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Senior School Certificate Examination

March 2017

Marking Scheme — Mathematics 65/2/1, 65/2/2, 65/2/3

General Instructions:

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage.

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question (s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

8. As per orders of the Hon'ble Supreme Court. The candidates would now be permitted to obtain photocopy of the Answer book on request on payment of the prescribed fee. All examiners/ Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

QUESTION PAPER CODE 65/2/1

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\text{adj } A = 3\begin{bmatrix} 3 & -1 \\ \frac{-5}{3} & \frac{2}{3} \end{bmatrix} = \begin{bmatrix} 9 & -3 \\ -5 & 2 \end{bmatrix}$ 1

2. $\lim_{x \to 0}\left(\frac{\sin 5x}{3x} + \cos x\right) = \frac{8}{3} \Rightarrow k = \frac{8}{3}$ $\frac{1}{2}+\frac{1}{2}$

3. $\int_0^{2\pi} \cos^5 x \, dx = 2\int_0^{\pi} \cos^5 x \, dx$ $\frac{1}{2}$

and $2\int_0^{\pi} \cos^5 x \, dx = 0 \Rightarrow \int_0^{2\pi} \cos^5 x \, dx = 0$ $\frac{1}{2}$

4. $\sqrt{(-5)^2 + (12)^2} = 13$ $\frac{1}{2}+\frac{1}{2}$

SECTION B

5. $|2AB| = 2^3 \times |A| \times |B|$ 1

$= 8 \times (-1) \times 3 = -24$ 1

6. $V = \pi r^2 h \Rightarrow \frac{dv}{dt} = \pi\left(r^2 \frac{dh}{dt} + 2r\frac{dr}{dt} h\right)$ 1

$= \frac{22}{7}[49 \times (+2) + 14(2)(-3)] = 44 \text{ cm}^3/\text{min}$ 1

$\therefore$ Volume is increasing at the rate of 44 cm^3/min

7. $\frac{dy}{dt} = 12 \sin t, \ \frac{dx}{dt} = 10 (1 - \cos t)$ $\frac{1}{2}+\frac{1}{2}$

$\therefore \frac{dy}{dx} = \frac{6}{5} \times \frac{\sin t}{1 - \cos t}$ $\frac{1}{2}$

$\left.\frac{dy}{dx}\right|_{t = \frac{2\pi}{3}} = \frac{6}{5\sqrt{3}}$ $\frac{1}{2}$

8. $f'(x) = \dfrac{\cos x - \sin x}{1 + (\sin x + \cos x)^2}$ 1

$1 + (\sin x + \cos x)^2 > 0, \ \forall x \in \mathbb{R}$

and $\dfrac{\pi}{4} < x < \dfrac{\pi}{2} \Rightarrow \cos x < \sin x \Rightarrow \cos x - \sin x < 0$ $\frac{1}{2}$

$\Rightarrow$ $f'(x) < 0 \Rightarrow f(x)$ is decreasing in $\left(\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$ $\frac{1}{2}$

9. Vector form: $\vec{r} = (2\hat{i} - 3\hat{j} + 4\hat{k}) + \lambda(3\hat{i} + 4\hat{j} - 5\hat{k})$ 1

Cartesian form: $\dfrac{x-2}{3} = \dfrac{y+3}{4} = \dfrac{z-4}{-5}$ 1

10. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $\frac{1}{2}$

$= P(A) + P(B) - P(A)\,P(B)$ as A and B are independent events $\frac{1}{2}$

$\therefore \ 0.6 = 0.4 + p - (0.4)p$ $\frac{1}{2}$

$\Rightarrow p = \dfrac{1}{3}$ $\frac{1}{2}$

11. Let number of goods A = x units, number of goods B = y units

LPP is: Maximize profit, $P = 40x + 50y$ $\frac{1}{2}$

subject to following:

$$\left.\begin{array}{l} 3x + y \le 9 \\ x + 2y \le 8 \\ x \ge 0, \ y \ge 0 \end{array}\right\}$$ $1\frac{1}{2}$

12. $I = \displaystyle\int \frac{dx}{\sqrt{(2)^2 - (x+1)^2}}$ 1

$= \sin^{-1}\left(\dfrac{x+1}{2}\right) + c$ 1

SECTION C

13. $\cot\frac{1}{2}\left[\cos^{-1}\left(\frac{2x}{1+x^2}\right)+\sin^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right]$

$= \cot\frac{1}{2}\left[\frac{\pi}{2}-\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\frac{\pi}{2}-\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right]$ 1

$= \cot\frac{1}{2}[\pi - 2\tan^{-1}x - 2\tan^{-1}y]$ 1

$= \cot\left[\frac{\pi}{2}-(\tan^{-1}x+\tan^{-1}y)\right]$ 1

$= \tan\left[\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right] = \frac{x+y}{1-xy}$ 1

14. **Note:** Since a negative sign is missing in the question, so the equality can not be proved. So, 4 marks may be given for genuine attempt.

OR

Let $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ 1

then, $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$ 1

equating and solving to get a = 1, b = –2, c = 2, d = 0 $1\frac{1}{2}$

$X = \begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix}$ $\frac{1}{2}$

15. $x\frac{dy}{dx}+y = e^{x-y}\left(1-\frac{dy}{dx}\right)$ 1

$= xy\left(1-\frac{dy}{dx}\right)$ 1

$$\Rightarrow \frac{dy}{dx} = \frac{xy-y}{x+xy} = \frac{y(x-1)}{x(1+y)}$$ 1+1

OR

Differentiating the given expression w.r.t. x, we get

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{1+x^2}$$ 1

$$\Rightarrow (1+x^2)\frac{dy}{dx} = y$$ 1

diff. again w.r.t. x,

$$(1+x^2)\frac{d^2y}{dx^2}+\frac{dy}{dx}(2x) = \frac{dy}{dx}$$ $1\frac{1}{2}$

$$\Rightarrow (1+x^2)\frac{d^2y}{dx^2}+(2x-1)\frac{dy}{dx} = 0$$ $\frac{1}{2}$

16. $I = \int\frac{dt}{(2+t)(4+t^2)}$ where $e^x = t$ $\frac{1}{2}$

Now, $\frac{1}{(2+t)(4+t^2)} = \frac{1}{8(2+t)}-\frac{1}{8}\left(\frac{t-2}{4+t^2}\right)$ $1\frac{1}{2}$

$$\Rightarrow \int\frac{dt}{(2+t)(4+t^2)} = \frac{1}{8}\log|2+t|-\frac{1}{16}\log|4+t^2|+\frac{1}{8}\tan^{-1}\left(\frac{t}{2}\right)+c$$ $1\frac{1}{2}$

$$\Rightarrow \int\frac{e^x dx}{(2+e^x)(4+e^{2x})} = \frac{1}{8}\log|2+e^x|-\frac{1}{16}\log|4+e^{2x}|+\frac{1}{8}\tan^{-1}\left(\frac{e^x}{2}\right)+c$$ $\frac{1}{2}$

17. $\int_{-2}^{1}|x^3-x|\,dx = \int_{-2}^{-1}-(x^3-x)\,dx+\int_{-1}^{0}(x^3-x)\,dx-\int_{0}^{1}(x^3-x)dx$ $1+\frac{1}{2}+\frac{1}{2}$

$$= \left|\frac{x^2}{2}-\frac{x^4}{4}\right|_{-2}^{-1}+\left|\frac{x^4}{4}-\frac{x^2}{2}\right|_{-1}^{0}-\left|\frac{x^4}{4}-\frac{x^2}{2}\right|_{0}^{1}$$ 1

$$= \frac{11}{4}$$ 1

OR

$$I = \int e^{2x} \sin(3x+1)\, dx$$

$$= \sin(3x+1)\cdot\frac{e^{2x}}{2} - \int 3\cos(3x+1)\cdot\frac{e^{2x}}{2}\, dx$$ $1\frac{1}{2}$

$$= \frac{e^{2x}}{2}\cdot\sin(3x+1) - \frac{3}{2}\left[\cos(3x+1)\cdot\frac{e^{2x}}{2} - \int -3\sin(3x+1)\cdot\frac{e^{2x}}{2}\, dx\right]$$ 1

$$= \frac{e^{2x}}{2}\sin(3x+1) - \frac{3}{4}\cos(3x+1)\cdot e^{2x} - \frac{9}{4}I + c$$

$$\Rightarrow \frac{13}{4}I = \frac{e^{2x}}{4}[2\sin(3x+1) - 3\cos(3x+1)] + c$$ 1

$$\Rightarrow I = \frac{e^{2x}}{13}[2\sin(3x+1) - 3\cos(3x+1)] + c$$ $\frac{1}{2}$

