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# aATIE 2017 

ELECTRONICS \& COMMUNICATION ENGINEERING SESSION - 1

## GATE-2017

## Electronics \& Communication Engineering Questions and Detailed Solution Session-1

1. Three fair cubical dice are thrown simultaneously. The probability that all three dice have the same number on the faces showing up is (up to third decimal place)

Sol. (0.0278)
The total no. of outcomes, $\mathrm{n}(\mathrm{s})=6 \times 6 \times 6=$ 216

The favourable outcomes, $\mathrm{n}(\mathrm{E})$ are ( $1,1,1$ ), (2, 2, 2), $\qquad$ $(6,6,6)$
So,

$$
\mathrm{n}(\mathrm{E})=6
$$

$\therefore$ required probability $=\frac{\mathrm{n}(\mathrm{E})}{\mathrm{n}(\mathrm{s})}=\frac{6}{216}$

$$
=0.0278
$$

2. Consider the following statements about the linear dependence of the real valued functions $y_{1}=1, y_{2}=x$ and $y_{3}=x^{2}$, over the field of real numbers.
I. $y_{1}, y_{2}$ and $y_{3}$ are linearly independent on $-1 \leq x \leq 0$
II. $y_{1}, y_{2}$ and $y_{3}$ are linearly dependent on $0 \leq x \leq 1$
III. $y_{1}, y_{2}$ and $y_{3}$ are linearly dependent on $0 \leq x \leq 1$
IV. $y_{1}, y_{2}$ and $y_{3}$ are linearly independent on $-1 \leq x \leq 1$

Which on among the following is correct?
(a) Both I and II are true
(b) Both I and III are true
(c) Both II and IV are true
(d) Both III and IV are true

Sol. (a)
$\mathrm{y}_{1}=1, \mathrm{y}_{2}=\mathrm{x}, \mathrm{y}_{3}=\mathrm{x}^{2}$
Linear combination is given by
$\mathrm{ay}_{1}+\mathrm{by}_{2}+\mathrm{cy}_{3}=0, \mathrm{a}, \mathrm{b}, \mathrm{c} \in \mathrm{R}$
$\Rightarrow \mathrm{a}+\mathrm{bx}+\mathrm{cx}^{2}=0, \mathrm{a}, \mathrm{b}, \mathrm{c}, \in \mathrm{R}$
Case : 1
If $\mathrm{x} \in[0,1]$
$a+b x+c x^{2}=0$
At $\mathrm{x}=0 \Rightarrow \mathrm{a}=0$
At $\mathrm{x}=\frac{1}{2} ; \frac{\mathrm{b}}{2}+\frac{\mathrm{c}}{4}=0$
At $\mathrm{x}=1, \mathrm{~b}+\mathrm{c}=0$
From equation (1) and (2), we get

$$
\begin{aligned}
& \mathrm{b}=\mathrm{c}=0 \\
& \mathrm{a}=\mathrm{b}=\mathrm{c}=0
\end{aligned}
$$

$\Rightarrow \mathrm{y}_{1}, \mathrm{y}_{2}$ and $\mathrm{y}_{3}$ are linearly independent for $0 \leq x \leq 1$.

## Case II :

If $x \in[-1,0]$
$\mathrm{a}+\mathrm{bx}+\mathrm{cx}^{2}=0$
At $\mathrm{x}=0 \Rightarrow \mathrm{a}=0$
At $\mathrm{x}=-1 \Rightarrow-\mathrm{b}+\mathrm{c}=0$
At $\mathrm{x}=-\frac{1}{2} \Rightarrow-\frac{\mathrm{b}}{2}+\frac{\mathrm{c}}{4}=0$
From equation (4) and (5), we get

$$
\begin{array}{ll} 
& \mathrm{b}=\mathrm{c}=0 \\
\therefore & \mathrm{a}=\mathrm{b}=\mathrm{c}=0
\end{array}
$$

$\Rightarrow y_{1}, y_{2}$ and $y_{3}$ are linearly independent for $-1 \leq \mathrm{x} \leq 0$.
3. Consider a wireless communication link between a transmitter and a receiver located in free space with finite and stricitly positive capacity. If the effective areas of the transmitter and the receiver antenna and the distance between them are all doubled and everything else remains unchanged, the maximum capacity of the wireless link
(a) increases by a factor of 2
(b) decrease by a factor of 2
(c) remains unchanged
(d) decreases by a factor of $\sqrt{2}$

Sol. (c)
As per friis free space propagation equation

$$
\mathrm{P}_{\mathrm{r}}=\frac{\mathrm{P}_{\mathrm{t}} \cdot \mathrm{~A}_{\mathrm{er}} \cdot \mathrm{~A}_{\mathrm{et}}}{(\lambda \mathrm{R})^{2}}
$$

where,
$\mathrm{P}_{\mathrm{r}}=$ Received power
$P_{t}=$ Transmitted power
$\mathrm{A}_{\text {er }}=$ Aperture area of receiver
$\mathrm{A}_{\mathrm{et}}=$ Aperture area of transmitter
$\lambda=$ Wave length
$\mathrm{R}=$ Distance between receiver and transmitter
Now, if $\mathrm{A}_{\mathrm{er}}, \mathrm{A}_{\mathrm{et}}$ and R are doubled, then
$\mathrm{P}_{\mathrm{r}}^{\prime}=\frac{\mathrm{P}_{\mathrm{t}}\left(2 \mathrm{~A}_{\mathrm{er}}\right)\left(2 \mathrm{~A}_{\mathrm{et}}\right)}{(\lambda 2 \mathrm{R})^{2}}$
$P_{r}^{\prime}=\frac{P_{t} A_{e r} A_{e t}}{(\lambda R)^{2}}$

$$
P_{r}^{\prime}=P_{r}
$$

Hence, maximum capacity of the wireless link will be the same.
4. A periodic signal $\mathrm{x}(\mathrm{t})$ has a trigonometric Fourier series expansion

$$
x(t)=a_{0}+\sum_{n=1}^{\infty}\left(a_{n} \cos n \omega_{0} t+b_{n} \sin n \omega_{0} t\right)
$$

If $x(t)=-x(-t)=-x\left(t-\pi / \omega_{0}\right)$. We can counclude that
(a) $a_{n}$ are zero for all $n$ and $b_{n}$ are zero for $n$ even
(b) $a_{n}$ are zero for all $n$ and $b_{n}$ are zero for n odd
(c) $a_{n}$ are zero for $n$ even and $b_{n}$ are zero for $n$ odd
(d) $a_{n}$ are zero for $n$ odd and $b_{n}$ are zero for $n$ even

Sol. (a)
Given, $\mathrm{x}(\mathrm{t})=-\mathrm{x}(-\mathrm{t})=-\mathrm{x}\left(\mathrm{t}-\frac{\pi}{\omega_{0}}\right)$
The given signal has

1. odd function symmetry

$$
\mathrm{a}_{\mathrm{n}}=0
$$

2. Half-wave symmetry
odd harmonics
$\therefore \mathrm{a}_{\mathrm{n}}$ are zero for all n and $\mathrm{b}_{\mathrm{n}}$ are zero for n even.
3. A bar of Gallium Arsenide (GaAs) is doped with Silicon such that the Silicon atoms occupy Gallium and Arsenic sites in the GaAs crystal. Which one of the followng statement is true?
(a) Silicon atoms act as p-type dopants in Arsenic sites and n-type dopants in Gallium sites.
(b) Silicon atoms act as n-type dopants in Arsenic sites and p-type dopants in Gallium sites.
(c) Silicon atoms act as p-type dopants in Arsenic as well as Gallium sites.
(d) Silicon atoms act as n-type dopants in Arsenic as well as Gallium sites.

Sol. (a)
When GaAs is doped with Silicon, the two possibilities arise

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(i) Silicon can replace Gallium
(ii) Silicon can replace Arsenic

So, if the Silicon replaces Gallium then Silicon has one more electron. So, the extra electron is available for conduction. It will make it n-type.
One the other hand, Silicon replaces Arsenic it has one less electron and it will make it p-type.
6. The miller effect in the context of a common Emitter amplifier explains :
(a) an increase in the low-frequency cutoff frequency
(b) an increase in the high-frequency cutoff frequency
(c) a decrease in the low-frequency cutoff frequency
(d) a decrease in the high-frequency cutoff frequency

Sol. (d)
A common emitter amplifier has a capacitance between the collector and the base, and the gain of CE amplifier is negative, so the Miller effect will occur which reduce the high-frequency response of the amplifier.
7. $\mathrm{An}_{\mathrm{n}}{ }^{+}{ }_{-\mathrm{n}}$ Silicon device is fabricated with uniform and non-degenerate donor doping concentrations of $\mathrm{N}_{\mathrm{D} 1}=1 \times 10^{18} \mathrm{~cm}^{-3}$ and $\mathrm{N}_{\mathrm{D} 2}=1 \times 10^{15} \mathrm{~cm}^{-3}$ corresponding to the $\mathrm{n}^{+}$ and n regions respectively. At the operational temperature T , assume complete impurity ionization $\mathrm{kT} / \mathrm{q}=25 \mathrm{mV}$ and intrinsic carrier concentration to be $n_{i}=1 \times 10^{10} \mathrm{~cm}^{-3}$. What is the magnitude of the bulit-in potential of this device ?
(a) 0.748 V
(b) 0.460 V
(c) 0.288 V
(d) 0.173 V

## Sol. (a)

The junction build-in voltage

$$
\begin{aligned}
\mathrm{V}_{0} & =\mathrm{V}_{\mathrm{T}} \ln \left(\frac{\mathrm{~N}_{\mathrm{A}} \mathrm{~N}_{\mathrm{D}}}{\mathrm{n}_{\mathrm{i}}^{2}}\right) \\
& =25 \times 10^{-3} \ln \left[\frac{10^{18} \times 10^{15}}{\left(1 \times 10^{10}\right)^{2}}\right] \\
& =25 \times 10^{-3} \times 29.9336 \mathrm{~V} \\
& =0.748 \text { Volts. }
\end{aligned}
$$

8. Consider a stable system with transfer function

$$
G(s)=\frac{s^{p}+b_{1} s^{p-1}+\ldots .+b_{p}}{s^{q}+a_{1} s^{q-1}+\ldots .+a_{q}}
$$

where $\mathrm{b}_{1} \ldots \ldots . \mathrm{b}_{\mathrm{p}}$ and $\mathrm{a}_{1} \ldots . \mathrm{a}_{\mathrm{q}}$ are real valued constants. The slope of the Bode log magnitude curve of $G$ (s) converges to -60 $\mathrm{dB} /$ decade as $\omega \rightarrow \infty$. A possible pair of values for $p$ and $q$ is:
(a) $\mathrm{p}=0$ and $\mathrm{q}=3$
(b) $\mathrm{p}=1$ and $\mathrm{q}=7$
(c) $\mathrm{p}=2$ and $\mathrm{q}=3$
(d) $\mathrm{p}=3$ and $\mathrm{q}=5$

Sol. (a)
Final slope $=-60 \mathrm{~dB} /$ decade, which indicates that $\mathrm{p}-\mathrm{q}=3$. option (a) satisfies this condition.
9. For a narrow base PNP BJT, the excess minority carrier concentrations ( $\Delta \mathrm{n}_{\mathrm{E}}$ for emitter, $\Delta p_{B}$ for base, $\Delta n_{C}$ for collector) normalized to equilibrium minority carrier concentrations $\mathrm{n}_{\mathrm{E} 0}$ for emitter, $\mathrm{p}_{\mathrm{B} 0}$ for base, $\mathrm{n}_{\mathrm{C} 0}$ for collector) in the quasi-nautral emitter, base and collector regions are shown below. Which one of the following biasing modes is the transistor operating in?

$X$ and $Y$ axes are not to scale
(a) Forward active
(b) Saturation
(c) Inverse active
(d) Cutoff

Sol. (c)

where,
$\Delta \mathrm{n}_{\mathrm{E}}, \Delta \mathrm{P}_{\mathrm{B}}, \Delta \mathrm{n}_{\mathrm{C}}$ are excess minority carrier concentration of emitter, base and collector region respectively.
$\mathrm{n}_{\mathrm{EO}}, \mathrm{P}_{\mathrm{BO}}, \mathrm{n}_{\mathrm{CO}}$ are thermally generated minority carrier of emitter, base and collector region.
At collector - Base region ratio of excess minority carrier concentration to equilibrium minority carrier concentration is in order of $10^{5}$ (very high). This is possible when the junction is forward bias (injection).
At emitter - Base region, ratio of excess minority carrier concentration to equilibrium minority carrier concentration is in order of 1 (negligible). This is possible, when junction is reverse bias (no injection).
Hence, collector-base junction is forward biased and emitter base junction is reverse biased. So it in inverse active mode.
10. Consider the following statements for continous-time linear time invariant (LTI) system :
I. There is no bounded input bounded output (BIBO) stable system with a pole in the right half of the complex plane.
II. There is no casual and BIBO stable system with a pole in the right half of the complex plane.
Which one among the following is correct?
(a) Both I and II are true
(b) Both I and II are not true
(c) Only I is true
(d) Only II is true

Sol. (d)
For stable system, ROC of pole must contain $\mathrm{j} \omega$-axis. It is not compulsory that right sided signal is stable. So statement (i) is wrong.
There is non-causal system, its pole start from left side of s-plane and for BIBO stable system, its pole must contain $\mathrm{j} \omega$-axis and it go right side. So statement (ii) is correct.
11. The clock frequency of an 8085 microprossor is 5 MHz . If the time required to execute an instruction is $1.4 \mu \mathrm{~s}$, then the number of Tstates needed for executing the instruction is
(a) 1
(b) 6
(c) 7
(d) 8

Sol. (c)
The number of T-states needed for executing the instruction $=$ (Execution time of instruction) $\times$ (Clock frequency)

$$
\begin{aligned}
& =1.4 \times 10^{-6} \times 5 \times 10^{6} \\
& =7
\end{aligned}
$$

12. Consider the D-Latch shown in the figure which is transparent when its clock input CK is high and has zero propagation delay.

