QUESTION PAPER CODE 65/1

EXPECTED ANSWER VALE POINTS

SECTION A

Marks

1. $3\vec{a}+2\vec{b} = 7\vec{i}-5\vec{j}+4\vec{k}$ ½

$\therefore$ D.R's are 7, –5, 4 ½

2. $(2\vec{i}+3\vec{j}+2\vec{k})\cdot(2\vec{i}+2\vec{j}+\vec{k}) = 12$ ½

$p = \frac{12}{|\vec{b}|} = \frac{12}{3} = 4$ ½

3. $\vec{r} = (\vec{i}+2\vec{j}+3\vec{k})+\lambda(\vec{i}+2\vec{j}-5\vec{k})$ 1

4. For singular matrix

$4\sin^2 x - 3 = 0$ ½

$\sin x = \pm\frac{\sqrt{3}}{2} \Rightarrow x = \frac{2\pi}{3}$ ½

5. $\int e^{2y}dy = \int x^3 dx$ ½

$\Rightarrow \frac{e^{2y}}{2} = \frac{x^4}{4}+c$ ½

6. I.F. $= e^{\int \frac{1}{\sqrt{x}}dx}$ ½

$= e^{2\sqrt{x}}$ ½

SECTION B

7. Let investment in first type of bonds be Rs x.

$\therefore$ Investment in 2nd type = Rs (35000 – x) ½

$$\begin{pmatrix} x & 35000-x \end{pmatrix}\begin{pmatrix} \frac{8}{100} \\ \frac{10}{100} \end{pmatrix} = (3200)$$ 1½

$$\Rightarrow \frac{8}{100}x + (35000-x)\frac{10}{100} = 3200$$

$$\Rightarrow x = \text{Rs } 15000$$ 1

$\therefore$ Investment in first = Rs 15000

and in 2nd = Rs 20000 1

8. Getting $A' = \begin{pmatrix} 2 & 7 & 1 \\ 4 & 3 & -2 \\ -6 & 5 & 4 \end{pmatrix}$ 1

Let $P = \frac{1}{2}(A + A') = \frac{1}{2}\begin{pmatrix} 4 & 11 & -5 \\ 11 & 6 & 3 \\ -5 & 3 & 8 \end{pmatrix} = \begin{pmatrix} 2 & 11/2 & -5/2 \\ 11/2 & 3 & 3/2 \\ -5/2 & 3/2 & 4 \end{pmatrix}$ 1

Since $P' = P$ $\therefore$ P is a symmetric matrix

Let $Q = \frac{1}{2}(A - A') = \frac{1}{2}\begin{pmatrix} 0 & -3 & -7 \\ 3 & 0 & 7 \\ 7 & -7 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -3/2 & -7/2 \\ 3/2 & 0 & 7/2 \\ 7/2 & -7/2 & 0 \end{pmatrix}$ 1

Since $Q' = -Q$ $\therefore$ Q is skew symmetric

Also

$$P + Q = \begin{pmatrix} 2 & 11/2 & -5/2 \\ 11/2 & 3 & 3/2 \\ -5/2 & 3/2 & 4 \end{pmatrix} + \begin{pmatrix} 0 & -3/2 & -7/2 \\ 3/2 & 0 & 7/2 \\ 7/2 & -7/2 & 0 \end{pmatrix} = A$$ 1

OR

$AB = \begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 5 \\ 5 & -14 \end{pmatrix}$ 1

$$\text{LHS} = (AB)^{-1} = -\frac{1}{11}\begin{pmatrix} -14 & -5 \\ -5 & -1 \end{pmatrix} \text{ or } \frac{1}{11}\begin{pmatrix} 14 & 5 \\ 5 & 1 \end{pmatrix}$$ 1

$$\text{RHS} = B^{-1}A^{-1} = 1\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}\frac{-1}{11}\begin{pmatrix} -4 & -3 \\ -1 & 2 \end{pmatrix} = \frac{1}{11}\begin{pmatrix} 14 & 5 \\ 5 & 1 \end{pmatrix}$$ 1+1

$\therefore$ LHS = RHS

9. $$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

$R_1 \to R_1 + R_2 + R_3$,

$$\Rightarrow \begin{vmatrix} 3a-x & 3a-x & 3a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$ 1