18. Given differential equation can be written as

$$\frac{dx}{dy} = \frac{x}{y} - \frac{1}{2e^{x/y}}$$ $\frac{1}{2}$

Put $x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$ $\frac{1}{2}$

$\therefore\ v + y\frac{dv}{dy} = v - \frac{1}{2e^{v}}$ $\frac{1}{2}$

$\Rightarrow \int \frac{dy}{y} = -2\int e^{v}dv$ $\frac{1}{2}$

$\Rightarrow \log|y| = -2e^{v} + c = -2e^{x/y} + c$ 1

when $x = 0,\ y = 1 \Rightarrow c = 2$ $\frac{1}{2}$

$\therefore \log|y| = 2(1 - e^{x/y})$ $\frac{1}{2}$

19. $\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB} = \hat{i} - \hat{j} + \hat{k}$ 1

Area $= \left|\overrightarrow{AB} \times \overrightarrow{BC}\right|$ = magnitude of $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix}$ 1

$= |5\hat{i} + \hat{j} - 4\hat{k}|$ 1

$= \sqrt{42}$ sq. units 1

20. Let $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$; $\vec{a} \cdot \vec{c} = 6 \Rightarrow 2x + y - z = 6$ 1

Now, $\vec{a} \times \vec{c} = \vec{b} \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ x & y & z \end{vmatrix} = 4\hat{i} - 7\hat{j} + \hat{k}$ $\frac{1}{2}$

$\Rightarrow \hat{i}(z + y) - \hat{j}(2z + x) + \hat{k}(2y - x) = 4\hat{i} - 7\hat{j} + \hat{k}$

$\Rightarrow z + y = 4,\ 2z + x = 7,\ 2y - x = 1$ 1

Solving and getting x = 3, y = 2, z = 2 $1\frac{1}{2}$

$\vec{c} = 3\hat{i} + 2\hat{i} + 2\hat{k}$

21. X can take the values 3, 4, 5, 6, 7 1

∴ X :	3	4	5	6	7	
P(X) :	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{4}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	1
=	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	
X.P(X) :	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{3}$	$\frac{6}{6}$	$\frac{7}{6}$	
X^2.P(X) :	$\frac{9}{6}$	$\frac{16}{6}$	$\frac{25}{3}$	$\frac{36}{6}$	$\frac{49}{6}$	

$\therefore$ Mean = $\Sigma X \cdot P(X) = \frac{30}{6} = 5$ 1

Variance = $\Sigma X^2 \cdot P(X) - [\Sigma X \cdot P(X)]^2 = \frac{160}{6} - 25 = \frac{5}{3}$ 1

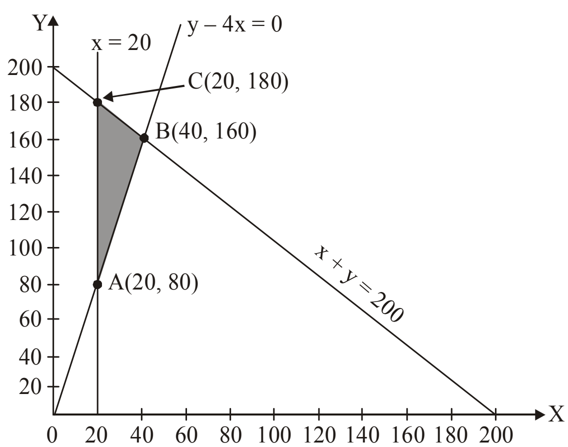
22. E_1 = Ghee purchased from shop X
E_2 = Ghee purchased from shop Y
A = Getting adultered ghee } 1

$P(E_1) = P(E_2) = \frac{1}{2}$, $P(A/E_1) = \frac{4}{7}$, $P(A/E_2) = \frac{6}{11}$ 1

$$P(E_2/A) = \frac{\frac{1}{2} \times \frac{6}{11}}{\frac{1}{2} \times \frac{6}{11} + \frac{1}{2} \times \frac{4}{7}} = \frac{21}{43}$$ 1

II part: Stringent punishment for the adultrators or any suitable measure 1

23.



For correct graphs 3 lines $1\frac{1}{2}$

For correct shading $1\frac{1}{2}$

Z(A) = 68,0000

Z(B) = 1,36,000

Z(C) = 1,28,000

$\therefore$ Maximum value of Z = 1,36,000 at x = 40, y = 160 1

SECTION D

24. Getting $AB = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$ $1\frac{1}{2}$

Given system of equations can be written as

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$ 1

i.e., $AX = C \Rightarrow X = A^{-1}C = \frac{1}{6}\cdot BC$ $\left(\because\ AB = 6I \Rightarrow A^{-1} = \frac{1}{6}B\right)$ $1+\frac{1}{2}$

$$= \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$ $1\frac{1}{2}$

$\Rightarrow x = 2,\ y = -1,\ z = 4$ $\frac{1}{2}$

25. Let $x_1, x_2 \in A$ and $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3} \Rightarrow (x_1 - 2)(x_2 - 3) = (x_1 - 3)(x_2 - 2)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow x_1 = x_2$

Hence f is a one-one function 2

Let $y = \frac{x - 2}{x - 3}$ for $y \in R - \{1\}$

$\Rightarrow x = \frac{3y - 2}{y - 1};\ y \neq 1$

$\therefore\ \forall\ y \in R - \{1\},\ x \in R - \{3\}$

i.e. Range of f = co-domain of f.

Hence f is onto and so bijective. 2

Also, $f^{-1}(x) = \frac{3x-2}{x-1};\ x \neq 1$ 1

Now, $f^{-1}(x) = 4 \Rightarrow \frac{3x-2}{x-1} = 4 \quad \Rightarrow \quad x = 2$ $\frac{1}{2}$

and $f^{-1}(7) = \frac{19}{6}$ $\frac{1}{2}$

OR

(i) (a, b) * (c, d) = (ad + bc, bd)

Now, (c, d) * (a, b) = (cb + da, db) = (ad + bc, bd) = (a, b) * (c, d)

$\Rightarrow$ * is commutative. 2

(ii) [(a, b) * (c, d)] * (e, f) = (ad + bc, bd) * (e, f) = (adf + bcf + bde, bdf)

(a,b) * [(c, d) * (e, f)] = (a, b) * (cf + de, df) = (adf + bcf + bde, bdf)

$\Rightarrow$ * is associative. 2

Let (e_1, e_2) be the identity element of A.

$\Rightarrow (a, b) * (e_1, e_2) = (a, b) = (e_1, e_2) * (a, b)$

$\Rightarrow (ae_2 + be_1, be_2) = (a, b) = (e_1b + e_2a, e_2b)$

$\Rightarrow ae_2 + be_1 = a$ and $be_2 = b \Rightarrow e_1 = 0,\ e_2 = 1$

$\Rightarrow$ (0, 1) is the identity on A. 2

26. Let length of one piece be x m, then length of the other piece = (34 – x) m

$\therefore$ Side of square is $\frac{x}{4}$ m then width of rectangle will be $\frac{34-x}{6}$ m. 1

Now, Area (A) = $\left(\frac{x}{4}\right)^2 + 2\left(\frac{34-x}{6}\right)^2$ 1

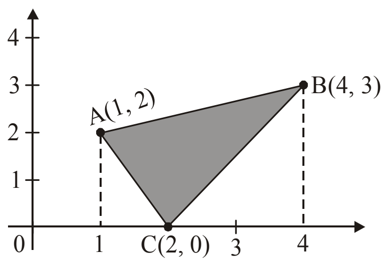
$\Rightarrow \frac{dA}{dx} = \frac{x}{8} - \frac{1}{9}(34-x)$ 1

$\frac{dA}{dx} = 0 \Rightarrow x = 16$ 1 also, $\frac{d^2A}{dx^2} = \frac{1}{8} + \frac{1}{9} > 0$ 1

so, A is minimum when x = 16

$\therefore$ Lengths of the two pieces are 16 m and 18 m. 1

27. Correct figure 1



Equation of AB : $y = \frac{x+5}{3}$

Equation of BC: $y = \frac{3x}{2} - 3$

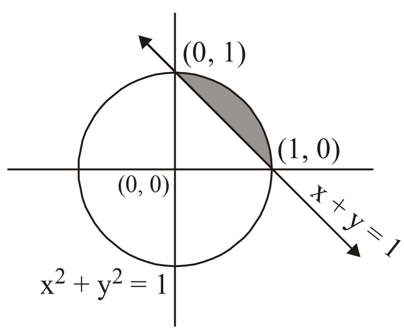
Equation of AC: $y = 4 - 2x$ $1\frac{1}{2}$

