QUESTION PAPER CODE 65/1/1

EXPECTED ANSWER/VALUE POINTS

SECTION A

Marks

1. $\hat{a} = \frac{1}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{j}$ then $7\hat{a} = \frac{7}{\sqrt{5}}\hat{i} - \frac{14}{\sqrt{5}}\hat{j}$ ½ + ½

2. $(\sqrt{2}\vec{a} - \vec{b}) \cdot (\sqrt{2}\vec{a} - \vec{b}) = 1 \Rightarrow \theta = \frac{\pi}{4}$ ½ + ½

3. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$ ½ + ½

4. $AB = \begin{bmatrix} -1 & 5 \\ 4 & 8 \end{bmatrix} \Rightarrow |AB| = -28$ ½ + ½

5. $1 \cdot y + x\frac{dy}{dx} = -c\sin x \Rightarrow x\frac{dy}{dx} + y + xy\tan x = 0$ ½ + ½

6. order = 2, degree = 3, sum = 2 + 3 = 5 ½ + ½

SECTION B

7. System of equation is

3x + y + 2z = 1100, x + 2y + 3z = 1400, x + y + z = 600 1½

(i) Matrix equation is

$$\begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1100 \\ 1400 \\ 600 \end{bmatrix}$$ 1

(ii) |A| = –3 ≠ 0, system of equations can be solved. ½

(iii) Any one value with reason. 1

8. $\begin{bmatrix}2x & 3\end{bmatrix}\begin{bmatrix}1 & 2\\ -3 & 0\end{bmatrix}\begin{bmatrix}x\\ 3\end{bmatrix}=0$

$\begin{bmatrix}2x-9 & 4x\end{bmatrix}\begin{bmatrix}x\\ 3\end{bmatrix}=0$ 1

$[2x^2-9x+12x]=[0] \Rightarrow 2x^2+3x=0,\ x=0 \text{ or } \frac{-3}{2}$ 1+1+1

9. $\begin{vmatrix}(a+1)(a+2) & a+2 & 1\\ (a+2)(a+3) & a+3 & 1\\ (a+3)(a+4) & a+4 & 1\end{vmatrix}$

$=\begin{vmatrix}(a+1)(a+2) & a+2 & 1\\ 2(a+2) & 1 & 0\\ 4a+10 & 2 & 0\end{vmatrix} \left.\begin{matrix}R_2 \to R_2-R_1\\ R_3 \to R_3-R_1\end{matrix}\right\}$ 1+1

$= 4a + 8 - 4a - 10 = -2.$ 1+1

10. $I=\int_0^{\pi/2}\frac{1}{1+\sqrt{\tan x}}dx$

$I=\int_0^{\pi/2}\frac{1}{1+\sqrt{\tan(\pi/2-x)}}dx=\int_0^{\pi/2}\frac{1}{1+\sqrt{\cot x}}dx=\int_0^{\pi/2}\frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}dx$ 1½

$\Rightarrow 2I=\int_0^{\pi/2}\frac{1+\sqrt{\tan x}}{1+\sqrt{\tan x}}dx=\int_0^{\pi/2}1\cdot dx=\frac{\pi}{2}$ 1½

$\Rightarrow I=\frac{\pi}{4}$ 1

11. $\frac{x}{(x-1)^2\,(x+2)}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+2}$ ½

$A=\frac{2}{9},\ B=\frac{1}{3},\ C=-\frac{2}{9}$ 1½

$$\int \frac{x}{(x-1)^2\,(x+2)}\,dx = \int \frac{2}{9(x-1)}\,dx + \int \frac{1}{3(x-1)^2}\,dx - \int \frac{2}{9(x+2)}\,dx$$ 1½

$$= \frac{2}{9}\log|x-1| - \frac{1}{3(x-1)} - \frac{2}{9}\log|x+2| + C$$ 1½

12. Let X be the number of defective bulbs. Then

$X = 0, 1, 2$ 1

$$P(X=0) = \frac{10_{C_2}}{15_{C_2}} = \frac{3}{7},\ P(X=1) = \frac{10_{C_1}\cdot 5_{C_1}}{15_{C_2}} = \frac{10}{21}$$ 1+1

$$P(X=2) = \frac{5_{C_2}}{15_{C_2}} = \frac{2}{21}$$ 1

X	0	1	2
P(X)	$\frac{3}{7}$	$\frac{10}{21}$	$\frac{2}{21}$

OR

E_1: Problem is solved by A.