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In the figure, the clock signal CLK1 has a $50 \%$ duty cycle and CLK2 is a one-fifth period delayed version of CLK1. The duty cycle at the output of the latch in percentage is


Sol. (30)


The output will be high only when both CLK1 and CLK2 are high:
So, Duty cycle $=\frac{\text { ON time }}{\text { Time Period }}$

$$
\begin{aligned}
& =\frac{\left(\frac{\mathrm{T}_{\mathrm{CLK}}}{2}-\frac{\mathrm{T}_{\mathrm{CLK}}}{5}\right)}{\mathrm{T}_{\mathrm{CLK}}} \\
& =\frac{5-2}{10} \\
& =0.3 \text { i.e., } 30 \%
\end{aligned}
$$

13. Which one of the following statements about differential pulse code modulation (DPCM) is true?
(a) The sum of message signal with its prediction is quantized
(b) The message signal sample is directly quantized and its prediction is not used
(c) The difference of message signal sample and a random signal is quantized
(d) The difference of message signal with its prediction is quantized.

Sol. (d)
Differential Pulse Code Modulation (DPCM) is a procedure of converting an analog into a digital signal in which an analog signal is sampled and then the difference between the actual sample value and its predicted value (predicted value is based on previous sample or samples) is quantized and then encoded forming a digital value.
14. The rank of the matrix $\mathbf{M}=\left[\begin{array}{ccc}5 & 10 & 10 \\ 1 & 0 & 2 \\ 3 & 6 & 6\end{array}\right]$ is
(a) 0
(b) 1
(c) 2
(d) 3

Sol. (c)

$$
\begin{aligned}
M & =\left[\begin{array}{ccc}
5 & 10 & 10 \\
1 & 0 & 2 \\
3 & 6 & 6
\end{array}\right] \\
\operatorname{det}\{M\} & =5(0-12)-1(60-60)+3(20-0) \\
& =-60-0+60=0
\end{aligned}
$$

The one of the minor of matrix $M$ has non zero determinant value. e.g.

$$
M_{11}=\left[\begin{array}{ll}
0 & 2 \\
6 & 6
\end{array}\right] \text { and }\left|M_{11}\right|=-12
$$

Hence rank of M is 2 .
15. In the circuit shown the positive angular frequency $\omega$ (in radians per second) at which the magnitude of the phase difference between the volage $\mathrm{V}_{1}$ and $\mathrm{V}_{2}$ equals $\frac{\pi}{4}$ radians is $\qquad$


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Sol. (1)
The given circuit can be redrawn as


Where, $Z_{1}=1 \Omega$ and $Z_{2}=1+j \omega 1$
and $\quad \mathrm{V}_{1}=\frac{\mathrm{Z}_{1}}{\mathrm{Z}_{1}+\mathrm{Z}_{2}} \mathrm{~V}_{\mathrm{i}}$ and $\mathrm{V}_{2}=\frac{\mathrm{Z}_{2}}{\mathrm{Z}_{1}+\mathrm{Z}_{2}} \mathrm{~V}_{\mathrm{i}}$
Or $\quad V_{1}=\frac{1}{1+1+j \omega} V_{i}$ and
$V_{2}=\frac{1+\mathrm{j} \omega}{1+1+\mathrm{j} \omega} \mathrm{V}_{\mathrm{i}}$
The magnitude of phase difference between $\mathrm{V}_{1}$ and $\mathrm{V}_{2}$
$\left|-\tan ^{-1} \frac{\omega}{2}-\left(\tan ^{-1} \omega-\tan ^{-1} \frac{\omega}{2}\right)\right|=\frac{\pi}{2}$
Or $\quad \tan ^{-1}=\frac{\pi}{4}$
Or $\quad \omega=1 \mathrm{rad} / \mathrm{sec}$.
16. The voltage of an electromagnetic wave propagating in a coaxial cable with uniform characteristic impedance is
$\mathrm{V}(l)=\mathrm{e}^{-\gamma l+\mathrm{j} \omega t}$ volts, where $l$ is the distance along the length of the cable in metres. $\gamma=(0.1+\mathrm{j} 40) \mathrm{m}^{-1}$ is the complex propagation constant and $\omega=2 \pi \times 10^{9} \mathrm{rad} / \mathrm{s}$ is the angular frequency. The absolute value of the attenuation in $\mathrm{dB} /$ metre is $\qquad$
Sol. (0.8686)
Given, $\gamma=(0.1+\mathrm{j} 40) \mathrm{m}^{-1}$
The propagation constant, $\gamma=\alpha+j \beta$
Where, $\quad \alpha=$ attenuation constant $\beta=$ phase constant

$$
\begin{aligned}
\therefore \quad \alpha & =0.1 \mathrm{~Np} / \mathrm{m} \\
& =8.686 \times 0.1 \mathrm{~dB} / \mathrm{m} \\
& =0.8686 \mathrm{~dB} / \mathrm{m}
\end{aligned}
$$

17. A good transconductance amplifier should have
(a) high input resistance and low output resistance
(b) low input resistance and high output resistance
(c) high input and output resistances
(d) low input and output resistances

Sol. (c)
For a transconductance amplifier
Input resistance, $R_{i}^{\prime}=\frac{R_{i}}{1+\beta A}$
and output resitance,

$$
\mathrm{R}_{0}^{\prime}=\frac{\mathrm{R}_{0}}{1+\mathrm{A} \beta}
$$

$\therefore$ for an ideal or good transconductance amplifier (where $A \beta \approx-1$ )
$\mathrm{R}_{\mathrm{i}}^{\prime} \rightarrow \infty$ and $\mathrm{R}_{0}^{\prime} \rightarrow \infty$
18. The open loop transfer function

$$
G(s)=\frac{(s+1)}{s^{p}(s+2)(s+3)}
$$

where $p$ is an integer is connected in unity feedback configuration as shown in the figure


Given that the steady state error is zero for unit step input and is 6 for unit ramp input. The value of the parameter $p$ is $\qquad$
Sol. (1)
Steady state error,

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$$
\mathrm{e}_{\mathrm{ss}}=\lim _{\mathrm{s} \rightarrow 0} \mathrm{~s} \cdot \mathrm{R}(\mathrm{~s}) \frac{1}{1+\mathrm{G}(\mathrm{~s}) \mathrm{H}(\mathrm{~s})}
$$

For unit ramp input

$$
\begin{array}{ll} 
& 6=\lim _{\mathrm{s} \rightarrow 0} \mathrm{~s} \cdot \frac{1}{\mathrm{~s}^{2}} \cdot \frac{1}{1+\frac{(\mathrm{s}+1)}{\mathrm{s}^{\mathrm{p}}(\mathrm{~s}+2)(\mathrm{s}+3)}} \\
\text { Or } & 6=\lim _{\mathrm{s} \rightarrow 0} \frac{1}{\mathrm{~s}+\frac{(\mathrm{s}+1)}{\mathrm{s}^{\mathrm{p}-1}(\mathrm{~s}+2)(\mathrm{s}+3)}} \\
\text { Or } & 6=\lim _{\mathrm{s} \rightarrow 0} \frac{1}{0+\frac{1}{6 \mathrm{~s}^{\mathrm{p}-1}}} \\
\therefore & \mathrm{p}
\end{array}
$$

No need to verify for unit step input.
19. In a digital communication system, the overall pulse shape $p(t)$ at the receiver before the sampler has the fourier transform $P(f)$. If the symbols are transmitted at the rate of 2000 symbols per second, for which of the following cases is the inter symbol interference zero?
(a)

(b)

(c)

(d)


Sol. 19 (b)
Condition for zero inter symbol interference

$$
\frac{1}{\mathrm{~T}_{\mathrm{S}}} \sum_{\mathrm{k}=-\infty}^{\infty} \mathrm{P}\left(\mathrm{f}-\frac{\mathrm{k}}{\mathrm{~T}_{\mathrm{s}}}\right)=1 \forall \mathrm{f}
$$

$\mathrm{P}(\mathrm{f})$ is fourier transform of $\mathrm{P}(\mathrm{t})$

$$
\begin{aligned}
\mathrm{f}_{\mathrm{s}} & =2000 \text { symbols } / \mathrm{sec} \\
& =2 \mathrm{k} \text { symbols } / \mathrm{sec}
\end{aligned}
$$

The above condition is satisfied by only option (b).

20. For the operational amplifier circuit shown, the output saturation voltages are $\pm 15 \mathrm{~V}$. The upper and lower threshold voltages for the circuit are respectively.

(a) +5 V and -5 V
(b) +7 V and -3 V
(c) -3 V and +7 V
(d) +3 V and -3 V

## Sol. (b)

For the given circuit, the upper and lower threshold voltage are given by

$$
\mathrm{UTP}=\frac{\mathrm{R}_{2}}{\mathrm{R}_{1}+\mathrm{R}_{2}} \cdot \mathrm{~V}_{\text {sat }}+\frac{\mathrm{R}_{1}}{\mathrm{R}_{1}+\mathrm{R}_{2}} \mathrm{~V}_{\mathrm{r}}
$$

and $\quad$ LTP $=-\frac{R_{2}}{R_{1}+R_{2}} V_{\text {sat }}+\frac{R_{1}}{R_{1}+R_{2}} V_{r}$
$\underset{\mathrm{V}_{\text {sat }}}{\text { Here, }}=15 \mathrm{~V} \mathrm{R}_{1}=10 \mathrm{k} \Omega, \mathrm{R}_{2}=5 \mathrm{k} \Omega, \mathrm{V}_{\mathrm{r}}=3 \mathrm{~V}$ and

$$
\begin{aligned}
\therefore \quad \mathrm{UTP} & =\frac{5}{5+10} \times 15+\frac{10}{5+10} \times 3 \\
& =5+2=7 \mathrm{~V}
\end{aligned}
$$

and

$$
\begin{aligned}
\text { LTP } & =-\frac{5}{5+10} \times 15+\frac{10}{5+10} \times 3 \\
& =-5+2=-3 \mathrm{~V}
\end{aligned}
$$

21. Consider the $5 \times 5$ matrix

$$
A=\left[\begin{array}{lllll}
1 & 2 & 3 & 4 & 5 \\
5 & 1 & 2 & 3 & 4 \\
4 & 5 & 1 & 2 & 3 \\
3 & 4 & 5 & 1 & 2 \\
2 & 3 & 4 & 5 & 1
\end{array}\right]
$$

It is given that A has only one real eigenvalue. Then the real eigenvalue of $A$ is
(a) -25
(b) 0
(c) 15
(d) 25

Sol. (c)
Characteristic equation is $|\mathrm{A}-\lambda \mathrm{I}|=0$
$\left[\begin{array}{ccccc}1-\lambda & 2 & 3 & 4 & 5 \\ 5 & 1-\lambda & 2 & 3 & 4 \\ 4 & 5 & 1-\lambda & 2 & 3 \\ 3 & 4 & 5 & 1-\lambda & 2 \\ 2 & 3 & 4 & 5 & 1-\lambda\end{array}\right]$

For real eigen value, sum of either one row or coloumn must be zero.
$\Rightarrow 1-\lambda+2+3+4+5=0, \therefore \lambda=15$
22. In the latch circuit shown, the NAND gates have non-zero but unequal propagation delays. The present input condition is $\mathrm{P}=$ $\mathrm{Q}=$ ' 0 '. if the input condition is changed simultaneously to $\mathrm{P}=\mathrm{Q}=$ ' 1 ' the outputs X and Y are

(a) $X=1, Y=1$
(b) either $\mathrm{X}=1, \mathrm{Y}={ }^{\prime} 0$ ' or $\mathrm{X}={ }^{\prime} 0$ ', $\mathrm{Y}=1$
(c) either $\mathrm{X}=1, \mathrm{Y}={ }^{\prime} 1$ ' or $\mathrm{X}={ }^{\prime} 0$ ', $\mathrm{Y}={ }^{\prime} 0$ '
(d) $\mathrm{X}={ }^{\prime} 0$ ', $\mathrm{Y}={ }^{\prime} 0^{\prime}$

Sol. (b)


NAND Gate

| A | B | Y |
| :--- | :--- | :--- |
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

Given,
Present input $\mathrm{P}=\mathrm{Q}=0$
then,

$$
\mathrm{X}=\mathrm{Y}=1
$$

Now, $\quad P=Q=1$,
then output X and Y will change as $\mathrm{X}=0$, $\mathrm{Y}=1$ or, $\mathrm{X}=1, \mathrm{Y}=0$ as per the propagation

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delay of NAND Gate.
If $\mathrm{P}=\mathrm{Q}=1$, then $\mathrm{X}=0, \mathrm{Y}=1$ if propagation delay of NAND gate-I is less than NAND Gate-II, because both the input of NAND Gate-I are 1.

If $\mathrm{P}=\mathrm{Q}=1$, then $\mathrm{X}=1, \mathrm{Y}=0$ if propagation delay of NAND Gate-II is less than NAND Gate-I because both the input of NAND Gate-II are 1.