$C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$$\Rightarrow \begin{vmatrix} 3a-x & 0 & 0 \\ a-x & 2x & 0 \\ a-x & 0 & 2x \end{vmatrix} = 0$$ 1+1

$\Rightarrow 4x^2(3a - x) = 0$

$\Rightarrow x = 0, x = 3a$ 1

10. $$I = \int_0^{\pi/4} \log(1+\tan x)\,dx \quad \text{...(i)}$$

$$= \int_0^{\pi/4} \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx = \int_0^{\pi/4} \log\left[1+\frac{1-\tan x}{1+\tan x}\right]dx$$ 1 + ½

$$= \int_0^{\pi/4} [\log 2 - \log(1+\tan x)]\,dx \quad \text{...(ii)}$$ 1

adding (i) and (ii) to get

$$2I = \log 2 \int_0^{\pi/4} 1 \cdot dx = \frac{\pi}{4} \log 2$$ 1

$$\Rightarrow I = \frac{\pi}{8} \log 2$$ ½

11. Writing $I = \int \frac{x}{(x^2+1)(x-1)} dx = \int \left(\frac{A}{x-1} + \frac{Bx+C}{x^2+1} \right) dx$ 1

$$= \int \frac{1/2}{x-1} dx + \int \frac{-\frac{1}{2}x + \frac{1}{2}}{x^2+1} dx$$ 1½

$$= \frac{1}{2} \log|x-1| - \frac{1}{4} \log(x^2+1) + \frac{1}{2} \tan^{-1} x + C$$ 1½

OR

$$I = \int_0^{1/\sqrt{2}} \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx$$

Putting $x = \sin\theta$, $\therefore$ $dx = \cos\theta\, d\theta$ and $x = 0$ then $\theta = 0$

$x \Rightarrow \frac{1}{\sqrt{2}}$ then $\theta = \frac{\pi}{4}$ 1

$$I = \int_0^{\pi/4} \theta \cdot \frac{\cos\theta}{\cos^3\theta} d\theta = \int_0^{\pi/4} \theta \cdot \sec^2\theta\, d\theta$$ 1

$$= [\theta \tan\theta - \log|\sec\theta|]_0^{\pi/4}$$ 1

$$= \frac{\pi}{4} - \frac{1}{2} \log 2$$ 1

12. (i) P (all four spades) = ${}^4C_4 \left(\frac{13}{52}\right)^4 \left(\frac{39}{52}\right)^0 = \frac{1}{256}$ 2

(ii) P (only 2 are spades) = ${}^4C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^2 = \frac{27}{128}$ 2

OR

$n = 4,\ p = \frac{1}{6},\ q = \frac{5}{6}$

No. of successes

x	0	1	2	3	4	½
P(x)	${}^4C_0\left(\frac{5}{6}\right)^4$	${}^4C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^3$	${}^4C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2$	${}^4C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)$	${}^4C_4\left(\frac{1}{6}\right)^4$	2½
	$=\frac{625}{1296}$	$=\frac{500}{1296}$	$=\frac{150}{1296}$	$=\frac{20}{1296}$	$=\frac{1}{1296}$	
xP(x)	0	$\frac{500}{1296}$	$\frac{300}{1296}$	$\frac{60}{1296}$	$\frac{4}{1296}$	

Mean = $\sum xP(x) = \frac{864}{1296} = \frac{2}{3}$. 1

13. LHS $= \vec{a}\cdot\{(\vec{b}+\vec{c})\times\vec{d}\} = \vec{a}\cdot\{\vec{b}\times\vec{d}+\vec{c}\times\vec{d}\}$ 1+1

$= \vec{a}\cdot(\vec{b}\times\vec{d}) + \vec{a}\cdot(\vec{c}\times\vec{d})$ 1

$= [\vec{a}, \vec{b}, \vec{d}] + [\vec{a}, \vec{c}, \vec{d}]$ 1

14. Here $\vec{a}_1 = 2\hat{i}-5\hat{j}+\hat{k},\ \vec{a}_2 = 7\hat{i}-6\hat{k}$

$\vec{b}_1 = 3\hat{i}+2\hat{j}+6\hat{k}, \quad \vec{b}_2 = \hat{i}+2\hat{j}+2\hat{k}$

