# 2 

# GATIE 2017 

# Detailed Solution 

ELECTRICAL ENGINEERING SESSION - 1

## GATE-2017 <br> Electrical Engineering Questions and Detailed Solution Session-1

1. The equivalent resistance between the terminals A and B is $\qquad$ $\Omega$.


Sol. (3 $\Omega$ )
Simplifying the circuit


Combining resistances $3 \Omega, 6 \Omega$, and $2 \Omega$ as these are parallel
$3 \Omega\|6 \Omega\| 2 \Omega=1 \Omega$
Circuit reduces to

2. Consider an electron a neutron and a proton initially at rest and placed along a straight line such that the neutron is exactly at the center of the line joining the electron and proton. At $t=0$, the particles are released but are constrained to move along the same straight line. Which of these will collide first?
(a) The particles will never collide
(b) All will collide together
(c) Proton and neutron
(d) Electron and neutron

Sol-2 : (d)


Mass of electron $=9.1094 \times 10^{-31} \mathrm{Kg}$
Mass of proton $=1.6726 \times 10^{-27} \mathrm{Kg}$
Electrostatic force will exist between electron and proton only. Let say force is ' F ' then by relation $\mathrm{F}=\mathrm{ma}$, where m is mass of particle and $a$ is acceleration.

Since mass of electron is lesser than proton so acceleration of electron will be more than proton.

By equation

$$
\begin{aligned}
\mathrm{s} & =\mathrm{ut}+\frac{1}{2} \mathrm{at}^{2} \\
\mathrm{~s} & =\text { distance travelled } \\
\mathrm{u} & =\text { inital speed }
\end{aligned}
$$

As $u=0$ so $s=\frac{1}{2} a t^{2}$
To travel distance 'd' electron will take lesser time so electron will collide with neutron first
3. The slope and level detector circuit in a CRO has a delay of 100 ns . The start-stop sweep generator has a response time of 50 ns. In order to display correctly a delay line of
(a) 150 ns has to be inserted into the $y$-channel
(b) 150 ns has to be inserted into the x channel
(c) 150 ns has to be inserted into both x and y channels
(d) 100 ns has to be inserted into both x and y channels

Sol. (a)
In a CRO during the sweep time the beam moves left to right across the CRT, during the retrace time the beam quickly moves to the left side of the CRT screen as shown in figure below.


Given data slope and level detector has delay time $\left(\mathrm{t}_{\mathrm{d}}\right)=100 \mathrm{~ns}$
response time $\left(\mathrm{t}_{\mathrm{re}}\right)=50 \mathrm{~ns}$
so total time taken for one sweep cycle of x-plate

$$
\begin{aligned}
& =\left(\mathrm{t}_{\mathrm{re}}+\mathrm{t}_{\mathrm{d}}+\mathrm{t}_{\mathrm{s}}+\mathrm{t}_{\mathrm{r}}\right) \\
& =150 \mathrm{~ns}+\left(\mathrm{t}_{\mathrm{s}}+\mathrm{t}_{\mathrm{r}}\right)
\end{aligned}
$$

In order to display correctly signal to $y$ channel has to be applied after a delay of 150 ns .
4. The following measurements are obtained on a single phase load $\mathrm{V}=220 \mathrm{~V} \pm 1 \%$. I $=50 \mathrm{~A} \pm 1 \%$ and $\mathrm{W}=555 \mathrm{~W} \pm 2 \%$. If the power factor is calculated using these measurements the worst case error in the calculated power factor in percent is (Give answer up to one decimal place)

Sol. (4\%)

$$
\begin{aligned}
\mathrm{V} & =220 \mathrm{~V} \pm 1 \% \\
\mathrm{I} & =5 \mathrm{~A} \pm 1 \% \\
\mathrm{~W} & =555 \mathrm{~W} \pm 2 \%
\end{aligned}
$$

Since,

$$
\mathrm{W}=\mathrm{VI} \cos \phi
$$

So,

$$
\begin{aligned}
& \frac{\delta \mathrm{W}}{\mathrm{~W}}= \pm\left(\frac{\delta \mathrm{V}}{\mathrm{~V}}+\frac{\Delta \mathrm{I}}{\mathrm{I}}+\frac{\delta(\cos \phi)}{\cos \phi}\right) \\
& 0.02= \pm\left(0.01+0.01+\frac{\delta(\text { p.f. })}{\text { p.f }}\right)
\end{aligned}
$$

Hence in worst case

$$
\begin{aligned}
\frac{\delta(\text { p.f. })}{\text { p.f. }} & =0.02+0.01+0.01 \\
& =0.04
\end{aligned}
$$

or in percent $4 \%$ or $4.0 \%$
5. A closed loop system has the characteristic equation given by $s^{3}+K s^{2}+(K+2) s+3$ $=0$. For this system to be stable, which one of the following conditions should be satisfied?
(a) $0<\mathrm{K}<0.5$
(b) $0.5<\mathrm{K}<1$
(c) $0<\mathrm{K}<1$
(d) $\mathrm{K}>1$

Sol. (d)
By Routh Hurwitz Criteria

| $s^{3}$ | 1 | $\mathrm{k}+2$ |
| :---: | :---: | :---: |
| $\mathrm{~s}^{2}$ | k | 3 |
| $\mathrm{~s}^{1}$ | $\frac{\mathrm{k}(\mathrm{k}+2)-3}{\mathrm{k}}$ |  |
| $\mathrm{s}^{0}$ | 3 |  |

For stability

$$
\begin{equation*}
\mathrm{k}>0 \tag{i}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\mathrm{k}(\mathrm{k}+2)-3}{\mathrm{k}}>0 \tag{ii}
\end{equation*}
$$

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$\Rightarrow \quad \frac{(\mathrm{k}-1)(\mathrm{k}+3)}{\mathrm{k}}>0$
$\Rightarrow$ Either $\mathrm{k}>1 ; \mathrm{k}>-3$ or $\mathrm{k}<1$; $\mathrm{k}<-3$
From equation (i) and (iii)
$\mathrm{k}>1$ hence option (d) is the correct answer.
6. The matrix $\mathrm{A}=\left[\begin{array}{ccc}\frac{3}{2} & 0 & \frac{1}{2} \\ 0 & -1 & 0 \\ \frac{1}{2} & 0 & \frac{3}{2}\end{array}\right]$ has three distinct eigenvalues and one of its eigenvectors is $\left[\begin{array}{l}1 \\ 0 \\ 1\end{array}\right]$. Which one of the following can be another eigenvector of A?
(a) $\left[\begin{array}{c}0 \\ 0 \\ -1\end{array}\right]$
(b) $\left[\begin{array}{c}-1 \\ 0 \\ 0\end{array}\right]$
(c)
$\left[\begin{array}{c}1 \\ -1 \\ 1\end{array}\right]$

Sol. (c)

$$
\mathrm{A}=\left[\begin{array}{ccc}
\frac{3}{2} & 0 & \frac{1}{2} \\
0 & -1 & 0 \\
\frac{1}{2} & 0 & \frac{3}{2}
\end{array}\right]
$$

To find the eigen values of $A$, $\operatorname{det}\left(\mathrm{A}-\lambda \mathrm{I}_{3}\right)=0$ i.e.,

$$
\left|\begin{array}{ccc}
\frac{3}{2}-\lambda & 0 & \frac{1}{2} \\
0 & -1-\lambda & 0 \\
\frac{1}{2} & 0 & \frac{3}{2}-\lambda
\end{array}\right|=0
$$

$$
\begin{aligned}
\left(\frac{3}{2}-\lambda\right)[ & \left.(-1-\lambda)\left(\frac{3}{2}-\lambda\right)\right] \\
+ & \frac{1}{2}\left[0-\frac{1}{2}(-1-\lambda)\right]=0
\end{aligned}
$$

$$
(-1-\lambda)\left[\left(\frac{3}{2}-\lambda\right)^{2}-\frac{1}{4}\right]=0
$$

$$
-(1+\lambda)\left[\lambda^{2}-3 \lambda+2\right]=0
$$

$$
\text { or } \quad-(1+\lambda)(\lambda-1)(\lambda-2)=0
$$

Hence eigen values of A are $-1,1$ and 2 .
Now, let X be an eigen vector of A associated to $\lambda$, then
AX $=\lambda \mathrm{X}$

$$
\text { So, }\left[\begin{array}{ccc}
\frac{3}{2} & 0 & \frac{1}{2} \\
0 & -1 & 0 \\
\frac{1}{2} & 0 & \frac{3}{2}
\end{array}\right]\left[\begin{array}{l}
1 \\
0 \\
1
\end{array}\right]=\lambda\left[\begin{array}{l}
1 \\
0 \\
1
\end{array}\right]
$$

On solving it; $\lambda=2$
Thus for $\lambda=+1$ by taking $X=\left[\begin{array}{l}x \\ y \\ z\end{array}\right]$

$$
\begin{aligned}
& {\left[\mathrm{A}+1 \mathrm{I}_{3}\right] \mathrm{X}=0} \\
& \frac{1}{2} \mathrm{x}+\frac{1}{2} \mathrm{z}=0 \\
& -2 \mathrm{y}=0 \\
& \frac{1}{2} \mathrm{x}+\frac{1}{2} \mathrm{z}=0 \\
& \text { or } \quad \frac{1}{2} \mathrm{x}+\frac{1}{2} \mathrm{z}=0 \\
& \mathrm{y}=0
\end{aligned}
$$

Hence option (c) satisfies.
7. A 10-bus power system consists of four generator buses indexed as G1, G2, G3, G4 and six load buses indexed as L1, L2, L3, $\mathrm{L} 4, \mathrm{~L} 5, \mathrm{~L} 6$. The generator-bus G1 is considered as slack bus and the load buses L3 and L4 are voltage controlled buses. The generator at bus G2 cannot supply the required reactive power demand and hence it is operating at its maximum reactive power limit. The number of non-linear equations required for solving the load flow problem using Newton-Raphson method in polar form is $\qquad$ _.

Sol. (14)
Given data
Total number of buses $(\mathrm{N})=10$
Number of PV buses $\left(x_{1}\right)=2$ (i.e. $G_{3}$ and $\mathrm{G}_{4}$ )
Number of voltage controlled buses $\left(x_{2}\right)=$ 2 (i.e. $\mathrm{L}_{3}$ and $\mathrm{L}_{4}$ )
Number of slack buses $=1$ (i.e. $G_{1}$ )
Number of load buses $=5$
The total number of equations to be solved

$$
\begin{aligned}
& =\left[2 \mathrm{~N}-2-\left(\mathrm{x}_{1}+\mathrm{x}_{2}\right)\right] \\
& =[2(10)-2-(2+2)] \\
& =20-2-4 \\
& =14
\end{aligned}
$$

The size of the jacobian matrix

$$
\begin{aligned}
& =\left[2 \mathrm{~N}-2-\left(\mathrm{x}_{1}+\mathrm{x}_{2}\right)\right] \times \\
& \quad\left[2 \mathrm{~N}-2-\left(\mathrm{x}_{1}+\mathrm{x}_{2}\right)\right] \\
& =14 \times 14
\end{aligned}
$$

8. A 3-bus power system is shown in the figure below, where the diagonal elements of Y bus matrix are : $\mathrm{Y}_{11}=-\mathrm{j} 12 \mathrm{pu}, \mathrm{Y}_{22}=-\mathrm{j} 15$ pu and $\mathrm{Y}_{33}=-\mathrm{j} 7 \mathrm{pu}$


The per unit values of the line reactances $p$, $q$ and $r$ shown in the figure are
(a) $\mathrm{p}=-0.2, \mathrm{q}=-0.1, \mathrm{r}=-0.5$
(b) $\mathrm{p}=0.2, \mathrm{q}=0.1, \mathrm{r}=0.5$
(c) $\mathrm{p}=-5, \mathrm{q}=-10, \mathrm{r}=-2$
(d) $\mathrm{p}=5, \mathrm{q}=10, \mathrm{r}=2$

Sol. (b)
From bus diagram

$$
\begin{aligned}
& \mathrm{y}_{12}=\mathrm{y}_{21}=\frac{1}{\mathrm{jq}} \\
& \mathrm{y}_{13}=\mathrm{y}_{31}=\frac{1}{\mathrm{jr}} \\
& \mathrm{y}_{23}=\mathrm{y}_{32}=\frac{1}{\mathrm{jp}}
\end{aligned}
$$

Since diagonal elements

$$
\mathrm{Y}_{11}=\mathrm{y}_{11}+\mathrm{y}_{12}+\mathrm{y}_{13}
$$

$$
\text { So, } \frac{1}{\mathrm{jq}}+\frac{1}{\mathrm{jr}}=-\mathrm{j} 12
$$

$$
\begin{equation*}
\frac{1}{\mathrm{q}}+\frac{1}{\mathrm{r}}=12 \tag{i}
\end{equation*}
$$

Similary for $Y_{22}=-j 15$

$$
\frac{1}{\mathrm{jq}}+\frac{1}{\mathrm{jp}}=-\mathrm{j} 15
$$

$$
\begin{equation*}
\text { or } \quad \frac{1}{q}+\frac{1}{p}=15 \tag{ii}
\end{equation*}
$$

For $Y_{33}=-j 7$

$$
\frac{1}{\mathrm{jr}}+\frac{1}{\mathrm{jp}}=-\mathrm{j} 7
$$

or $\quad \frac{1}{\mathrm{r}}+\frac{1}{\mathrm{p}}=7$
On solving equations (i), (ii) and (iii)

$$
\frac{1}{\mathrm{p}}=5 ; \frac{1}{\mathrm{q}}=10 ; \frac{1}{\mathrm{r}}=2
$$

Hence,

$$
\mathrm{p}=0.2, \mathrm{q}=0.1, \mathrm{r}=0.5
$$

9. For the power semiconductor devices IGBT, MOSFET, Diode and Thyristor, which one of the following statement is TRUE?
(a) All the four are majority carrier devices
(b) All the four are minority carrier devices
(c) IGBT and MOSFET are majority carrier devices, whereas Diode and Thyristor are minority carrier devices
(d) MOSFET is majority carrier device, whereas IGBT, Diode, Thyristor are minority carrier devices
Sol. (d)
MOSFET is the only majority carrier device among MOSFET, DIODE, Thyristor and IGBT. In majority carrier devices conduction is only because of majority carriers whereas in minority carrier devices conduction is due to both majority and minority carriers.
10. Consider the unity feedback control system shown. The value of $K$ that results in a phase margin of the system to be $30^{\circ}$ is ___. (Give the answer up to two decimal places.)