$\therefore$ Area (A) $= \int_1^4 \frac{x+5}{3}dx - \int_1^2 (4-2x)dx - \int_2^4 \left(\frac{3x}{2} - 3\right)dx$ 1

$= \frac{1}{3}\left[\frac{x^2}{2} + 5x\right]_1^4 - \left[4x - x^2\right]_1^2 - \left[\frac{3x^2}{4} - 3x\right]_2^4$ $1\frac{1}{2}$

$= \frac{15}{2} - 1 - 3 = \frac{7}{2}$ sq.units 1

OR



For correct figure

For correct shading $1 + \frac{1}{2}$

$A = \int_0^1 \left(\sqrt{1-x^2} - (1-x)\right)dx$ $1\frac{1}{2}$

$= \left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right]_0^1 - \left[x - \frac{x^2}{2}\right]_0^1$ 2

$= \frac{1}{2}\sin^{-1}(1) - \frac{1}{2} = \left(\frac{\pi}{4} - \frac{1}{2}\right)$ sq.units 1

28. Here, I.F. $= e^{\int -3\cot x\, dx} = \dfrac{1}{\sin^3 x}$ $1\frac{1}{2}$

Solution is given by, $y\left(\dfrac{1}{\sin^3 x}\right) = \displaystyle\int \frac{\sin 2x}{\sin^3 x}\, dx = 2\int \frac{\cos x}{\sin^2 x}\, dx$ $1\frac{1}{2}$

$\Rightarrow \dfrac{y}{\sin^3 x} = \dfrac{-2}{\sin x} + c$ $1\frac{1}{2}$

when $x = \dfrac{\pi}{2}$, $y = 2 \Rightarrow c = 4$ 1

$\therefore \dfrac{y}{\sin^3 x} = \dfrac{-2}{\sin x} + 4$ or $y = -2\sin^2 x + 4\sin^3 x$ $\frac{1}{2}$

29. Equation of the plane through the intersection of planes is

$(x + y + z - 1) + \lambda(2x + 3y + 4z - 5) = 0$ 1

$\Rightarrow (1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z - (1 + 5\lambda) = 0$ ....(i)

This plane is perpendicular to $x - y + z = 0$

$\therefore 1(1 + 2\lambda) - 1\,(1 + 3\lambda) + 1(1 + 4\lambda) = 0 \Rightarrow \lambda = \dfrac{-1}{3}$ 2

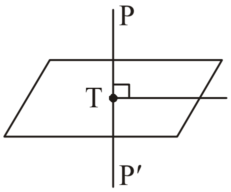
$\therefore$ Equation of plane is

$(x + y + z - 1) - \dfrac{1}{3}(2x + 3y + 4z - 5) = 0 \Rightarrow x - z + 2 = 0.$ 1

Vector form of plane is $\vec{r}\cdot(\hat{i} - \hat{k}) + 2 = 0$ 1

Yes, line lies on the plane as (–2, 3, 0) satisfies $\vec{r}\cdot(\hat{i} - \hat{k}) + 2 = 0$ and normal to plane is perpendicular to the given line as $1(5) + 0(4) - 1(5) = 0$ 1

OR



Let PT is perpendicular to given plane.

Let p.v. of T is $\vec{b}_1 = a\hat{i} + b\hat{j} + c\hat{k}$

$\therefore\ \overrightarrow{PT} = (a-1)\hat{i} + (b-3)\hat{j} + (c-4)\hat{k}$ 1

$\overrightarrow{PT} \,||\, \vec{n}$ (nromal) $\therefore\ \dfrac{a-1}{-2} = \dfrac{b-3}{1} = \dfrac{c-4}{-1} = \lambda$

$\Rightarrow a = -2\lambda + 1,\ b = \lambda + 3,\ c = -\lambda + 4$ 1

$\therefore\ \vec{b}_1 = (-2\lambda+1)\hat{i} + (\lambda+3)\hat{j} + (-\lambda+4)\hat{k}$ $\frac{1}{2}$

$\vec{b}_1$ lies on plane

$\therefore\ [(-2\lambda+1)\hat{i} + (\lambda+3)\hat{j} + (-\lambda+4)\hat{k}]\cdot(-2\hat{i} + \hat{j} - \hat{k}) = 3$

$\Rightarrow \lambda = 1$ 1

$\therefore\ \vec{b}_1 = -\hat{i} + 4\hat{j} + 3\hat{k}$ $\frac{1}{2}$

Let p.v. of P′ is $\vec{c}_1 = x\hat{i} + y\hat{j} + z\hat{k}$

Using section formula, $\vec{c}_1 = -3\hat{i} + 5\hat{j} + 2\hat{k}$ 1

Also, $PP' = \sqrt{24}$ or $2\sqrt{6}$ 1

QUESTION PAPER CODE 65/2/2

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\int_0^{2\pi} \cos^5 x \, dx = 2\int_0^{\pi} \cos^5 x \, dx$ $\frac{1}{2}$

and $2\int_0^{\pi} \cos^5 x \, dx = 0 \Rightarrow \int_0^{2\pi} \cos^5 x \, dx = 0$ $\frac{1}{2}$

2. $\sqrt{(-5)^2 + (12)^2} = 13$ $\frac{1}{2}+\frac{1}{2}$

3. $\text{adj } A = 3\begin{bmatrix} 3 & -1 \\ \frac{-5}{3} & \frac{2}{3} \end{bmatrix} = \begin{bmatrix} 9 & -3 \\ -5 & 2 \end{bmatrix}$ 1

4. $\lim_{x \to 0}\left(\frac{\sin 5x}{3x} + \cos x\right) = \frac{8}{3} \Rightarrow k = \frac{8}{3}$ $\frac{1}{2}+\frac{1}{2}$

SECTION B

5. $\frac{dy}{dt} = 12 \sin t, \ \frac{dx}{dt} = 10\ (1 - \cos t)$ $\frac{1}{2}+\frac{1}{2}$

$\therefore \ \frac{dy}{dx} = \frac{6}{5} \times \frac{\sin t}{1 - \cos t}$ $\frac{1}{2}$

$\left.\frac{dy}{dx}\right|_{t = \frac{2\pi}{3}} = \frac{6}{5\sqrt{3}}$ $\frac{1}{2}$

6. $f'(x) = \frac{\cos x - \sin x}{1 + (\sin x + \cos x)^2}$ 1

$1 + (\sin x + \cos x)^2 > 0, \ \forall x \in \mathbb{R}$

and $\frac{\pi}{4} < x < \frac{\pi}{2} \Rightarrow \cos x < \sin x \Rightarrow \cos x - \sin x < 0$ $\frac{1}{2}$

$\Rightarrow$ $f'(x) < 0 \Rightarrow f(x)$ is decreasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ $\frac{1}{2}$

7. Vector form: $\vec{r} = (2\hat{i} - 3\hat{j} + 4\hat{k}) + \lambda(3\hat{i} + 4\hat{j} - 5\hat{k})$ 1

Cartesian form: $\frac{x-2}{3} = \frac{y+3}{4} = \frac{z-4}{-5}$ 1

8. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $\frac{1}{2}$

$= P(A) + P(B) - P(A)\,P(B)$ as A and B are independent events $\frac{1}{2}$

$\therefore\ 0.6 = 0.4 + p - (0.4)p$ $\frac{1}{2}$

$\Rightarrow p = \frac{1}{3}$ $\frac{1}{2}$

9. Let number of goods A = x units, number of goods B = y units

LPP is: Maximize profit, $P = 40x + 50y$ $\frac{1}{2}$

subject to following:

$$\left.\begin{array}{l} 3x + y \le 9 \\ x + 2y \le 8 \\ x \ge 0,\ y \ge 0 \end{array}\right\}$$ $1\frac{1}{2}$