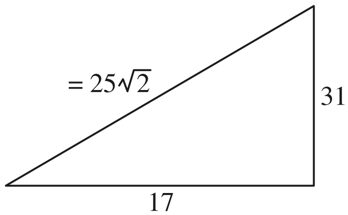
$\vec{a}_2-\vec{a}_1 = 5\hat{i}+5\hat{j}-7\hat{k}$ 1

$\vec{b}_1\times\vec{b}_2 = -8\hat{i}+4\hat{k}$ 1

SD = $\frac{|(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)|}{|\vec{b}_1\times\vec{b}_1|}$ 1

$$= \frac{|-40-28|}{\sqrt{64+16}} = \frac{68}{\sqrt{80}} = \frac{17}{\sqrt{5}}$$ 1

15.



$$\text{LHS} = 2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\frac{2\cdot\frac{1}{2}}{1-\frac{1}{4}} + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\frac{4}{3} + \tan^{-1}\frac{1}{7}$$ 1½

$$= \tan^{-1}\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\cdot\frac{1}{7}} = \tan^{-1}\frac{31}{17}$$ 1½

$$= \sin^{-1}\left(\frac{31}{25\sqrt{2}}\right) = \text{RHS}$$ 1

OR

$$\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x$$

$$\Rightarrow \tan^{-1}1 - \tan^{1}x = \frac{1}{2}\tan^{-1}x$$ 1½

$$\Rightarrow \frac{3}{2}\tan^{-1}x = \frac{\pi}{4} \Rightarrow \tan^{-1}x = \frac{\pi}{6}$$ 1½

$$x = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}.$$ 1

16. $\text{LHL} = \lim_{x\to 0^-} f(x) = 2\lambda$

$\text{RHL} = \lim_{x\to 0^+} f(x) = 6$

$f(0) = 2\lambda$

$\Rightarrow 2\lambda = 6 \Rightarrow \lambda = 3$ 2

Differentiability

$$\text{LHD} = \lim_{h\to 0}\frac{f(0)-f(0-h)}{h} = \lim_{h\to 0}\frac{3(2)-3((-h)^2+2)}{h} = \lim_{h\to 0} 3h = 0$$ 1

$$\text{RHD} = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0}\frac{(4h+6)-3(2)}{h} = \lim_{h\to 0} 4 = 4$$ ½

LHD ≠ RHD $\quad\therefore$ f(x) is not differentiable at x = 0 ½

17. $x = ae^t(\sin t + \cos t)$ and $y = ae^t(\sin t - \cos t)$

$$\frac{dx}{dt} = a[e^t(\cos t - \sin t) + e^t(\sin t + \cos t)] = -y + x$$ 1½

$$\frac{dy}{dt} = a[e^t(\cos t + \sin t) + e^t(\sin t - \cos t) = x + y$$ 1½

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{x+y}{x-y}$$ 1

18. $y = Ae^{mx} + Be^{nx} \Rightarrow mAe^{mx} + nBe^{nx}$ 1

$$\frac{d^2y}{dx^2} = m^2Ae^{mx} + n^2Be^{nx}$$ 1

$$\text{LHS} = \frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny$$

$$= m^2Ae^{mx} + n^2Be^{nx} - (m+n)\{mAe^{mx} + nBe^{nx}\} + mn\{Ae^{mx} + Be^{nx}\}$$ 1

$$= Ae^{mx}(m^2 - m^2 - mn + mn) + Be^{nx}(n^2 - mn - n^2 + mn)$$

$= 0 =$ RHS. 1

19. $I = \int \frac{x+3}{\sqrt{5-4x-2x^2}}\,dx = \int \frac{-\frac{1}{4}(-4-4x)+2}{\sqrt{5-4x-2x^2}}\,dx$ 1

$= -\frac{1}{4}\cdot 2\cdot\sqrt{5-4x-2x^2} + \frac{2}{\sqrt{2}}\int \frac{dx}{\sqrt{\left(\sqrt{\frac{7}{2}}\right)^2 - (x+1)^2}}$ 1+1

$= -\frac{1}{2}\sqrt{5-4x-2x^2} + \sqrt{2}\sin^{-1}\left(\frac{x+1}{\sqrt{7/2}}\right) + C$ 1

SECTION C

20. Here,

$$R = \begin{Bmatrix} (1,1),(2,2),(3,3),(4,4),(5,5) \\ (1,3),(1,5),(2,4),(3,5) \\ (3,1),(5,1),(4,2),(5,3) \end{Bmatrix}$$ 2