So, option (b)
23. Consider a single input single output discrete-time system with $\mathrm{x}[\mathrm{n}]$ as input and $y[n]$ as output, where the two are related as

$$
y(n)=\left\{\begin{array}{cc}
n \mid x[n], & \text { for } 0 \leq n \leq 10 \\
x[n]-x[n-1], & \text { Otherwise }
\end{array}\right.
$$

which one of the following statements is true about the system.
(a) It is causal and stable
(b) It is causal but not stable
(c) It is not causal but stable
(d) It is neither causal not stable

Sol. (a)
$y[n]= \begin{cases}n|\times[n]| & \text { for } 0 \leq n \leq 10 \\ x[n]-x[n-1] & \text { otherwise }\end{cases}$

- Present output depends on present input and past input, so it is a causal system.
- For a bounded input, bounded output yields, so it is a stable system.

24. Which of the following can be the pole-zero configuration of a phase-lag controller (lag compensation)?



Sol. (a)
For phase-lag controller, the transfer function is

$$
G(s)=\frac{1+s T}{1+s \beta T}
$$

Where, $\quad \beta>1$
$\therefore$ The pole-zero configuration will be

25. Let $\left(X_{1}, X_{2}\right)$ be independent random variables, $X_{1}$ has mean 0 and variance 1 , while $X_{2}$ has mean 1 and variance 4 . The mutual information $\mathrm{I}\left(\mathrm{X}_{1} ; \mathrm{X}_{2}\right)$ between $\mathrm{X}_{1}$ and $\mathrm{X}_{2}$ in bits is. $\qquad$
Sol. (0)
Mutual information of two discrete random variable $\mathrm{X}_{1}$ and $\mathrm{X}_{2}$ can be defined as:
$\mathrm{I}\left(\mathrm{X}_{1}, \mathrm{X}_{2}\right)$


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$$
=\sum_{\mathrm{x}_{2} \in \mathrm{X}_{2}} \sum_{\mathrm{x}_{1} \in \mathrm{X}_{1}} \mathrm{P}\left(\mathrm{x}_{1}, \mathrm{x}_{2}\right) \log \left[\frac{\mathrm{P}\left(\mathrm{x}_{1}, \mathrm{x}_{2}\right)}{\mathrm{P}\left(\mathrm{x}_{1}\right) \mathrm{P}\left(\mathrm{x}_{2}\right)}\right]
$$

If $\mathrm{X}_{1}$ and $\mathrm{X}_{2}$ are independent then $\mathrm{P}\left(\mathrm{x}_{1}, \mathrm{x}_{2}\right)$ $=\mathrm{P}\left(\mathrm{x}_{1}\right) \mathrm{P}\left(\mathrm{x}_{2}\right)$

$$
\begin{gathered}
\log \left[\frac{\mathrm{P}\left(\mathrm{x}_{1}, \mathrm{x}_{2}\right)}{\mathrm{P}\left(\mathrm{x}_{1}\right) \mathrm{P}\left(\mathrm{x}_{2}\right)}\right]=\log \left[\frac{\mathrm{P}\left(\mathrm{x}_{1}\right) \mathrm{P}\left(\mathrm{x}_{2}\right)}{\mathrm{P}\left(\mathrm{x}_{1}\right) \mathrm{P}\left(\mathrm{x}_{2}\right)}\right] \\
=\log 1=0 \\
\mathrm{I}\left(\mathrm{X}_{1}, \mathrm{X}_{2}\right)=0
\end{gathered}
$$

26. In binary frequency shift keying (FSK), the given signal wavefonus are
$\mathrm{u}_{0}(\mathrm{t})=5 \cos (20000 \pi \mathrm{t}) ; 0 \leq \mathrm{t} \leq \mathrm{T}$, and
$\mathrm{u}_{1}(\mathrm{t})=5 \cos (22000 \pi \mathrm{t}) ; 0 \leq \mathrm{t} \leq \mathrm{T}$,
where $T$ is the bit-duration interval and $t$ is in seconds. Both $u_{0}(t)$ and $u_{1}(t)$ are zero outside the interval $0 \leq \mathrm{t} \leq \mathrm{T}$. With a matched filter (correlator) based receiver, the smallest positive value of $T$ (in milli seconds) required to have $u_{0}(t)$ and $u_{1}(t)$ uncorrelated is
(a) 0.25 ms
(b) 0.5 ms
(c) 0.75 ms
(d) 1.0 ms

Sol. (b)

$$
\begin{array}{r}
\mathrm{u}_{0}(\mathrm{t})=5 \cos (20000 \pi \mathrm{t}) ; 0 \leq+\leq \mathrm{T} \\
\mathrm{u}_{1}(\mathrm{t})=5 \cos (22000 \pi \mathrm{t}) ; 0 \leq+\leq \mathrm{T} \\
\mathrm{f}_{1}=11000 \mathrm{~Hz} \\
\mathrm{f}_{2}=10000 \mathrm{~Hz}
\end{array}
$$

For FSK wave form to be uncorrelated.

$$
\begin{aligned}
& \mathrm{f}_{1}-\mathrm{f}_{2}=\frac{\mathrm{nR}}{\mathrm{~b}} \\
& 2
\end{aligned} \mathrm{n}=1,2,3, \ldots .
$$

$\therefore$ minimum value of $\mathrm{n}=1$,

$$
\mathrm{T}_{\mathrm{b}(\min .)}=\frac{1}{\mathrm{R}_{\mathrm{b}(\max .)}}=0.5 \mathrm{~ms}
$$

27. Two discrete-time signals $x[n]$ and $h[n]$ are both non-zero only for $\mathrm{n}=0,1,2$ and are zero otherwise. It is given that
$\mathrm{x}[0]=1, \mathrm{x}[1]=2, \mathrm{x}[2]=1, \mathrm{~h}(0)=1$
Let $y[n]$ be the linear convolution of $x[n]$ and $\mathrm{h}[\mathrm{n}]$. Given that $\mathrm{y}[1]=3$ and $\mathrm{y}[2]=4$, the value of the expression $(10 y[3]+y[4])$ is.....
Sol. (31)
Given,

$$
\begin{aligned}
\mathrm{x}[\mathrm{n}]= & {[1,2,1] } \\
& \uparrow \\
\mathrm{h}[\mathrm{n}] & =[1, a, b]
\end{aligned}
$$

and

We know that,

$$
\begin{aligned}
& \mathrm{y}[\mathrm{n}]=\mathrm{x}[\mathrm{n}] * \mathrm{~h}[0] \\
& =\begin{array}{l|lll} 
& 1 & 2 & 1 \\
\hline 1 & 1 & 2 & 1 \\
\mathrm{a} & \mathrm{a} & 2 \mathrm{a} & \mathrm{a} \\
\mathrm{~b} & \mathrm{~b} & 2 \mathrm{~b} & \mathrm{~b} \\
\hline
\end{array} \\
& y[n]=[1,(2+a),(2 a+b+1),(a+2 b), b] \\
& \uparrow
\end{aligned}
$$

Given,

$$
\begin{aligned}
& \mathrm{y}[1]=3=2+\mathrm{a} \\
& \text { Or } \quad \mathrm{a}=1 \\
& \\
& \text { Or } \quad \begin{aligned}
\mathrm{y}[2] & =4=2 \mathrm{a}+\mathrm{b}+1 \\
\mathrm{~b} & =1
\end{aligned} \\
& \mathrm{y}[3]=\mathrm{a}+2 \mathrm{~b}=1+2=3 \\
& \mathrm{y}[4]=\mathrm{b}=1 \\
& \therefore(10 \mathrm{y}[3]+\mathrm{y}[4]=10 \times 3+1 \\
&=31
\end{aligned}
$$

28. The figure shows an RLC circuit excited by the sumsoidal voltage $100 \cos (3 \mathrm{t})$ Volts, where $t$ is in seconds. The ratio $\frac{\text { amplitude of } v_{2}}{\text { amplitude of } v_{1}}$ is $\qquad$


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## Sol. (0.3846)

The given circuit can be redrawn as


From voltage division rule

$$
\mathrm{V}_{1}=\frac{\mathrm{Z}_{1}}{\mathrm{Z}_{1}+\mathrm{Z}_{2}} \mathrm{~V}_{\mathrm{in}}
$$

and

$$
\mathrm{V}_{2}=\frac{\mathrm{Z}_{2}}{\mathrm{Z}_{1}+\mathrm{Z}_{2}} \mathrm{~V}_{\mathrm{in}}
$$

$$
\therefore \quad \frac{\left|\mathrm{V}_{1}\right|}{\left|\mathrm{V}_{2}\right|}=\frac{\left|\mathrm{Z}_{1}\right|}{\left|\mathrm{Z}_{2}\right|}=\frac{\sqrt{16+9}}{\sqrt{25+144}}
$$

$$
=\frac{5}{13}=0.3846
$$

29. In the circuit shown the voltage $\mathrm{V}_{\text {IN }}(\mathrm{t})$ is described by

$$
V_{\text {IN }}(t)=\left\{\begin{array}{cc}
0, & \text { for } t \leq 0 \\
15 \text { Volts, }, & \text { for } t \geq 0
\end{array}\right.
$$

Where $t$ is in seconds. The time (in seconds) at which the current I in the circuit will reach the value 2 Amperes is $\qquad$


Sol. (0.0954)
Equivalent inductance, $\mathrm{L}_{\mathrm{eq}}=\frac{2 \times 1}{2+1}=\frac{2}{3} \mathrm{H}$ and equivalent resistance, $\mathrm{R}_{\mathrm{eq}}=1 \Omega$
$\therefore$ Time constant, $\tau=\frac{\mathrm{L}}{\mathrm{R}}=\frac{2}{3}$ sec. and current at time, t ,

$$
\begin{aligned}
\mathrm{i}(\mathrm{t}) & =\frac{V_{\text {in }}}{R}\left[1-\mathrm{e}^{-\mathrm{t} / \tau}\right] \\
& =\frac{15}{1}\left[1-\mathrm{e}^{-3 \mathrm{t} / 2}\right] \\
& =15\left(1-\mathrm{e}^{-1.5 \mathrm{t}}\right) \\
\mathrm{i}\left(\mathrm{t}_{0}\right) & =2=15\left(1-\mathrm{e}^{-1.5 \mathrm{t}_{0}}\right) \\
\text { Or } \mathrm{e}^{-1.5 \mathrm{t}_{0}} & =0.8667 \\
\therefore \quad \mathrm{t}_{0} & =0.0954 \mathrm{sec} .
\end{aligned}
$$

30. As shown, two Silicon (Si) abrupt p-n junction diodes are fabricated with uniform donor doping concentrations of $\mathrm{N}_{\mathrm{D} 1}=10^{14}$ $\mathrm{cm}^{-3}$ and $\mathrm{N}_{\mathrm{D} 2}=10^{16} \mathrm{~cm}^{-3}$ in the n-regions of the diodes, and uniform acceptor doping concentration of $\mathrm{N}_{\mathrm{A} 1}=10^{14} \mathrm{~cm}^{-3}$ and $\mathrm{N}_{\mathrm{A} 2}=$ $10^{16} \mathrm{~cm}^{-3}$ in the p-regions of the diodes, respectively. Assuming that the reverse bias voltage is in built-in potentials of the diodes, the ratio $\mathrm{C}_{2} / \mathrm{C}_{1}$ of then reverse bias capacitances for the same applied reverse bias is $\qquad$

$\mathrm{C}_{1}$
Diode 1

$\mathrm{C}_{2}$
Diode 2

Sol. (10)
Given that:
Donor doping concentration,

$$
\begin{aligned}
& \mathrm{N}_{\mathrm{D}_{1}}=10^{14} \mathrm{~cm}^{-3} \\
& \mathrm{~N}_{\mathrm{D}_{2}}=10^{16} \mathrm{~cm}^{-3}
\end{aligned}
$$

Acceptor doping concentration,

$$
\mathrm{N}_{\mathrm{A}_{1}}=10^{14} \mathrm{~cm}^{-3}
$$

$$
\mathrm{N}_{\mathrm{A}_{2}}=10^{16} \mathrm{~cm}^{-3}
$$

Since,

$$
\begin{array}{lr}
\text { Since, } & V_{0} \ll V_{R} \\
\Rightarrow & V_{0}+V_{R}=V_{R}
\end{array}
$$

$$
\begin{aligned}
\mathrm{C} & =\frac{\in \mathrm{A}}{\mathrm{w}} \\
\frac{\mathrm{C}_{2}}{\mathrm{C}_{1}} & =\frac{\frac{\in \mathrm{A}}{\mathrm{w}_{2}}}{\frac{\in \mathrm{~A}}{\mathrm{w}_{1}}}=\frac{\mathrm{w}_{1}}{\mathrm{w}_{2}}
\end{aligned}
$$

$$
=\frac{\sqrt{\frac{2 \in \mathrm{~V}_{\mathrm{R}}}{\mathrm{q}}\left[\frac{1}{\mathrm{~N}_{\mathrm{A}_{1}}}+\frac{1}{\mathrm{~N}_{\mathrm{D}_{1}}}\right]}}{\sqrt{\frac{2 \in \mathrm{~V}_{\mathrm{R}}}{\mathrm{q}}\left[\frac{1}{\mathrm{~N}_{\mathrm{A}_{2}}}+\frac{1}{\mathrm{~N}_{\mathrm{D}_{2}}}\right]}}
$$

$$
=\sqrt{ } \sqrt{\frac{\frac{\mathrm{N}_{\mathrm{D}_{1}}+\mathrm{N}_{\mathrm{A}_{1}}}{\mathrm{~N}_{\mathrm{A}_{1}} \cdot \mathrm{~N}_{\mathrm{D}_{1}}}}{\frac{\mathrm{~N}_{\mathrm{D}_{2}}+\mathrm{N}_{\mathrm{A}_{2}}}{\mathrm{~N}_{\mathrm{D}_{2}} \cdot \mathrm{~N}_{\mathrm{A}_{2}}}}}
$$

$$
\begin{aligned}
& =\sqrt{\frac{\frac{10^{14}+10^{14}}{\frac{10^{14} \cdot 10^{14}}{10^{16}+10^{16}}} 10^{16} \cdot 10^{16}}{}}=\sqrt{\frac{\frac{2 \times 10^{14}}{\frac{10^{28}}{2\left(10^{36}\right)}}}{10^{32}}}
\end{aligned}
$$

$$
=\sqrt{\frac{2 \times 10^{14}}{10^{28}} \times \frac{10^{32}}{2\left(10^{16}\right)}}
$$

$$
=\sqrt{\frac{10^{46}}{10^{44}}}
$$

$$
=\sqrt{10^{2}}=10
$$

31. Let $\mathrm{x}(\mathrm{t})$ be a continuous time periodic signal with fundamental period $\mathrm{T}=1$ seconds. Let $\left\{\mathrm{a}_{\mathrm{k}}\right\}$ be the complex Fourier series
coefficients of $\mathrm{x}(\mathrm{t})$. Where k is integer valued. Consider the following statements about $\mathrm{x}(3 \mathrm{t})$.
I. The complex Fourier series coefficients of $x(3 t)$ are $\left\{a_{k}\right\}$ where $k$ is integer valued
II. The complex Fourier series coefficients of $x(3 t)$ are $\left\{3 \mathrm{a}_{\mathrm{k}}\right\}$ where k is integer valued
III. The fundamental angular frequency of $\mathrm{x}(3 \mathrm{t})$ is $6 \pi \mathrm{rad} / \mathrm{s}$
For the three statements above, which one of the following is correct?
(a) Only II and III are true
(b) Only I and III are true
(c) Only III is true
(d) Only I is true