10. $I = \int \frac{dx}{\sqrt{(2)^2 - (x+1)^2}}$ 1

$= \sin^{-1}\left(\frac{x+1}{2}\right) + c$ 1

11. $|2AB| = 2^3 \times |A| \times |B|$ 1

$= 8 \times (-1) \times 3 = -24$ 1

12. $V = \frac{1}{3}\pi r^2 h$

$\Rightarrow \frac{dV}{dt} = \frac{1}{3}\pi\left[r^2\frac{dh}{dt} + h \cdot 2r\frac{dr}{dt}\right]$ 1

$= \frac{1}{3} \times \frac{22}{7} \times [(3.5)^2 \times 3 - 2(3.5)(6)(2)]$

$= -49.5 \text{ cm}^3/\text{min}$ 1

$\therefore$ Volume is decreasing at the rate of $49.5 \text{ cm}^3/\text{min}$

SECTION C

13. X can take the values 3, 4, 5, 6, 7 1

$\therefore$ X :	3	4	5	6	7
P(X) :	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{4}{12}$	$\frac{2}{12}$	$\frac{2}{12}$
=	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$
X.P(X) :	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{3}$	$\frac{6}{6}$	$\frac{7}{6}$
X^2.P(X) :	$\frac{9}{6}$	$\frac{16}{6}$	$\frac{25}{3}$	$\frac{36}{6}$	$\frac{49}{6}$

1

$\therefore$ Mean $= \Sigma X \cdot P(X) = \frac{30}{6} = 5$ 1

Variance $= \Sigma X^2 \cdot P(X) - [\Sigma X \cdot P(X)]^2 = \frac{160}{6} - 25 = \frac{5}{3}$ 1

14. Given differential equation can be written as

$\frac{dx}{dy} = \frac{x}{y} - \frac{1}{2e^{x/y}}$ $\frac{1}{2}$

Put $x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$ $\frac{1}{2}$

$\therefore \; v + y\frac{dv}{dy} = v - \frac{1}{2e^v}$ $\frac{1}{2}$

$\Rightarrow \int \frac{dy}{y} = -2\int e^v dv$ $\frac{1}{2}$

$\Rightarrow \log|y| = -2e^v + c = -2e^{x/y} + c$ 1 when x = 0, y = 1 ⇒ c = 2 $\frac{1}{2}$

$\therefore$ log |y| = 2(1 – $e^{x/y}$) $\frac{1}{2}$

15. E_1 = Ghee purchased from shop X
E_2 = Ghee purchased from shop Y
A = Getting adultered ghee } 1

$P(E_1) = P(E_2) = \frac{1}{2}, P(A/E_1) = \frac{4}{7}, P(A/E_2) = \frac{6}{11}$ 1

$P(E_2/A) = \frac{\frac{1}{2}\times\frac{6}{11}}{\frac{1}{2}\times\frac{6}{11}+\frac{1}{2}\times\frac{4}{7}} = \frac{21}{43}$ 1

II part: Stringent punishment for the adultrators or any suitable measure 1

16. Let $\vec{c} = x\hat{i}+y\hat{j}+z\hat{k}$; $\vec{a}\cdot\vec{c} = 6 \Rightarrow 2x+y-z = 6$ 1

Now, $\vec{a}\times\vec{c} = \vec{b} \Rightarrow \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -1\\ x & y & z\end{vmatrix} = 4\hat{i}-7\hat{j}+\hat{k}$ $\frac{1}{2}$

$\Rightarrow \quad \hat{i}(z+y)-\hat{j}(2z+x)+\hat{k}(2y-x) = 4\hat{i}-7\hat{j}+\hat{k}$

$\Rightarrow \quad z + y = 4,\ 2z + x = 7,\ 2y - x = 1$ 1

Solving and getting x = 3, y = 2, z = 2 $1\frac{1}{2}$

$\vec{c} = 3\hat{i}+2\hat{i}+2\hat{k}$

17. $x\frac{dy}{dx}+y = e^{x-y}\left(1-\frac{dy}{dx}\right)$ 1

$= xy\left(1-\frac{dy}{dx}\right)$ 1

$\Rightarrow \frac{dy}{dx} = \frac{xy-y}{x+xy} = \frac{y(x-1)}{x(1+y)}$ 1+1

OR

Differentiating the given expression w.r.t. x, we get

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{1+x^2}$$ 1

$$\Rightarrow (1+x^2)\frac{dy}{dx} = y$$ 1

diff. again w.r.t. x,

$$(1+x^2)\frac{d^2y}{dx^2}+\frac{dy}{dx}(2x) = \frac{dy}{dx}$$ $1\frac{1}{2}$

$$\Rightarrow (1+x^2)\frac{d^2y}{dx^2}+(2x-1)\frac{dy}{dx} = 0$$ $\frac{1}{2}$

18. $$\int_{-2}^{1}|x^3-x|\,dx = \int_{-2}^{-1}-(x^3-x)\,dx+\int_{-1}^{0}(x^3-x)\,dx-\int_{0}^{1}(x^3-x)dx$$ $1+\frac{1}{2}+\frac{1}{2}$

$$= \left|\frac{x^2}{2}-\frac{x^4}{4}\right|_{-2}^{-1}+\left|\frac{x^4}{4}-\frac{x^2}{2}\right|_{-1}^{0}-\left|\frac{x^4}{4}-\frac{x^2}{2}\right|_{0}^{1}$$ 1

$$= \frac{11}{4}$$ 1

OR

$$I = \int e^{2x}\sin(3x+1)\,dx$$

$$= \sin(3x+1)\cdot\frac{e^{2x}}{2}-\int 3\cos(3x+1)\cdot\frac{e^{2x}}{2}dx$$ $1\frac{1}{2}$

$$= \frac{e^{2x}}{2}\cdot\sin(3x+1)-\frac{3}{2}\left[\cos(3x+1)\cdot\frac{e^{2x}}{2}-\int -3\sin(3x+1)\cdot\frac{e^{2x}}{2}dx\right]$$ 1

$$= \frac{e^{2x}}{2}\sin(3x+1)-\frac{3}{4}\cos(3x+1)\cdot e^{2x}-\frac{9}{4}I+c$$

$$\Rightarrow \frac{13}{4}I = \frac{e^{2x}}{4}[2\sin(3x+1)-3\cos(3x+1)]+c$$ 1

$$\Rightarrow I = \frac{e^{2x}}{13}[2\sin(3x+1)-3\cos(3x+1)]+c$$ $\frac{1}{2}$

19. **Note:** Since a negative sign is missing in the question, so the equality can not be proved. So, 4 marks may be given for genuine attempt.

OR

Let $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ 1

then, $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$ 1

equating and solving to get a = 1, b = –2, c = 2, d = 0 $1\frac{1}{2}$

$X = \begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix}$ $\frac{1}{2}$

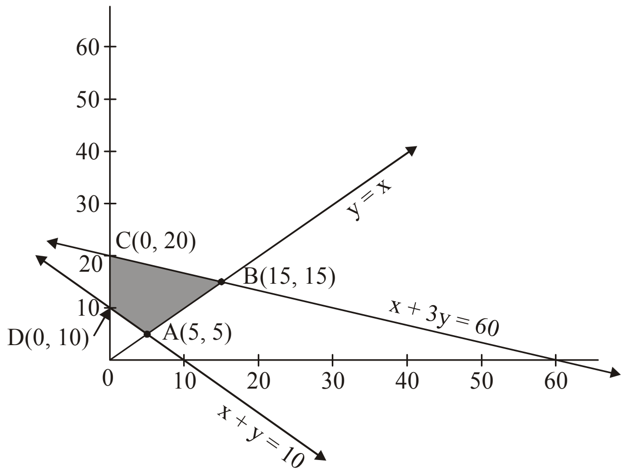
20. $I = \frac{1}{2}\int \frac{1}{(2+t)(4+t^2)}\,dt$ where $t = x^2$ $\frac{1}{2}$

Now, $\frac{1}{2}\left[\frac{1}{(2+t)(4+t^2)}\right] = \frac{1}{2}\left[\frac{1}{8(2+t)} - \frac{1}{8}\left(\frac{t-2}{4+t^2}\right)\right]$ $1\frac{1}{2}$

$\Rightarrow \int \frac{1}{(2+t)(4+t^2)}\,dt = \frac{1}{16}\log|2+t| - \frac{1}{32}\log|4+t^2| + \frac{1}{16}\tan^{-1}\left(\frac{t}{2}\right) + c$ $1\frac{1}{2}$

$\Rightarrow \int \frac{x}{(2+x^2)(4+x^4)}\,dx = \frac{1}{16}\log|2+x^2| - \frac{1}{32}\log|4+x^4| + \frac{1}{16}\tan^{-1}\left(\frac{x^2}{2}\right) + c$ $\frac{1}{2}$

21.



