# M 

# aATIE 2017 

# Detailed Solution 

## MECHANICAL ENGINEERING SESSION - 1

## GATE—2017

## Mech. Engineering Questions and Details Solution Session-1

1. A particle of unit mass is moving on a plane. Its trajectory in polar coordinates is given by $\mathrm{r}(\mathrm{t})=\mathrm{t}^{2}, \phi(\mathrm{t})=\mathrm{t}$ where t is time. The kinetic of the particle at time $t=2$ is
(a) 4
(b) 12
(c) 16
(d) 24

Sol. (c)

2. Cylindrical pins of diameter $15^{ \pm 0.020} \mathrm{~mm}$ are being produced on a machine. Statistical quality control tests show a mean of 14.995 mm and standard deviation of 0.04 mm . The process capability index $\mathrm{C}_{\mathrm{p}}$ is
(a) 0.833
(b) 1.667
(c) 3.333
(d) 3.750

Sol. (b)

$$
\mathrm{C}_{\mathrm{P}}=\frac{\mathrm{USL}-\mathrm{LSL}}{6 \sigma}
$$

$$
\begin{aligned}
& =\frac{15.02-14.98}{6 \times 0.004} \\
& =1.667
\end{aligned}
$$

3. Which one of the following is NOT a rotating machine?
(a) Centrifugal pump
(b) Gear pump
(c) Jet pump
(d) Vane pump

Sol. (c)
Centrifugal pump: It has rotating part eg., impeller, Vane.
Gear Pump: In this pump there is gear mechanism which is rotating part.
Jet Pump: Here pump utilizing ejecter principle which have nozzle and difusses not rotating parts.
Vane Pump: It consist of rotating disc which called as rotor in which number of radial slots are there where sliding vanes is inserted
4. A six-face fair dice is rolled a large number of times. The mean valueof the outcomes is ___.4. A six-face fair dice is rolled a large number of times. The mean value of the outcomes is $\qquad$ .

Sol. (3.5)

$$
\begin{aligned}
\text { Mean outcome } & =\sum_{\mathrm{i}=1}^{6} \mathrm{n}_{\mathrm{i}} \mathrm{p}_{\mathrm{i}} \\
& =\frac{1+2+3+4+5+6}{6}\left[\mathrm{p}_{\mathrm{i}}=\frac{1}{6}\right] \\
& =3.5
\end{aligned}
$$

5. In an arc welding process, welding speed is doubled. Assuming all other process parameters to be constant, the cross sectional area of the weld bead will
(a) increase by $25 \%$
(b) increase by $50 \%$
(c) reduce by $25 \%$
(d) reduce by $50 \%$

Sol. (d)
Since, all process parameter are constant
Material deposition rate $=$ constant

$$
\begin{array}{rlrl}
= & \text { Area of weld }\left(\mathrm{A}_{\mathrm{w}}\right) \\
& & \times \text { welding speed }\left(\mathrm{V}_{\mathrm{w}}\right) \\
\because \quad & \mathrm{V}_{\mathrm{w}}^{\prime}= & 2 \mathrm{~V}_{\mathrm{w}} \\
\therefore \quad & \mathrm{~A}_{\mathrm{w}}^{\prime}= & \mathrm{A}_{\mathrm{w}} \times \frac{\mathrm{V}_{\mathrm{w}}}{\mathrm{~V}_{\mathrm{w}}^{\prime}}=\frac{\mathrm{A}_{\mathrm{w}}}{2}
\end{array}
$$

$$
\% \text { change }=\frac{\mathrm{A}_{\mathrm{w}}^{\prime}-\mathrm{A}_{\mathrm{w}}}{\mathrm{~A}_{\mathrm{w}}} \times 100=-50 \%
$$

6. Saturated steam at $100^{\circ} \mathrm{C}$ condenses on the outside of a tube. Cold fluid enters the tube at $20^{\circ} \mathrm{C}$ and exits at $50^{\circ} \mathrm{C}$. The value of the Log Mean Temperature Difference (LMTD) is $\qquad$ ${ }^{\circ} \mathrm{C}$.