Sol. (1.05)
For unity feedback system with

$$
G(j \omega) H(j \omega)=\frac{\mathrm{Ke}^{-\mathrm{s}}}{\mathrm{~s}}
$$

Phase margin is given by

$$
\begin{aligned}
& \text { P.M. }=180^{\circ}+\phi \\
& \text { where, } \phi=|\mathrm{G}(\mathrm{j} \omega) \cdot \mathrm{H}(\mathrm{j} \omega)|_{\omega=\omega_{\mathrm{gc}}} \\
& \text { and }|\mathrm{G}(\mathrm{j} \omega) \cdot \mathrm{H}(\mathrm{j} \omega)|_{\omega=\omega_{\mathrm{gc}}}=1
\end{aligned}
$$

Since, $\left|\mathrm{e}^{-\mathrm{j} \omega}\right|=1$
So, $\frac{\mathrm{k}}{\omega}=1$ at $\omega=\omega_{\mathrm{gc}}$

So, $\omega_{\mathrm{gc}}=\mathrm{k}$
Now, $\phi=G(\mathrm{j} \omega) \mathrm{H}(\mathrm{j} \omega)=-90^{\circ}-57.3 \omega$
at $\omega_{\mathrm{gc}}=\mathrm{k}$

$$
\phi=-90^{\circ}-57.3 \mathrm{k}
$$

$\mathrm{PM}=180^{\circ}+\phi=30^{\circ}$
$\Rightarrow \phi=-150^{\circ}$
$\Rightarrow-90^{\circ}-57.3 \mathrm{k}=-150^{\circ}$
$\Rightarrow \mathrm{k}=1.047$
Upto two decimal places

$$
\mathrm{k}=1.05
$$

11. The transfer function of a system is given by

$$
\frac{V_{0}(s)}{V_{i}(s)}=\frac{1-s}{1+s}
$$

Let the output of the system be $\mathrm{v}_{0}(\mathrm{t})=$ $\mathrm{V}_{\mathrm{m}} \sin (\omega \mathrm{t}+\phi)$ for the input, $\mathrm{v}_{\mathrm{i}}(\mathrm{t})=$ $\mathrm{V}_{\mathrm{m}} \sin (\omega \mathrm{t})$. Then the minimum and maximum values of $\phi$ (in radians) are respectively
(a) $-\frac{\pi}{2}$ and $\frac{\pi}{2}$
(b) $-\frac{\pi}{2}$ and 0
(c) 0 and $\frac{\pi}{2}$
(d) $-\pi$ and 0

Sol. (d)
For transfer function

$$
\begin{aligned}
\frac{\mathrm{V}_{0}(\mathrm{~s})}{\mathrm{V}_{\mathrm{i}}(\mathrm{~s})} & =\frac{1-\mathrm{s}}{1+\mathrm{s}} \\
\left|\frac{\mathrm{~V}_{0}(\mathrm{j} \omega)}{\mathrm{V}_{\mathrm{i}}(\mathrm{j} \omega)}\right| & =1 \\
\left\lvert\, \frac{\mathrm{V}_{0}(\mathrm{j} \omega)}{\mathrm{V}_{\mathrm{i}}(\mathrm{j} \omega)}\right. & =-2 \tan ^{-1} \omega
\end{aligned}
$$

Here,

$$
\begin{aligned}
\mathrm{v}_{\mathrm{i}}(\mathrm{t}) & =\mathrm{V}_{\mathrm{m}} \sin (\omega \mathrm{t}) \\
\mathrm{v}_{0}(\mathrm{t}) & =\mathrm{V}_{\mathrm{m}} \sin (\omega \mathrm{t}+\phi)
\end{aligned}
$$

So, for

$$
\omega=0 \text { to } \omega=\infty
$$

# M IES MASTER 

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$-2 \tan ^{-1} \omega$ varies from $-180^{\circ}$ to $0^{\circ}$
Hence, option (d) is the correct answer.
12. For the circuit shown in the figure below, assume that diodes $\mathrm{D}_{1}, \mathrm{D}_{2}$ and $\mathrm{D}_{3}$ are ideal.


The DC components of voltages $\mathrm{v}_{1}$ and $\mathrm{v}_{2}$, respectively are
(a) 0 V and 1 V
(b) -0.5 V and 0.5 V
(c) 1 V and 0.5 V
(d) 1 V and 1 V

Sol. (b)
During positive half cycle $\mathrm{D}_{1} \mathrm{ON}, \mathrm{D}_{2}$ and $\mathrm{D}_{3}$ will be OFF During negative half cycle $\mathrm{D}_{2}$ and $\mathrm{D}_{3} \mathrm{ON}$ but $\mathrm{D}_{1} \mathrm{OFF}$


$$
\begin{aligned}
& \mathrm{V}_{1(\text { avg })}=\frac{1}{2 \pi}\left[\int_{0}^{\pi} \frac{\pi}{2} \sin (100 \pi \mathrm{t}) \mathrm{d}(\omega \mathrm{t})\right. \\
& \left.+\int_{\pi}^{2 \pi} \pi \sin (100 \pi \mathrm{t}) \mathrm{d}(\omega \mathrm{t})\right] \\
& =\frac{1}{2 \pi}[\pi-2 \pi]=-\frac{1}{2}=-0.5 \mathrm{~V} \\
& \mathrm{~V}_{2 \text { (avg.) }}=\frac{1}{2 \pi}\left[\int_{0}^{\pi} \frac{\pi}{2} \sin (100 \pi \mathrm{t}) \mathrm{d}(\omega \mathrm{t})\right] \\
& =\frac{1}{2 \pi}\left[\frac{\pi}{2}(-\cos \pi+\cos 0)\right] \\
& =\frac{1}{2}=0.5
\end{aligned}
$$

13. The power supplied by the 25 V source in the figure shown below is $\qquad$ -W


Sol. (250)


Using Kirchoffs current law at node 'a'

$$
\begin{array}{rlrl}
\mathrm{I}+0.4 \mathrm{I} & =14 \\
\Rightarrow \quad & \quad \mathrm{I} & =10 \mathrm{~A}
\end{array}
$$

Power supplied by 25 V source;

$$
\begin{aligned}
& \mathrm{P}=25 \mathrm{~V} \times 10 \mathrm{~A} \\
& \mathrm{P}=250 \mathrm{watt}
\end{aligned}
$$

14. A three-phase, 50 Hz , star-connected cylindrical-rotor synchronous machine is running as a motor. The machine is operated from a 6.6 kV grid and draws current at unity power factor (UPF). The synchronous reactance of the motor is $30 \Omega$ per phase. The load angle is $30^{\circ}$. The power
deliver to the motor in kW is $\qquad$ (Give the answer up to one decimal place).
Sol. (838.3 kW)
Given that,

$$
\begin{aligned}
\mathrm{V} & =6.6 \mathrm{kV} \\
\delta & =30^{\circ} \\
\text { P.f } & =1(\mathrm{UPF})
\end{aligned}
$$

Synchronoces reactance $\left(\mathrm{X}_{\mathrm{s}}\right)=30 \Omega$

$$
\mathrm{P}=\frac{\mathrm{VE}_{\mathrm{f}}}{\mathrm{X}_{\mathrm{s}}} \sin \delta
$$

For unity P.f. for synchronous motor.


From above phasor diagram,

$$
\mathrm{E}_{\mathrm{f}} \cos \delta=\mathrm{V}
$$

or, $\quad E_{f}=\frac{V}{\cos \delta}$

$$
=\frac{6.6 \mathrm{kV}}{\cos 30^{\circ}}
$$

$$
=7.62 \mathrm{kV}
$$

Hence,

$$
\mathrm{P}=\frac{7.62 \times 6.6}{30} \sin 30^{\circ} \mathrm{MW}
$$

$$
0.8383 \mathrm{MW}
$$

or,

$$
\mathrm{P}=838.3 \mathrm{~kW}
$$

15. A source is supplying a load through a 2 phase, 3 -wire transmission system as shown in figure below. The instantaneous voltage and current in phase-a are $\mathrm{v}_{\mathrm{an}}=$ $220 \sin (100 \pi t) \mathrm{V}$ and $\mathrm{i}_{\mathrm{a}}=10 \sin (100 \pi \mathrm{t}) \mathrm{A}$, respectively. Similarly for phase-b, the instantaneous voltage and current are $\mathrm{V}_{\mathrm{bn}}$ $=220 \cos (100 \pi t) V$ and $\mathrm{i}_{\mathrm{b}}=10 \cos (100 \pi \mathrm{t}) \mathrm{A}$


The total instantaneous power flowing from the source to the load is
(a) 2200 W
(b) $2200 \sin ^{2}(100 \pi \mathrm{t}) \mathrm{W}$
(c) 4400 W
(d) $2200 \sin (100 \pi \mathrm{t}) \cos (100 \pi \mathrm{t}) \mathrm{W}$

## Sol. (a)

Instantaneous power;

$$
\begin{aligned}
& P=v \cdot i \\
& P=v_{a n} \cdot i_{a}+v_{b n} \cdot i_{b}
\end{aligned}
$$

$=220 \sin (100 \pi \mathrm{t}) \cdot 10 \sin (100 \pi \mathrm{t})$
$+220 \cos (100 \pi \mathrm{t}) \cdot 10 \cos (100 \pi \mathrm{t})$

$$
\begin{gathered}
=2200 \sin ^{2}(100 \pi \mathrm{t})+2200 \cos ^{2}(100 \pi \mathrm{t}) \\
\mathrm{P}=2200 \mathrm{~W}
\end{gathered}
$$

16. For a complex number $z$,
$\lim _{z \rightarrow i} \frac{z^{2}+1}{z^{3}+2 z-i\left(z^{2}+2\right)}$ is
(a) -2 i
(b) -i
(c) i
(d) 2 i

Sol. (d)

$$
\lim _{z \rightarrow i} \frac{z^{2}+1}{z^{3}+2 z-i\left(z^{2}+2\right)}
$$

This is $\frac{0}{0}$ form, so on differentiating both numerator and denominator
$\lim _{z \rightarrow i} \frac{2 z}{3 z^{2}+2-2 z i}$

$$
=\frac{2 \mathrm{i}}{3\left(\mathrm{i}^{2}\right)+2-2(\mathrm{i}) \mathrm{i}}=\frac{2 \mathrm{i}}{-3+2+2}=2 \mathrm{i}
$$

17. Let $I=c \iint_{R} x y^{2} d x d y$, where $R$ is the region shown in the figure and $c=6 \times 10^{-4}$. The value of I equals $\qquad$ . (Give the answer up to two decimal places).


Sol. (0.99)
$I=c \iint_{R} x y^{2} d x d y$


Region R is bounded by $\mathrm{y}=0$ and $\mathrm{y}=2 \mathrm{x}$

$$
\begin{aligned}
I & =c\left[\int_{1}^{5} \int_{0}^{2 x} x y^{2} d y d x\right] \\
& =c\left[\left.\int_{1}^{5}\left(\frac{x y^{3}}{3}\right)\right|_{0} ^{2 x} d x\right] \\
& =c\left[\int_{1}^{5} \frac{8}{3} x^{4} d x\right] \\
& =c\left[\left.\frac{8}{15} x^{5}\right|_{1} ^{5}\right]=c\left[\frac{8}{15}\left(5^{5}-1\right)\right]
\end{aligned}
$$

$$
=6 \times 10^{-4} \times\left[\frac{8}{15}\left(5^{5}-1\right)\right]
$$

$\mathrm{I}=0.99$ (upto two decimal places)
18. A 4 pole induction machine is working as an induction generator. The generator supply frequency is 60 Hz . The rotor current frequency is 5 Hz . The mechanical speed of the rotor in RPM is
(a) 1350
(b) 1650
(c) 1950
(d) 2250

Sol. (c)
For 4 pole, 60 Hz induction machine synchronous speed;

$$
\begin{aligned}
\mathrm{N}_{\mathrm{s}} & =\frac{120 \times \mathrm{f}}{\mathrm{P}} \\
& =\frac{120 \times 60}{4} \\
& =1800 \text { r.p.m. } \\
\mathrm{s} & =\frac{\mathrm{f}_{\mathrm{r}}}{\mathrm{f}_{\mathrm{s}}} \\
& =\frac{5}{60}
\end{aligned}
$$

For induction generator slip is negative, So,

$$
\frac{\mathrm{N}_{\mathrm{s}}-\mathrm{N}_{\mathrm{r}}}{\mathrm{~N}_{\mathrm{s}}}=-\frac{5}{60}
$$

$$
\begin{array}{ll}
\Rightarrow & \frac{1800-\mathrm{N}_{\mathrm{r}}}{1800}=\frac{-5}{60} \\
\Rightarrow & 1800-\mathrm{N}_{\mathrm{r}}=-\frac{5}{60} \times 1800
\end{array}
$$

$$
\Rightarrow \quad \mathrm{N}_{\mathrm{r}}=1800+\frac{5}{60} \times 1800
$$

$$
\mathrm{N}_{\mathrm{r}}=1950 \text { r.p.m. }
$$

19. A 3-phase voltage source inverter is supplied from a 600 V DC source as shown in the figure below. For a star connected resistive load of $20 \Omega$ per phase, the load

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power for $120^{\circ}$ device conduction, in kW , is
$\qquad$


Sol. ( 9 kW)
In $120^{\circ}$ device canduction mode and star connected load:

At any instant only 2 IGBTs will conduct so, when IGBT 1 and 6 are conducting in $0-60^{\circ}$ cycle, equivalent ckt can be given as


So power, $\quad \mathrm{P}=\frac{\left(\frac{\mathrm{V}_{\mathrm{s}}}{2}\right)^{2}}{\mathrm{R}} \times 2$
$=\frac{\mathrm{V}_{\mathrm{s}}^{2}}{2 \mathrm{R}}$
$P=\frac{(600)^{2}}{2 \times 20}$
$=9000$ watt
or,
$=9 \mathrm{~kW}$
20. A solid iron cylinder is placed in a region containing a uniform magnetic field such that the cylinder axis is parallel to the magnetic field direction. The magnetic field lines inside the cylinder will
(a) Bend closer to the cylinder axis
(b) Bend farther away from the axis
(c) Remain uniform as before
(d) Cease to exist inside the cycliner

Sol. (a)
The magnetic field lines will bend closer to the cylinder axis to find a minimum reluctance path.
21. Consider the system with following inputoutput relation

$$
\mathrm{y}[\mathrm{n}]=\left(1+(-1)^{\mathrm{n}}\right) \mathrm{x}[\mathrm{n}]
$$

where, $x[n]$ is the input and $y[n]$ is the output. The system is
(a) Invertible and time invariant
(b) Invertible and time varying
(c) Non-invertible and time invariant
(d) Non-invertible and time varying

Sol. (d)

$$
\mathrm{y}[\mathrm{n}]=\left(1+(-1)^{\mathrm{n}}\right) \mathrm{x}[\mathrm{n}]
$$

A system is said to be invertible if there is a one-to-one correspondence between its input and output signals.
for $\mathrm{n}=1$,

$$
y[1]=\left(1+(-1)^{1}\right) x[1]=0
$$

for $\mathrm{n}=2$,

$$
y[2]=\left(1+(-1)^{2}\right) x[2]=0
$$

for $\mathrm{n}=3$,

$$
y[3]=\left(1+(-1)^{3}\right) x[3]=0
$$

Here for odd values of ' $n$ ' output will always be zero so system is non-invertible.