For correct graphs of 3 lines $1\frac{1}{2}$

For correct shading $1\frac{1}{2}$

Z(A) = 60

Z(B) = 180

Z(C) = 180

Z(D) = 90

$\therefore$ Minimum value of z is 60 when x = 5, y = 5 1

22. $\overrightarrow{AD} = \overrightarrow{AB} - \overrightarrow{DB} = 3\hat{i} - 2\hat{j} + 4\hat{k}$ 1

$$\text{Area} = \left|\overrightarrow{AB} \times \overrightarrow{AD}\right| = \text{magnitude of} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 0 & 7 \\ 3 & -2 & 4 \end{vmatrix}$$ 1

$$= \left|14\hat{i} + \hat{j} - 10\hat{k}\right|$$ 1

$$= \sqrt{297} \text{ sq.units or } 3\sqrt{33} \text{ sq.units}$$ 1

23. $$\text{LHS} = \tan^{-1}\left(\frac{2x + \frac{4x}{1-4x^2}}{1 - 2x\left(\frac{4x}{1-4x^2}\right)}\right)$$ 2

$$= \tan^{-1}\left(\frac{2x - 8x^3 + 4x}{1 - 4x^2 - 8x^2}\right)$$

$$= \tan^{-1}\left(\frac{6x - 8x^3}{1 - 12x^2}\right) = \text{RHS}$$ 2

SECTION D

24. Equation of the plane through the intersection of planes is

$(x + y + z - 1) + \lambda(2x + 3y + 4z - 5) = 0$ 1

$\Rightarrow (1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z - (1 + 5\lambda) = 0$ ....(i)

This plane is perpendicular to $x - y + z = 0$

$\therefore\ 1(1 + 2\lambda) - 1\ (1 + 3\lambda) + 1(1 + 4\lambda) = 0 \Rightarrow \lambda = \frac{-1}{3}$ 2

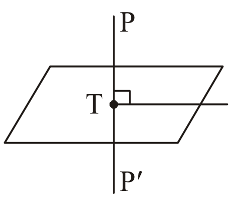
$\therefore$ Equation of plane is

$(x + y + z - 1) - \frac{1}{3}(2x + 3y + 4z - 5) = 0 \Rightarrow x - z + 2 = 0.$ 1

Vector form of plane is $\vec{r} \cdot (\hat{i} - \hat{k}) + 2 = 0$ 1

Yes, line lies on the plane as (–2, 3, 0) satisfies $\vec{r} \cdot (\hat{i} - \hat{k}) + 2 = 0$ and normal to plane is perpendicular to the given line as $1(5) + 0(4) - 1(5) = 0$ 1

OR



Let PT is perpendicular to given plane.

Let p.v. of T is $\vec{b}_1 = a\hat{i} + b\hat{j} + c\hat{k}$

$\therefore\ \overrightarrow{PT} = (a-1)\hat{i} + (b-3)\hat{j} + (c-4)\hat{k}$ 1

$\overrightarrow{PT} \parallel \vec{n}$ (nromal) $\therefore\ \frac{a-1}{-2} = \frac{b-3}{1} = \frac{c-4}{-1} = \lambda$

$\Rightarrow a = -2\lambda + 1,\ b = \lambda + 3,\ c = -\lambda + 4$ 1

$\therefore\ \vec{b}_1 = (-2\lambda + 1)\hat{i} + (\lambda + 3)\hat{j} + (-\lambda + 4)\hat{k}$ $\frac{1}{2}$

$\vec{b}_1$ lies on plane

$\therefore\ [(-2\lambda + 1)\hat{i} + (\lambda + 3)\hat{j} + (-\lambda + 4)\hat{k}] \cdot (-2\hat{i} + \hat{j} - \hat{k}) = 3$

$\Rightarrow \lambda = 1$ 1

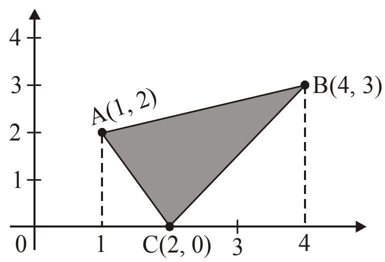
$\therefore\ \vec{b}_1 = -\hat{i} + 4\hat{j} + 3\hat{k}$ $\frac{1}{2}$

Let p.v. of P′ is $\vec{c}_1 = x\hat{i} + y\hat{j} + z\hat{k}$

Using section formula, $\vec{c}_1 = -3\hat{i} + 5\hat{j} + 2\hat{k}$ 1

Also, $PP' = \sqrt{24}$ or $2\sqrt{6}$ 1

25. Correct figure 1



Equation of AB : $y = \frac{x+5}{3}$

Equation of BC: $y = \frac{3x}{2} - 3$

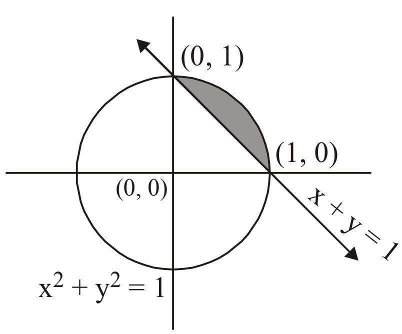
Equation of AC: $y = 4 - 2x$ $1\frac{1}{2}$

$\therefore$ Area (A) $= \int_1^4 \frac{x+5}{3}dx - \int_1^2 (4-2x)dx - \int_2^4 \left(\frac{3x}{2} - 3\right)dx$ 1

$= \frac{1}{3}\left[\frac{x^2}{2} + 5x\right]_1^4 - \left[4x - x^2\right]_1^2 - \left[\frac{3x^2}{4} - 3x\right]_2^4$ $1\frac{1}{2}$

$= \frac{15}{2} - 1 - 3 = \frac{7}{2}$ sq.units 1

OR



For correct figure

For correct shading $1 + \frac{1}{2}$

$A = \int_0^1 \left(\sqrt{1-x^2} - (1-x)\right)dx$ $1\frac{1}{2}$

$= \left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right]_0^1 - \left[x - \frac{x^2}{2}\right]_0^1$ 2

$= \frac{1}{2}\sin^{-1}(1) - \frac{1}{2} = \left(\frac{\pi}{4} - \frac{1}{2}\right)$ sq.units 1

26. Let $x_1, x_2 \in A$ and $f(x_1) = f(x_2)$

$\Rightarrow \quad \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3} \Rightarrow (x_1 - 2)(x_2 - 3) = (x_1 - 3)(x_2 - 2)$

$\Rightarrow \quad x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow \quad x_1 = x_2$

Hence f is a one-one function 2

Let $y = \frac{x - 2}{x - 3}$ for $y \in R - \{1\}$

$\Rightarrow \quad x = \frac{3y - 2}{y - 1}; \ y \neq 1$

$\therefore \ \forall \ y \in R - \{1\}, x \in R - \{3\}$

i.e. Range of f = co-domain of f.

Hence f is onto and so bijective. 2

Also, $f^{-1}(x) = \frac{3x - 2}{x - 1}; \ x \neq 1$ 1

Now, $f^{-1}(x) = 4 \Rightarrow \frac{3x - 2}{x - 1} = 4 \ \Rightarrow \ x = 2$ $\frac{1}{2}$

and $f^{-1}(7) = \frac{19}{6}$ $\frac{1}{2}$

OR

(i) (a, b) * (c, d) = (ad + bc, bd)

Now, (c, d) * (a, b) = (cb + da, db) = (ad + bc, bd) = (a, b) * (c, d)

$\Rightarrow$ * is commutative. 2

(ii) [(a, b) * (c, d)] * (e, f) = (ad + bc, bd) * (e, f) = (adf + bcf + bde, bdf)

(a,b) * [(c, d) * (e, f)] = (a, b) * (cf + de, df) = (adf + bcf + bde, bdf)

$\Rightarrow$ * is associative. 2

Let (e_1, e_2) be the identity element of A.