Sol. (63.82 ${ }^{\circ}$ C)


LMTD is given by

$$
\left(\Delta \mathrm{T}_{\mathrm{m}}\right)=\frac{\theta_{1}-\theta_{2}}{\ln \left(\frac{\Delta \theta_{1}}{\Delta \theta_{2}}\right)}
$$

For parallel as well as counter flow heat exchanger.
Considering it as parallel flour heat exchanger.

$$
\begin{aligned}
\Delta \mathrm{T}_{\mathrm{i}} & =100-20=80^{\circ} \mathrm{C} \\
\Delta \mathrm{~T}_{\mathrm{e}} & =100-50=50^{\circ} \mathrm{C} \\
\left(\Delta \mathrm{~T}_{\mathrm{m}}\right) & =\frac{80-50}{\ln \left(\frac{80}{50}\right)} \\
\left(\Delta \mathrm{T}_{\mathrm{m}}\right) & =63.82^{\circ} \mathrm{C}
\end{aligned}
$$

7. The damping ratio for a viscously damped spring mass system, governed by the relationship $m \frac{d^{2} x}{d t^{2}}+c \frac{d x}{d t}+k x=F(t)$, is given by
(a) $\sqrt{\frac{\mathrm{c}}{\mathrm{mk}}}$
(b) $\frac{\mathrm{c}}{2 \sqrt{\mathrm{~km}}}$
(c) $\frac{\mathrm{c}}{\sqrt{\mathrm{km}}}$
(d) $\sqrt{\frac{c}{2 m k}}$

Sol. (b)
$\frac{\mathrm{md}^{2} \mathrm{x}}{\mathrm{dt}^{2}}+\frac{\mathrm{Cdx}}{\mathrm{dt}}+\mathrm{kx}=\mathrm{F}(\mathrm{t})$
or, $m \ddot{x}+c \dot{x}+k x=0$
(By considering sum of the inertia force and external forces on a body in a direction in to be zero)
or,

$$
\mathrm{k}=\mathrm{Ae}^{\alpha \mathrm{t}}+\mathrm{Be}^{\alpha \mathrm{t}}
$$

i.e., $\alpha^{2}+\frac{c}{m} \alpha^{2}+\frac{k}{m}=0$

$$
\alpha_{1,2}=-\frac{\mathrm{C}}{2 \mathrm{~m}} \pm \sqrt{\left(\frac{\mathrm{C}}{2 \mathrm{~m}}\right)^{2}-\left(\frac{\mathrm{k}}{\mathrm{~m}}\right)}
$$

The ratio of $\left(\frac{C}{2 m}\right)^{2}$ to $\frac{s}{m}$ gives the degree of dumpness and square root of those termed as damping ratio.

$$
\varepsilon=\sqrt{\frac{\left(\frac{\mathrm{C}}{2 \mathrm{~m}}\right)^{2}}{\frac{K}{\mathrm{~m}}}}=\frac{\mathrm{C}}{2 \sqrt{\mathrm{~km}}}
$$

Institute for Engineers (IES/GATE/PSUs)

$$
\frac{\mathrm{T}}{\mathrm{~J}}=\frac{\mathrm{T}_{\max }}{\mathrm{r}_{\mathrm{a}}}
$$

8. A motor driving a solid circular steel shaft transmits 40 kW of power at 500 rpm . If the diameter of the shaft is 40 mm , the maximum shear stress in the shaft is $\qquad$ MPa.

Sol. (60.792 MPa)


Given
Power transmitted P, 40 KW
Speed of shaft, N = 500 rpm
Diameter, $\mathrm{a}=40 \mathrm{~mm}$
We know

$$
\mathrm{P}=\frac{2 \pi \mathrm{NT}}{60}[\text { Where T-Torque }]
$$

So, $\quad \mathrm{T}=\frac{60 \mathrm{P}}{2 \pi \mathrm{~N}}$

$$
\begin{aligned}
& \mathrm{T}=\frac{60 \times 40 \times 10^{3}}{2 \times \pi \times 500} \mathrm{~N}-\mathrm{m} \\
& \mathrm{~T}=763.44 \mathrm{~N}-\mathrm{m}
\end{aligned}
$$

Maximum shear stress after applying Torque, $T$ will be at a distance $d / 2$ from neutral axis and will be given by

$$
\frac{\mathrm{T}}{\mathrm{~J}}=\frac{\tau_{\max }}{\mathrm{r}_{\max }}
$$

[Where $\mathrm{T}_{\text {min }}=$ Shear Stress
$\mathrm{J}=$ Polar moment of inerita
$\mathrm{r}_{\text {max }}=\mathrm{d} / 2$

$$
\begin{aligned}
\tau_{\max } & =\frac{\mathrm{T} \times \mathrm{d} \times 32}{\pi \mathrm{~d}^{4} \times 2} \\
\tau_{\min } & =60.792 \mathrm{MPa}
\end{aligned}
$$