To check time invariancy
For delayed input,

$$
\begin{equation*}
\mathrm{y}\left[\mathrm{n}_{1} \mathrm{n}_{0}\right]=\left(1+(-1)^{\mathrm{n}}\right) \mathrm{x}\left[\mathrm{n}-\mathrm{n}_{0}\right] \tag{i}
\end{equation*}
$$

For delayed response,

$$
\begin{equation*}
\mathrm{y}\left[\mathrm{n}_{1} \mathrm{n}_{0}\right]=\left(1+(-1)^{\mathrm{n}-\mathrm{n}_{0}}\right) \mathrm{x}\left[\mathrm{n}-\mathrm{n}_{0}\right] \tag{ii}
\end{equation*}
$$



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For time invariant system output for delayed input should be equal to delayed response.
Hence this system is time varying.
22. Consider $\mathrm{g}(\mathrm{t})=$
$\left\{\begin{array}{ll}\mathrm{t}-\lfloor\mathrm{t}\rfloor, & \mathrm{t} \geq 0 \\ \mathrm{t}-\Gamma \mathrm{t} 7, & \text { otherwise }\end{array}\right.$, where $\mathrm{t} \in \mathrm{R}$
Here, $\lfloor\mathrm{t} \perp$ represents the largest integer less than or equal to t and $\lceil\mathrm{t}\rceil$ denotes the smallest integer greater than or equal to $t$. The coefficient of the second harmonic component of the Fourier series representing $g(t)$ is $\qquad$
Sol. (0.0796)

$$
\mathrm{g}(\mathrm{t})=\left\{\begin{array}{lc}
\mathrm{t}-\lfloor\mathrm{t}\rfloor, & \mathrm{t}>0 \\
\mathrm{t}-\lceil\mathrm{t}, & \text { otherwise }
\end{array}\right.
$$



$$
\mathrm{T}=1
$$

$$
\begin{aligned}
& \omega_{0}=\frac{2 \pi}{\mathrm{~T}}=2 \pi \\
& \mathrm{~g}(\mathrm{t})=\mathrm{t}
\end{aligned} \quad 0 \leq \mathrm{t} \leq 1 .
$$

$$
g(t)=\sum_{n=-\infty}^{\infty} G_{n} e^{-j n \omega_{0} t}
$$

$$
\mathrm{G}_{\mathrm{n}}=\frac{1}{\mathrm{~T}} \int_{-\mathrm{T} / 2}^{\mathrm{T} / 2} \mathrm{~g}(\mathrm{t}) \mathrm{e}^{-\mathrm{jn} \omega_{0} \mathrm{t}} \mathrm{dt}
$$

$$
=\frac{1}{1} \int_{0}^{1}(\mathrm{t}) \mathrm{e}^{-\mathrm{j} 2 \pi \mathrm{nt}} \mathrm{dt}
$$

$$
=\int_{0}^{1}(\mathrm{t}) \mathrm{e}^{-\mathrm{j} 2 \pi n \mathrm{n}} \mathrm{dt}
$$

$$
\mathrm{G}_{\mathrm{n}}=\int_{0}^{1} \mathrm{t}^{-\mathrm{j} 2 \pi \mathrm{nt}} \mathrm{dt}
$$

$$
=\left.\mathrm{t} \cdot \frac{\mathrm{e}^{-\mathrm{j} 2 \pi \mathrm{nt}}}{-\mathrm{j} 2 \pi \mathrm{n}}\right|_{0} ^{1}-\int_{0}^{1} 1 \cdot \frac{\mathrm{e}^{-\mathrm{j} 2 \pi \mathrm{nt}}}{-\mathrm{j} 2 \pi \mathrm{n}} d \mathrm{t}
$$

$$
\begin{aligned}
& =\frac{-1}{j 2 \pi n}-\frac{1}{j^{2} 4 \pi^{2} \mathrm{n}^{2}}\left(\mathrm{e}^{-\mathrm{j} 2 \pi \mathrm{n}}-1\right) \\
\mathrm{G}_{\mathrm{n}} & =\frac{-1}{\mathrm{j} 2 \pi \mathrm{n}} \\
\mathrm{G}_{2} & =\frac{-1}{\mathrm{j} 4 \pi} \\
\left|\mathrm{G}_{2}\right| & =\frac{1}{4 \pi}=0.0796
\end{aligned}
$$

23. The boolean expression $A B+A \bar{C}+B C$ simplifies to
(a) $\mathrm{BC}+\mathrm{A} \overline{\mathrm{C}}$
(b) $\mathrm{AB}+\mathrm{A} \overline{\mathrm{C}}+\mathrm{B}$
(c) $\mathrm{AB}+\mathrm{A} \overline{\mathrm{C}}$
(d) $\mathrm{AB}+\mathrm{BC}$

Sol. (a)

$$
\mathrm{f}=\mathrm{AB}+\mathrm{A} \overline{\mathrm{C}}+\mathrm{BC}
$$

Using k-map,


$$
\mathrm{f}=\mathrm{A} \overline{\mathrm{C}}+\mathrm{BC}
$$

24. Let $z(t)=x(t) * y(t)$, where $n * "$ denotes convolution. Let c be a positive real-valued constant. Choose the correct expression for z(ct)
(a) $\quad c . x(c t) * y(c t)$
(b) $x(c t) * y(c t)$
(c) $\quad c . x(t) * y(c t)$
(d) $\quad$ c. $x(c t) * y(t)$

Sol. (a)

$$
\mathrm{z}(\mathrm{t})=\mathrm{x}(\mathrm{t}) * \mathrm{y}(\mathrm{t})
$$

taking fourier transform

$$
\begin{align*}
& Z(j \omega)=X(j \omega) \cdot Y(j \omega)  \tag{1}\\
& Z(t) \longrightarrow \frac{1}{c} Z\left(\frac{j \omega}{c}\right) \tag{2}
\end{align*}
$$

Also, byusing eq. (1)

$$
\begin{aligned}
Z\left(\frac{j \omega}{c}\right) & =X\left(\frac{j \omega}{c}\right) \cdot Y\left(\frac{j \omega}{c}\right) \\
\therefore \quad \frac{1}{c} Z\left(\frac{j \omega}{c}\right) & =\frac{1}{c} X\left(\frac{j \omega}{c}\right) \cdot Y\left(\frac{j \omega}{c}\right)
\end{aligned}
$$

Multiplying and dividing R.H.S. by c

$$
\begin{aligned}
\frac{1}{c} Z\left(\frac{j \omega}{c}\right) & =c\left[\frac{1}{c} \cdot X\left(\frac{j \omega}{c}\right) \cdot \frac{1}{c} Y\left(\frac{j \omega}{c}\right)\right] \\
z(t) & =c \cdot x(c t) * y(c t)
\end{aligned}
$$

25. In the converter circuit shown below, the switches are controlled such that the load voltage $\mathrm{v}_{0}(\mathrm{t})$ is a 400 Hz square wave.


The RMS value of the fundamental component of $\mathrm{v}_{0}(\mathrm{t})$ in volts is
Sol. (198.069)
For single phase full bridge inverter,

$$
v_{0}=\sum_{\mathrm{n}=1,3,5 \ldots . .}^{\mathrm{n}} \frac{4 \mathrm{~V}_{\mathrm{s}}}{\mathrm{n} \pi} \sin n \omega \mathrm{t}
$$

Volts
RMS value of fundamental component is given by:

$$
\begin{aligned}
\frac{4 \mathrm{~V}_{\mathrm{s}}}{\pi} \times \frac{1}{\sqrt{2}} & =\frac{4 \times 220}{\pi} \times \frac{1}{\sqrt{2}} \\
& =198.069 \text { Volts. }
\end{aligned}
$$

26. The positive, negative and zero sequence reactances of a wye-connected synchronous generator are $0.2 \mathrm{pu}, 0.2 \mathrm{pu}$ and 0.1 pu respectively. The generator is on open circuit with a terminal voltage of 1 pu . The minimum value of the inductive reactance, in pu, required to be connected between neutral and ground so that the fault current does not exceed 3.75 pu if a single line to ground fault occurs at the terminals is (assume fault impedance to be zero). (Give the answer up to one decimal place).

## Sol. (0.1 pu)

Positive sequence reactance $\mathrm{X}_{1}=0.2 \mathrm{pu}$
Negative sequence reactance $\mathrm{X}_{2}=0.2 \mathrm{pu}$
Zero sequence reactance $X_{0}=0.1 \mathrm{pu}$
Fault current $\mathrm{i}_{\mathrm{f}}=3.75 \mathrm{pu}$
For single line to ground fault,

$$
\mathrm{i}_{\mathrm{f}}=\frac{3 \mathrm{E}}{\mathrm{X}_{1}+\mathrm{X}_{2}+\mathrm{X}_{0}+3 \mathrm{X}_{\mathrm{n}}}
$$

Where $\mathrm{X}_{\mathrm{n}}$ is the rectance connected between neutral and ground.
Therefore,for $\mathrm{E}=1 \mathrm{pu}$

$$
\mathrm{i}_{\mathrm{f}}=\frac{3}{0.2+0.2+0.1+3 \mathrm{X}_{\mathrm{n}}}=3.75
$$

On solying,

$$
\mathrm{X}_{\mathrm{n}}=0.1 \mathrm{pu}
$$

27. Two passive two-port networks are connected in cascade as shown in figure. A voltage source is connected at port 1


Given

$$
\begin{aligned}
& \mathrm{V}_{1}=\mathrm{A}_{1} \mathrm{~V}_{2}+\mathrm{B}_{1} \mathrm{I}_{2} \\
& \mathrm{I}_{1}=\mathrm{C}_{1} \mathrm{~V}_{2}+\mathrm{D}_{1} \mathrm{I}_{2} \\
& \mathrm{~V}_{2}=\mathrm{A}_{2} \mathrm{~V}_{3}+\mathrm{B}_{2} \mathrm{I}_{3} \\
& \mathrm{I}_{2}=\mathrm{C}_{2} \mathrm{~V}_{3}+\mathrm{D}_{2} \mathrm{I}_{3}
\end{aligned}
$$

$\mathrm{A}_{1}, \mathrm{~B}_{1}, \mathrm{C}_{1}, \mathrm{D}_{1}, \mathrm{~A}_{2}, \mathrm{~B}_{2}, \mathrm{C}_{2}$ and $\mathrm{D}_{2}$ are the generalized circuit constants. If the Thevenin equivalent circuit at port 3 consists of a voltage source $\mathrm{V}_{\mathrm{T}}$ and an impedance $\mathrm{Z}_{\mathrm{T}}$, connected in series, then
(a) $\mathrm{V}_{\mathrm{T}}=\frac{\mathrm{V}_{1}}{\mathrm{~A}_{1} \mathrm{~A}_{2}}, \mathrm{Z}_{\mathrm{T}}=\frac{\mathrm{A}_{1} \mathrm{~B}_{2}+\mathrm{B}_{1} \mathrm{D}_{2}}{\mathrm{~A}_{1} \mathrm{~A}_{2}+\mathrm{B}_{1} \mathrm{C}_{2}}$
(b) $\quad \mathrm{V}_{\mathrm{T}}=\frac{\mathrm{V}_{1}}{\mathrm{~A}_{1} \mathrm{~A}_{2}+\mathrm{B}_{1} \mathrm{C}_{2}}, \mathrm{Z}_{\mathrm{T}}=\frac{\mathrm{A}_{1} \mathrm{~B}_{2}+\mathrm{B}_{1} \mathrm{D}_{2}}{\mathrm{~A}_{1} \mathrm{~A}_{2}}$
(c) $\mathrm{V}_{\mathrm{T}}=\frac{\mathrm{V}_{1}}{\mathrm{~A}_{1} \mathrm{~A}_{2}}, \mathrm{Z}_{\mathrm{T}}=\frac{\mathrm{A}_{1} \mathrm{~B}_{2}+\mathrm{B}_{1} \mathrm{D}_{2}}{\mathrm{~A}_{1} \mathrm{~A}_{2}}$
(d) $\quad \mathrm{V}_{\mathrm{T}}=\frac{\mathrm{V}_{1}}{\mathrm{~A}_{1} \mathrm{~A}_{2}+\mathrm{B}_{1} \mathrm{C}_{2}}, \mathrm{Z}_{\mathrm{T}}=\frac{\mathrm{A}_{1} \mathrm{~B}_{2}+\mathrm{B}_{1} \mathrm{D}_{2}}{\mathrm{~A}_{1} \mathrm{~A}_{2}+\mathrm{B}_{1} \mathrm{C}_{2}}$

## Sol. (d)

For cascaded network, equivalent ABCD parameters are given by,

$$
\begin{aligned}
& \quad\left[\begin{array}{ll}
\mathrm{A} & \mathrm{~B} \\
\mathrm{C} & \mathrm{D}
\end{array}\right]=\left[\begin{array}{ll}
\mathrm{A}_{1} & \mathrm{~B}_{1} \\
\mathrm{C}_{1} & \mathrm{D}_{1}
\end{array}\right]\left[\begin{array}{ll}
\mathrm{A}_{2} & \mathrm{~B}_{2} \\
\mathrm{C}_{2} & \mathrm{D}_{2}
\end{array}\right] \\
& =\left[\begin{array}{ll}
\mathrm{A}_{1} \mathrm{~A}_{2}+\mathrm{B}_{1} \mathrm{C}_{2} & \mathrm{~A}_{1} \mathrm{~B}_{2}+\mathrm{B}_{1} \mathrm{D}_{2} \\
\mathrm{C}_{1} \mathrm{~A}_{2}+\mathrm{D}_{1} \mathrm{C}_{2} & \mathrm{C}_{1} \mathrm{~B}_{2}+\mathrm{D}_{1} \mathrm{D}_{2}
\end{array}\right]
\end{aligned}
$$

Therefore,

$$
\left[\begin{array}{l}
\mathrm{V}_{1} \\
\mathrm{I}_{1}
\end{array}\right]=\left[\begin{array}{ll}
\mathrm{A} & \mathrm{~B} \\
\mathrm{C} & \mathrm{D}
\end{array}\right]\left[\begin{array}{l}
\mathrm{V}_{3} \\
\mathrm{I}_{3}
\end{array}\right]
$$

Now, open circuit voltage $V_{T}=V_{3}$ when $\mathrm{I}_{3}=0$.

$$
\begin{array}{rlrl}
\text { So, } & & \mathrm{V}_{1} & =\mathrm{AV}_{3} \\
& & =\mathrm{AV}_{\mathrm{T}} \\
\Rightarrow \quad & & \mathrm{~V}_{\mathrm{T}} & =\frac{\mathrm{V}_{1}}{\mathrm{~A}} \\
\Rightarrow \quad & \mathrm{~V}_{\mathrm{T}} & =\frac{\mathrm{V}_{1}}{\mathrm{~A}_{1} \mathrm{~A}_{2}+\mathrm{B}_{1} \mathrm{C}_{2}}
\end{array}
$$