Sol. (b)
$\mathrm{x}(\mathrm{t})$ be a continuous time periodic signal, Fundamental time period ( T ) $=1$

So, $\quad \omega_{0}=2 \pi \mathrm{rad} / \mathrm{sec}$.
We know, $\mathrm{x}(\mathrm{at}) \rightarrow \mathrm{a}_{\mathrm{k}}$, $\mathrm{a} \omega_{0}$
So, when $\mathrm{x}(\mathrm{t})$ is compressed by 3 , frequency will expand by 3 .

$$
\mathrm{x}(3 \mathrm{t}) \rightarrow \mathrm{a}_{\mathrm{k}}, 3 \omega_{0}=6 \pi
$$

So, both statement I and II are correct.
32. For the DC analysis of the Common-Emitter amplifier shown, neglect the base current and assume that the emitter and collector currents are equal. Given that $\mathrm{V}_{\mathrm{T}}=25 \mathrm{mV}$, $\mathrm{V}_{\mathrm{BE}}=0.7 \mathrm{~V}$, and the BJT output resistance $\mathrm{r}_{0}$ is practically infinite. Under these conditons the midband voltage gain magnitude $A v=\left|\mathrm{v}_{0} / \mathrm{v}_{1}\right| \mathrm{V} / \mathrm{V}$, is. $\qquad$


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Sol. (128)
DC analysis: all capacitor are open circuit. Now, Redrawing circuit,


$$
\begin{aligned}
& \mathrm{V}_{\mathrm{th}}=\frac{12 \times 47}{73+47}=4.7 \mathrm{~V} \\
& \mathrm{R}_{\mathrm{th}}=\frac{73 \times 47}{47+73}=28.59 \mathrm{k} \Omega
\end{aligned}
$$



Fig.: DC Analysis

$$
\mathrm{I}_{\mathrm{B}}=0(\text { Given })
$$

then

$$
\mathrm{I}_{\mathrm{C}}=\mathrm{I}_{\mathrm{E}}
$$

Applying KVL in loop1

$$
\begin{aligned}
& 4.7-28.59 \mathrm{~K} \cdot \mathrm{I}_{\mathrm{B}}-0.7-2 \mathrm{~K} \cdot \mathrm{I}_{\mathrm{E}}=0 \\
& \Rightarrow 4.7-0.7-2 \mathrm{~K} \cdot \mathrm{I}_{\mathrm{E}}=0 \\
& \mathrm{I}_{\mathrm{E}}=\frac{4.7-0.7}{2 \mathrm{~K}} \\
& \\
& =\frac{4}{2 \mathrm{~K}}=2 \mathrm{mAmp} \\
& \therefore \quad \begin{aligned}
\mathrm{I}_{\mathrm{c}} & =\mathrm{I}_{\mathrm{E}} \\
& =2 \mathrm{~m} \mathrm{Amp} \\
\mathrm{~g}_{\mathrm{m}} & =\frac{\mathrm{I}_{\mathrm{c}}}{\mathrm{~V}_{\mathrm{T}}}
\end{aligned}
\end{aligned}
$$

$$
=\frac{2 \mathrm{~m}}{25 \mathrm{~m}}=80 \mathrm{~V}
$$

Now from AC analysis,

$$
\begin{aligned}
\mathrm{A}_{\mathrm{v}} & =-\mathrm{g}_{\mathrm{m}} \cdot \mathrm{R}_{\mathrm{L}}^{\prime} \\
& =-80(2| | 8) \\
& =-80\left(\frac{2 \times 8}{2+8}\right) \\
& =\frac{-80 \times 16}{10}=-128 \\
\left|A_{\mathrm{v}}\right| & =128
\end{aligned}
$$

33. A continuous time signal $x(t)=4$ $\cos (200 \pi \mathrm{t})+8, \cos (400 \pi \mathrm{t})$, where t is in seconds, is the input to a linear time invariant (LTI) filter with the impulse response
$h(t)=\left\{\begin{array}{cc}\frac{2 \sin (300 \pi t)}{\pi t} & t \neq 0 \\ 600, & t=0\end{array}\right.$
Let $y(t)$ be the output of this filter. The maximum value of $|y(t)|$ is $\qquad$
Sol. (8)
Given continuous time signal,
$\mathrm{X}(\mathrm{t})=4 \cos (200 \pi \mathrm{t})+8 \cos (400 \pi \mathrm{t})$
Impulse response, $h(t)=$
$\begin{cases}\frac{2 \sin (300 \pi \mathrm{t})}{\pi \mathrm{t}} & ; \mathrm{t} \neq 0 \\ 600 & ; \mathrm{t}=0\end{cases}$
So, its fourier transform $\rightarrow \mathrm{H}(\omega)$


Given input signal frequencies are 100, 200 Hz .
So, the o/p signal

$$
\begin{aligned}
\mathrm{y}(\mathrm{t}) & =2 \times 4 \cos (200 \pi \mathrm{t}) \\
& =8 \cos (200 \pi \mathrm{t})
\end{aligned}
$$

So, maximum value $|y(t)|=8$

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34. Which one of the following is the general solution of the first order differential equation

$$
\frac{d y}{d x}=(x+y=1)^{2},
$$

where $\mathrm{x}, \mathrm{y}$ are real?
(a) $y=1+x+\tan ^{-1}(x+c)$, where $c$ is a constant
(b) $\mathrm{y}=1+\mathrm{x}+\tan (\mathrm{x}+\mathrm{c})$, where c is a constant
(c) $\mathrm{y}=1-\mathrm{x}+\tan ^{-1}(\mathrm{x}+\mathrm{c})$, where c is a constant
(d) $\mathrm{y}=1-\mathrm{x}+\tan (\mathrm{x}+\mathrm{c})$, where c is a constant

Sol. (c)
Given,

$$
\begin{equation*}
\frac{d y}{d x}=(x+y-1)^{2} \tag{1}
\end{equation*}
$$

Let, $\mathrm{x}+\mathrm{y}-1=\mathrm{t}$
Then, $1+\frac{d y}{d x}-0=\frac{d t}{d x}$
Or $\quad \frac{d y}{d x}=\frac{d t}{d x}-1$
From equation (1)

$$
\frac{\mathrm{dt}}{\mathrm{dx}}-1=\mathrm{t}^{2}
$$

Or $\frac{\mathrm{dt}}{1+\mathrm{t}^{2}}=\mathrm{dx}$
Or $\int \frac{d t}{1+t^{2}}=\int d x$
Or $\tan ^{-1} t=x+c$
Or $\quad t=\tan (x+c)$
Or $\quad x+y-1=\tan (x+c)$
Or $\quad y=1-x+\tan (x+c)$
35. The amplifier circuit shown in the figure is implemented using a compensated operational amplifier ( $\mathrm{Op}-\mathrm{amp}$ ) and has an open-loop voltage gain $\mathrm{A}_{0}=10^{5} \mathrm{~V} / \mathrm{V}$ and an open-loop cut-off frequency, $\mathrm{f}_{\mathrm{c}}=8 \mathrm{~Hz}$ The voltage gain of the amplifier at 15 kHz in V/V is


Sol. (44.4)
In the given circuit
Feed back factor,

$$
\begin{aligned}
\beta & =\frac{\mathrm{R}_{1}}{\mathrm{R}_{1}+\mathrm{R}_{2}} \\
& =\frac{1}{1+79} \\
& =\frac{1}{80}
\end{aligned}
$$

Closeloop gain, $A_{o_{f}}=\frac{A_{0}}{1+A_{0} \beta} \cong 80$

$$
\begin{aligned}
\mathrm{f}_{\mathrm{c}}{ }^{\prime} & =\mathrm{f}_{\mathrm{c}}\left(1+\mathrm{A}_{0} \beta\right) \\
& =8\left(1+\frac{10^{5}}{80}\right) \mathrm{Hz} \\
& =10,008 \mathrm{~Hz} .
\end{aligned}
$$

Voltage gain at frequency

$$
\begin{aligned}
\mathrm{f} & =15 \mathrm{kHz} \text { is } \\
\mathrm{A}_{\mathrm{f}} & =\frac{\mathrm{A}_{0_{\mathrm{f}}}}{\sqrt{1+\left(\frac{\mathrm{f}}{\left.\mathrm{f}_{\mathrm{c}}\right)^{2}}\right.}} \\
& =\frac{80}{\sqrt{\left[1+\left(\frac{15000}{10,008}\right)^{2}\right]}} \\
& =44.4
\end{aligned}
$$

36. For the circuit shown, assume that the NMOS transistor is in saturation. Its threshold voltage $\mathrm{V}_{\text {in }}=1 \mathrm{~V}$ and its transconductance parameter
$\mu_{\mathrm{n}} \mathrm{C}_{0 \mathrm{x}}\left(\frac{\mathrm{W}}{\mathrm{L}}\right)=1 \mathrm{~mA} / \mathrm{V}^{2}$. Neglect channel

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length modulation and body bias effects. Under these conditons, the drain current l in mA is $\qquad$


Sol. (2)
Given,
Threshold voltage, $\mathrm{V}_{\text {th }}=1 \mathrm{~V}$
Transconductance parameter,

$$
\mu_{\mathrm{n}} \mathrm{C}_{\mathrm{ox}}\left(\frac{\mathrm{w}}{\mathrm{~L}}\right)=
$$

$1 \mathrm{~mA} / \mathrm{V}^{2}$

NMOS Transistor is in saturation,

$$
\begin{equation*}
\mathrm{I}_{\mathrm{D}}=\frac{1}{2} \mu_{\mathrm{n}} \mathrm{C}_{\mathrm{ox}}\left(\frac{\mathrm{w}}{\mathrm{~L}}\right) \cdot\left(\mathrm{V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{T}}\right)^{2} \tag{i}
\end{equation*}
$$

From circuit,

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{G}}=\mathrm{V}_{\mathrm{th}}=\frac{8 \times 5}{3+5}=5 \mathrm{~V} \\
& \mathrm{R}_{\mathrm{G}}=\mathrm{R}_{\mathrm{th}}=\frac{3 \times 5}{3+5}=1.875 \mathrm{M} \Omega
\end{aligned}
$$



Applying KVL in LOOP I.

$$
\begin{aligned}
& \Rightarrow \quad 5-1.875 \mathrm{M} \cdot \mathrm{I}_{\mathrm{G}}-\mathrm{V}_{\mathrm{GS}}-1 \cdot \mathrm{I}_{\mathrm{D}}=0 \quad\left[\therefore \mathrm{I}_{\mathrm{G}}=0\right] \\
& \Rightarrow \quad 5-\mathrm{V}_{\mathrm{GS}}-1 \cdot \mathrm{I}_{\mathrm{D}}=0
\end{aligned}
$$

$$
\begin{equation*}
\mathrm{I}_{\mathrm{D}}=5-\mathrm{V}_{\mathrm{GS}} \tag{ii}
\end{equation*}
$$

Put the value of (ii) in (i),

$$
\begin{aligned}
5-\mathrm{V}_{\mathrm{GS}} & =\frac{1}{2}\left(\mathrm{~V}_{\mathrm{GS}}-1\right)^{2} \\
\Rightarrow \quad 10-2 \mathrm{~V}_{\mathrm{GS}} & =\mathrm{V}_{\mathrm{GS}}^{2}+1-2 \mathrm{~V}_{\mathrm{GS}} \\
\Rightarrow \quad \mathrm{~V}_{\mathrm{GS}}^{2} & =10-1 \\
& =9 \\
\mathrm{~V}_{\mathrm{GS}} & =3 \mathrm{~V}
\end{aligned}
$$