9. Consider the following partial differential equation $\mathrm{u}(\mathrm{x}, \mathrm{y})$ with the constant $\mathrm{c}>1$ :

$$
\frac{\partial u}{\partial y}+c \frac{\partial u}{\partial x}=0
$$

Solution of this equation is
(a) $u(x, y)=f(x+c y)$
(b) $u(x, y)=f(x-c y)$
(c) $u(x, y)=f(c x+y)$
(d) $u(x, y)=f(c x-y)$

Sol. (b)
Let $u=f(a x+b y)$
$\therefore \quad \frac{\partial u}{\partial(a x+b y)}=f^{\prime}(a x+b y)$
Now, $\frac{\partial u}{\partial y}+C \frac{\partial u}{\partial x}=0$

$$
\begin{gathered}
\frac{\partial u}{\partial(\mathrm{ax}+\mathrm{by})} \times \frac{\partial(\mathrm{ax}+\mathrm{by})}{\partial \mathrm{y}}+\mathrm{C} \frac{\mathrm{du}}{\partial(\mathrm{ax}+\mathrm{by})} \times \frac{\partial(\mathrm{ax}+\mathrm{by})}{\partial \mathrm{x}} \\
=0
\end{gathered}
$$

$$
\Rightarrow \quad b+c \times a=0
$$

$$
\Rightarrow \quad \mathrm{b}=-\mathrm{ac}
$$

If $\mathrm{a}=1$
$\mathrm{b}=-\mathrm{c}$

$$
\begin{aligned}
\therefore \quad u & =\mathrm{f}(1 . \mathrm{x}-\mathrm{C} . \mathrm{y}) \\
& =\mathrm{f}(\mathrm{x}-\mathrm{cy})
\end{aligned}
$$

10. Consider the two-dimensional velocity field given by $\vec{V}=\left(5+a_{1} x+b_{1} y\right)$ $\hat{i}+\left(4+a_{2} x+b_{2} y\right) \hat{j}$. where $a_{1}, b_{1} a_{2}$ and $b_{2}$ are constants. Which one of the following conditins needs to be satisfied for the flow to be incompressible?
(a) $\mathrm{a}_{1}+\mathrm{b}_{1}=0$
(b) $\mathrm{a}_{1}+\mathrm{b}_{2}=0$
(c) $\mathrm{a}_{2}+\mathrm{b}_{2}=0$
(d) $\mathrm{a}_{2}+\mathrm{b}_{2}=0$
11. 

## Sol. (b)

$$
\begin{gathered}
\overrightarrow{\mathrm{V}}=\left(5+\mathrm{a}_{1} \mathrm{x}+\mathrm{b}_{1} \mathrm{y}\right) \hat{\mathrm{i}}+\left(\overline{4}+\mathrm{a}_{2} \mathrm{x}+\mathrm{b}_{2} \mathrm{y}\right) \hat{\mathrm{j}} \\
=\mathrm{u} \hat{\mathrm{i}}+\mathrm{V} \hat{\mathrm{j}}
\end{gathered}
$$

For, incompressible flow,

$$
\begin{gathered}
\frac{\partial \mathrm{u}}{\partial \mathrm{x}}+\frac{\partial \mathrm{v}}{\partial \mathrm{y}}=0 \\
\mathrm{a}_{1}+\mathrm{b}_{2}=0
\end{gathered}
$$

11. The product of eignvalues of the matrix $P$ is

$$
P=\left[\begin{array}{ccc}
2 & 0 & 1 \\
4 & -3 & 3 \\
0 & 2 & -1
\end{array}\right]
$$

(a) -6
(b) 2
(c) 6
(d) -2

Sol. (b)
Product of eigen value $=|P|$

$$
\left|\begin{array}{ccc}
2 & 0 & 1 \\
4 & -3 & 3 \\
0 & 2 & -1
\end{array}\right|
$$

$$
\begin{gathered}
=2(3-6)+1(8-0) \\
=2
\end{gathered}
$$

12. For steady flow of a viscous incompressible fluid through a circular pipe of constant diameter, the average velocity in the fully developed region is constant. Which one of the folloiwng statements about the average velocityin the developing region is TRUE?
(a) It increases until the flow is fully developed.
(b) It is constant and is equal to the average velocity in the fully developed region.
(c) It decreases until the flow is fully developed
(d) It is constant but is always lower than the average velocity in the fully developed region.