E_2: Problem is solved by B.

$$P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{3}, P(\bar{E}_1) = \frac{1}{2}, P(\bar{E}_2) = \frac{2}{3}$$ 1

$$P(E_1 \cap E_2) = P(E_1)\cdot P(E_2) = \frac{1}{6}$$

P(problem is solved) $= 1 - P(\bar{E}_1)\cdot P(\bar{E}_2) = 1 - \frac{1}{2}\cdot\frac{2}{3} = \frac{2}{3}$ 1½

P(one of them is solved) $= P(E_1)P(\bar{E}_2) + P(\bar{E}_1)P(E_2)$

$$= \frac{1}{2}\cdot\frac{2}{3} + \frac{1}{2}\cdot\frac{1}{3} = \frac{1}{2}$$ 1½

13. $\overrightarrow{AB} = -4\hat{i} - 6\hat{j} - 2\hat{k}$

$\overrightarrow{AC} = -\hat{i} + (\lambda - 5)\hat{j} + 3\hat{k}$ 1½

$\overrightarrow{AD} = -8\hat{i} - \hat{j} + 3\hat{k}$

$$\overrightarrow{AB} \cdot (\overrightarrow{AC} \times \overrightarrow{AD}) = \begin{vmatrix} -4 & -6 & -2 \\ -1 & \lambda - 5 & 3 \\ -8 & -1 & 3 \end{vmatrix} = 0$$ 1

$-4(3\lambda - 12) + 6(21) - 2(8\lambda - 39) = 0 \Rightarrow \lambda = 9$ 1½

14. $\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}, \vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$

$\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k}, \vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$ 1

$$\vec{a}_2 - \vec{a}_1 = \hat{i} - 3\hat{j} - 2\hat{k}, \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix} = -3\hat{i} + 3\hat{k}$$ ½ + 1

$|\vec{b}_1 \times \vec{b}_2| = 3\sqrt{2}$ ½

$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 - \vec{b}_2) = -3 - 6 = -9$

Shortest distance $= \left| \dfrac{-9}{3\sqrt{2}} \right| = \dfrac{3\sqrt{2}}{2}$ 1

15. $\tan^{-1} 2x + \tan^{-1} 3x = \dfrac{\pi}{4}$

$\tan^{-1}\left(\dfrac{5x}{1 - 6x^2}\right) = \dfrac{\pi}{4}$ 1

$\dfrac{5x}{1 - 6x^2} = 1$ 1

$\Rightarrow 6x^2 + 5x - 1 = 0$ 1

$\Rightarrow x = \frac{1}{6}, x = -1$ (rejected) 1

OR

$\sin^{-1}\frac{5}{13} = \tan^{-1}\frac{5}{12}$ 1

$\cos^{-1}\frac{3}{5} = \tan^{-1}\frac{4}{3}$ 1

$$\text{R.H.S.} = \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5} = \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{4}{3}$$

$$= \tan\left(\frac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{12}\cdot\frac{4}{3}}\right)$$ 1

$$= \tan^{-1}\frac{63}{16}$$ 1

16. $y = \frac{x\cos^{-1}x}{\sqrt{1-x^2}} - \log\sqrt{1-x^2}$

$$\frac{dy}{dx} = \frac{\sqrt{1-x^2}\cdot\left(1\cos^{-1}x - \frac{x}{\sqrt{1-x^2}}\right) - \frac{x\cos^{-1}x(-2x)}{2\sqrt{1-x^2}}}{1-x^2} + \frac{2x}{2(1-x^2)}$$ 1+1

$$= \frac{\sqrt{1-x^2}\,\cos^{-1}x - x + \frac{x^2\cos^{-1}x}{1-x^2}}{1-x^2} + \frac{x}{1-x^2}$$ 1

$$= \frac{(1-x^2)\cos^{-1}x + x^2\cos^{-1}x}{(1-x^2)^{3/2}} = \frac{\cos^{-1}x}{(1-x^2)^{3/2}}$$ 1

17. $y = (\sin x)^x + \sin^{-1}\sqrt{x}$

$\Rightarrow y = e^{x \log \sin x} + \sin^{-1}\sqrt{x}$ 1

$\Rightarrow \frac{dy}{dx} = e^{x \log \sin x}[\log \sin x + x \cot x] + \frac{1}{2\sqrt{x}\ \sqrt{1-x}}$ 1½

$\Rightarrow \frac{dy}{dx} = (\sin x)^x\ (\log \sin x + x \cot x) + \frac{1}{2\sqrt{x}\ \sqrt{1-x}}$ 1½