$\Rightarrow (a, b) * (e_1, e_2) = (a, b) = (e_1, e_2) * (a, b)$

$\Rightarrow (ae_2 + be_1, be_2) = (a, b) = (e_1b + e_2a, e_2b)$

$\Rightarrow ae_2 + be_1 = a$ and $be_2 = b \Rightarrow e_1 = 0, e_2 = 1$

$\Rightarrow (0, 1)$ is the identity on A. 2

27. Let length of one piece be x m, then length of the other piece = (34 – x) m

$\therefore$ Side of square is $\frac{x}{4}$ m then width of rectangle will be $\frac{34-x}{6}$ m. 1

Now, Area (A) = $\left(\frac{x}{4}\right)^2 + 2\left(\frac{34-x}{6}\right)^2$ 1

$\Rightarrow \frac{dA}{dx} = \frac{x}{8} - \frac{1}{9}(34-x)$ 1

$\frac{dA}{dx} = 0 \Rightarrow x = 16$ 1

also, $\frac{d^2A}{dx^2} = \frac{1}{8} + \frac{1}{9} > 0$ 1

so, A is minimum when x = 16

$\therefore$ Lengths of the two pieces are 16 m and 18 m. 1

28. $|A| = 27 \neq 0, A^{-1}$ exist 1

$$\left.\begin{array}{l} A_{11} = 4, A_{21} = 17, A_{13} = 3 \\ A_{12} = -1, A_{22} = -11, A_{32} = 6 \\ A_{13} = 5, A_{23} = 1, A_{33} = -3 \end{array}\right\}$$ 2

$$\therefore A^{-1} = \frac{1}{27}\begin{pmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{pmatrix}$$ $\frac{1}{2}$

Given system of equations can be written as AX = B where

$$A = \begin{pmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{pmatrix}, \ X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \ B = \begin{pmatrix} 10 \\ -2 \\ -11 \end{pmatrix}$$ $\frac{1}{2}$

Now, $AX = B \Rightarrow X = A^{-1}B = \frac{1}{27}\begin{pmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{pmatrix}\begin{pmatrix} 10 \\ -2 \\ -11 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix}$ $1\frac{1}{2}$

$\therefore\ x = -1,\ y = -2,\ z = 3$ $\frac{1}{2}$

29. Given differential equation can be written as

$\frac{dy}{dx} + y\cot x = 2\cos x$ 1

I.F. $= e^{\int \cot x\, dx} = e^{\log \sin x} = \sin x$ 1

Solution is given by

$y\sin x = \int 2\sin x \cos x\, dx = \int \sin 2x\, dx$ 1

$= \frac{-\cos 2x}{2} + c$ 1

when $x = \frac{\pi}{2},\ y = 2,\ \Rightarrow c = \frac{3}{2}$ 1

Solution is given by $y\sin x = -\frac{1}{2}\cos 2x + \frac{3}{2}$ or $y = \operatorname{cosec} x + \sin x$ 1

QUESTION PAPER CODE 65/2/3

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\sqrt{(-5)^2 + (12)^2} = 13$ $\frac{1}{2} + \frac{1}{2}$

2. $\int_0^{2\pi} \cos^5 x \, dx = 2\int_0^{\pi} \cos^5 x \, dx$ $\frac{1}{2}$

and $2\int_0^{\pi} \cos^5 x \, dx = 0 \Rightarrow \int_0^{2\pi} \cos^5 x \, dx = 0$ $\frac{1}{2}$

3. $\lim_{x \to 0} \left(\frac{\sin 5x}{3x} + \cos x \right) = \frac{8}{3} \Rightarrow k = \frac{8}{3}$ $\frac{1}{2} + \frac{1}{2}$

4. $\text{adj } A = 3\begin{bmatrix} 3 & -1 \\ \frac{-5}{3} & \frac{2}{3} \end{bmatrix} = \begin{bmatrix} 9 & -3 \\ -5 & 2 \end{bmatrix}$ 1

SECTION B

5. $I = \int \frac{dx}{\sqrt{(2)^2 - (x+1)^2}}$ 1

$= \sin^{-1}\left(\frac{x+1}{2}\right) + c$ 1

6. Let number of goods A = x units, number of goods B = y units

LPP is: Maximize profit, P = 40x + 50y $\frac{1}{2}$

subject to following:

$$\left.\begin{aligned} 3x + y &\le 9 \\ x + 2y &\le 8 \\ x \ge 0, \ y &\ge 0 \end{aligned}\right\}$$ $1\frac{1}{2}$

7. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $\frac{1}{2}$

$= P(A) + P(B) - P(A)\,P(B)$ as A and B are independent events $\frac{1}{2}$

$\therefore\ 0.6 = 0.4 + p - (0.4)p$ $\frac{1}{2}$

$\Rightarrow p = \frac{1}{3}$ $\frac{1}{2}$

8. Vector form: $\vec{r} = (2\hat{i} - 3\hat{j} + 4\hat{k}) + \lambda(3\hat{i} + 4\hat{j} - 5\hat{k})$ 1

Cartesian form: $\frac{x-2}{3} = \frac{y+3}{4} = \frac{z-4}{-5}$ 1

9. $f'(x) = \frac{\cos x - \sin x}{1 + (\sin x + \cos x)^2}$ 1

$1 + (\sin x + \cos x)^2 > 0,\ \forall x \in \mathbb{R}$

and $\frac{\pi}{4} < x < \frac{\pi}{2} \Rightarrow \cos x < \sin x \Rightarrow \cos x - \sin x < 0$ $\frac{1}{2}$

$\Rightarrow$ $f'(x) < 0 \Rightarrow f(x)$ is decreasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ $\frac{1}{2}$

10. $\frac{dy}{dt} = 12 \sin t,\ \frac{dx}{dt} = 10\,(1 - \cos t)$ $\frac{1}{2} + \frac{1}{2}$

$\therefore\ \frac{dy}{dx} = \frac{6}{5} \times \frac{\sin t}{1 - \cos t}$ $\frac{1}{2}$

$\left.\frac{dy}{dx}\right|_{t = \frac{2\pi}{3}} = \frac{6}{5\sqrt{3}}$ $\frac{1}{2}$

11. $|2AB| = 2^3 \times |A| \times |B|$ 1

$= 8 \times (-1) \times 3 = -24$ 1

12. CSA of cylinder, $A = 2\pi rh$

$\Rightarrow \frac{dA}{dt} = 2\pi\left[r\frac{dh}{dt} + h\frac{dr}{dt}\right]$ 1

$= 2 \times \frac{22}{7}\left[3.5 \times (-0.4) + 7(0.3)\right] = 4.4 \text{ cm}^2/\text{s}$ 1

$\therefore$ CSA is increasing at the rate of 4.4 cm^2/s

SECTION C

13. X can take the values 3, 4, 5, 6, 7 1

$\therefore$ X :	3	4	5	6	7
P(X) :	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{4}{12}$	$\frac{2}{12}$	$\frac{2}{12}$
=	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$
X.P(X) :	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{3}$	$\frac{6}{6}$	$\frac{7}{6}$
X^2.P(X) :	$\frac{9}{6}$	$\frac{16}{6}$	$\frac{25}{3}$	$\frac{36}{6}$	$\frac{49}{6}$

(P(X) rows) 1

$\therefore$ Mean $= \Sigma X \cdot P(X) = \frac{30}{6} = 5$ 1

Variance $= \Sigma X^2 \cdot P(X) - [\Sigma X \cdot P(X)]^2 = \frac{160}{6} - 25 = \frac{5}{3}$ 1

14. Let $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$; $\vec{a} \cdot \vec{c} = 6 \Rightarrow 2x + y - z = 6$ 1

Now, $\vec{a} \times \vec{c} = \vec{b} \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ x & y & z \end{vmatrix} = 4\hat{i} - 7\hat{j} + \hat{k}$ $\frac{1}{2}$

$\Rightarrow \quad \hat{i}(z + y) - \hat{j}(2z + x) + \hat{k}(2y - x) = 4\hat{i} - 7\hat{j} + \hat{k}$

$\Rightarrow \quad z + y = 4,\ 2z + x = 7,\ 2y - x = 1$ 1

Solving and getting x = 3, y = 2, z = 2 $1\frac{1}{2}$

$\vec{c} = 3\hat{i} + 2\hat{i} + 2\hat{k}$

15. $\int_{-2}^{1}|x^3 - x|\,dx = \int_{-2}^{-1} -(x^3 - x)\,dx + \int_{-1}^{0}(x^3 - x)\,dx - \int_{0}^{1}(x^3 - x)dx$ $1+\frac{1}{2}+\frac{1}{2}$

$= \left|\frac{x^2}{2} - \frac{x^4}{4}\right|_{-2}^{-1} + \left|\frac{x^4}{4} - \frac{x^2}{2}\right|_{-1}^{0} - \left|\frac{x^4}{4} - \frac{x^2}{2}\right|_{0}^{1}$ 1