Sol. (c)


As the distance from leading edge increases, retradation goes on increasing and hence average velocity goes on decreasing.
13. The Poisson's rati for a perfectly incompressible linear elastic material is
(a) 1
(b) 0.5
(c) 0
(d) infinity

Sol. (b)
Volumetric strain for linear elastic material,

$$
\varepsilon_{\mathrm{v}}=\frac{\Delta \mathrm{V}}{\mathrm{~V}}=\frac{(1-2 \mu)}{\mathrm{E}}\left(\sigma_{\mathrm{x}}+\sigma_{\mathrm{y}}+\sigma_{\mathrm{z}}\right)
$$

For incompressible flow

14. In the engineering stress-strain curve for mild steel, the Ultimate Tensile Strength (UTS) refers to
(a) Yield stress
(b) Proportional limit
(c) Maximum stress
(d) Fracture stress

Sol. (c)
For mild, steel stress-strain curve is :

15. The molar specific heat at constant volume of an ideal gas is equal to 2.5 times the universal gas constant ( $8.314 \mathrm{~J} / \mathrm{mol} . \mathrm{K}$ ). When the temperature increases by 100 K , the change in molar specific enthalpy is $\qquad$ J/ mol.

# M IES MASTER 

## ANNOUNCES NEW BATCHES FOR IES/GATE/PSUs

## BRANCHES ।




## ADMISSION OPEN FOR SESSION 2017-18

F-126, Katwaria Sarai, New Dehi-16

Sol. (2909.9 J/mol)
$\Delta \mathrm{h}=$ specific enthalpy $=\mathrm{C}_{\mathrm{P}} \Delta \mathrm{T}$

$$
\begin{aligned}
& =\left(\mathrm{C}_{\mathrm{V}}+\mathrm{R}\right) \Delta \mathrm{T} \\
& =(2.5 \mathrm{R}+\mathrm{R}) \Delta \mathrm{T} \\
& =3.5 \times 8.314 \times 100 \mathrm{~J} / \mathrm{mol} \\
& =2909.9
\end{aligned}
$$

16. A heat pump absorbs 10 kW of heat from outside environment at 250 K while absorbing 15 kW of work. It delivers the heat to a room that must be kept warm at 300 K . The Coefficient of Performance (COP) of the heat pump is $\qquad$ _.

Sol. (1.67)

$$
\begin{aligned}
\text { C.O.P. } & =\frac{\text { Head delivered to room }}{\text { work input }} \\
& =\frac{25 \mathrm{Kw}}{15 \mathrm{Kw}}=1.67
\end{aligned}
$$

Here,
Heat delivered $=$ Heat taken + work input
17. The following figure shows the velocity-time plot for a particle travelling along a straight line. The distance covered by the particle from $\mathrm{t}=0$ to $5=5 \mathrm{~s}$ is _ m .


Sol. (10)
Since, $\frac{D}{t}=V$


Distance covered
$=$ Area under the curve from $t=0$ to $t=5 \mathrm{sec}$.


$$
\begin{aligned}
& +\frac{1}{2} \times(4+2) \times(5-3) \\
& =10
\end{aligned}
$$

18. The differential equation $\frac{d^{2} y}{d x^{2}}+16 y=0$ for $y(x)$ with the two boundary conditions
$\left.\frac{d y}{d x}\right|_{x=0}=1$ and $\left.\frac{d y}{d x}\right|_{x=\frac{\pi}{2}}=-1$ has
(a) no solution
(b) exactly two solutions
(c) exactly one solution
(d) infinitely many solutions

Sol. (a)

$$
\begin{aligned}
\frac{\mathrm{d}^{2} \mathrm{y}}{\mathrm{dx}^{2}}+16 \mathrm{y} & =0 \\
\left(\mathrm{D}^{2}+16\right) \mathrm{y} & =0 \\
\text { Let } \quad \mathrm{D}^{2} & =\mathrm{m}^{2} \\
\mathrm{~m}^{2}+16 & =0 \text { (this is a complex equation) } \\
\mathrm{m} & = \pm 4 \mathrm{i}=0 \pm 4 \mathrm{i}
\end{aligned}
$$

Regd. office : F-126, (Upper Basement), Katwaria Sarai, New Delhi-110016 Phone : 011-41013406
Mob. : 8010009955, 9711853908 - E-mail: ies_master@yahoo.co.in, info@iesmaster.org

$$
\begin{aligned}
\mathrm{y} & =\left(\mathrm{C}_{1} \cos 4 \mathrm{x}+\mathrm{C}_{2} \sin 4 \mathrm{x}\right) \mathrm{e}^{\mathrm{ox}} \\
\Rightarrow \quad \mathrm{y} & =\mathrm{C}_{1} \cos 4 \mathrm{x}+\mathrm{C}_{2} \sin 4 \mathrm{x} \\
\Rightarrow \quad \mathrm{y}^{\prime} & =-4 \mathrm{C}_{1} \sin 4 \mathrm{x}+4 \mathrm{C}_{2} \cos 4 \mathrm{x} \\
\mathrm{y}^{\prime}(0) & =4 \mathrm{C}_{2}=1 \\
\mathrm{C}_{2} & =\frac{1}{4} \\
\mathrm{y}^{\prime}\left(\frac{\pi}{2}\right) & =-1=-4 \mathrm{C}_{1} \sin 2 \pi+4 \mathrm{C}_{2} \cos 2 \pi \\
-1 & =4 \mathrm{C}_{2} \\
\mathrm{C}_{2} & =-\frac{1}{4}
\end{aligned}
$$