18. $x = a \sec^3 \theta$

$\frac{dx}{d\theta} = 3a \sec^3 \theta \tan \theta$ ½

$y = a \tan^3 \theta$

$\frac{dy}{d\theta} = 3a \tan^2 \theta \sec^2 \theta$ ½

$\frac{dy}{dx} = \frac{3a \tan^2 \theta \sec^2 \theta}{3a \sec^3 \theta \tan \theta} = \sin \theta$ 1

$\frac{d^2y}{dx^2} = \cos \theta \cdot \frac{d\theta}{dx} = \frac{\cos \theta}{3a \sec^3 \theta \tan \theta} = \frac{\cos^4 \theta}{3a \tan \theta}$ 1

$\left.\frac{d^2y}{dx^2}\right]_{\theta = \frac{\pi}{4}} = \frac{1}{12a}$ 1

19. $\int \frac{e^x (x^2+1)}{(x+1)^2}\, dx$

$= \int e^x \left[\frac{(x^2-1)+2}{(x+1)^2}\right] dx$ 1

$= \int e^x \left[\frac{x-1}{x+1} + \frac{2}{(x+1)^2}\right] dx$ 1

$$= \frac{x-1}{x+1} \cdot e^x - \int \frac{2}{(x+1)^2} e^x \, dx + \int \frac{2}{(x+1)^2} e^x \, dx$$ 1

$$= \frac{e^x (x-1)}{x+1} + C$$ 1

SECTION C

20. (a, b) * (c, d) = (a + c, b + d) = (c + a, d + b) = (c, d) * (a, b) $\therefore$ * is commutative 1½

[(a, b) * (c, d)] * (e, f) = (a + c, b + d) * (e, f)

= (a + c + e, b + d + f) = (a, b) * (c + e, d + f) 1

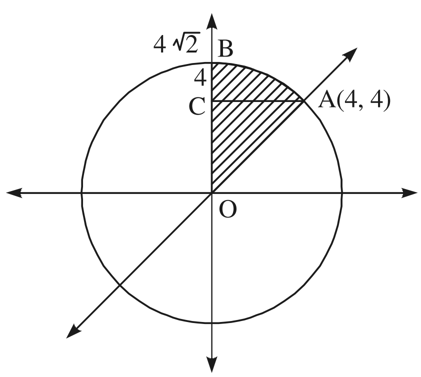
= (a, b) * [(c, d) * (e, f) $\therefore$ * is associate 1½

Let (e, e′) be the identity

(a, b) * (e, e′) = (a, b) $\Rightarrow$ (a + e, b + e′) = (a, b) $\Rightarrow$ e = 0, e′ = 0

$\Rightarrow$ Identity element is (0, 0) 2

21.



$x^2 + y^2 = 32$; $y = x$ point of intersection is $y = 4$ ½

Correct figure 1

$$\text{Required Area} = \int_0^4 y \, dy + \int_4^{4\sqrt{2}} \sqrt{32 - y^2} \, dy$$ 1½

$$= \left[\frac{y^2}{2}\right]_0^4 + \left[\frac{y}{2}\sqrt{32 - y^2} + 16 \sin^{-1} \frac{y}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$$ 1½

$$= 8 + \left(0 + 16 \cdot \frac{\pi}{2}\right) - \left(8 + 16 \cdot \frac{\pi}{2}\right) = 4\pi$$ 1½

22. $$x \frac{dy}{dx} + y - x + xy \cot x = 0 \Rightarrow \frac{dy}{dx} + \left(\frac{1}{x} + \cot x\right) y = 1$$ ½

$$\text{I.F.} = e^{\int \left(\frac{1}{x} + \cot x\right) dx} = x \sin x$$ 1

Solution: $y \cdot x \sin x = \int 1 \cdot x \sin x \, dx$ 1½

$\Rightarrow yx \sin x = -x \cos x + \sin x + C$ 1

when

$x = \frac{\pi}{2}, y = 0$, we have $C = -1$ 1

$yx \sin x + x \cos x - \sin x = 1$ 1

OR

$x^2 dy + (xy + y^2)dx = 0 \Rightarrow \frac{dy}{dx} = \frac{-(xy + y^2)}{x^2}$ 1

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1

$\Rightarrow v + x\frac{dv}{dx} = -(v + v^2) \Rightarrow \frac{dv}{v^2 + 2v} = -\frac{dx}{dx}$