To calculate Thevenin equivalent impedance $Z_{T}$ voltage source is short circuited. So for voltage source of $\mathrm{V}_{3}$ feeding current $\left(-\mathrm{I}_{3}\right)$

$$
\begin{aligned}
\mathrm{Z}_{\mathrm{T}} & =\frac{\mathrm{V}_{3}}{-\mathrm{I}_{3}} \\
0 & =\mathrm{AV}_{3}+\mathrm{BI}_{3} \\
\mathrm{Z}_{\mathrm{T}} & =\frac{\mathrm{B}}{\mathrm{~A}} \\
\mathrm{Z}_{\mathrm{T}} & =\frac{\mathrm{A}_{1} \mathrm{~B}_{2}+\mathrm{B}_{1} \mathrm{D}_{2}}{\mathrm{~A}_{1} \mathrm{~A}_{2}+\mathrm{B}_{1} \mathrm{C}_{2}}
\end{aligned}
$$

Hence, option (d) is the correct answer.
28. A 220 V DC series motor runs drawing a current of 30 A from the supply. Armature and field circuit resistances are $0.4 \Omega$ and $0.1 \Omega$, respectively. The load torque varies as the square of the speed. The flux in the motor may be taken as being proportional
to the armature current. To reduce the speed of the motor by $50 \%$, the resistance in ohms the should be added in series with the armature is $\qquad$ . (Give the answer up to two decimal places).
Sol. (10.75 $\Omega$ )


Back E.m.f. $\mathrm{E}_{1}=220-30(0.1+0.4)$

$$
=205 \mathrm{~V}
$$

Torque,

$$
\tau=\phi I_{a}
$$

\{where $\phi=$ flux, $\mathrm{Ia}=$ armature current $\}$

$$
\begin{equation*}
\tau=I_{a}^{2} \tag{i}
\end{equation*}
$$

as in series motor $\phi \propto I_{a}$
Also, $\quad \tau \propto N^{2}$
here N is the speed of motor
From eq. (i) \& (ii)
$\mathrm{I}_{\mathrm{a}}^{2} \propto \mathrm{~N}^{2}$ or $\mathrm{I}_{\mathrm{a}} \propto \mathrm{N}$
Therefore to reduce speed by $50 \%, \mathrm{I}_{\mathrm{a}}$ will reduce to $50 \%$ i.e., 15 A .
Now back emf will change to

$$
\mathrm{E}_{2}=220-15(\mathrm{R}+0.1+0.4)
$$

Where $R$ is the external resistance added in series with armature.
Since, $E \propto \phi \cdot N$
So, $E_{1} \propto \phi_{1} \cdot N_{1}$

$$
\begin{aligned}
& \mathrm{E}_{2} \propto \phi_{2} \cdot \mathrm{~N}_{2} \\
& \phi_{2}=\frac{\phi_{1}}{2} \\
& \mathrm{~N}_{2}=\frac{\mathrm{N}_{1}}{2}
\end{aligned}
$$

Thus,

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$$
\begin{array}{r}
205=\phi_{1} \mathrm{~N}_{1} \\
220-15(\mathrm{R}+0.5)=\phi_{2} \mathrm{~N}_{2}
\end{array}
$$

dividing eq. (v) by eq. (iv),

$$
\begin{aligned}
& \frac{220-15(\mathrm{R}+0.5)}{205}=\frac{\phi_{2} \mathrm{~N}_{2}}{\phi_{1} \mathrm{~N}_{1}}=\frac{\left(\phi_{1} / 2\right) \cdot\left(\mathrm{N}_{1} / 2\right)}{\phi_{1} \cdot \mathrm{~N}_{1}} \\
& \frac{220-15(\mathrm{R}+0.5)}{205}=\frac{1}{4}
\end{aligned}
$$

on solving,

$$
\mathrm{R}=10.75 \Omega
$$

29. The logical gate implemented using the circuit shown below where, $\mathrm{V}_{1}$ and $\mathrm{V}_{2}$ are inputs (with 0 V as digital 0 and 5 V as digital 1) and $V_{\text {OUT }}$ is the output, is

(a) NOT
(b) NOR
(c) NAND
(d) XOR

Sol. (b)
From the given circuit it can be deduced that $Q_{1}$ will be $O N$ when $V_{1}$ is high, $Q_{2}$ is ON when $V_{2}$ is high

## Truth Table

| $\mathrm{V}_{1}$ | $\mathrm{Q}_{1}$ | $\mathrm{~V}_{2}$ | $\mathrm{Q}_{2}$ | $\mathrm{~V}_{\text {out }}$ |
| :--- | :--- | :--- | :--- | :--- |
| High | ON | High | ON | Low |
| High | ON | Low | OFF | Low |
| Low | OFF | High | ON | Low |
| Low | OFF | Low | OFF | High |

This is the truth table of NOR gate.
30. A function $f(x)$ is defined as $f(x)=$
$\left\{\begin{array}{cc}e^{x}, & x<1 \\ \ln x+\mathrm{ax}^{2}+\mathrm{bx} & \mathrm{x} \geq 1\end{array}\right\}, \quad$ where $\quad \mathrm{x} \in \mathbb{R}$.
Which one of the following statement is TRUE?
(a) $f(x)$ is NOT differentiable at $x=1$ for any values of $a$ and $b$.
(b) $f(x)$ is differentiable at $x=1$ for the unique value of $a$ and $b$.
(c) $f(x)$ is differentiable at $x=1$ for all values of $a$ and $b$ such that $a+b=e$.
(d) $f(x)$ is differentiable at $x=1$ for all values of $a$ and $b$

Sol. (b)

$$
f(x)=\left\{\begin{array}{cc}
e^{x}, & x<1 \\
\ln x+a x^{2}+b x, & x \geq 1
\end{array}\right.
$$

Left hand derivative $=\lim _{x \rightarrow 1} \frac{f(x)-f(1)}{x-1}$
where $f(1)=\ln (1)+a(1)^{2}+b(1)=a+b$ so,
Left hand derivatives (LHD)

$$
=\lim _{x \rightarrow 1} \frac{e^{x}-(a+b)}{x-1}
$$

this limit exists if $\mathrm{e}=\mathrm{a}+\mathrm{b}$
then

$$
\mathrm{LHD}=\lim _{\mathrm{x} \rightarrow 1} \frac{\mathrm{e}^{\mathrm{x}}}{1}=\mathrm{e}
$$

Right hand derivative (RHD)

$$
\begin{aligned}
& =\lim _{x \rightarrow 1} \frac{f(x)-f(1)}{x-1} \\
& =\lim _{x \rightarrow 1} \frac{\ln x+a x^{2}+b x-(a+b)}{x-1}
\end{aligned}
$$

this limit exist if $1+2 \mathrm{a}+\mathrm{b}=\mathrm{a}+\mathrm{b}$
or $\quad a=-1$
Hence

$$
\mathrm{RHD}=\lim _{\mathrm{x} \rightarrow 1} \frac{\frac{1}{\mathrm{x}}+2 \mathrm{ax}+\mathrm{b}}{1}=1+2 \mathrm{a}+\mathrm{b}
$$

LHD will be equal to RHD if $\mathrm{a}=-1$ and b $=\mathrm{e}+1$
Hence, $\mathrm{f}(\mathrm{x})$ is differentiable at $\mathrm{x}=1$ for unique values of $a$ and $b$.
31. The switch in the figure below was closed for a long time. It is opened at $t=0$. The current in the inductor of 2 H for $\mathrm{t} \geq 0$, is

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10
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(a) $2.5 \mathrm{e}^{-4 \mathrm{t}}$
(b) $5 e^{-4 t}$
(c) $2.5 \mathrm{e}^{-0.25 \mathrm{t}}$
(d) $5 \mathrm{e}^{-0.25 \mathrm{t}}$

Sol. (a)
For $t=0^{-}$circuit can be represented as below


Inductor can be taken as short circuit at steady state.
So current in inductor at $\mathrm{t}=0^{-}$will be

$$
\begin{aligned}
\mathrm{i}_{\mathrm{L}}\left(0^{-}\right) & =\frac{50}{6+(8118)} \times \frac{8}{(8+8)} \\
\Rightarrow \quad \mathrm{i}_{\mathrm{L}} & =2.5 \mathrm{~A}
\end{aligned}
$$

On opening of switch at $t=0, i_{L}$ can be given by $i_{L}(t)=i_{L}(0) e^{-t / \tau}$ where $\tau=L / R$ $R$ is $R_{\text {equivalent }}$ across $L$


Hence

$$
\mathrm{i}_{\mathrm{L}}(\mathrm{t})=2.5 \mathrm{e}^{-\frac{\mathrm{t}}{1 / 4}}
$$

or

$$
\mathrm{i}_{\mathrm{L}}(\mathrm{t})=2.5 \mathrm{e}^{-4 \mathrm{t}}
$$

32. A three-phase, three winding $\Delta / \Delta / \mathrm{Y}$ (1.1 $\mathrm{kV} / 6.6 \mathrm{kV} / 400 \mathrm{~V}$ ) transformer is energized from AC mains at the 1.1 kV side. It
supplies 900 kVA load at 0.8 power factor lag from the 6.6 kV winding and 300 kVA load at 0.6 power factor lag from the 400 V winding. The RMS line current in ampere drawn by the 1.1 kV winding from the mains is $\qquad$ . (Give the answer up to one decimal place).

## Sol. (625.1 A)

## $\Delta / \Delta / \mathrm{Y}(1.1 \mathrm{kV} / 6.6 \mathrm{kV} / 400 \mathrm{~V})$

For secondary winding $(\Delta)$

$$
\begin{aligned}
900 \mathrm{kVA} & =3 \mathrm{~V}_{\mathrm{ph}_{2}} \mathrm{I}_{\mathrm{ph}_{2}} \\
\Rightarrow \quad \mathrm{I}_{\mathrm{ph}_{2}} & =\frac{900 \mathrm{kVA}}{3 \times 6.6 \mathrm{kV}}=45.45 \mathrm{~A}
\end{aligned}
$$

For P.f of 0.8 lag .
$\mathrm{I}_{\mathrm{ph}_{2}}=45.45\left|-\cos ^{-1} 0.8=45.45\right|-36.87^{\circ} \mathrm{A}$
For tertiary winding (Y)

$$
\begin{aligned}
300 \mathrm{kVA} & =\frac{\sqrt{3} \times 400 \times \mathrm{I}_{\mathrm{ph} 3}}{1000} \\
\mathrm{I}_{\mathrm{ph} 3} & =433.01 \mathrm{~A}
\end{aligned}
$$

For p.f. of $0.6 \log$
$\mathrm{I}_{\mathrm{ph}_{3}}=433.01 \underline{-\cos ^{-1} 0.6}=433.01 \underline{-53.13^{\circ}} \mathrm{A}$
Corresponding currents in primary winding for $\mathrm{I}_{\mathrm{ph}_{2}}$ is given by

$$
\begin{aligned}
& \qquad \begin{aligned}
\mathrm{I}_{\mathrm{ph}_{2}}^{\prime} & =\left(\frac{6.6}{1.1}\right) \mathrm{I}_{\mathrm{ph}_{2}}=6 \times \mathrm{I}_{\mathrm{ph}_{2}} \\
=6 \times 45.45-36.87 & =272.73 \mid-36.87^{\circ} \mathrm{A} \\
\text { similarly } \quad \mathrm{I}_{\mathrm{ph}_{3}}^{\prime} & =\left(\frac{400 / \sqrt{3}}{1.1 \times 1000}\right) \times \mathrm{I}_{\mathrm{ph}_{3}} \\
& =90.91 \mid-53.13^{\circ} \mathrm{A}
\end{aligned}
\end{aligned}
$$

Total RMS phase current $=\mathrm{I}_{\mathrm{ph}_{2}}^{\prime}+\mathrm{I}_{\mathrm{ph}_{3}}^{\prime}$
$=272.73 \underline{-36.87^{\circ}}+90.91 \underline{-53.13^{\circ}}$
$=360.90-40.91$
Total RMS line current = $360.87 \times \sqrt{3}=625.101$
or $\mathrm{I}_{1}=625.1 \mathrm{~A}$ (upto one decimal place)
33. Consider the line integral $I=\int_{c}\left(x^{2}+i y^{2}\right) d z$, where $z=x+i y$. The line $c$ is shown in the figure below


The value of I is
(a) $\frac{1}{2} \mathrm{i}$
(b) $\frac{2}{3} \mathrm{i}$
(c) $\frac{3}{4} \mathrm{i}$
(d) $\frac{4}{5} \mathrm{i}$

Sol. (b)
Given integral $I=\int_{c}\left(x^{2}+i y^{2}\right) d z, z=x+i y$ the line C is a straight line passing through origin and its equation is given by $y=x$

substituting $\mathrm{y}=\mathrm{x}$ in given integral we get

$$
\begin{aligned}
I & =\int_{c}\left(x^{2}+i x^{2}\right)(d x+i d x) \\
& =\int_{c} x^{2}(1+i)(1+i) d x \\
& =\int_{c} x^{2}(1-1+2 i) d x \\
& =\int_{c} 2 i x^{2} d x
\end{aligned}
$$

$$
\begin{aligned}
& =2 \mathrm{i} \int_{0}^{1} x^{2} d x \\
& =2 \mathrm{i}\left[\frac{x^{3}}{3}\right]_{0}^{1} \\
& =2 \mathrm{i}\left[\frac{1}{3}-\frac{0}{3}\right] \\
& =\frac{2 \mathrm{i}}{3}
\end{aligned}
$$

34. The figure shows the single line diagram of a power system with a double circuit transmission line. The expression for electrical power is $1.5 \sin \delta$, where $\delta$ is the rotor angle. The system is operating at the stable equilibrium point with mechanical power equal to 1 pu . If one of the transmission line circuits is removed, the maximum value of $\delta$, as the rotor swings, is 1.221 radian. If the expression for electrical power with one transmission line circuit removed is $\mathrm{P}_{\text {,max }} \sin \delta$, the value of $P_{\text {max }}$, in pu is $\qquad$ (Give the answer up to three decimal places.)