$= \frac{11}{4}$ 1

OR

$I = \int e^{2x}\sin(3x+1)\,dx$

$= \sin(3x+1)\cdot\frac{e^{2x}}{2} - \int 3\cos(3x+1)\cdot\frac{e^{2x}}{2}\,dx$ $1\frac{1}{2}$

$= \frac{e^{2x}}{2}\cdot\sin(3x+1) - \frac{3}{2}\left[\cos(3x+1)\cdot\frac{e^{2x}}{2} - \int -3\sin(3x+1)\cdot\frac{e^{2x}}{2}\,dx\right]$ 1

$= \frac{e^{2x}}{2}\sin(3x+1) - \frac{3}{4}\cos(3x+1)\cdot e^{2x} - \frac{9}{4}I + c$

$\Rightarrow \frac{13}{4}I = \frac{e^{2x}}{4}[2\sin(3x+1) - 3\cos(3x+1)] + c$ 1

$\Rightarrow I = \frac{e^{2x}}{13}[2\sin(3x+1) - 3\cos(3x+1)] + c$ $\frac{1}{2}$

16. E_1 = Ghee purchased from shop X
E_2 = Ghee purchased from shop Y
A = Getting adultered ghee 1

$P(E_1) = P(E_2) = \frac{1}{2}$, $P(A/E_1) = \frac{4}{7}$, $P(A/E_2) = \frac{6}{11}$ 1

$P(E_2/A) = \frac{\frac{1}{2}\times\frac{6}{11}}{\frac{1}{2}\times\frac{6}{11} + \frac{1}{2}\times\frac{4}{7}} = \frac{21}{43}$ 1

II part: Stringent punishment for the adultrators or any suitable measure 1

17. $I = \int \frac{dt}{(2+t)(4+t^2)}$ where $e^x = t$ $\frac{1}{2}$

Now, $\frac{1}{(2+t)(4+t^2)} = \frac{1}{8(2+t)} - \frac{1}{8}\left(\frac{t-2}{4+t^2}\right)$ $1\frac{1}{2}$

$\Rightarrow \int \frac{dt}{(2+t)(4+t^2)} = \frac{1}{8}\log|2+t| - \frac{1}{16}\log|4+t^2| + \frac{1}{8}\tan^{-1}\left(\frac{t}{2}\right) + c$ $1\frac{1}{2}$

$\Rightarrow \int \frac{e^x dx}{(2+e^x)(4+e^{2x})} = \frac{1}{8}\log|2+e^x| - \frac{1}{16}\log|4+e^{2x}| + \frac{1}{8}\tan^{-1}\left(\frac{e^x}{2}\right) + c$ $\frac{1}{2}$

18. $x\frac{dy}{dx} + y = e^{x-y}\left(1 - \frac{dy}{dx}\right)$ 1

$= xy\left(1 - \frac{dy}{dx}\right)$ 1

$\Rightarrow \frac{dy}{dx} = \frac{xy - y}{x + xy} = \frac{y(x-1)}{x(1+y)}$ 1+1

OR

Differentiating the given expression w.r.t. x, we get

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{1+x^2}$ 1

$\Rightarrow (1+x^2)\frac{dy}{dx} = y$ 1

diff. again w.r.t. x,

$(1+x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx}(2x) = \frac{dy}{dx}$ $1\frac{1}{2}$

$\Rightarrow (1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$ $\frac{1}{2}$

19. **Note:** Since a negative sign is missing in the question, so the equality can not be proved. So, 4 marks may be given for genuine attempt.

OR

Let $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ 1

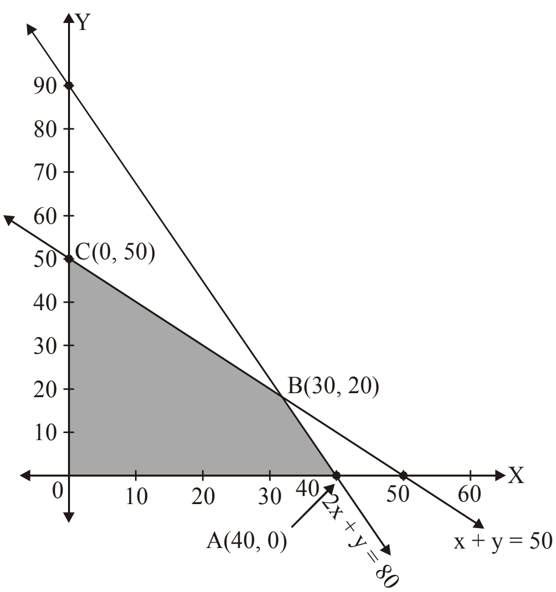
then, $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$ 1

equating and solving to get a = 1, b = –2, c = 2, d = 0 $1\frac{1}{2}$

$X = \begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix}$ $\frac{1}{2}$

20.



For correct graph of 2 lines 2

For correct shading 1

Z(O) = 0

Z(A) = 4200

Z(B) = 4950

Z(C) = 4500

$\therefore$ Maximum value of Z is 4950

at x = 30, y = 20 1

21. Given differential equation can be written as

$$\frac{dy}{dx} = \frac{y}{x} + \frac{1}{\cos\left(\frac{y}{x}\right)}$$ $\frac{1}{2}$

Put $y = vx \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ 1

$\therefore\ v + x\dfrac{dv}{dx} = v + \dfrac{1}{\cos v}$ $\frac{1}{2}$

$\Rightarrow \int \cos v\, dv = \int \dfrac{dx}{x}$ $\frac{1}{2}$

$\Rightarrow \sin v = \log|x| + c$ 1

$\Rightarrow \sin\left(\dfrac{y}{x}\right) = \log|x| + c$ $\frac{1}{2}$

22. Put $x^2 = \cos 2\theta \Rightarrow \theta = \dfrac{1}{2}\cos^{-1} x^2$ 1

$$\text{LHS} = \tan^{-1}\left(\frac{\sqrt{2\cos^2\theta} + \sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta} - \sqrt{2\sin^2\theta}}\right)$$

$= \tan^{-1}\left(\dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}\right) = \tan^{-1}\left(\dfrac{1+\tan\theta}{1-\tan\theta}\right)$ $1\frac{1}{2}$

$= \tan^{-1}\left(\tan\left(\dfrac{\pi}{4} + \theta\right)\right) = \dfrac{\pi}{4} + \theta$ 1

$= \dfrac{\pi}{4} + \dfrac{1}{2}\cos^{-1}x^2,\ -1 < x < 1 = \text{RHS}$ $\frac{1}{2}$

23. $\overrightarrow{AB} = \hat{i} - 3\hat{j} + \hat{k},\ \overrightarrow{AC} = 3\hat{i} + 3\hat{j} - 4\hat{k}$ $\frac{1}{2} + \frac{1}{2}$

Area of $\Delta ABC = \dfrac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$

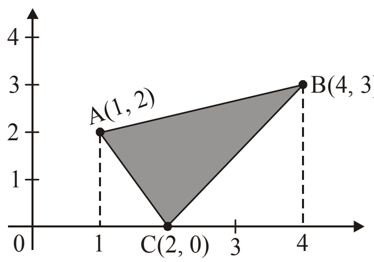
$= \dfrac{1}{2}$ magnitude of $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 3 & 3 & -4 \end{vmatrix}$ 1

$= \frac{1}{2}|9\hat{i}+7\hat{j}+12\hat{k}|$ 1

$= \frac{1}{2}\sqrt{81+49+144} = \frac{1}{2}\sqrt{274}$ sq.units 1

SECTION D

24. Correct figure 1



Equation of AB : $y = \frac{x+5}{3}$

Equation of BC: $y = \frac{3x}{2} - 3$

Equation of AC: $y = 4 - 2x$

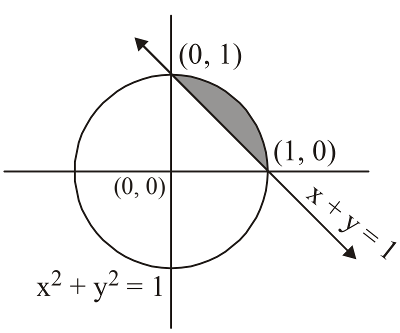
$1\frac{1}{2}$

$\therefore$ Area (A) $= \int_1^4 \frac{x+5}{3}dx - \int_1^2 (4-2x)dx - \int_2^4 \left(\frac{3x}{2} - 3\right)dx$ 1