Clearly

(i) $\forall a \in A, (a, a) \in R \quad \therefore$ R is reflexive 1

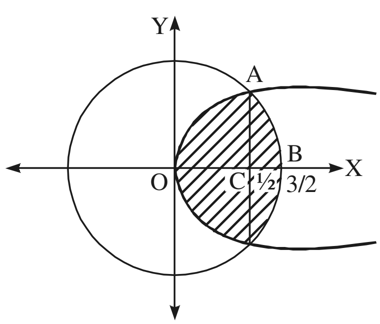
(ii) $\forall (a, b) \in A, (b, a) \in R \quad \therefore$ R is symmetric 1

(iii) $\forall (a, b), (b, c) \in R, (a, c) \in R \quad \therefore$ R is transitive 1

$\therefore$ R is an equivalence relation.

[1] = {1, 3, 5}, [2] = {2, 4} 1

21. $\{(x, y) : y^2 \le 4x,\ 4x^2 + 4y^2 \le 9\}$



Correct figure 1

Getting $x = \frac{1}{2}$ as point of intersection ½

$A = 2\left[2\int_0^{1/2}\sqrt{x}\,dx + \int_{1/2}^{3/2}\sqrt{\frac{9}{4}-x^2}\,dx\right]$ 1

$$= 2\left[\left(\frac{4}{3}x^{3/2}\right)_0^{1/2} + \left(\frac{x}{2}\sqrt{\frac{9}{4}-x^2} + \frac{9}{8}\sin^{-1}\frac{2x}{3}\right)_{1/2}^{3/2}\right]$$ 1½

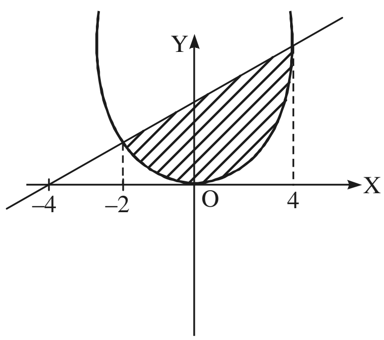
$$= 2\left[\frac{2}{3\sqrt{2}} + \frac{9\pi}{16} - \frac{\sqrt{2}}{4} - \frac{9}{8}\sin^{-1}\frac{1}{3}\right]$$ 1

$$= \frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3} \text{ sq. unit}$$ 1

OR

Correct figure 1

Getting x = 4, –2 as points of intersection ½



$$A = \int_{-2}^{4} \frac{1}{2}(3x+12)\,dx - \int_{-2}^{4} \frac{3}{4}x^2 dx$$ 1

$$= \frac{1}{2}\left(\frac{3x^2}{2} + 12x\right)_{-2}^{4} - \frac{1}{4}(x^3)\Big]_{-2}^{4}$$ 1½

$$= \frac{1}{2}(24+48-6+24) - \frac{1}{4}(64+8)$$ 1½

= 45 – 18 = 27 sq. units ½

22. $\left(x\sin^2\left(\frac{y}{x}\right) - y\right)dx + x\,dy = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{y - x\sin^2(y/x)}{x} = \frac{y}{x} - \sin^2\left(\frac{y}{x}\right)$$ 1

$$v + x\frac{dv}{dx} = v - \sin^2 v \quad \text{where } \frac{y}{x} = v.$$ 1

$$\Rightarrow \int -\frac{dv}{\sin^2 v} = \int \frac{dx}{x} \quad \text{or} \quad \int -\text{cosec}^2 v\,dv = \int \frac{dx}{x}$$ 1½