So, from equation (ii),

$$
\mathrm{I}_{\mathrm{D}}=5-\mathrm{V}_{\mathrm{GS}}=5-3=2 \mathrm{~V}
$$

37. A linear time invariant (LTI) system with the transfer function
$G(s)=\frac{K\left(s^{2}+2 s+2\right)}{\left(s^{2}-3 s+2\right)}$ is connected in unity feedback configuration as shown in the figure


For the closed loop system shown, the root locus for $0<\mathrm{K}<\infty$ interesects the imaginary axis for $K=1.5$. The closed loop system is stable for
(a) $\mathrm{K}>1.5$
(b) $1<\mathrm{K}<1.5$
(c) $0<\mathrm{K}<1$
(d) no positive value of $K$

Sol. (a)
The characteristic equation of given feedback system is
$1+\frac{\mathrm{k}\left(\mathrm{s}^{2}+2 \mathrm{~s}+2\right)}{\left(\mathrm{s}^{2}-3 \mathrm{~s}+2\right)}=0$
Or $s^{2}(k+1)+s(2 k-3)+2(k+1)=0$
The Routh Hurwitz table is

$$
\mathrm{s}^{2} \quad \mathrm{k}+1 \quad 2(\mathrm{k}+1)
$$

$$
\begin{array}{ccc}
\mathrm{s}^{1} & 2 \mathrm{k}-3 & 0 \\
\mathrm{~s}^{0} & 2(\mathrm{k}+1) &
\end{array}
$$

For system to be stable

$$
\mathrm{k}+1>0 \text { and } 2 \mathrm{k}-3>0
$$

Or $\quad \mathrm{k}>-1$ and $\mathrm{k}>1.5$
$\therefore \quad \mathrm{k}>1.5$
38. Let $\mathrm{X}(\mathrm{t})$ be a wide sense stationary random process with the power special density $S_{x}(f)$ as shown in Figure (a), where f is in Hertz $(\mathrm{Hz})$. The random process $\mathrm{X}(\mathrm{t})$ is input to an ideal lowpass filter with the frequency response
$\mathrm{H}(\mathrm{f})= \begin{cases}1 & |\mathrm{f}| \leq \frac{1}{2} \mathrm{~Hz} \\ 0, & |\mathrm{f}|>\frac{1}{2} \mathrm{~Hz}\end{cases}$
as shown in Figure (b). The output of the lowpass filter is $\mathrm{Y}(\mathrm{t})$.


Let $E$ be the expectation operator and consider the following statements
I. $\quad \mathrm{E}(\mathrm{X}(\mathrm{t}))=\mathrm{E}(\mathrm{Y}(\mathrm{t}))$
II. $\quad \mathrm{E}\left(\mathrm{X}^{2}(\mathrm{t})\right)=\mathrm{E}\left(\mathrm{Y}^{2}(\mathrm{t})\right)$
II. $\mathrm{E}\left(\mathrm{Y}^{2}(\mathrm{t})\right)=2$

Select the connect option
(a) only I is true
(b) only II and II are true
(c) only I and II are true
(d) only I and III are true

Sol. (a)
Given input power spectrum density $\mathrm{S}_{\mathrm{x}}(\mathrm{f})$


Ideal low pass filter have frequency response.

$\because \quad \mathrm{E}[\mathrm{y}(\mathrm{t})]=\mathrm{H}(0) \mathrm{E}[\mathrm{X}(\mathrm{t})]$
and $\quad \mathrm{H}(0)=1$
so,

$$
\begin{aligned}
\mathrm{E}[\mathrm{y}(\mathrm{t})] & =\mathrm{Ex}(\mathrm{t})] \\
\mathrm{E}\left[\mathrm{x}^{2}(\mathrm{t})\right] & =\mathrm{E}\left[\mathrm{y}^{2}(\mathrm{t})\right]
\end{aligned}
$$

Since, Ideal LPF does not allow total power from input to output.
So, $\quad E\left[x^{2}(t)\right] \neq E\left[y^{2}(t)\right]$

$$
\mathrm{E}\left[\mathrm{y}^{2}(\mathrm{t})\right]=2
$$

$$
\begin{aligned}
E\left[y^{2}(t)\right] & =\int_{0}^{\infty} S_{x}(f) d f \\
& =2
\end{aligned}
$$

$\because$ from (II),

$$
\mathrm{E}\left[\mathrm{x}^{2}(\mathrm{t})\right] \neq \mathrm{E}\left[\mathrm{y}^{2}(\mathrm{t})\right]
$$

So, $\quad \mathrm{E}\left[\mathrm{y}^{2}(\mathrm{t})\right] \neq 2$
Hence, only statement-(I) is correct.
39. The dependence of drift velocity of electrons on electric field in a semiconductor is shown below. The semiconductor has a uniform electron concentration of $\mathrm{n}=1 \times 10^{16} \mathrm{~cm}^{-3}$ and electronic charge $q=1.6 \times 10^{-19} \mathrm{C}$. If a bias of 5 V is applied across a 1 mm region of this semiconductor, the resulting current density in this region in $\mathrm{kA} / \mathrm{cm}^{2}$, is $\qquad$


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Sol. (1.6)
Electric field in the semiconductor
$\mathrm{E}=\frac{\mathrm{V}}{\mathrm{t}}=\frac{5}{1 \times 10^{-4}}=5 \times 10^{4} \mathrm{~V} / \mathrm{cm}$
Now,

$$
\begin{aligned}
& \text { Mobility }=\frac{\text { Drift velocity }}{\text { Electric field Intensity }} \\
&=\frac{10^{7} \mathrm{~cm} / \mathrm{s}}{5 \times 10^{5} \mathrm{~V} / \mathrm{cm}} \\
& \quad\left[\because \mathrm{E}<5 \times 10^{5} \mathrm{~V} / \mathrm{cm}\right] \\
&=20 \mathrm{~cm}^{2} / \mathrm{V} \cdot \mathrm{sec}
\end{aligned}
$$

Conductivity, $\sigma=$ ne $\mu$

$$
\begin{aligned}
& =10^{16} \times 1.6 \times 10^{-19} \times 20 \\
& =32 \times 10^{-3}
\end{aligned}
$$

So, current density

$$
\begin{aligned}
\mathrm{J} & =\sigma \mathrm{E} \\
& =32 \times 10^{-3} \times 5 \times 10^{4} \mathrm{~A} / \mathrm{cm}^{2} \\
& =1600 \mathrm{~A} / \mathrm{cm}^{2} \\
& =1.6 \mathrm{kA} / \mathrm{cm}^{2}
\end{aligned}
$$

40. Let $f(x)=e^{x+x^{2}}$ for real $x$. From among the following, choose the Taylor series approximation of $f(x)$ around $x=0$, which includes all powers of $x$ less than or equal to 3 .
(a) $1+x+x^{2}+x^{3}$
(b) $1+x+\frac{3}{2} x^{2}+x^{3}$
(c) $1+x+\frac{3}{2} x^{2}+\frac{7}{6} x^{3}$
(d) $1+x+3 x^{2}+7 x^{3}$

Sol. (c)
Given,

$$
\begin{aligned}
\mathrm{f}(\mathrm{x}) & =\mathrm{e}^{\mathrm{x}+\mathrm{x}^{2}} \\
\mathrm{f}^{\prime}(\mathrm{x}) & =\mathrm{e}^{\mathrm{x}+\mathrm{x}^{2}}(1+2 \mathrm{x}) \\
\mathrm{f}^{\prime \prime}(\mathrm{x}) & =(1+2)^{2} e^{x+x^{2}}+e^{x+x^{2} \times 2}
\end{aligned}
$$

$$
\begin{aligned}
= & e^{x+x^{2}}\left(3+4 x^{2}+4 x\right) \\
f^{\prime \prime \prime}(x)= & \left(3+4 x+4 x^{2}\right)(1+2 x) \\
& e^{x+x^{2}}+e^{x+x^{2}}(4+8 x)
\end{aligned}
$$

Now,

$$
\begin{aligned}
f^{\prime}(0) & =1, \mathrm{f}^{\prime \prime}(0)=3, \\
f^{\prime \prime \prime}(0) & =3+4=7
\end{aligned}
$$

Taylor series of $f(x)$ around $x=0$ is

$$
\begin{aligned}
f(x)= & f(0)+\frac{f^{\prime}(0)}{1!} x+\frac{f^{\prime \prime}(0)}{2!} x^{2} \\
& +\frac{f^{\prime \prime \prime}(0)}{3!} x^{3}+\ldots \ldots . \\
\therefore \quad f(x)= & 1+x+\frac{3}{2} x^{2}+\frac{7}{6} x^{3}
\end{aligned}
$$

41. In the figure shown the upon transistor acts as a swich


For the input $V_{m}(t)$ as shown in the figure, the transistor switches between the cut-off and saturation regions of operation. When T is large, Assume collector-to-emitter voltage at saturation $\mathrm{V}_{\mathrm{CE}(\text { sat })}=0.2 \mathrm{~V}$ and base-to-emitter voltage $\mathrm{V}_{\mathrm{BE}}=0.7 \mathrm{~V}$. The minimum value of the common-base current grain ( $\alpha$ ) of the transistor for the switching should be
Sol. (0.902)
Redrawn the ckt.


## ESE-2017 Conventional Test Schedule, Electronics Engineering

| Date | Topic |
| :---: | :---: |
| 5th Mar 2017 | N.T. : BEE-1, MI-1, CS-1 |
|  | R.T. : |
| 11th Mar 2017 | N.T. : BEX-1, NT-1, EMT-1 |
|  | R.T. : BEE-1, CS-1, MI-1 |
| 19th Mar 2017 | N.T. : BEE-2, NT-2, EMT-2, CO-1 |
|  | R.T. : BEX-1, EMT-1, NT-1 |
| 26th Mar 2017 | N.T. : MI-2, NT-3, MAT-1, CS-2 |
|  | R.T. : BEE-2, NT-2, CS-1, EMT-2 |
| 2nd Apr 2017 | N.T. : BEX-2, CS-3, CO-2 |
|  | R.T. : MI-2, CO-1, MAT-1, NT-2 |
| 9th Apr 2017 | N.T. : ADC-1, EMT-3, COMM-1 |
|  | R.T. : CS-2, NT-1, EMT-1, BEX-1, EMT-2 |
| 16th Apr 2017 | N.T. : ADC-2 BEX-3, ACT-1 |
|  | R.T. : BEE-2, MI-2, EMT-3, ADC-1, NT-2, CS-2, CS-3 |
| 23rd Apr 2017 | N.T. : AET-1, MAT-2, ADC-3 |
|  | R.T. : ADC-2, BEX-2, BEE-1, MI-1, CS-2, ACT-1, NT-3, CO-2, COMM-1 |
| 30th Apr 2017 | N.T. : AET-2, ACT-2, COMM-2 |
|  | R.T. : ADC-1, ADC-3, AET-1, CS-3, BEX-1, MAT-2, MAT-1 |
| 3rd May 2017 | N.T. : COMM $3, \mathrm{MI}-3, \mathrm{CO}-3$ |
|  | R.T. : ADC-3, AET-2, ACT-1, CO-1, CO-3, COMM-2, NT-3, MAT-2, ACT-2, MI-3 |
| 7th May 2017 | N.T. : AET-3, ADC-4, MAT-3 |
|  | R.T. : CO-3, ACT-2, MAT-3, BEX-2, CS-2, EMT-3, BEX-3, AET-1 AET-2, COMM-2, ADC-4 |
| 9th May 2017 | Full Length (Test Paper-1 + Test Paper-2) |
| Test Type Timing Day |  |
| Conventional Test 10:00 A.M. to 1:00 P.M. ___ Sunday |  |
| Conventional Full Length Test Paper-1 _ 10:00 A.M. to 1:00 P.M. __ Tuesday |  |
| Note : The timing of the test may change on certain dates. Prior information will be given in this regard <br> *N.T. : New Topic. *R.T. : Revision Topic <br> Call us : 8010009955, 011-41013406 or Mail us : info@iesmaster.org |  |