Sol. (1.220pu)
With double circuit transmission line

$$
\mathrm{P}_{\mathrm{eI}}=1.5 \sin \delta
$$

with single line
$\mathrm{P}_{\mathrm{eII}}=\mathrm{P}_{\max } \sin \delta$


Here,
$\delta_{1}=\sin ^{-1}\left(\frac{1}{1.5}\right)=41.81^{\circ}$
or 0.7297 radian
$\delta_{m}=1.221$ radian or $69.96^{\circ}$
For stability

$$
\mathrm{A}_{1}=\mathrm{A}_{2}
$$

$$
\int_{\delta_{1}}^{\delta_{2}}\left(1-\mathrm{P}_{\max } \sin \delta\right) \mathrm{d} \delta=\int_{\delta_{2}}^{\delta_{\mathrm{m}}}\left(\mathrm{P}_{\max } \sin \delta-1\right) \mathrm{d} \delta
$$

$$
\left.\delta\right|_{\delta_{1}} ^{\delta_{2}}-\left.\mathrm{P}_{\max }(-\cos \delta)\right|_{\delta_{1}} ^{\delta_{2}}
$$

$$
=\left.\mathrm{P}_{\max }(-\cos \delta)\right|_{\delta_{2}} ^{\delta_{\mathrm{m}}}-\left.\delta\right|_{\delta_{2}} ^{\delta_{\mathrm{m}}}
$$

$$
\mathrm{P}_{\max }\left(\cos \delta_{\mathrm{m}}-\cos \delta_{1}\right)=\delta_{1}-\delta_{\mathrm{m}}
$$

substituting values and solving for $\mathrm{P}_{\max }$ $\mathrm{P}_{\max }\left(\cos 69.96^{\circ}-\cos 41.81^{\circ}\right)=41.81^{\circ}-$ $69.96^{\circ}$
$\mathrm{P}_{\text {max }}=1.220 \mathrm{pu}$
35. The approximate transfer characteristic for the circuit shown below with an ideal operational amplifier and diode will be


Sol. (a)
For $\mathrm{V}_{\mathrm{in}}<0$
Output of operational amplifier will be negative hence due to presence of diode
(reversed biased) in this case output will be zero

For $\mathrm{V}_{\text {in }}>0$
Diode will be forward biased so $\mathrm{V}_{\text {in }}=\mathrm{V}_{0}$
Thus transfer characteristics

36. A load is supplied by a $230 \mathrm{~V}, 50 \mathrm{~Hz}$ source. The active power $P$ and the reactive power $Q$ consumed by the load are such that $1 \mathrm{~kW} \leq \mathrm{P} \leq 2 \mathrm{~kW}$ and $1 \mathrm{kVAR} \leq \mathrm{Q} \leq 2 \mathrm{kVAR}$. A capacitor connected across the load for power factor correction generates 1 kVAR reactive power. The worst case power factor after power factor correction is
(a) 0.447 lag
(b) 0.707 lag
(c) 0.894 lag
(d) 1

Sol. (b)
For worst case power factor

$$
\begin{aligned}
\mathrm{P} & =1 \mathrm{~kW}, \\
\mathrm{Q} & =2 \mathrm{kVAR}
\end{aligned}
$$

After addition of capacitor for power factor correction Q becomes $2-1=1 \mathrm{kVAR}$ new

$$
\begin{aligned}
\text { P.f } & =\cos \left(\tan ^{-1} \frac{\mathrm{Q}}{\mathrm{P}}\right) \\
& =\cos \left(\tan ^{-1} \frac{1}{1}\right) \\
& =\cos 45^{\circ} \\
\text { or } \quad \text { P.f } & =0.707 \mathrm{lag}
\end{aligned}
$$

37. A separately excited DC generator supplies 150 A to a 145 V DC grid. The generator is running at 800 RPM . The armature resistance of the generator is $0.1 \Omega$. If the speed of the generator is increased to 1000 RPM, the current in amperes supplied by the generator to the DC grid is $\qquad$ . (Give the answer up to one decimal place).

Sol. (550A)


Since back emf $E=V_{t}+I_{a} R_{a}$
and

$$
\begin{equation*}
\mathrm{E} \propto \phi \mathrm{~N} \tag{i}
\end{equation*}
$$

For separately excited generator $\phi$ remains constant so $\mathrm{E} \propto \mathrm{N}$
For

$$
\begin{align*}
\mathrm{N} & =800 \mathrm{rpm}  \tag{iii}\\
\mathrm{E}_{1} & =145+150 \times 0.1=160 \mathrm{~V}
\end{align*}
$$

Using equation (iii)

$$
\begin{equation*}
\mathrm{E}_{1} \propto \mathrm{~N}_{1} \tag{iv}
\end{equation*}
$$

or $\quad 160 \propto 800$
For $\mathrm{N}=1000 \mathrm{rpm}$

$$
\mathrm{E}_{2} \propto 1000
$$

(v)

On solving equation (iv) and (v)

Thus

$$
\mathrm{E}_{2}=200 \mathrm{~V}
$$

$$
200=145+\mathrm{I}_{\mathrm{a}} \times 0.1
$$

or

$$
\begin{gathered}
\mathrm{I}_{\mathrm{a}}=\frac{200-145}{0.1} \\
\mathrm{I}_{\mathrm{a}}=550 \mathrm{~A}
\end{gathered}
$$

38. Consider the differential equation
$\left(\mathrm{t}^{2}-81\right) \frac{d y}{d t}+5 t y=\sin (\mathrm{t})$ with $\mathrm{y}(1)=2 \pi$.
There exists a unique solution for this differential equation when $t$ belongs to the interval
(a) $(-2,2)$
(b) $(-10,10)$
(c) $(-10,2)$
(d) $(0,10)$

## Sol. (a)

Given differential equation is

$$
\begin{equation*}
\left(\mathrm{t}^{2}-81\right) \frac{\mathrm{dy}}{\mathrm{dt}}+5 \mathrm{ty}=\sin \mathrm{t} \tag{1}
\end{equation*}
$$

initial condition $y(1)=2 \pi$
Converting the given equation into standard form

$$
\begin{equation*}
\frac{d y}{d t}+\left(\frac{5 t}{t^{2}-81}\right) y=\frac{\sin t}{t^{2}-81} \tag{2}
\end{equation*}
$$

this is of the form

$$
\frac{d y}{d t}+p y=Q
$$

where

$$
\mathrm{P}=\frac{5 \mathrm{t}}{\mathrm{t}^{2}-81}, \mathrm{Q}=\frac{\sin \mathrm{t}}{\mathrm{t}^{2}-81}
$$

we know integrating factor (IF) $=e^{\int \rho d t}$

$$
\begin{aligned}
& =\mathrm{e}^{\int \frac{5 \mathrm{t}}{\mathrm{t}^{2}-81} \mathrm{dt}} \\
& =\mathrm{e}^{\int \frac{5}{2}\left(\frac{2 \mathrm{t}}{\mathrm{t}^{2}-81}\right)} \\
& =\mathrm{e}^{\frac{5}{2} \ln \left(\mathrm{t}^{2}-81\right)^{5 / 2}} \quad\left[\because \mathrm{e}^{\ln \mathrm{x}}=\mathrm{x}\right] \\
\mathrm{IF} & =\left(\mathrm{t}^{2}-81\right)^{5 / 2} \\
y(\text { IF }) & =\int Q \text { IF } \mathrm{dt}+c
\end{aligned}
$$

$$
\mathrm{y}\left(\mathrm{t}^{2}-81\right)^{5 / 2}=\int \frac{\sin \mathrm{t}}{\left(\mathrm{t}^{2}-81\right)}\left(\mathrm{t}^{2}-81\right)^{5 / 2} \mathrm{dt}+\mathrm{c}
$$

$$
y=\int \frac{\sin t\left(t^{2}-81\right)^{3 / 2}}{\left(t^{2}-81\right)^{5 / 2}} d t+c\left(t^{2}-81\right)^{-5 / 2}
$$

$$
\mathrm{y}=\int \sin \mathrm{t}\left(\mathrm{t}^{2}-81\right)^{-1} \mathrm{dt}+\mathrm{c}\left(\mathrm{t}^{2}-81\right)^{-5 / 2}
$$

and solving from the options by verifying initial condition we get unique solution
If $t= \pm 9$ then solution is not unique hence range $(-10,10),(-10,2),(0,10)$ can be eliminated, then left option is $(-2,2)$
39. The input voltage $\mathrm{V}_{\mathrm{DC}}$ for the buck-boost converter shown below varies from 32 V to 72 V. Assume that all components are ideal, inductor current is continuous and output voltage is ripple free. The range of duty ratio D of the convector for which the magnitude of the steady-state output voltage remains constant at 48 V is

# TM IES MASTER <br> IES MASTER <br> Institute for Engineers (IES/GATE/PSUs) 

## ESE-2017 Conventional Test Schedule, Electrical Engineering

| Date | Topic |
| :---: | :---: |
| 5th Mar 2017 | N.T. : ECF-1, MC-1, MC-2, ADE-2 |
|  | R.T. |
| 11th Mar 2017 | N.T. : ECF-2, MI-1, CS-1, CS-2 |
|  | R.T. : ECF-1, MC-1, MC-2, ADE-2 |
| 19th Mar 2017 | N.T. : ECF-3, MI-2, MC-3, MC-4 |
|  | R.T. : ECF-2, MI-1, CS-1, CS-2 |
| 26th Mar 2017 | N.T. : ECF-4, BEX-1, ADE-1, ADE-3 |
|  | R.T. : ECF-3, MI-2, MC-3, MC-4 |
| 2nd Apr 2017 | N.T. : EM-1, MATH-1, PS-1, SSP-1 |
|  | R.T. : ECF-4, BEX-1, ADE-1, ADE-3 |
| 9th Apr 2017 | N.T. : CF-1, MATH-2, PS-2, PE-1 |
|  | R.T. : EM-1, MATH-1, PS-1, SSP-1 |
| 16th Apr 2017 | N.T. : BEX-2, MI-3, CS-3, SSP-2 |
|  | R.T. : CF-1, MATH-2, PS-2, PE-1 |
| 23rd Apr 2017 | N.T. : EM-2, PS-3 |
|  | R.T. : BEX-2, MI-1, MI-3,. CS-3, SSP-2, ADE-3, MC-1, MC-2 |
| 30th Apr 2017 | N.T. : CF-2, PE-2 |
|  | R.T. : EM-2, ECF-1, ECF-3, MI-2, PS-2, PS-3, ADE-2, CS-2 |
| 3rd May 2017 | N.T. : CF-3, MATH-3 |
|  | R.T. : CF-2, ECF-2, MI-1, BEX-1, EM-1, CS-1, MI-3, CS-3, ADE-3, PE-2, SSP-1 |
| 7th May 2017 | N.T. |
|  | R.T. : MATH-1, MATH-3, EM-1, EM-2, ECF-1, ECF-4, BEX-2, CF-3, ADE-2, CS-2, PS-1, PS-3 PE-1, SSP-2 |
| 9th May 2017 | Full Length (Test Paper-1 + Test Paper-2) |
| Test Type | Timing Day |
| Conventional Test __ 10:00 A.M. to 1:00 P.M. ___ Sunday |  |
| Conventional Full Length Test Paper-1 $\qquad$ 10:00 A.M. to 1:00 P.M. $\qquad$ Tuesday Conventional Full Length Test Paper-2 $\qquad$ 02:00 P.M. to 5:00 P.M. $\qquad$ Tuesday |  |
| Note : The timing of the test may change on certain dates. Prior information will be given in this regard. <br> *N.T. : New Topic. *R.T. : Revision Topic <br> Call us : 8010009955, 011-41013406 or Mail us : info@iesmaster.org |  |

Subject Code Details


(a) $\frac{2}{5} \leq \mathrm{D} \leq \frac{3}{5}$
(b) $\frac{2}{3} \leq \mathrm{D} \leq \frac{3}{4}$
(c) $0 \leq \mathrm{D} \leq 1$
(d) $\frac{1}{3} \leq \mathrm{D} \leq \frac{2}{3}$

Sol. (a)
For buck-boost converter

$$
\mathrm{V}_{0}=\frac{\alpha}{1-\alpha} \mathrm{V}_{\mathrm{s}}
$$

where $\alpha$ is duty cycle of converter

$$
\mathrm{V}_{\mathrm{s}}=\text { supply voltage }
$$

$$
\mathrm{V}_{0}=\text { output voltage }
$$

For $\mathrm{V}_{\mathrm{s}}=32 \mathrm{~V}$ and $\mathrm{V}_{0}=48 \mathrm{~V}$
40. The load shown in the figure is supplied by a 400 V (line-to-line), 3-phase source (RYB sequence). The load is balanced and inductive, drawing 3464 VA. When the switch $S$ is in position $N$, the three wattmeters $\mathrm{W}_{1}, \mathrm{~W}_{2}$ and $\mathrm{W}_{3}$ read 577.35 W each. If the switch is moved to position Y , the readings of the wattmeters in watts will be:

(a) $\mathrm{W}_{1}=1732$ and $\mathrm{W}_{2}=\mathrm{W}_{3}=0$
(b) $\mathrm{W}_{1}=0, \mathrm{~W}_{2}=1732$ and $\mathrm{W}_{3}=0$
(c) $\mathrm{W}_{1}=866, \mathrm{~W}_{2}=0, \mathrm{~W}_{3}=866$
(d) $\mathrm{W}_{1}=\mathrm{W}_{2}=0$ and $\mathrm{W}_{3}=1732$

## Sol. (d)

Apparent power $=3464 \mathrm{VA}$
Real power $=3 \times 577.35 \mathrm{~W}=1732.05 \mathrm{Watts}$

$$
\begin{aligned}
\text { P.f. } & =\frac{\text { Real power }}{\text { Apperent power }} \\
& =\frac{1732.05}{3464} \\
\text { P.f. } & =0.5
\end{aligned}
$$

When switch is moved to position Y
$\Rightarrow$ Voltage across potential coil of wattmeter two is zero so $\mathrm{W}_{2}=0$
For RYB phase sequence


Voltage across potential coil of wattmeter one is $\mathrm{V}_{\mathrm{RY}}$.
Voltage across potential coil of wattmeter two is $\mathrm{V}_{\mathrm{BY}}$.