$\cot v = \log x + C$ i.e., $\cot\frac{y}{x} = \log x + C$ 1½

$y = \frac{\pi}{4}, x = 1, \Rightarrow C = 1$ ½

$\Rightarrow \cot\frac{y}{x} = \log x + 1$ ½

OR

$\frac{dy}{dx} - 3\cot x \cdot y = \sin 2x$

$\text{IF} = \int_e -3\cot x\, dx = -3\log_e \sin x = \text{cosec}^3 x$ 1

$\therefore$ Solution is

$y \cdot \text{cosec}^3 x = \int \sin 2x\, \text{cosec}^3 x\, dx$ 1½

$= \int 2\text{cosec}\, x \cot x\, dx$ ½

$y \cdot \text{cosec}^3 x = -2\, \text{cosec}\, x + C$ 1½

or $y = -2\sin^2 x + C\sin^3 x$

$x = \frac{\pi}{2}, y = 2 \Rightarrow C = 4$ 1

$\Rightarrow y = -2\sin^2 x + 4\sin^3 x$ ½

23. Equation of plane is

$\left\{\vec{r}\cdot(2\hat{i}+2\hat{j}-3\hat{k})-7\right\}+\lambda\left\{\vec{r}\cdot(2\hat{i}+5\hat{j}+3\hat{k})-9\right\} = 0$ 1½

$\Rightarrow \vec{r}\cdot\left\{(2+2\lambda)\hat{i}+(2+5\lambda)\hat{j}(-3+3\lambda)\hat{k}\right\} = (7+9\lambda)$ 1½

x-intercept = y-intercept $\Rightarrow \frac{7+9\lambda}{2+2\lambda} = \frac{7+9\lambda}{-3+3\lambda}$ 1

$\Rightarrow \lambda = 5$ ½

$\therefore$ Eqn. of plane is

$\vec{r} \cdot (12\hat{i} + 27\hat{j} + 12\hat{k}) = 52$ ½

and $12x + 27y + 12z - 52 = 0$ 1

24. E_1: student knows the answer

E_2: student guesses the answer

A: answers correctly.

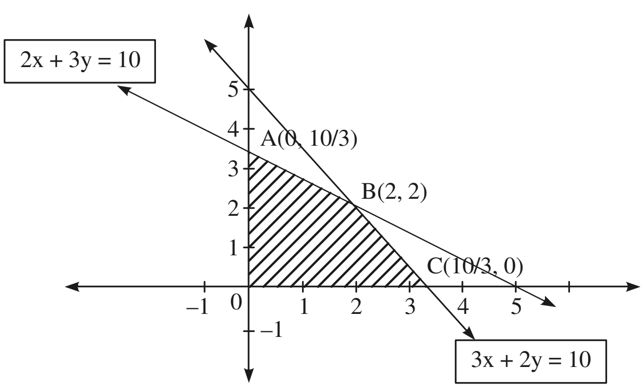
$P(E_1) = \frac{3}{5}, \; P(E_2) = \frac{2}{5}$ 1

$P\left(\frac{A}{E_1}\right) = 1, \quad P\left(\frac{A}{E_2}\right) = \frac{1}{3}$ 1+1

$P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$ 1

$= \frac{\frac{3}{5} \cdot 1}{\frac{3}{5} \cdot 1 + \frac{2}{5} \cdot \frac{1}{3}} = \frac{9}{11}$ 1+1

25.



L.P.P. is Maximise P = 24x + 18y ½

s.t. $2x + 3y \le 10$

$3x + 2y \le 10$ 2

$x, y \ge 0$

Correct figure 2

P(A) = Rs 60

P(B) = Rs 84 ½

P(C) = Rs 80

$\therefore$ Max. = 84 at (2, 2) 1

26. Given: $s = 4\pi r^2 + 2\left[\frac{x^2}{3} + 2x^2 + \frac{2x^2}{3}\right]$

$$= 4\pi r^2 + 6x^2 \quad 1$$

$$V = \frac{4}{3}\pi r^3 + \frac{2x^3}{3}$$

$$V = \frac{4}{3}\pi r^3 + \frac{2}{3}\left(\frac{S - 4\pi r^2}{6}\right)^{3/2} \quad 1$$

$$\frac{dv}{dr} = 4\pi r^2 + \left(\frac{S - 4\pi r^2}{6}\right)^{1/2}\left(\frac{-8\pi r}{6}\right) \quad 1$$

$$\frac{dv}{dr} = 0 \Rightarrow r = \sqrt{\frac{S}{54 + 4\pi}} \quad 1$$

showing $\frac{d^2v}{dr^2} > 0$ 1

$\therefore$ For $r = \sqrt{\frac{S}{54 + 4\pi}}$ volume is minimum

i.e., $(54 + 4\pi)r^2 = 4\pi r^2 + 6x^2$

$6x^2 = 54r^2 \Rightarrow x^2 = 9r^2 \Rightarrow x = 3r$ 1