Subject Code Details

| Basic Electronics Engineering (BEX) | BEX-1 |  | BEX-2 |  | BEX-3 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | - Basics of Semiconductors <br> - Diode : Basics, Characteristics \& its types <br> - BJT, JFET, MOSFET-Basic Structure \& Characteristics |  | - Transistor Amplifiers <br> - Oscillators \& Other circuits <br> - Basic of Linear ICs <br> - Operational Amplifier \& their applications |  | - Basics of ICs; Bipolar, MOS \& CMOS ICs <br> - Optical Sources / Detectors <br> - Basics of Optoelectronics \& Applications |  |
| Basic Electrical Engineering (BEE) | BEE-1 |  |  | BEE-2 |  |  |
|  | - Basics of Circuit Theory and Electromagnetic Field Theory <br> -Single Phase AC circuits Transformer DC Machine |  |  | - Induction Machine Synchronous Machine <br> - Electrical Power Sources, Basics of Batteries \& its uses |  |  |
| Material Science (MAT) | MAT-1 |  | MAT-2 |  | MAT-3 |  |
|  | - Crystalline Structure - Dielectric properties of matter <br> - Ceramic materials |  | - Magnetic properties of materials <br> - Insulating laminates for electronics <br> - Conductors \& Superconductors |  | - Semiconductor \& Optical materials <br> - Nano materials Nano-optical / Magnetic / Electronic materials |  |
| Electronic Measurement and Instrumentation (MI) | MI-1 |  | MI-2 |  | MI-3 |  |
|  | - Error analysis \& basics of measurement <br> - Basic measuring instruments <br> - Measurement of Energy \& Power |  | - Measurement of Resistance <br> - AC Bridges Potentiometer <br> - Cathode Ray Oscilloscope (CRO) <br> - Q-meter |  | - Basics of electronic measurements <br> - Digital \& electronic voltmeter $\downarrow$ Digital frequency meter Transducers \& Displays <br> - Basics of Telemetry <br> - Data Acquisition System |  |
| Network Theory (NT) | NT-1 |  | NT-2 |  | NT-3 |  |
|  | - Network elements $\leqslant$ Network theorems - 2-port networks |  | - Transient and Steady State Response <br> - Steady State Sinusoidal analysis <br> - Resonance |  | - Network Functions <br> - Graph Theory Filters <br> - State equations for networks |  |
| Analog and Digital Circuits (ADC) | ADC-1 ADC-2 |  |  | ADC-3 |  | ADC-4 |
|  | - Small Signal equivalent of Diodes, BJTs and FETs <br> - Different Diode Circuits <br> - Biasing and Stability of BJTs <br> \& JFET amplifier circuits | Analysis / Design of amplifiers signal \& multi-stage <br> - Feedback \& its uses <br> - Active filters, timers, multipliers, wave shaping, <br> $A / D \& D / A$ converters |  | - Boolean Algebra \& Logic Gates <br> - Combinational circuits : <br> Design \& Applications <br> - Memories and <br> Microprocessor : Design \& Applications |  | - Sequential circuits : Design \& Applications <br> - Design IC Logic families |
| Among and Digital Communication (COMM) | COMM-1 |  | COMM-2 |  | COMM-3 |  |
|  | - Probability Theory <br> - Analog Communication Systems |  | - Random Signals and Noise <br> - Digital Communication Systems |  | - Information Theory <br> - Multiple Access-TDMA, FDMA, CDMA <br> - Optical Communication |  |
| Control Systems (CS) | CS-1 |  | CS-2 |  | CS-3 |  |
|  | - Signals and Systems <br> - System Realization <br> - Transforms \& their Applications |  | - Basics of Control Systems <br> - Block Diagram \& Signal Flow Graphs <br> - Time Response Analysis <br> - Routh Hurwitz criteria \& Root Locus Technique |  | - Frequency Response Analysis <br> - Stability in Frequency Domain <br> - Controllers and compensators <br> - State Space Analysis |  |
| Computer Organization and Architecture (CO) | CO-1 |  | CO-2 |  | CO-3 |  |
|  | - Basics of Computer Organization |  | - Operating Systems |  | - Database Management Systems <br> - Data Structure and Programming |  |
| Electromagnetics (EMT) | EMT-1 |  | EMT-2 |  | EMT-3 |  |
|  | - Elements of Vector Calculus <br> - Electrostatics <br> - Magnetostatics |  | - Maxwell's Equations <br> - Electromagnetic Wave propagation through different media <br> - Transmission Lines |  | - Waveguides <br> - antenna Theory |  |
| Advanced Electronics Topics (AET) | AET-1 |  | AET-2 |  | AET-3 <br> - Microprocessors and Microcontrollers <br> - Embedded Systems |  |
|  | - VLSI Technology $\downarrow$ VLSI Design <br> - Mealy and Moore circuit design <br> - Pipeline concept and functions <br> - Designs for tesatblity and examples |  | - Digital Signals Processing <br> - Digital Filters <br> - Speech / Audio / Radar Signal Processing |  |  |  |
|  | ACT-1 |  |  | ACT-2 |  |  |
| Advanced communication Topics (ACT) | - Communication Networks : Principles / Practices / Technologies / <br> Uses / OSI Model / Security <br> - Basic packet multiplexed streams / scheduling <br> - Protocols (TCP / TCP-IP) |  |  | - Microwave \& Satellite Communication <br> - Fiber Optic Communication <br> - Cellular Networks : Types, Analysis |  |  |

In base loop;

$$
\begin{aligned}
-2 \mathrm{~V}+12 \mathrm{I}_{\mathrm{B}}+0.7 & =0 \\
\Rightarrow \quad \mathrm{I}_{\mathrm{B}} & =\frac{2-0.7}{12} \\
& =\frac{1.3}{12}=0.10833 \mathrm{~mA}
\end{aligned}
$$

From collector loop,
Transistor is in saturation region

$$
\begin{aligned}
-5 \mathrm{~V}+4.8 \mathrm{I}_{\mathrm{C}}+0.2 & =0 \\
\mathrm{I}_{\mathrm{C}} & =\frac{5-0.2}{4.8 \times 10^{3}} \\
& =\frac{4.8}{4.8 \times 10^{3}}=1 \mathrm{~mA}
\end{aligned}
$$

We know that,
In saturation,

$$
\begin{array}{rlrl} 
& & \mathrm{I}_{\mathrm{B}} \geq \mathrm{I}_{\mathrm{B} \min .} & =\frac{\mathrm{I}_{\mathrm{csat}}}{\beta} \\
\Rightarrow & & \mathrm{I}_{\mathrm{B}} & \geq \frac{1 \mathrm{~mA}}{\beta} \\
\Rightarrow & & & \geq \frac{1}{0.10833} \\
\text { and, } & & \beta_{\mathrm{mm}} & =9.23 \\
\alpha_{\min } & =\frac{\beta_{\min .}}{1+\beta_{\min .}} \\
& & =0.902 \\
\alpha_{\min } & =0.902
\end{array}
$$

42. The expression for an electric field in free space
$E=E_{0}(\hat{x}+\hat{y}+j 2 \hat{z}) e^{-j(\omega t-k x+k y)}$ where $x, y, z$ represent the spatial coordinates, $t$ represents time and $\omega, \mathrm{k}$ are constants. This electric field
(a) does not represent a plane wave
(b) represents a circularly polarized plane wave propagating normal to the z -axis
(c) represents and elliptically polarized plane wave propagating along the $x-y$ plane
(d) represents a linearly polarized plane wave

Sol. (c)

$$
E=E_{0}(\hat{x}+\hat{y}+j 2 \hat{z}) e^{-j(\omega t-k x+k y)}
$$

We know,

$$
\begin{array}{rlrl} 
& & \mathrm{e}^{-\mathrm{jkr}} & =\mathrm{e}^{-\mathrm{j}(-\mathrm{kx}+\mathrm{ky})} \\
\therefore & \mathrm{kr} & =\mathrm{k}(-\mathrm{x}+\mathrm{y})
\end{array}
$$

Propagation vector $\hat{a}_{P}$

$$
\begin{aligned}
& =\frac{\nabla(\mathrm{kr})}{|\nabla(\mathrm{kr})|} \\
\nabla(\mathrm{kr}) & =\mathrm{k}(-\hat{\mathrm{x}}+\hat{\mathrm{y}}) \\
|\nabla(\mathrm{kr})| & =\mathrm{k} \sqrt{2} \\
\hat{\mathrm{a}}_{\mathrm{P}} & =\frac{\nabla(\mathrm{kr})}{|\nabla(\mathrm{kr})|} \\
& =\frac{-\hat{\mathrm{x}}+\hat{\mathrm{y}}}{\sqrt{2}}
\end{aligned}
$$

For plane wave,

$$
\begin{aligned}
\hat{\mathrm{a}}_{\mathrm{P}} \hat{\mathrm{E}} & =\left[\frac{-\hat{\mathrm{x}}+\hat{\mathrm{y}}}{\sqrt{2}}\right] \cdot \mathrm{E}_{0}[\hat{\mathrm{x}}+\hat{\mathrm{y}}+\mathrm{j} 2 \hat{\mathrm{z}}] \\
\hat{\mathrm{a}}_{\mathrm{P}} \mathrm{E}_{\mathrm{P}} & =0
\end{aligned}
$$

$\therefore$ Given is a plane wave,
As, $E=E_{0}(\hat{\mathrm{x}}+\hat{\mathrm{y}}+\hat{\mathrm{j}} 2 \hat{\mathrm{z}}) \mathrm{e}^{-\mathrm{j}(\omega \mathrm{t}-\mathrm{kx}+\mathrm{ky})}$
For the given, plane of incidence is xy plane $E$ in $x y$ plane is parallel polarized, $\mathrm{E}_{| |}=|\mathrm{E}|_{\mathrm{xy}}=\sqrt{1+1}=\sqrt{2}$ and along z is perpendicular polarize $\mathrm{E}_{| |}=\left|\mathrm{E}_{\mathrm{z}}\right|_{\mathrm{z}}=\sqrt{2^{2}}$ $=2$.

$$
\begin{aligned}
\left|\mathrm{E}_{\mathrm{T}}\right| & =\left|\mathrm{E}_{| |}\right|+\left|\mathrm{E}_{\mathrm{r}}\right| \\
& =\sqrt{2}+2 \\
\left|\mathrm{E}_{| |}\right| & \neq\left|\mathrm{E}_{\mathrm{r}}\right|
\end{aligned}
$$

and phase difference is $90^{\circ}$; i.e., given is a elliptically polarized plane wave.
43. The following FIVE instructions were executed on an 8085 microprocessor

MVI A, 33H

MVI B, 78 H
ADD B
CMA
ANI 32H
The Accumulator value immediately after the execution of the fifth instruction is
(a) 00 H
(b) 10 H
(c) 11 H
(d) 32 H

Sol. (b)
MVI A $33 \mathrm{H} \Rightarrow[\mathrm{A}]=33 \mathrm{H}$
MVI B $78 \mathrm{H} \Rightarrow[\mathrm{B}]=78 \mathrm{H}$
$\mathrm{ADD} \mathrm{B} \Rightarrow[\mathrm{A}] \Leftarrow[\mathrm{A}]+[\mathrm{B}]$

$$
\begin{aligned}
& \Rightarrow \begin{array}{r}
00110011 \\
+01111000 \\
+\underbrace{1010}_{\mathrm{A}} \underbrace{1011}_{\mathrm{B}}
\end{array} \Rightarrow \mathrm{ABH}
\end{aligned}
$$

i.e. $[\mathrm{A}]=\mathrm{AB} \mathrm{H}$

CMA $\Rightarrow$ complement accumulator
i.e. $\quad[\mathrm{A}] \Leftarrow \overline{\mathrm{A}}$
where, $\quad \overline{\mathrm{A}}=\overline{10101011}=01010100$
ANI $32 \mathrm{H} \Rightarrow 0011 \quad 0010$

$$
\frac{01010100}{00010000} \Rightarrow 10 \mathrm{H}
$$

$[\mathrm{A}]=10 \mathrm{H}$
i.e. option (B)
44. Which one of the following options correctly describes the location of the roots of the equation $s^{4}+s^{2}+1=0$ on the complex plane?
(a) Four left half plane (LHP) roots
(b) One right half plane (RHP) root, one LHP root and two roots on the imaginary axis
(c) Two RHP roots and two LHP roots
(d) All four roots are on the imaginary axis

Sol. (c)
Given equation is,
$s^{4}+s^{2}+1=0$
Let $\quad s^{2}=y$
then $\quad y^{2}+y+1=0$
$\Rightarrow \quad y=\frac{-1 \pm \sqrt{1-4}}{2}=\frac{-1}{2} \pm j \frac{\sqrt{3}}{2}$
$\Rightarrow \mathrm{y}=1 \angle 120^{\circ}$ and $1 \angle-120^{\circ}$
for $\mathrm{y}=\mathrm{s}^{2}=1 \angle 120^{\circ}$
$\Rightarrow \quad \mathrm{s}= \pm\left(1 \angle 60^{\circ}\right)$
and, for $y=s^{2}=1 \angle-120^{\circ}$

$$
\Rightarrow \mathrm{s}= \pm\left(1 \angle-60^{\circ}\right)
$$


i.e. equation has two right half plane (RHP) roots and two left half plane (LHP) roots. i.e. option (c).
45. As shown a uniformly doped Silicon (Si) bar of length $\mathrm{L}=0.1 \mu \mathrm{~m}$ with a donor concentration $\mathrm{N}_{\mathrm{D}}=10^{16} \mathrm{~cm}^{-3}$ is illuminated at $\mathrm{x}=0$ such that electron and hole pairs are generated at the rate of $G_{L}=G_{L 0}$
$\left(1-\frac{\mathrm{x}}{\mathrm{L}}\right), 0 \leq \mathrm{x} \leq \mathrm{L}$. where, $\mathrm{G}_{\mathrm{L} 0}=10^{17} \mathrm{~cm}^{-3} / \mathrm{s}$.
Hole lifetime $10^{-4} \mathrm{~s}$, electronic change $\mathrm{q}=$ $1.6 \times 10^{-19} \mathrm{C}$, hole diffusion coefficient $\mathrm{D}_{\mathrm{p}}=$ $100 \mathrm{~cm}^{2} / \mathrm{s}$ and low level injection condition prevails. Assuming a linearly decaying steady state excess hole concentration that goes to 0 at $\mathrm{x}=\mathrm{L}$, the magnitude of the diffusion current density at $\mathrm{x}=\mathrm{L} / 2$, in $\mathrm{A} /$ $\mathrm{cm}^{2}$, is $\qquad$ —.