So

$$
\begin{aligned}
& \mathrm{W}_{1}=\mathrm{V}_{\mathrm{RY}} \cdot \mathrm{I}_{\mathrm{R}} \cdot \cos (30+\phi) \\
& \mathrm{W}_{2}=\mathrm{V}_{\mathrm{BY}} \cdot \mathrm{I}_{\mathrm{B}} \cos \left(30^{\circ}-\phi\right)
\end{aligned}
$$

$$
\begin{aligned}
& 48=\frac{\alpha}{1-\alpha} \times 32 \\
& \Rightarrow \quad \alpha=\frac{3}{5} \\
& \text { For } \quad \mathrm{V}_{\mathrm{s}}=72 \mathrm{~V} \text { and } \mathrm{V}_{0}=48 \mathrm{~V} \\
& 48=\frac{\alpha}{1-\alpha} \times 72 \\
& \alpha=\frac{2}{5} \\
& \frac{2}{5} \leq \alpha \leq \frac{3}{5}
\end{aligned}
$$

For P.f $=0.5 ; \phi=\cos ^{-1}(0.5)=60^{\circ}$
Thus
$\mathrm{W}_{1}=\mathrm{V}_{\mathrm{RY}} \cdot \mathrm{I}_{\mathrm{R}} \cdot \cos \left(30+60^{\circ}\right)=0$
$\mathrm{W}_{3}=\mathrm{V}_{\mathrm{BY}} \cdot \mathrm{I}_{\mathrm{B}} \cdot \cos \left(30-60^{\circ}\right)=1732 \mathrm{~W}$
41. The bus admittance matrix for a power system network is

$$
\left[\begin{array}{lll}
-\mathrm{j} 39.9 & \mathrm{j} 20 & \mathrm{j} 20 \\
\mathrm{j} 20 & -\mathrm{j} 39.9 & \mathrm{j} 20 \\
\mathrm{j} 20 & \mathrm{j} 20 & -\mathrm{j} 39.9
\end{array}\right] \mathrm{pu}
$$

There is a transmission line, connected between buses 1 and 3 , which is represented by the circuit shown in figure


If this transmission line is removed from service, what is the modified bus admittance matrix?
(a) $\left[\begin{array}{lll}-j 19.9 & j 20 & 0 \\ j 20 & -j 39.9 & j 20 \\ 0 & j 20 & -j 19.9\end{array}\right] \mathrm{pu}$
(b)
$\left[\begin{array}{lll}-j 39.95 & j 20 & 0 \\ j 20 & -j 39.9 & j 20 \\ 0 & j 20 & -j 39.9\end{array}\right] \mathrm{pu}$
(c)

(d)
$\left[\begin{array}{lll}-j 19.95 & j 20 & 0 \\ j 20 & -j 39.9 & j 20 \\ j 20 & j 20 & -j 19.95\end{array}\right] p u$

Sol. (c)
Given y-bus $[\mathrm{Y}]_{3 \times 3}$

$$
=\left[\begin{array}{ccc}
-\mathrm{j} 39.9 & \mathrm{j} 20 & \mathrm{j} 20 \\
\mathrm{j} 20 & -\mathrm{j} 39.9 & \mathrm{j} 20 \\
\mathrm{j} 20 & \mathrm{j} 20 & -\mathrm{j} 39.9
\end{array}\right]
$$

Converting the given transmission line parameters into Y parameters we get

whenever we remove the transmission line between Bus 1 and Bus 3 the parameters $\mathrm{Y}_{11}, \mathrm{Y}_{13}, \mathrm{Y}_{31}, \mathrm{Y}_{33}$ will get affected
$\mathrm{Y}_{11}=-\mathrm{j} 39.9-\mathrm{y}_{11}-\mathrm{y}_{13}=-\mathrm{j} 39.9-(\mathrm{j} 0.05)-(-\mathrm{j} 20)$
$=-\mathrm{j} 39.9-\mathrm{j} 0.05+\mathrm{j} 20=-\mathrm{j} 19.95$
$Y_{13}=j 20+y_{13}=j 20-j 20=0$
$\mathrm{Y}_{31}=\mathrm{j} 20+\mathrm{y}_{13}=\mathrm{j} 20-\mathrm{j} 20=0$
$\mathrm{Y}_{33}=-\mathrm{j} 39.9-\mathrm{y}_{33}-\mathrm{y}_{31}$
$=-\mathrm{j} 39.9-\mathrm{j} 0.05-(-\mathrm{j} 20)=-\mathrm{j} 19.95$
New Y bus matrix $[\mathrm{Y}]_{3 \times 3}$

$$
=\left[\begin{array}{ccc}
-\mathrm{j} 19.95 & \mathrm{j} 20 & 0 \\
\mathrm{j} 20 & -\mathrm{j} 39.9 & \mathrm{j} 20 \\
0 & \mathrm{j} 20 & -\mathrm{j} 19.95
\end{array}\right]
$$

42. In the system whose signal flow graph is shown in the figure, $\mathrm{U}_{1}(\mathrm{~s})$ and $\mathrm{U}_{2}(\mathrm{~s})$ are inputs. The transfer function $\frac{\mathrm{Y}(\mathrm{s})}{\mathrm{U}_{1}(\mathrm{~s})}$ is

(a) $\frac{\mathrm{k}_{1}}{\mathrm{JLs}^{2}+\mathrm{JRs}+\mathrm{k}_{1} \mathrm{k}_{2}}$
(b) $\frac{\mathrm{k}_{1}}{\mathrm{JLs}^{2}-\mathrm{JRs}-\mathrm{k}_{1} \mathrm{k}_{2}}$
(c) $\frac{\mathrm{k}_{1}-\mathrm{U}_{2}(\mathrm{R}+\mathrm{sL})}{\mathrm{JLs}^{2}+\left(\mathrm{JR}-\mathrm{U}_{2} \mathrm{~L}\right) \mathrm{s}+\mathrm{k}_{1} \mathrm{k}_{2}-\mathrm{U}_{2} \mathrm{R}}$
(d) $\frac{\mathrm{k}_{1}-\mathrm{U}_{2}(\mathrm{sL}-\mathrm{R})}{\mathrm{JLs}^{2}-\left(\mathrm{JR}+\mathrm{U}_{2} \mathrm{~L}\right) \mathrm{s}-\mathrm{k}_{1} \mathrm{k}_{2}+\mathrm{U}_{2} \mathrm{R}}$

Sol. (a)
Using mason's gain formula

$$
\begin{aligned}
& \text { T.F. }=\frac{\Sigma \mathrm{P}_{\mathrm{k}} \Delta_{\mathrm{k}}}{\Delta} \\
& \mathrm{P}_{1}=\frac{1}{\mathrm{~L}} \cdot \frac{1}{\mathrm{~s}} \cdot \mathrm{k}_{1} \cdot \frac{1}{\mathrm{~J}} \cdot \frac{1}{\mathrm{~s}}=\frac{\mathrm{k}_{1}}{\mathrm{JLs}^{2}} \\
& \Delta_{1}=1 \\
& \Delta=1-\left[\left(\frac{-\mathrm{R}}{\mathrm{Ls}}\right)+\left(\frac{-\mathrm{k}_{1} \mathrm{k}_{2}}{\mathrm{JLs}^{2}}\right)\right] \\
&=1+\frac{\mathrm{R}}{\mathrm{Ls}}+\frac{\mathrm{k}_{1} \mathrm{k}_{2}}{\mathrm{JLs}^{2}} \\
& \Delta=\frac{\mathrm{JLs}^{2}+\mathrm{JRs}^{2} \mathrm{k}_{1} \mathrm{k}_{2}}{\mathrm{JLs}^{2}} \\
& \text { T.F. }=\frac{\frac{\mathrm{k}_{1}}{\mathrm{JLs}^{2}}}{\mathrm{JLs}^{2}+\mathrm{JRs}^{2} \mathrm{k}_{1} \mathrm{k}_{2}} \\
& \mathrm{JLs}^{2}
\end{aligned} \quad \begin{aligned}
& \text { T.F. }
\end{aligned}
$$

43. The circuit shown in the figure uses matched transistors with a thermal voltage $\mathrm{V}_{\mathrm{T}}=25 \mathrm{mV}$. The base currents of the transistors are negligible. The value of the resistance $R$ in $k \Omega$ that is required to provide $1 \mu \mathrm{~A}$ bias current for the differential amplifier block shown in $\qquad$ _. (Give the answer up to one decimal place).


## Sol. (12.2 M $\Omega$ )

$$
\begin{aligned}
& \text { Given data } \mathrm{V}_{\mathrm{T}}=25 \mathrm{mV} \\
& \text { given } \mathrm{I}_{\mathrm{B}_{1}}=\mathrm{I}_{\mathrm{B}_{2}} \approx 0 \mathrm{~A} \\
& \mathrm{I}_{\mathrm{C}_{1}}=1 \mathrm{~mA} \\
& \mathrm{I}_{\mathrm{C}_{2}}=1 \mu \mathrm{~A} \text { (Bias current) }
\end{aligned}
$$

Applying the KVL for the given circuit we get

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JATIN KUMAR RACHIT JAIN ADARSH R. SRIVASTAV

SHIVAM DWIVEDI
AMRIT ANAND AVDHESH MEENA

| AIR | AIR | AIR |
| :---: | :---: | :---: |
| $\begin{aligned} & 10 \\ & C E \end{aligned}$ | $\begin{aligned} & 11 \\ & C E \end{aligned}$ | $\begin{aligned} & 12 \\ & \text { CE } \end{aligned}$ |


BHARAT BHUSHAN DIXIT

MOHAMMAD IDUL AHMED CHIRAG SRIVASTAV



$$
=\frac{25 \mathrm{~m} \ln \left(\frac{1 \mathrm{~m}}{1 \mu}\right)+12}{(1 \mu)}
$$

$\mathrm{R}=12.172 \mathrm{M} \Omega \simeq 12.2 \mathrm{M} \Omega$
44. For a system having transfer function $\mathrm{G}(\mathrm{s})$ $=\frac{-\mathrm{s}+1}{\mathrm{~s}+1}$ a unit step input is applied at time $t=0$. The value of the response of the system at $\mathrm{t}=1.5 \mathrm{sec}$ (rounded off to three decimal places) is $\qquad$
Sol. (0.554)

$$
\mathrm{G}(\mathrm{~s})=\frac{-\mathrm{s}+1}{\mathrm{~s}+1}
$$

For unit step input $R(s)=\frac{1}{s}$
So output $\mathrm{y}(\mathrm{s})=\mathrm{R}(\mathrm{s}) . \mathrm{G}(\mathrm{s})=\frac{1}{\mathrm{~s}} \cdot \frac{(-\mathrm{s}+1)}{\mathrm{s}+1}$
$y(t)=L^{-1}\left(\frac{-1}{s+1}\right)+L^{-1}\left(\frac{1}{s(s+1)}\right)$
$=-\mathrm{e}^{-\mathrm{t}}+\int_{0}^{\mathrm{t}} \mathrm{e}^{-\mathrm{t}} \mathrm{dt}$
$=-e^{-t}+\left(-e^{-t}\right)+1$

$$
y(t)=1-2 e^{-t}
$$

at $\mathrm{t}=1.5 \mathrm{sec}$

$$
\mathrm{y}(1.5)=1-2 \mathrm{e}^{-1.5}
$$

$$
=0.5537
$$

or $\quad \mathrm{y}(1.5)=0.554$
45. Two parallel connected, three-phase, 50 Hz , 11 kV , star-connected synchronous machines A and B are operating as synchronous condensers. They together supply 50 MVAR to a 11 kV grid. Current supplied by both the machines are equal. Synchronous reactances of machine A and machine B are $1 \Omega$ and $3 \Omega$, respectively. Assuming the magnetic circuit to be linear, the ratio of excitation current of machine A to that of machine $B$ is $\qquad$ . (Give the answer up to two decimal places).

Sol. (0.74)
Total current supplied by both machine to grid.

$$
\begin{aligned}
& \mathrm{I}_{\mathrm{T}}=\mathrm{I}_{\mathrm{A}}+\mathrm{I}_{\mathrm{B}} \\
& \mathrm{I}_{\mathrm{T}}=\frac{50 \mathrm{MVAR}}{11 \mathrm{kV}} \\
&=4.54545 \mathrm{kA} \\
& \mathrm{I}_{\mathrm{A}}=\mathrm{I}_{\mathrm{B}}=\frac{\mathrm{I}_{\mathrm{T}}}{2}=2.27273 \mathrm{kA} \\
& \mathrm{E}_{\mathrm{A}}=\mathrm{V}_{\mathrm{t}_{\mathrm{t}}}+\mathrm{I}_{\mathrm{A}} \mathrm{X}_{\mathrm{S}}=11 \mathrm{kV}+ \\
& 2.2723 \times 1=13.273 \mathrm{kV} \\
& \mathrm{E}_{\mathrm{B}}=\mathrm{V}_{\mathrm{t}}+\mathrm{I}_{\mathrm{B}} \mathrm{X}=11 \mathrm{kV}+ \\
& 2.2723 \times 3=17.817 \mathrm{kV} \\
& \frac{\mathrm{E}_{\mathrm{A}}}{}=\frac{13.273}{17.817}=0.745 \simeq 0.74
\end{aligned}
$$

Therefore, the ratio of excitation current of machine A to that of machine B will be 0.74 .
46. Let the signal

$$
x(t)=\sum_{k=-\infty}^{+\infty}(-1)^{k} \delta\left(t-\frac{k}{2000}\right)
$$

be passed through an LTI system with frequency response $H(\omega)$, as given in the figure below


The Fourier series representation of the output is given as
(a) $4000+4000 \cos (2000 \pi t)+4000 \cos (4000 \pi t)$
(b) $2000+2000 \cos (2000 \pi \mathrm{t})+2000 \cos (4000 \pi \mathrm{t})$
(c) $4000 \cos (2000 \pi t)$
(d) $\mathrm{A} 2000 \cos (2000 \pi \mathrm{t})$

Sol. (c)
Given function in time domain

$$
x(t)=\sum_{k=-\infty}^{\infty}(-1)^{k} \delta\left(t-\frac{k}{2000}\right)
$$

This function looks like $\mathrm{f}(\mathrm{t}-\mathrm{T})$ delayed by time T
here $\quad \delta\left(\mathrm{t}-\frac{\mathrm{k}}{2000}\right)$ is compared with $\delta(\mathrm{t}-\mathrm{kT})$

Where $T=\frac{1}{2000}$
the values of $x(t)$ for $k=0,1,2, \ldots$ are $\mathrm{k}=-1,-2,-3, \ldots$.

$$
\begin{aligned}
& x(t)=\delta(t) \text { for } k=0 \\
& x(t)=(-1) \delta\left(t-\frac{1}{2000}\right) \text { for } k=1 \\
& x(t)=(-1)^{-1} \delta\left(t-\frac{1}{2000}\right) \text { for } k=-1
\end{aligned}
$$

Drawing the function $\mathrm{x}(\mathrm{t})$ for various values of $k$ we get,


The figure shown above passess even half wave symmetry with time period

$$
\mathrm{T}_{0}=2 \mathrm{~T}=\frac{2}{2000}=\frac{1}{1000}
$$

$\omega_{0}=\frac{2 \pi}{T}=\frac{2 \pi}{\left(\frac{1}{1000}\right)}=2000 \pi$
In the case of even half wave symmetry $b_{n}$ $=0$ and consists of only odd harmonics of $\mathrm{a}_{\mathrm{n}}$.
The frequency components are $\omega_{0}, 3 \omega_{0} \ldots$
i.e. $2000 \pi, 6000 \pi$.....
and $2000 \pi$ is the only frequency available
in the above range or $-5000 \pi$ to $5000 \pi$

$$
\begin{aligned}
\therefore \quad \mathrm{a}_{\mathrm{n}} & =\frac{2 \pi}{\mathrm{~T}} \int_{-\mathrm{T}_{0} / 2}^{\mathrm{T}_{0} / 2} \mathrm{f}(\mathrm{t}) \cos n \omega_{0} \mathrm{t} d \mathrm{t} \\
& =\frac{4}{\mathrm{~T}_{0}} \int_{0}^{\mathrm{T}_{0} / 2} \mathrm{f}(\mathrm{t}) \cos n \omega_{0} \mathrm{tdt}
\end{aligned}
$$

$$
\mathrm{a}_{2}=\frac{4}{\mathrm{~T}_{0}} \int_{0}^{\mathrm{T}_{0} / 2} \delta(\mathrm{t}) \cos 2 \omega_{0}(0) \mathrm{dt}
$$

$$
=\frac{4}{T_{0}} \int_{0}^{\mathrm{T}_{0} / 2} \delta(\mathrm{t}) \mathrm{dt}
$$

$$
=\frac{4}{\mathrm{~T}_{0}}(1)=4000
$$

$\therefore$ The output

$$
y(t)=4000 \cos \omega_{0}+4000 \cos \left(3 \omega_{0} t\right)+\ldots
$$

$$
=4000 \cos 2000 \pi t
$$

$$
+4000 \cos 6000 \pi t+\ldots
$$

Hence, $4000 \cos 2000 \pi \mathrm{t}$ is in the range of $-5000 \pi$ to $5000 \pi$.
47. The output expression for the Karnaugh map shown below is

| CD |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{AB} \triangle 00011110$ |  |  |  |  |
| 00 | 0 | 0 | 0 | 0 |
| 01 | 1 | 0 | 0 | 1 |
| 11 | 1 | 0 | 1 | 1 |
| 10 | 0 | 0 | 0 | 0 |