$\Rightarrow \int \frac{dv}{(v+1)^2 - (1)^2} - \int \frac{dx}{x} \Rightarrow \frac{1}{2}\log\frac{v}{v+2} = -\log x + \log C$ 1

$\Rightarrow \frac{C}{x} = \sqrt{\frac{y}{y+x}}$ 1

If $x = 1, y = 1$, then $C = \frac{1}{\sqrt{3}}$ 1

$\Rightarrow \frac{1}{\sqrt{3}\,x} = \sqrt{\frac{y}{y+x}}$ 1

23. Plane passing through the intersection of given planes:

$(x + y + z - 1) + \lambda(2x + 3y + 4z - 5) = 0$ 1

$(1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z + (-1 - 5\lambda) = 0$ 1½

Now $(1 + 2\lambda)\,1 + (1 + 3\lambda)\,(-1) + (1 + 4\lambda)1 = 0$ 1½

$\Rightarrow \lambda = -\frac{1}{3}$ 1

Equation of required plane is

$\Rightarrow x - z + 2 = 0$ 1

24. E_1: First bag is selected.

E_2: Second bag is selected.

A: both balls are red. 1

$P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{2}, P\left(\frac{A}{E_1}\right) = \frac{12}{56}, P\left(\frac{A}{E_2}\right) = \frac{2}{56}$ ½ + ½ + 1 + 1

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\,P\left(\frac{A}{E_1}\right)}{P(E_1)\,P\left(\frac{A}{E_1}\right) + P(E_2)\,P\left(\frac{A}{E_2}\right)} = \frac{\frac{1}{2}\times\frac{12}{56}}{\frac{1}{2}\times\frac{12}{56} + \frac{1}{2}\cdot\frac{2}{56}} = \frac{6}{7}$$ ½ + 1½

25. Let x and y be the number of takes. Then

Maximise:

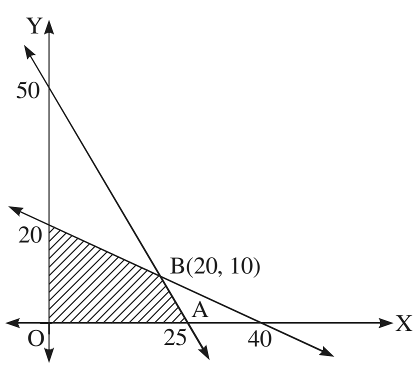
$z = x + y$ 1

Subject to:

$200x + 100y \le 5000$

$25x + 50y \le 1000$

$x \ge 0,\ y \ge 0$ 2



Correct figure 2

at (20, 10), z = 20 + 10 = 30 is maximum.

at (25, 0), z = 25 + 0 = 25

at (0, 20), z = 20 1

26. $l \times b \times 3 = 75 \Rightarrow l \times b = 25$ 1

Let C be the cost. Then

$C = 100\ (l \times b) + 100\ h(b + l)$ 1

$C = 100\left(l \times \frac{25}{l}\right) + 300\left(\frac{25}{l} + l\right)$ 1

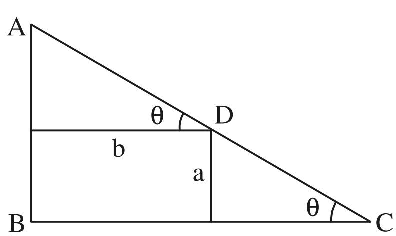
$\frac{dC}{dl} = 0 + 300\left(\frac{-25}{l^2} + 1\right)$

$\frac{dC}{dl} = 0 \Rightarrow l = 5$ 1

$\frac{d^2C}{dl^2} > 0 \Rightarrow$ C is maximum when $l = 5 \Rightarrow b = 5$ 1

C = 100 (25) + 300(10)) = Rs. 5500 1

OR



Correct figure 1

AD = b sec θ, DC = a cosec θ 1

L = AC = b sec θ + a cosec θ 1

$\frac{dL}{d\theta} = b \sec\theta \tan\theta - a\operatorname{cosec}\theta \cot\theta$ 1

$\frac{dL}{d\theta} = 0 \Rightarrow \tan^3\theta = \frac{a}{b}$ 1

$\frac{d^2L}{d\theta^2} > 0 \Rightarrow$ minima

$L = \frac{b \cdot \sqrt{a^{2/3} + b^{2/3}}}{b^{1/3}} + \frac{a\sqrt{a^{2/3} + b^{2/3}}}{a^{1/3}}$

$\Rightarrow L = (a^{2/3} + b^{2/3})^{2/3}$ 1