Sol. (16)
Given,

$$
\mathrm{L}=0.1 \mu \mathrm{~m}
$$

so,

$$
\mathrm{N}_{\mathrm{D}}=10^{16} / \mathrm{cm}^{3} \text { at } \mathrm{x}=0
$$

Hole pair generated rate,

$$
G_{L}=G_{L_{0}}\left(1-\frac{x}{L}\right)
$$

$0 \leq \mathrm{x} \leq \mathrm{L}$
Where,

$$
\begin{aligned}
\mathrm{G}_{\mathrm{L}_{0}} & =10^{17} \mathrm{~cm}^{-3} \mathrm{~s}^{-1} \\
\tau & =10^{-4} \mathrm{~s}, \\
\mathrm{q} & =1.6 \times 10^{-19} \mathrm{c} \\
\mathrm{D}_{\mathrm{P}} & =100 \mathrm{~cm}^{2} / \mathrm{s} \\
\mathrm{~J}_{\mathrm{p}_{\text {diff. }}} & =?
\end{aligned}
$$

$\therefore$ Net hole density varying in the direction of $x$ is:

$$
P_{n}(x)=P_{n_{0}}+P P
$$

$$
=\mathrm{P}_{\mathrm{n}_{0}}+\mathrm{G}_{\mathrm{L}} \mathrm{I}_{\mathrm{P}}
$$

$$
=\mathrm{P}_{\mathrm{n}_{0}}+\mathrm{G}_{\mathrm{L} 0} \mathrm{I}_{\mathrm{P}}\left(1-\frac{\mathrm{x}}{\mathrm{~L}}\right)
$$

$$
\mathrm{J}_{\text {Pdiff. }}=-\mathrm{eD}_{\mathrm{P}} \frac{\mathrm{dp}}{\mathrm{dx}}
$$

$$
=-\mathrm{eD} \mathrm{D}_{\mathrm{P}} \frac{\mathrm{dp}}{\mathrm{dx}}
$$

$$
=-\mathrm{eD} \mathrm{D}_{\mathrm{p}}\left[\frac{-\mathrm{G}_{\mathrm{L} 0} \cdot \mathrm{I}_{\mathrm{P}}}{\mathrm{~L}}\right]
$$

$=\frac{1.6 \times 10^{-19} \times 100 \times 10^{17} \times 10^{-4}}{0.1 \times 10^{-4}}$
$=16 \mathrm{~A} / \mathrm{cm}^{2}$
46. Let $\mathrm{I}=\int_{\mathrm{C}}(2 \mathrm{zdx}+2 \mathrm{y} d \mathrm{y}+2 \mathrm{xdz})$ where $\mathrm{x}, \mathrm{y}$, z are real and let C be the straight line segment from point A: $(0,2,1)$ to point B: $(4,1-1)$. The value of I is $\qquad$
Sol. (-11)

$$
I=\int_{c}(2 z d x+2 y d y+2 x d z)
$$

The equation of straight line joining $\mathrm{A}(0,2$, $1)$ and $B(4,1,-1)$ is given by

$$
\begin{aligned}
\frac{x-0}{4-0} & =\frac{y-2}{1-2}=\frac{z-1}{-1-1} \\
\Rightarrow \quad \frac{x}{4} & =\frac{y-2}{-1}=\frac{z-1}{-2} \\
x & =-4 y+8=-2 z+2 \\
\Rightarrow y & =\frac{8-x}{4} \\
\therefore d y & =\frac{-1}{4} d x \\
\text { and, } z & =\frac{2-x}{2} \\
\therefore d z & =\frac{-1}{2} d x
\end{aligned}
$$

$$
\text { then, } \mathrm{I}=\int_{0}^{4}\left[2\left(\frac{2-\mathrm{x}}{2}\right) \mathrm{dx}+2\left(\frac{8-\mathrm{x}}{4}\right)\left(\frac{-1}{4} \mathrm{dx}\right)\right.
$$

$$
\left.+2 \mathrm{x}\left(\frac{-1}{2} \mathrm{dx}\right)\right]
$$

$$
=\int_{0}^{4}\left(2-x-1+\frac{x}{8}-x\right) d x
$$

$$
=\int_{0}^{4}\left(1-\frac{15}{8} x\right) d x=\left[x-\frac{15}{16} x^{2}\right]_{0}^{4}
$$

$$
=4-15=-11
$$

47. An optical fiber is kept along the $z$ direction. The refractive indices for the electric fields along $\hat{x}$ and $\hat{y}$ directions in the fiber are $n_{x}$ $=1.50000$ and $\mathrm{n}_{\mathrm{y}}=1.5001$, respectively ( $\mathrm{n}_{\mathrm{x}} \neq \mathrm{n}_{\mathrm{y}}$ due to the imperfection in the fiber cross-section). The free space wavelength of a light wave propagating in the fiber is $1.5 \mu \mathrm{~m}$. If the lightwave is circularly polarized at the input of the fiber, the minimum propagation distance after which it becomes linearly polarized in centimeters, is $\qquad$


Sol. (0.375)
For to have linear polarization, phase difference has to be $0^{\circ}$ or $180^{\circ}$. Given the light wave is circularly polarized that is initial phase difference is $90^{\circ}$.

$$
\text { so, } \begin{aligned}
\beta_{1} \mathrm{z} \sim \beta_{2} \mathrm{z} & =\frac{\pi}{2} \\
\Rightarrow \frac{\mathrm{w}}{\mathrm{~V}_{\mathrm{Px}}} \mathrm{z} \sim \frac{\mathrm{w}}{\mathrm{~V}_{\mathrm{Py}}} \mathrm{z} & =\frac{\pi}{2} \\
\Rightarrow \frac{2 \pi \mathrm{f}}{\mathrm{c}}\left(\eta_{\mathrm{x}} \sim \eta_{\mathrm{y}}\right) \mathrm{z} & =\frac{\pi}{2} \\
\Rightarrow \frac{2 \pi}{\lambda}\left(\eta_{\mathrm{x}} \sim \eta_{\mathrm{y}}\right) \mathrm{z} & =\frac{\pi}{2} \\
\Rightarrow & =\frac{\pi}{2} \times \frac{\lambda}{2 \pi\left(\eta_{\mathrm{x}} \sim \eta_{\mathrm{y}}\right)} \\
& =\frac{\lambda}{4\left(\eta_{\mathrm{x}} \sim \eta_{\mathrm{y}}\right)} \\
& =\frac{1.5 \times 10^{-6}}{4 \times 0.0001} \\
& =0.375 \mathrm{~cm}
\end{aligned}
$$

48. The Nyquist plot of the transfer function

$$
G(s)=\frac{K}{\left(s^{2}+2 s+2\right)(s+2)}
$$

does not encircle the point $(-1+j 0)$ for $\mathrm{K}=$ 10 but does encircle the point $(-1+j 0)$ for $\mathrm{K}=100$. Then the closed loop system (having unity gain feedback) is
(a) stable for $\mathrm{K}=10$ and stable for $\mathrm{K}=$ 100
(b) stable for $\mathrm{K}=10$ and unstable for $\mathrm{K}=$ 100
(c) unstable for $\mathrm{K}=10$ and sable for $\mathrm{K}=$ 100
(d) unstable for $\mathrm{K}=10$ and unstable for K $=100$

Sol. (b)
Given open-loop transfer function

$$
\mathrm{G}(\mathrm{~s})=\frac{\mathrm{k}}{\left(\mathrm{~s}^{2}+2 \mathrm{~s}+2\right)(\mathrm{s}+2)}
$$

poles: $\mathrm{s}^{2}+2 \mathrm{~s}+2=0$

$$
\mathrm{s}=\frac{-2 \pm \sqrt{4-8}}{2}=\frac{-2 \pm \mathrm{j} 2}{2}=-1 \pm \mathrm{j}
$$

and $\mathrm{s}=-2$
i.e. none of the poles of open-loop system lies on right half of s-plane.
i.e. $P=0$.

Now, for $\mathbf{k}=10$ :
No. of encirclement $=0$

$$
\text { i.e. } N=0
$$

since, $Z=N+P=0+0=0$
i.e. none of the poles of closed loop system lies in right half of s-plane. So, system will be stable.
For $\mathbf{K}=100$ :
No. of encirclement $=1$
i.e. $\quad N=1$
since, $\quad \mathrm{Z}=\mathrm{N}+\mathrm{P}=1+0=1$.
i.e. one pole of closed loop system lies in right half of s-plane. So, system will be unstable.
i.e. option (b).
49. A three dimensional region $R$ of finite volume is described by

$$
\mathrm{x}^{2}+\mathrm{y}^{2} \leq \mathrm{z}^{3} ; 0 \leq \mathrm{z} \leq 1 .
$$

where $\mathrm{x}, \mathrm{y}, \mathrm{z}$ are real. The volume of R (upto two decimal place) is $\qquad$
Sol. $\left(\frac{\pi}{4}\right)$

$$
\begin{array}{lrl}
\because & 0 \leq \mathrm{z} \leq 1 \\
\Rightarrow & \mathrm{z}_{\min } & =0 \\
& \& \mathrm{z}_{\max } & =1
\end{array}
$$

$$
\text { so, } x^{2}+y^{2} \leq z^{3} \Rightarrow\left\{\begin{array}{l}
\operatorname{Min}\left(x^{2}+y^{2}\right)=0 \\
\operatorname{Max}\left(x^{2}+y^{2}\right)=0
\end{array}\right.
$$

$\Rightarrow \quad \min \mathrm{x}=0 \& \min \mathrm{y}=0$ $\max y=\sqrt{1-y^{2}} \& \max x=1$
so, $0 \leq \mathrm{z} \geq 1,0 \leq \mathrm{y} \leq \sqrt{1-\mathrm{x}^{2}}, 0 \leq \mathrm{x} \leq 1$
so, $\quad$ Volume $=\iiint(1) d x d y d z$

$$
\begin{aligned}
= & \int_{x=0}^{1} \int_{z=0}^{1} \int_{y=0}^{\sqrt{1-x^{2}}}(1) d y d x d x \\
& =\int_{z=0}^{1}\left[\int_{x=0}^{1} \sqrt{1-x^{2}} d x\right] d z \\
& =\int_{z=0}^{1}\left(\frac{\pi}{4}\right) d z \\
& =\frac{\pi}{4}
\end{aligned}
$$

Method II: In cylindrical coordinates ( $x, y$, z) $\approx(\mathrm{r}, \theta, \mathrm{z})$

Where $\mathrm{x}^{2}+\mathrm{y}^{2} \leq \mathrm{z}^{3}=0 \leq \mathrm{z} \leq 1$

$$
0 \leq \theta \leq 2 \pi
$$

$$
0 \leq r \leq z^{\frac{3}{2}}
$$

$$
\left[\because \mathrm{x}^{2}+\mathrm{y}^{2}=\mathrm{r}^{2}\right]
$$

$V=\iiint(1) d x d y d z$
$=\iiint \mathrm{r} \mathrm{dr} \mathrm{d} \theta \mathrm{dz}$
$=\int_{z=0}^{1} \int_{\theta=0}^{2 \pi} \int_{r=0}^{z^{3 / 2}} r \cdot d r \cdot d \theta d z$
$=\int_{z=0}^{1} \int_{\theta=0}^{2 \pi}\left(\frac{z^{3}}{2}\right) d \theta d z$
$=\int_{\theta=0}^{2 \pi}\left(\frac{\mathrm{z}^{4}}{8}\right)_{0}^{1} \mathrm{~d} \theta$
$=\frac{1}{8} \int_{0}^{2 \pi}(1) \mathrm{d} \theta$
$=\frac{\pi}{4}$
50. Which one of the following gives the simplified sum of products expression for the Boolean function $\mathrm{F}=\mathrm{m}_{0}+\mathrm{m}_{2}+\mathrm{m}_{3}+\mathrm{m}_{5}$, where $\mathrm{m}_{0}, \mathrm{~m}_{2}, \mathrm{~m}_{3}$ and $\mathrm{m}_{5}$ are minterms corresponding to the inputs $\mathrm{A}, \mathrm{B}$ and C with A is the MSB and C as the LSB?
(a) $\overline{\mathrm{A}} \mathrm{B}+\overline{\mathrm{A}} \overline{\mathrm{B}} \overline{\mathrm{C}}+\mathrm{A} \overline{\mathrm{B}} \mathrm{C}$
(b) $\overline{\mathrm{A}} \overline{\mathrm{C}}+\overline{\mathrm{A}} \mathrm{B}+\mathrm{A} \overline{\mathrm{B}} \mathrm{C}$
(c) $\overline{\mathrm{A}} \overline{\mathrm{C}}+\mathrm{A} \overline{\mathrm{B}}+\mathrm{A} \overline{\mathrm{B}} \mathrm{C}$
(d) $\overline{\mathrm{A}} \mathrm{BC}+\overline{\mathrm{A}} \overline{\mathrm{C}}+\mathrm{A} \overline{\mathrm{B}} \mathrm{C}$

Sol. (b)

$$
\begin{aligned}
& \mathrm{F}=\left(\mathrm{m}_{0}+\mathrm{m}_{2}\right)+\left(\mathrm{m}_{2}+\mathrm{m}_{3}\right)+\mathrm{m}_{5} \\
& \bar{A} \bar{C}+\bar{A} B+A \bar{B} C
\end{aligned}
$$

51. A finite state machine (FSM) is implemented using the D flip-flops A and B and logic gates as shown in the figure below. The four possible states of the $F$ SM are $Q_{A} Q_{B}=00$, 01,10 , and 11


Assume that $\mathrm{X}_{\mathrm{IN}}$ is held at a constant logic level throughout the operation of the FSM. When the FSM is initialized to the state $Q_{A} Q_{B}=00$ and clocked, after a few clock cycle, it starts cycling through
(a) all of the four possible states if $\mathrm{X}_{\mathrm{IN}}=1$
(b) three of the four possible states if $\mathrm{X}_{\text {IN }}$ $=0$
(c) only two of the four possible states if $\mathrm{X}_{\mathrm{IN}}=1$

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(d) only two of the four possible states if $\mathrm{X}_{\mathrm{IN}}=0$