(a) $\mathrm{B} \overline{\mathrm{D}}+\mathrm{BCD}$
(b) $\mathrm{B} \overline{\mathrm{D}}+\mathrm{AB}$
(c) $\overline{\mathrm{B}} \mathrm{D}+\mathrm{ABC}$
(d) $\mathrm{B} \overline{\mathrm{D}}+\mathrm{ABC}$

Sol. (d)

$\mathrm{f}=\mathrm{B} \overline{\mathrm{D}}+\mathrm{ABC}$
48. The transfer function of the system $\mathrm{Y}(\mathrm{s}) /$ $\mathrm{U}(\mathrm{s})$ whose state-space equations are given below is:

$$
\begin{aligned}
& {\left[\begin{array}{l}
\dot{x}_{1}(\mathrm{t}) \\
\dot{x}_{2}(\mathrm{t})
\end{array}\right]=\left[\begin{array}{ll}
1 & 2 \\
2 & 0
\end{array}\right]\left[\begin{array}{l}
\mathrm{x}_{1}(\mathrm{t}) \\
\mathrm{x}_{2}(\mathrm{t})
\end{array}\right]+\left[\begin{array}{l}
1 \\
2
\end{array}\right] \mathrm{u}(\mathrm{t})} \\
& \mathrm{y}(\mathrm{t})=\left[\begin{array}{ll}
1 & 0
\end{array}\right]\left[\begin{array}{l}
\mathrm{x}_{1}(\mathrm{t}) \\
\mathrm{x}_{2}(\mathrm{t})
\end{array}\right]
\end{aligned}
$$

(a) $\frac{(\mathrm{s}+2)}{\left(\mathrm{s}^{2}-2 \mathrm{~s}-2\right)}$
(b) $\frac{(\mathrm{s}-2)}{\left(\mathrm{s}^{2}+\mathrm{s}-4\right)}$
(c) $\frac{(\mathrm{s}-4)}{\left(\mathrm{s}^{2}+\mathrm{s}-4\right)}$
(d) $\frac{(\mathrm{s}+4)}{\left(\mathrm{s}^{2}-\mathrm{s}-4\right)}$

Sol. (d)
Transfer function T.F $=\mathrm{C}(\mathrm{sI}-\mathrm{A})^{-1} \mathrm{~B}$

$$
\begin{aligned}
\mathrm{sI}-\mathrm{A} & =\left[\begin{array}{cc}
\mathrm{s}-1 & -2 \\
-2 & \mathrm{~s}
\end{array}\right] \\
|\mathrm{sI}-\mathrm{A}| & =\mathrm{s}(\mathrm{~s}-1)-4=\mathrm{s}^{2}-\mathrm{s}-4 \\
{[\mathrm{sI}-\mathrm{A}]^{-1} } & =\frac{1}{\mathrm{~s}^{2}-\mathrm{s}-4}\left[\begin{array}{cc}
\mathrm{s} \\
+2 & \mathrm{~s}-1
\end{array}\right] \\
{[\mathrm{sI}-\mathrm{A}]^{-1} \mathrm{~B} } & =\frac{1}{\mathrm{~s}^{2}-\mathrm{s}-4}\left[\begin{array}{cc}
\mathrm{s} & +2 \\
2 & \mathrm{~s}-1
\end{array}\right]\left[\begin{array}{l}
1 \\
2
\end{array}\right] \\
& =\frac{1}{\mathrm{~s}^{2}-\mathrm{s}-4}\left[\begin{array}{c}
\mathrm{s}+4 \\
2 \mathrm{~s}
\end{array}\right] \\
\mathrm{C}[\mathrm{sI}-\mathrm{A}]^{-1} \mathrm{~B} & =\frac{1}{\mathrm{~s}^{2}-\mathrm{s}-4}\left[\begin{array}{ll}
1 & 0]
\end{array}\right]\left[\begin{array}{c}
\mathrm{s}+4 \\
25
\end{array}\right] \\
\mathrm{T} . \mathrm{F} . & =\frac{\mathrm{s}+4}{\mathrm{~s}^{2}-\mathrm{s}-4}
\end{aligned}
$$

49. The magnitude of magnetic flux density (B) in micro Teslas ( $\mu \mathrm{T}$ ), at the center of a loop of wire wound as a regular hexagon of side length 1 m carrying a current ( $\mathrm{I}=1 \mathrm{~A}$ ) and placed in vacuum as shown in the
figure is $\qquad$ . (Give the answer up to two decimal places).


Sol. (0.69)
Magnetic field due to finite length of current carrying conductor is given by
$\mathrm{H}=\frac{\mathrm{I}}{4 \pi \rho}\left(\cos \alpha_{2}-\cos \alpha_{1}\right) \hat{\mathrm{a}}_{\phi . .}$

$\alpha_{2}=60^{\circ}$
$\alpha_{1}=120^{\circ}$
In case of regular hexagon

$\rho=\sqrt{a^{2}-\frac{a^{2}}{4}}$ where $a=1$
$=\sqrt{1-\frac{1}{4}}=\frac{\sqrt{3}}{2} \mathrm{~m}$
Using formula in eq-(i) magnetic field intensity at centre due to one side of regular Hexagon
$\mathrm{H}^{\prime}=\frac{1}{\left.4 \pi\left(\frac{\sqrt{3}}{2}\right)^{\left[\cos 60^{\circ}\right.}-\cos 120^{\circ}\right] \alpha_{\phi}}$
$=0.091888 \mathrm{H} / \mathrm{m}$

Magnetic field intensity due to all six sides of regular hexagon will be
$\mathrm{H}=6 \times \mathrm{H}^{\prime}=6 \times 0.091888$
$=0.551329 \mathrm{H} / \mathrm{m}$
Magnetic flux density $\mathrm{B}=\mu \mathrm{H}$
Hence $\mathrm{B}=\mu_{0} \mathrm{H} \quad$ (in vaccum)
$=4 \pi \times 10^{-7} \times 0.551329$
$=6.9282 \times 10^{-7}$ Tesla
or $B=0.6928 \mu \mathrm{~T}$
$\mathrm{B}=0.69 \mu \mathrm{~T}$ upto two decimal places
50. The figure below shows an uncontrolled diode bride rectifier supplied from a 220 V , 50 Hz , 1-phase ac source. The load draws a constant current $\mathrm{I}_{0}=14 \mathrm{~A}$. The conduction angle of the diode $D_{1}$ in degrees (rounded off to two decimal places) is $\qquad$


Sol. (210.84 ${ }^{\circ}$ )
For single phase controlled bridge rectifier effect of source inducfance will modify the average output voltage as,
$\mathrm{V}_{0}=\frac{\mathrm{V}_{\mathrm{m}}}{\pi}[\cos \alpha+\cos (\alpha+\mu)]$
where $\mu$ is overlap angle
But for diode (uncontrolled) bridge, $\alpha=0$
so $\mathrm{V}_{0}=\frac{\mathrm{V}_{\mathrm{m}}}{\pi}[1+\cos \mu]$
Also
$\mathrm{V}_{0}=\frac{2 \mathrm{~V}_{\mathrm{m}}}{\pi}-\frac{\omega \mathrm{L}_{\mathrm{s}}}{\pi} \mathrm{I}_{0} \ldots$
where $\mathrm{L}_{\mathrm{s}}=$ source inductance
From eq. (1) and eq (2).
$\frac{2 \mathrm{~V}_{\mathrm{m}}}{\pi}-\frac{\omega \mathrm{L}_{\mathrm{s}}}{\pi} \mathrm{I}_{0}=\frac{\mathrm{V}_{\mathrm{m}}}{\pi}[1+\cos \mu]$
Substituting all the values in above equation
$\frac{2 \times 220 \times \sqrt{2}}{\pi}-\frac{2 \pi \times 50 \times 10 \times 10^{-3} \times 14}{\pi}$
$=\frac{220 \times \sqrt{2}}{\pi}[1+\cos \mu]$
Solving for $\cos \mu$
$\cos \mu=0.8586$
$\Rightarrow \mu=30.836^{\circ}$
Conduction angle for diode will be $180^{\circ}+$

Hence conduction angle $\gamma=180^{\circ}+\mu$
$=180^{\circ}+30.836$
$\gamma=210.84^{\circ}$ upto two decimal places.
51. In the circuit shown below, the maximum power transferred to the resistor $R$ is $\qquad$ W


Sol. (3.025 W)
$\mathrm{P}_{\text {max }}$ across R will be given by
$\mathrm{P}_{\text {max }}=\frac{\mathrm{V}_{\mathrm{Th}}^{2}}{4 \mathrm{R}_{\mathrm{Th}}}$
Where $\mathrm{V}_{\mathrm{Th}}=$ Thevenin's voltage across ' R '
$R_{T h}=$ Thevenin's resistance across ' $R$ '

## To Calculate $\mathbf{R}_{\text {th }}$

Short cirauiting all voltage sources and open circuiting all current sources, circuit reduces to


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Hence $\mathrm{R}_{\mathrm{Th}}=5 \Omega| | 5 \Omega=2.5 \Omega$

## To calculate $\mathrm{V}_{\text {th }}$

Using superposition theorem
Taking 5 V source only
Circuit reduces to

$\mathrm{V}_{1}=\frac{5}{2}=2.5$
Taking 6V source only, circuit reduces to

$\mathrm{V}_{2}=-3 \mathrm{~V}$
Taking 2A current source only,

$\mathrm{V}_{3}=-5 \mathrm{~V}$
$\mathrm{V}_{\mathrm{Th}}=\mathrm{V}_{1}+\mathrm{V}_{2}+\mathrm{V}_{3}=2.5-3-5=-5.5 \mathrm{~V}$
$P_{\max }=\frac{(5.5)^{2}}{4 \times 2.5}$
or $\mathrm{P}_{\text {max }}=3.025 \mathrm{~W}$
52. A $375 \mathrm{~W}, 230 \mathrm{~V}, 50 \mathrm{~Hz}$, capacitor start single-phase induction motor has the following constants for the main and auxiliary windings (at starting):
$\mathrm{Z}_{\mathrm{m}}=(12.50+\mathrm{j} 15.75) \Omega$ (main winding),
$\mathrm{Z}_{\mathrm{a}}=(24.50+\mathrm{j} 12.75) \Omega$ (auxiliary winding).
Neglecting the magnetizing branch, the value of the capacitance (in $\mu \mathrm{F}$ ) to be added in series with the auxiliary winding to obtain maximum torque at starting is $\qquad$

## Sol. ( $98.87 \mu \mathrm{~F}$ )

Capacitor start single phase mduction motor


Torque will be maximum when $\phi=90^{\circ}$ between currents of auxiliary winding and mains winding.
$\mathrm{I}_{\mathrm{M}}=\frac{230}{(12.50+\mathrm{j} 15.75)}$
$\phi_{\mathrm{M}}=-\tan ^{-1}\left(\frac{15.75}{12.50}\right)$
Taking $\mathrm{X}_{\mathrm{c}}$ as reactance of capacitor $\mathrm{C}_{\mathrm{s}}$
$I_{A}=\frac{230}{\left(24.50+\mathrm{j} 12.75-\mathrm{jX}_{\mathrm{c}}\right)}$
$\phi_{\mathrm{A}}=-\tan ^{1}\left(\frac{12.75-\mathrm{X}_{\mathrm{C}}}{24.50}\right)$
Taking $\phi_{\mathrm{m}}+90^{\circ}=\phi_{\mathrm{A}}$
$\tan ^{-1}\left(\frac{15.75}{12.50}\right)+90^{\circ}=\tan ^{-1}\left(\frac{12.75-\mathrm{X}_{\mathrm{c}}}{24.50}\right)$
$\tan ^{-1}\left(\frac{15.75}{12.5}\right)-\tan ^{-1}\left(\frac{12.75-\mathrm{X}_{\mathrm{c}}}{24.50}\right)=90^{\circ}$
Taking tan on both sides
$\frac{\left(\frac{15.75}{12.5}\right)-\left(\frac{12.75-\mathrm{X}_{\mathrm{c}}}{24.50}\right)}{1+15.75)}$
$\overline{1+\left(\frac{15.75}{12.5}\right)\left(\frac{12.75-\mathrm{X}_{\mathrm{c}}}{24.50}\right)}=\tan 90^{\circ}=\infty$
$\therefore 1+\left(\frac{15.75}{12.5}\right)\left(\frac{12.75-\mathrm{X}_{\mathrm{c}}}{24.50}\right)=0$
Solving for $\mathrm{X}_{\mathrm{c}}$
$\mathrm{X}_{\mathrm{c}}=32.194$
also $\mathrm{X}_{\mathrm{c}}=\frac{1}{\omega \mathrm{C}_{\mathrm{s}}}$
$\Rightarrow \mathrm{C}_{\mathrm{s}}=\frac{1}{\omega \mathrm{X}_{\mathrm{c}}}=\frac{1}{2 \pi \times 100 \times 32.194}$
or $\mathrm{C}_{\mathrm{s}}=98.87 \mu \mathrm{~F}$
53. Consider a causal and stable LTI system with rational transfer function $\mathrm{H}(\mathrm{z})$, whose corresponding impulse response begins at $\mathrm{n}=0$. Furthermore, $\mathrm{H}(1)=\frac{5}{4}$. The poles of $H(z)$ are $p_{k}=\frac{1}{\sqrt{2}} \exp \left(j \frac{(2 k-1) \pi}{4}\right)$ for $k=1$,

