}$

Area $ABCD = \frac{1}{2}(20+2x)\sqrt{100-x^2}$

$A = (10+x)\sqrt{100-x^2}$

let $S = (10+x)^2(100-x^2)$ 1

$\frac{ds}{dx} = (10+x)^2 \cdot (-2x) + 2(10+x)(100-x^2)$

$= 2(10+x)(-10x-x^2+100-x^2)$ 1

$= 2(10+x)(100-10x-2x^2)$ 1

$\frac{ds}{dx} = 0 \Rightarrow x = 5$ cm.

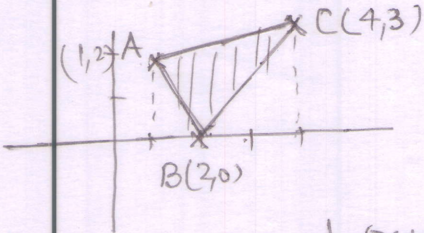
$\frac{d^2s}{dx^2} = 2(10+x)(-10-4x) + 2(100-10x-2x^2)$ 1

$= 2(15)(-30) < 0$ (at $x=5$) 1

$\therefore$ Max. area $= 15\sqrt{75} = 75\sqrt{3}$ sq. units.

24. let the line AB, BC and CA have equations $2x+y=4$, $3x-2y=6$ and $x-3y+5=0$ resp.

$\therefore$ $B(2,0)$, $C(4,3)$ and $A(1,2)$



Area $= \int_1^4 \frac{1}{3}(x+5)dx - \int_1^2 (4-2x)dx - \int_2^4 \frac{1}{2}(3x-6)\,dx$

$= \frac{1}{3}\frac{(x+5)^2}{2}\Big]_1^4 + 2\frac{(2-x)^2}{2}\Big]_1^2 - \frac{3}{2}\frac{(x-2)^2}{2}\Big]_2^4$

$= \left(\frac{81}{6} - \frac{36}{6}\right) + (0-1) - \frac{3}{4}\cdot 4$

$= \frac{15}{2} - 1 - 3 = \frac{7}{2}$ sq. u.

25. Same as Q29 in 65|1

26. same as Q28 in 65|1

27. Same as Q27 in 65|1

28. Same as Q26 in 65|1

29. Same as Q25 in 65|1

1. [illegible] 4. Raj Pal Singh

2. [illegible] 5. [illegible]