$= \frac{1}{3}\left[\frac{x^2}{2}+5x\right]_1^4 - \left[4x - x^2\right]_1^2 - \left[\frac{3x^2}{4} - 3x\right]_2^4$ $1\frac{1}{2}$

$= \frac{15}{2} - 1 - 3 = \frac{7}{2}$ sq.units 1

OR



For correct figure

For correct shading

$1 + \frac{1}{2}$

$A = \int_0^1 \left(\sqrt{1-x^2} - (1-x)\right)dx$ $1\frac{1}{2}$

$= \left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right]_0^1 - \left[x - \frac{x^2}{2}\right]_0^1$ 2

$= \frac{1}{2}\sin^{-1}(1) - \frac{1}{2} = \left(\frac{\pi}{4} - \frac{1}{2}\right)$ sq.units 1

25. Let length of one piece be x m, then length of the other piece = (34 – x) m

$\therefore$ Side of square is $\frac{x}{4}$ m then width of rectangle will be $\frac{34-x}{6}$ m. 1

Now, Area (A) = $\left(\frac{x}{4}\right)^2 + 2\left(\frac{34-x}{6}\right)^2$ 1

$\Rightarrow \frac{dA}{dx} = \frac{x}{8} - \frac{1}{9}(34-x)$ 1

$\frac{dA}{dx} = 0 \Rightarrow x = 16$ 1

also, $\frac{d^2A}{dx^2} = \frac{1}{8} + \frac{1}{9} > 0$ 1

so, A is minimum when x = 16

$\therefore$ Lengths of the two pieces are 16 m and 18 m. 1

26. Let $x_1, x_2 \in A$ and $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3} \Rightarrow (x_1 - 2)(x_2 - 3) = (x_1 - 3)(x_2 - 2)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow x_1 = x_2$

Hence f is a one-one function 2

Let $y = \frac{x-2}{x-3}$ for $y \in R - \{1\}$

$\Rightarrow x = \frac{3y-2}{y-1}; y \neq 1$

$\therefore \forall y \in R - \{1\}, x \in R - \{3\}$

i.e. Range of f = co-domain of f.

Hence f is onto and so bijective. 2

Also, $f^{-1}(x) = \frac{3x-2}{x-1}; x \neq 1$ 1

Now, $f^{-1}(x) = 4 \Rightarrow \frac{3x-2}{x-1} = 4 \Rightarrow x = 2$ $\frac{1}{2}$

and $f^{-1}(7) = \frac{19}{6}$ $\frac{1}{2}$

OR

(i) (a, b) * (c, d) = (ad + bc, bd)

Now, (c, d) * (a, b) = (cb + da, db) = (ad + bc, bd) = (a, b) * (c, d)

⇒ * is commutative. 2

(ii) [(a, b) * (c, d)] * (e, f) = (ad + bc, bd) * (e, f) = (adf + bcf + bde, bdf)

(a,b) * [(c, d) * (e, f)] = (a, b) * (cf + de, df) = (adf + bcf + bde, bdf)

⇒ * is associative. 2

Let (e_1, e_2) be the identity element of A.

$\Rightarrow (a, b) * (e_1, e_2) = (a, b) = (e_1, e_2) * (a, b)$

$\Rightarrow (ae_2 + be_1, be_2) = (a, b) = (e_1b + e_2a, e_2b)$

$\Rightarrow ae_2 + be_1 = a$ and $be_2 = b \Rightarrow e_1 = 0, e_2 = 1$

⇒ (0, 1) is the identity on A. 2

27. Equation of the plane through the intersection of planes is

$(x + y + z - 1) + \lambda(2x + 3y + 4z - 5) = 0$ 1

$\Rightarrow (1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z - (1 + 5\lambda) = 0$ ....(i)

This plane is perpendicular to $x - y + z = 0$

$\therefore\ 1(1 + 2\lambda) - 1\ (1 + 3\lambda) + 1(1 + 4\lambda) = 0 \Rightarrow \lambda = \frac{-1}{3}$ 2

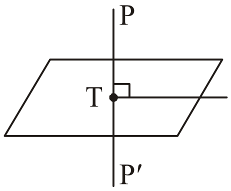
∴ Equation of plane is

$(x+y+z-1)-\frac{1}{3}(2x+3y+4z-5) = 0 \Rightarrow x - z + 2 = 0.$ 1

Vector form of plane is $\vec{r}\cdot(\hat{i}-\hat{k})+2 = 0$ 1

Yes, line lies on the plane as (–2, 3, 0) satisfies $\vec{r}\cdot(\hat{i}-\hat{k})+2 = 0$ and normal to plane is perpendicular to the given line as $1(5) + 0(4) - 1(5) = 0$ 1

OR



Let PT is perpendicular to given plane.

Let p.v. of T is $\vec{b}_1 = a\hat{i} + b\hat{j} + c\hat{k}$

$\therefore\ \overrightarrow{PT} = (a-1)\hat{i} + (b-3)\hat{j} + (c-4)\hat{k}$ 1

$\overrightarrow{PT} \parallel \vec{n}$ (nromal) $\therefore\ \dfrac{a-1}{-2} = \dfrac{b-3}{1} = \dfrac{c-4}{-1} = \lambda$

$\Rightarrow a = -2\lambda + 1,\ b = \lambda + 3,\ c = -\lambda + 4$ 1

$\therefore\ \vec{b}_1 = (-2\lambda+1)\hat{i} + (\lambda+3)\hat{j} + (-\lambda+4)\hat{k}$ $\frac{1}{2}$

$\vec{b}_1$ lies on plane

$\therefore\ [(-2\lambda+1)\hat{i} + (\lambda+3)\hat{j} + (-\lambda+4)\hat{k}]\cdot(-2\hat{i} + \hat{j} - \hat{k}) = 3$

$\Rightarrow \lambda = 1$ 1

$\therefore\ \vec{b}_1 = -\hat{i} + 4\hat{j} + 3\hat{k}$ $\frac{1}{2}$

Let p.v. of P′ is $\vec{c}_1 = x\hat{i} + y\hat{j} + z\hat{k}$

Using section formula, $\vec{c}_1 = -3\hat{i} + 5\hat{j} + 2\hat{k}$ 1

Also, $PP' = \sqrt{24}$ or $2\sqrt{6}$ 1

28. $|A| = 11 \neq 0$, A^{-1} will exist 1

$$\left.\begin{array}{lll} A_{11} = 7, & A_{21} = 2, & A_{31} = -6 \\ A_{12} = -2, & A_{22} = 1, & A_{32} = -3 \\ A_{13} = -4, & A_{23} = 2, & A_{33} = 5 \end{array}\right\}$$ 2

$$A^{-1} = \frac{1}{11}\begin{pmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{pmatrix}$$ $\frac{1}{2}$

Given sysem of equations can be written as AX = B, where

$$A = \begin{pmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{pmatrix}, X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, B = \begin{pmatrix} 10 \\ 8 \\ 7 \end{pmatrix}$$ $\frac{1}{2}$

$$X = A^{-1}B = \frac{1}{11}\begin{pmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{pmatrix}\begin{pmatrix} 10 \\ 8 \\ 7 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}$$ $1\frac{1}{2}$

$\therefore\ x = 4,\ y = -3,\ z = 1$ $\frac{1}{2}$

29. Given differential equation can be written as

$$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{e^{\tan^{-1} y}}{1+y^2}$$ 1

$$\text{I.F.} = e^{\int \frac{dy}{1+y^2}} = e^{\tan^{-1} y}$$ 1

Solution is given by

$$xe^{\tan^{-1} y} = \int \frac{e^{\tan^{-1} y}}{1+y^2} \times e^{\tan^{-1} y}\, dy = \int \frac{e^{2\tan^{-1} y}}{1+y^2}\, dy$$ 1

$$\Rightarrow\ xe^{\tan^{-1} y} = \frac{e^{2\tan^{-1} y}}{2} + c$$ $1\frac{1}{2}$

when $x = 1,\ y = 0 \Rightarrow c = \frac{1}{2}$ 1

$\therefore$ Solution is given by $xe^{\tan^{1} y} = \frac{1}{2}e^{2\tan^{-1} y} + \frac{1}{2}$ or $x = \frac{1}{2}(e^{\tan^{-1} y} + e^{-\tan^{-1} y})$ $\frac{1}{2}$