## $2,3,4$. The zeros of $H(z)$ are all at $z=0$.

 Let $g[n]=j^{n h}[n]$. The value of $g[8]$ equals$\qquad$ . (Give the answer up to three decimal places).
Sol. (0.098)
(0.098)

$$
\begin{aligned}
\mathrm{P}_{\mathrm{k}} & =\frac{1}{\sqrt{2}} \exp \left(\mathrm{j} \frac{(2 \mathrm{k}-1) \pi}{4}\right), \\
\mathrm{k} & =1,2,3,4 \\
\mathrm{P}_{1} & =\frac{1}{\sqrt{2}} \mathrm{e}^{\frac{\mathrm{j} \pi}{4}} \\
& =\frac{1}{\sqrt{2}}\left[\cos \left(\frac{\pi}{4}\right)+\mathrm{j} \sin \left(\frac{\pi}{4}\right)\right] \\
& =\frac{(1+\mathrm{j})}{2} \\
\mathrm{P}_{2} & =\frac{1}{\sqrt{2}} \mathrm{e}^{\frac{\mathrm{j} 3 \pi}{4}} \\
& =\frac{1}{\sqrt{2}}\left[\cos \left(\frac{3 \pi}{4}\right)+\mathrm{j} \sin \left(\frac{3 \pi}{4}\right)\right] \\
& =\frac{(-1+\mathrm{j})}{2}
\end{aligned}
$$

$$
\begin{aligned}
P_{3} & =\frac{1}{\sqrt{2}} e^{\frac{j 5 \pi}{4}} \\
& =\frac{1}{\sqrt{2}}\left[\cos \left(\frac{5 \pi}{4}\right)+j \sin \left(\frac{5 \pi}{4}\right)\right] \\
& =\frac{(-1-j)}{2} \\
P_{4} & =\frac{1}{\sqrt{2}} e^{\frac{j 7 \pi}{4}} \\
& =\frac{1}{\sqrt{2}}\left[\cos \left(\frac{7 \pi}{4}\right)+j \sin \left(\frac{7 \pi}{4}\right)\right] \\
& =\left(\frac{1-j}{2}\right)
\end{aligned}
$$

System is causal so order of numerator can not be greater than order of denominator. Therefore,

$$
\mathrm{H}(\mathrm{z})=\frac{\mathrm{K} \cdot \mathrm{Z}^{4}}{\left(\mathrm{Z}-\mathrm{P}_{1}\right)\left(\mathrm{Z}-\mathrm{P}_{2}\right)\left(\mathrm{Z}-\mathrm{P}_{3}\right)\left(\mathrm{Z}-\mathrm{P}_{4}\right)}
$$

$$
=\frac{K \cdot Z^{4}}{\left[Z-\left(\frac{1+j}{2}\right)\right]\left[Z-\left(\frac{-1+j}{2}\right)\right]\left[Z-\left(\frac{-1-j}{2}\right)\right]\left[Z-\frac{(1-j)}{2}\right]}
$$

$$
\begin{aligned}
& \mathrm{H}(\mathrm{z})=\frac{\mathrm{KZ}^{4}}{\mathrm{Z}^{4}+\frac{1}{4}} \\
& \mathrm{H}(1)=\frac{5}{4} \\
& \frac{\mathrm{~K}}{1+\frac{1}{4}}=\frac{5}{4} \\
& \frac{4}{5} \mathrm{~K}=\frac{5}{4} \\
& \mathrm{H}(\mathrm{z})=\frac{25}{16} \frac{\mathrm{Z}^{4}}{\mathrm{Z}^{4}+\frac{1}{4}} \\
& \mathrm{H}(\mathrm{z})=\frac{25}{16}\left[1-\frac{1}{4} \mathrm{Z}^{-4}+\frac{1}{16} \mathrm{Z}^{-8}+\ldots . .\right]
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{h}[\mathrm{n}]= \frac{25}{16}\left[\delta(\mathrm{n})-\frac{1}{4} \delta(\mathrm{n}-4)+\frac{1}{16} \delta(\mathrm{n}-8) \ldots . .\right] \\
& \mathrm{h}[8]=\frac{25}{16}\left[\delta(8)-\frac{1}{4} \delta(4)+\frac{1}{16} \delta(0) \ldots . .\right] \\
&=\frac{25}{16}\left[0-\frac{1}{4} \times 0+\frac{1}{16} \times 1+\ldots . .0\right] \\
& \mathrm{h}[8]=\frac{25}{16} \times \frac{1}{16}=\frac{25}{256}=0.098 \\
& \mathrm{~g}[\mathrm{n}]=\mathrm{j}^{\mathrm{n}} \mathrm{~h}[\mathrm{n}] \\
& \mathrm{g}[8]=\mathrm{j}^{8} \mathrm{~h}[8]=\mathrm{h}[8]=0.098 \\
& \mathrm{~g}[8]=0.098
\end{aligned}
$$

54. Let a causal LTI system be characterized by the following differential equation, with initial rest condition

$$
\frac{d^{2} y}{d t^{2}}+7 \frac{d y}{d t}+10 y(t)=4 x(t)+5 \frac{d x(t)}{d t}
$$

where, $x(t)$ and $y(t)$ are the input and output respectively. The impulse response of the system is $[u(t)$ is the unit step function]
(a) $2 \mathrm{e}^{-2 \mathrm{t}} \mathrm{u}(\mathrm{t})-7 \mathrm{e}^{-5 \mathrm{t}} \mathrm{u}(\mathrm{t})$
(b) $-2 \mathrm{e}^{-2 \mathrm{t}} \mathrm{u}(\mathrm{t})+7 \mathrm{e}^{-5 \mathrm{t}} \mathrm{u}(\mathrm{t})$
(c) $7 \mathrm{e}^{-2 \mathrm{t}} \mathrm{u}(\mathrm{t})-2 \mathrm{e}^{-5 \mathrm{t}} \mathrm{u}(\mathrm{t})$
(d) $-7 \mathrm{e}^{-2 \mathrm{t}} \mathrm{u}(\mathrm{t})+2 \mathrm{e}^{-5 \mathrm{t}} \mathrm{u}(\mathrm{t})$

Sol. (b)
Differential equation
$\frac{\mathrm{d}^{2} \mathrm{y}}{\mathrm{dt}^{2}}+7 \frac{\mathrm{dy}}{\mathrm{dt}}+10 \mathrm{y}(\mathrm{t})=4 \mathrm{x}(\mathrm{t})+5 \frac{\mathrm{dx}(\mathrm{t})}{\mathrm{dt}}$
Taking Laplace on both sides (initial rest condition)
$\mathrm{s}^{2} \mathrm{Y}(\mathrm{s})+7 \mathrm{sY}(\mathrm{s})+10 \mathrm{Y}(\mathrm{s})=4 \mathrm{X}(\mathrm{s})+5 \mathrm{~s} \mathrm{X}(\mathrm{s})$
$H(s)=\frac{Y(s)}{X(s)}=\frac{5 s+4}{s^{2}+7 s+10}$
Impulse response $h(t)=L^{-1}(H(s))$
$h(t)=L^{-1}\left(\frac{5 s+4}{(s+2)(s+5)}\right)$
$=\mathrm{L}^{-1}\left(\frac{-2}{\mathrm{~s}+2}+\frac{7}{\mathrm{~s}+5}\right)$
$h(t)=-2 \mathrm{e}^{-2 \mathrm{t}} 4(\mathrm{t})+7 \mathrm{e}^{-5 \mathrm{t}} 4(\mathrm{t})$
55. Only one of the real roots of $f(x)=x^{6}-x$ - 1 lies in the interval $1 \leq x \leq 2$ and bisection method is used to find its value. For achieving an accuracy of 0.001 , the required minimum number of iterations is

Sol. (10)
In bisection method, the minimum number of iterations is given by $\frac{|b-a|}{2^{\text {n }}}<\varepsilon$ where
a: lower limit of interval
b: upper limit of interval
$\varepsilon$ : Error in approximation
n : Number of iteration
Thus

$$
\begin{aligned}
& \frac{|2-1|}{2^{\mathrm{n}}}<0.001 \\
\Rightarrow & 2^{\mathrm{n}}>1000 \\
\Rightarrow & \mathrm{n}=10
\end{aligned}
$$

## Aptitude

1. The probability that a k-digit number does NOT contain the digits 0,5 or 9 is
(a) $0.3^{\mathrm{k}}$
(b) $0.06^{\mathrm{k}}$
(c) $0.7^{\mathrm{k}}$
(d) $0.9^{\mathrm{k}}$

Sol. (c)
k- digit number


Excluding digits 0, 5 or 9

Probability

$$
\begin{aligned}
& \mathrm{P}=\frac{{ }^{7} \mathrm{C}_{1} \cdot{ }^{7} \mathrm{C}_{1} \cdot{ }^{7} \mathrm{C}_{1} \ldots \cdot{ }^{7} \mathrm{C}_{1}}{{ }^{10} \mathrm{C}_{1}{ }^{10} \mathrm{C}_{1}{ }^{10} \mathrm{C}_{1} \ldots{ }^{10} \mathrm{C}_{1}}(\mathrm{k} \text { - times }) \\
&=\frac{7.7 .7 \ldots . .7}{10.10 .10 \ldots . .10}(\mathrm{k} \text { - times }) \\
&(\mathrm{k}-\text { times })
\end{aligned}
$$

or $\mathrm{P}=(0.7)^{\mathrm{k}}$
2. Rahul, Murali, Srinivas and Arul are seated around a square table. Rahul is sitting to the left of Murali. Srinivas is sitting to the right of Arul. Which of the following pairs are seated opposite each other?
(a) Rahul and Murali
(b) Srinivas and Arul
(c) Srinivas and Murali
(d) Srinvas and Rahul

Sol. (c)

3. Research in the workplace reveals that people work for many reasons $\qquad$
(a) money beside
(b) beside money
(c) money besides
(d) besides money

Sol. (d)
Beside $\rightarrow$ 'next to'
Besides $\rightarrow$ 'Except'
4. After Rajendra Chola returned from his voyage to Indonesia, he $\qquad$ to visit the temple in Thanjavur.
(a) was wishing
(b) is wishing
(c) wished
(d) had wished

Sol. (c)
5. Find the smallest number y such that $\mathrm{y} \times$ 162 is a perfect cube
(a) 24
(b) 27
(c) 32
(d) 36

Sol. (d)
$\mathrm{y} \times 162$ as perfact cube $16^{2}=2 \times 3 \times 3 \times 3 \times 3$
to make it perfect cube $y=2 \times 2 \times 3 \times 3$ or $\mathrm{y}=36$
6. The expression $\frac{(\mathrm{x}+\mathrm{y})-|\mathrm{x}-\mathrm{y}|}{2}$ is equal to
(a) The maximum of $x$ and $y$
(b) The minimum of $x$ and $y$
(c) 1
(d) None of the above

Sol. (b)
Given expression is $\frac{(x+y)-|x-y|}{2}$
we know modulus of any number should be a positive value
Case (1): $\mathrm{x}>\mathrm{y}$ (here y is minimum)
then $|x-y|=(x-y)$ positive value

$$
\text { then } \begin{aligned}
\frac{(x+y)-(x-y)}{2} & =\frac{(x+y)-(x-y)}{2} \\
& =\frac{2 y}{2} \\
=y & \text { (minimum of } x \text { and } y)
\end{aligned}
$$

Case (2): $y>x$ (here $x$ is minimum)
then $|x-y|=(y-x)$ (positive value)
then $\frac{(x+y)-|x-y|}{2}=\frac{(x+y)-(y-x)}{2}$

$$
\begin{gathered}
=\frac{2 \mathrm{x}}{2} \\
=\mathrm{x}(\text { minimum of } \mathrm{x} \text { and } \mathrm{y})
\end{gathered}
$$

In both the cases we get the minimum of x and y and the corretion option is (b).
7. "The hold of the nationalist inagination on our colonial past is such that anything inadequately or improperly nationlist is just not history."

Which of the following statements best reflects the author's opinion?
(a) Nationalists are highly imaginative
(b) History is viewed through the filter of nationalism
(c) Our colonial past never happened
(d) Nationalism has to be both adequately and properly imagined

Sol. (b)
8. A contour line joins locations having the same height above the mean sea level. The following is a contour plot of a geographical region. Contour lines are shown at 25 m intervals in this plot. If in a flood, the water level rises to 525 m . Which of the villages P, Q, R, S, T get submerged?

(a) $\mathrm{P}, \mathrm{Q}$
(b) $P, Q, T$
(c) $R, S, T$
(d) $\mathrm{Q}, \mathrm{R}, \mathrm{S}$

Sol. (c)
Height above mean sea level for
$\mathrm{P} \Rightarrow \mathrm{H}_{\mathrm{P}}=575 \mathrm{~m}$
$\mathrm{Q} \Rightarrow \mathrm{H}_{\mathrm{Q}}=525 \mathrm{~m}$
$\mathrm{R} \Rightarrow \mathrm{H}_{\mathrm{R}}=475 \mathrm{~m}$
$\mathrm{S} \Rightarrow \mathrm{H}_{\mathrm{s}}=475 \mathrm{~m}$
$\mathrm{T} \Rightarrow \mathrm{H}_{\mathrm{T}}=500 \mathrm{~m}$
if water level in a flood is 252 m then $\mathrm{R}, \mathrm{S}, \mathrm{T}$ will be submerged.
9. Six people are seated around a circular table. There are at least two men and two women. There are at least three righthanded persons. Every woman has a lefthanded person to her immediate right. None of the women are right-handed. The number of women at the table is
(a) 2
(b)
(c) 4
(d) Cannot be determined

Sol. (a)
Total perosns-6

## Conditions:

1. Atleast two men and two women
2. Atleast 3-right handed persosn
3. Every women has a left handed person to her immediate right and all women are left handed.

Let us choose at least two women (minimum) then total left handed persons $=2+1(1 \mathrm{man}$ is immediate right of one woman when both woman are sitting together) $=3$
Remaining three will be right handed
Hence correct answer is (a).
10. Arun, Gulab, Neel and Shweta must choose one shift each from a pile of four shirt coloured red, pink, blue and white respectively. Arun dislike the colour red and Shweta dislikes the colour white. Gulab and Neel like all the colours. In how many different ways can they choose the shirts so that no one has a shirt with a colour he or she dislikes?
(a) 21
(b) 18
(c) 16
(d) 14

Sol. (d)
Colour - Red, Pink, Blue, White
Arun $\leftarrow$ (hed
Shweta $\leftarrow$ Nite
Case-Arun chooses pink shirt then Shweta will have two options Red and blue so number of ways
$\mathrm{n}_{1}={ }^{1} \mathrm{C}_{1} \cdot{ }^{2} \mathrm{C}_{1} \cdot{ }^{2} \mathrm{C}_{1} \cdot{ }^{1} \mathrm{C}_{1}=4$

## Case-2

Arun chooses bule shirt, Shweta will have two options Red and Pink, so
$\mathrm{n}_{2}={ }^{1} \mathrm{C}_{1} \cdot{ }^{2} \mathrm{C}_{1} \cdot{ }^{2} \mathrm{C}_{1} \cdot{ }^{1} \mathrm{C}_{1}=4$
Case-3
Arun chooses white, then Shweta will have three options, so
$\mathrm{n}_{3}={ }^{1} \mathrm{C}_{1} \cdot{ }^{3} \mathrm{C}_{1} \cdot{ }^{2} \mathrm{C}_{1} \cdot{ }^{1} \mathrm{C}_{1}=6$
So total number of ways $=4+4+6=14$