Sol. (d)
When $\mathrm{X}_{\mathrm{IN}}=0$ :

|  | $\mathrm{Q}_{\mathrm{A}}$ | $\mathrm{Q}_{\mathrm{B}}$ | $\mathrm{D}_{\mathrm{A}}$ | $\mathrm{D}_{\mathrm{B}}$ |
| :---: | :--- | :--- | :--- | :--- |
| CLK 0 | 0 | 0 | 0 | 1 |
| CLK 1 | 0 | 1 | 1 | 1 |
| CLK 2 | 1 | 1 | 0 | 1 |
| CLK 3 | 0 | 1 | 1 | 1 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |

i.e. output $Q_{A} Q_{B}$ starts cycling through only two of the four possible states.
When $\mathbf{X}_{\text {IN }}=1$ :

|  | $\mathrm{Q}_{\mathrm{A}}$ | $\mathrm{Q}_{\mathrm{B}}$ | $\mathrm{D}_{\mathrm{A}}$ | $\mathrm{D}_{\mathrm{B}}$ |
| :---: | :--- | :--- | :--- | :--- |
| CLK 0 | 0 | 0 | 0 | 1 |
| CLK 1 | 0 | 1 | 1 | 1 |
| CLK 2 | 1 | 1 | 0 | 0 |
| CLK 3 | 0 | 0 | 0 | 1 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |

i.e. output $Q_{A} Q_{B}$ starts cycling though three of the four possible states.
52. Let $\mathrm{h}[\mathrm{n}]$ be the impulse response of a discrete-time linear time invariant (LTI) filter. The impulse response is given by

$$
\begin{aligned}
& \mathrm{h}[0]=\frac{1}{3} ; \mathrm{h}[1]=\frac{1}{3} ; \mathrm{h}[2]=\frac{1}{3} ; \text { and } \\
& \mathrm{h}[\mathrm{n}]=0 \text { for } \mathrm{n}<0 \text { and } \mathrm{n}>2 .
\end{aligned}
$$

let $H(\omega)$ be the discrete-time Fourier transform (DTFT) of h[n]. where $\omega$ is the normalized angular frequency in radians. Given that $H\left(\omega_{0}\right)=0$ and $0<\omega_{0}<\pi$, the value of $\omega_{0}$ (in radian) is equal to $\qquad$
Sol. (2.094)

$$
\begin{aligned}
& \mathrm{h}(0)=\frac{1}{3} ; \\
& \mathrm{h}(1)=\frac{1}{3} ;
\end{aligned}
$$

$$
h(2)=\frac{1}{3} \text {; }
$$

and $\quad \mathrm{h}[\mathrm{n}]=0$
for and also given that $\mathrm{H}\left(\omega_{0}\right)=0$ and $0<\omega_{0}<\pi$
$\begin{aligned} \mathrm{h}[\mathrm{n}] & =\left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right] \\ \text { Since, } & \mathrm{h}[\mathrm{n}]\end{aligned}=\frac{1}{3} \delta[\mathrm{n}]+\frac{1}{3} \delta[\mathrm{n}-1]$.

So, $\quad H\left(e^{j \omega}\right)=\frac{1}{3}+\frac{1}{3} \mathrm{e}^{-\mathrm{j} \omega}+\frac{1}{3} \mathrm{e}^{-2 \mathrm{j} \omega}$

$$
=\frac{1}{3} \mathrm{e}^{-\mathrm{j} \omega}+\frac{1}{3} \mathrm{e}^{-\mathrm{j} \omega}\left(\mathrm{e}^{\mathrm{j} \omega}+\mathrm{e}^{-\mathrm{j} \omega}\right)
$$

$$
\left[\frac{\mathrm{e}^{\mathrm{j} \omega}+\mathrm{e}^{-\mathrm{j} \omega}}{2}=\cos \omega\right]
$$

$$
\mathrm{H}\left(\mathrm{e}^{\mathrm{j} \omega}\right)=\frac{1}{3} \mathrm{e}^{-\mathrm{j} \omega}[1+2 \cos \omega]
$$

$$
\mathrm{H}\left(\mathrm{e}^{\mathrm{j} \omega_{0}}\right)=0 \quad \text { when }
$$

$$
\Rightarrow \quad 1+2 \cos \omega_{0}=0
$$

$$
\Rightarrow \quad \cos \omega_{0}=\frac{-1}{2}
$$

$$
\Rightarrow \quad \omega_{0}=\cos ^{-1}\left(\frac{1}{2}\right)=120^{\circ}
$$

$$
=\frac{2 \pi}{3}=2.094 \mathrm{rad} .
$$

53. A half wavelength dipole is kept in the $x-y$ plane and oriented along $45^{\circ}$ from the x axis. Determine the direction of null in the radiation pattern for $0 \leq \phi \leq \pi$. Here the angle $\theta(0 \leq \theta \leq \pi)$ is measured from the z axis, and the angle $\phi(0 \leq \phi \leq 2 \pi$. $)$ is measured from the x -ais in the $\mathrm{x}-\mathrm{y}$ plane.
(a) $\theta=90^{\circ}, \phi=45^{\circ}$
(b) $\theta=45^{\circ}, \phi=90^{\circ}$
(c) $\theta=90^{\circ}, \phi=135^{\circ}$
(d) $\theta=45^{\circ}, \phi=135^{\circ}$

## Sol. (a)

As the antenna is placed is xy-plane which is horizontal plane

$$
\text { i.e., } \theta=\frac{\pi}{2}
$$



As there is no field along antenna i.e. null along antenna,

$$
\theta=45^{\circ}
$$

as

$$
0 \leq \phi \leq \pi \text { given }
$$

$\therefore$ for the given antenna null is at $\theta=90^{\circ}$, $\phi=45^{\circ}$.
54. Starting with $x=1$, the solution of the equation $x^{3}+x=1$, after two iterations of Newton-Raphson's method (upto two decimal places) is $\qquad$
Sol. (0.686)
Let,

$$
\begin{aligned}
f(x) & =x^{3}+x-1 \\
f^{\prime}(x) & =3 x^{2}+1
\end{aligned}
$$

Using Newton - Raphson formula

$$
\mathrm{x}_{\mathrm{n}+1}=\mathrm{x}_{\mathrm{n}}-\frac{\mathrm{f}\left(\mathrm{x}_{\mathrm{n}}\right)}{\mathrm{f}^{\prime}\left(\mathrm{x}_{\mathrm{n}}\right)}
$$

Starting with $\mathrm{x}_{\mathrm{n}}=1$

$$
\begin{aligned}
\mathrm{x}_{\mathrm{n}+1} & =1-\frac{\mathrm{f}(1)}{\mathrm{f}^{\prime}(1)}=1-\frac{1+1-1}{3 \times 1+1} \\
& =1-\frac{1}{4}=0.75
\end{aligned}
$$

Now, $\quad x_{n}^{\prime}=0.75$

$$
\begin{aligned}
\mathrm{x}_{\mathrm{n}+1}^{\prime} & =0.75-\frac{\mathrm{f}(0.75)}{\mathrm{f}^{\prime}(0.75)} \\
& =0.75-\frac{(0.75)^{3}+(0.75)-1}{3 \times(0.75)^{2}+1}
\end{aligned}
$$

$$
\begin{aligned}
& =0.75-\frac{0.171875}{2.6871} \\
& =0.686
\end{aligned}
$$

55. A 4-bit shift register circuit configured for right-shift operation i.e. $D_{\text {in }} \rightarrow A, A \rightarrow B, B \rightarrow C, C \rightarrow D$, is shown. If the present state of the shift register is $\mathrm{ABCD}=1101$, the number of clock cycles required to reach the state $\mathrm{ABCD}=1111$ is
$\qquad$


Sol. (10)

| ClK | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{D}$ | $\mathbf{D}_{\text {in }}=\mathbf{A} \oplus \mathbf{D}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 | 0 |
| 2 | 0 | 0 | 1 | 1 | 1 |
| 3 | 1 | 0 | 0 | 1 | 0 |
| 4 | 0 | 1 | 0 | 0 | 0 |
| 5 | 0 | 0 | 1 | 0 | 0 |
| 6 | 0 | 0 | 0 | 1 | 1 |
| 7 | 1 | 0 | 0 | 0 | 1 |
| 8 | 1 | 1 | 0 | 0 | 1 |
| 9 | 1 | 1 | 1 | 0 | 1 |
| 10 | 1 | 1 | 1 | 1 | 0 |
| RRequired |  |  |  |  |  |
| State |  |  |  |  |  |

$\therefore$ The number of Clock Cycles required $=10$.

## Aptitude

1. $40 \%$ of deaths on city roads may be attributed to drunken driving. The number of degrees needs to represent this as a slice of a pie chart is
(a) 120
(b) 144
(c) 160
(d) 212

Sol. (b)
In pie-chart, $100 \%$ represents $360^{\circ}$

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$$
\therefore \quad 40 \%=\frac{40}{100} \times 360=144^{\circ}
$$

2. Some tables are shelves. Some shelves are chains. All chairs are benches. Which of the following conclusions can be deduced from the preceding sentences?
(I) At least one bench is a table
(II) At least one shelf is a bench
(III) At least one shelf is a table
(IV) All benches are chairs
(a) Only I
(b) Only II
(c) Only II and III
(d) Only IV

Sol. (b)

i.e. at least one shelf is a bench.
3. In the summer, water consumption is known to decrease overall by $25 \%$. A Water Board official states that in the summer household consumption decreases by $20 \%$ while other consumption increased by $70 \%$.
Which of the following statements is correct?
(a) The ratio of household to other consumption is $8 / 17$
(b) The ratio of household to other consumption is $1 / 17$
(c) The ratio of household to other consumption is $17 / 8$
(d) There are errors in the official's statement.

Sol. (d)
As the household consumption decreases by $20 \%$ and other consumption increase by $70 \%$ then the overall decrease must be less than $20 \%$.

Hence, there are errors in the official's statement.
4. She has a sharp tongue and it can occasionally turn
(a) hurtful
(b) left
(c) methodical
(d) vital

Sol. (a)
Have a sharp tongue: To be someone who often criticizes and speaks in a severe way.

So, it can occasionally turn hurtful.
5. I $\qquad$ made arrangements had I $\qquad$ informed earlier.
(a) could have, been
(b) would have, being
(c) had, have
(d) had been, been

## Sol. (a)

6. Truck ( 10 m long) and cars ( 5 m long) go on a single lane bridge. There must be gap of atleast 20 m after each truck and a gap of at least 15 m after each car. Truck and cars travel at a speed of $36 \mathrm{~km} / \mathrm{h}$. If cars and trucks go alternately, what is the maximum number of vehicles that can use the bridge in one hour?
(a) 1440
(b) 1200
(c) 720
(d) 600

Sol. (a)
Let the number of vehicles that can use the bridge in one hour are 2 x (i.e. x cars and x trucks).
Speed of Cars and Trucks $=36 \mathrm{Km} / \mathrm{h}$

$$
=\frac{36 \times 5}{18}=10 \mathrm{~m} / \mathrm{sec}
$$

$\therefore$ In one hour,
$10 \times 60 \times 60=$ (length of Cars + gap after each cars) $\times x+$ (length of Truck + gap after each Truck) $\times \mathrm{x}$
Or $36000=(5+15) \mathrm{x}+(10+20) \mathrm{x}$
Or $50 \mathrm{x}=36000$
Or $2 \mathrm{x}=1440$
7. A contour line joins locations having the same height above the mean sea level. The following is a contour plot of a geographical region. Contour lines are shown at 25 m intervals in this plot.


The path from $P$ to $Q$ is best described by
(a) Up-Down-Up-Down
(b) Down-Up-Down-Up
(c) Down-Up-Down
(d) Up-Down-Up

Sol. (c)

$$
\begin{aligned}
& \underbrace{(>500) \rightarrow(550) \rightarrow(500)}_{\text {Down }} \rightarrow \\
& \underbrace{(500) \rightarrow(550) \rightarrow(>550)}_{\text {Up }} \rightarrow \underbrace{(500) \rightarrow(<500)}_{\text {Down }}
\end{aligned}
$$

8. "If you are looking for a history of India or for an account of the rise and fall of the British Raj, or for the reason of the cleaving of the subcontinent into two mutually antagonistic parts and the effects this mutilation will have in the respective sections, and ultimately on Asia, you will not find it in these pages for though I have spent a lifetime in the country. I lived too near the seat of events, and was too intimately associated with the actors, to get the perspective needed for the impartial recording of these matters".
Here, the word 'antagonistic' is closest in meaning to
(a) impartial
(b) argumentative
(c) separated
(d) hostile

Sol. (*)
9. $S, T, U, V, W, X, Y$, and $Z$ are seated around a circular table. T's neighbours are Y and $\mathrm{V}, \mathrm{Z}$ is seated third to the left of T and second to the right of S. U's neighbours are S and Y ; and T and W are not seated opposite each other. Who is third to the left of V?
(a) X
(b) W
(c) U
(d) T

Sol. (a)
The seating arrangement of different people according to questions is shown below

$\therefore \mathrm{X}$ is third to the left of V .
10. There are 3 Indians and 3 Chinese in a group of 6 people. How many subgroups of this group can we choose so that every subgroup has at least one Indian?
(a) 56
(b) 52
(c) 48
(d) 44

Sol. (a)
The total number of required subgroups
$=\left({ }^{3} \mathrm{C}_{1}+{ }^{3} \mathrm{C}_{2}+{ }^{3} \mathrm{C}_{3}\right) \times\left({ }^{3} \mathrm{C}_{0}+{ }^{3} \mathrm{C}_{1}+{ }^{3} \mathrm{C}_{2}+{ }^{3} \mathrm{C}_{3}\right)$
$=\left(2^{3}-1\right)\left(2^{3}\right)$
$=7 \times 8$
$=56$.