19. In a metal forming operation when the material has just started yielding, the principal stresses are $\sigma_{1}=+180 \mathrm{MPa}, \sigma_{2}=-100 \mathrm{MPa}, \sigma_{3}=0$. Following von Mises' criterion the yield stress is
$\qquad$ MPa .

Sol. (245.76)
According to Von-misces, yield stress ( $\sigma_{y t}$ ) is given by

$$
\left(\sigma_{1}-\sigma_{2}\right)^{2}+\left(\sigma_{2}-\sigma_{3}\right)^{2}+\left(\sigma_{3}-\sigma_{1}\right)^{2} \leq 2\left(\frac{\sigma_{\mathrm{yt}}}{\mathrm{~N}}\right)^{2}
$$

Given,

$$
\begin{aligned}
\sigma_{1} & =+180 \mathrm{MPa} \\
\sigma_{2} & =-100 \mathrm{MPa} \\
\sigma_{3} & =0 \\
\mathrm{~N} & =1 \\
\sigma_{\mathrm{yt}} & =\frac{\sqrt{\left(\sigma_{1}-\sigma_{2}\right)+\sigma_{2}^{2}+\sigma_{1}^{2}}}{\sqrt{2}} \\
& =245.76 \mathrm{MPa}
\end{aligned}
$$

20. The value of $\lim _{x \rightarrow 0} \frac{x^{3}-\sin (x)}{x}$ is
(a) 0
(b) 3
(c) 1
(d) -1

Sol. (d)

$$
\operatorname{Lim}_{x \rightarrow 0} \frac{x^{3}-\sin x}{x}=\lim _{x \rightarrow 0} \frac{3 x^{2}-\cos }{1}
$$

[Using L Hospital Rule]

$$
=-1
$$

21. Consider the schematic of a riveted lap joint subjected to tensile load F , as shown below. Let $d$ be the diameter of the rivets, and $S_{f}$ be the maximum permissible tensile stress in the paltes. What should be the minimum value for the thickness of the plates to guard against tensile failure of the plates? Assume the plates to be identical.

(a) $\frac{\mathrm{F}}{\mathrm{S}_{\mathrm{f}}(\mathrm{W}-2 \mathrm{~d})}$
(b) $\frac{\mathrm{F}}{\mathrm{S}_{\mathrm{f}} \mathrm{W}}$
(c) $\frac{\mathrm{F}}{\mathrm{S}_{\mathrm{f}}(\mathrm{W}-\mathrm{d})}$
(d) $\frac{2 \mathrm{~F}}{\mathrm{~S}_{\mathrm{f}} \mathrm{W}}$

Sol. (a)

$\frac{\mathrm{F}}{\text { (Area of shear) }}$
$=$ Max. permissible tensile stress $\left(S_{f}\right)$
$\Rightarrow \frac{\mathrm{F}}{(\mathrm{w}-2 \mathrm{~d}) \times \mathrm{t}}=\mathrm{S}_{\mathrm{f}}$
$\Rightarrow \quad t=\frac{F}{S_{f}(w-2 d)}$

Regd. office : F-126, (Upper Basement), Katwaria Sarai, New Delhi-110016 Phone : 011-41013406
22. Water (density $=1000 \mathrm{~kg} / \mathrm{m}^{3}$ ) at ambient temperature flows through a horizontal pipe of uniform corss section at the rate of $1 \mathrm{~kg} /$ s . If the pressure drop across the pipe is 100 kPa , the minimum power required to pump the water across the pipe, in watts, is $\qquad$ _.

Sol. (100 Watt)

$$
\begin{aligned}
& \Delta \mathrm{P}=100 \mathrm{kPa}=100 \times 10^{3} \mathrm{~N} / \mathrm{m}^{2} \\
& \mathrm{Q}=1 \mathrm{~kg} / \mathrm{sec} \\
& \text { or, } \\
& \rho A V=1 \mathrm{~kg} / \mathrm{sec} \\
& \text { or } \\
& A=\frac{1}{\rho V}=\frac{1}{\rho} \\
& \text { Power }=\frac{\Delta \mathrm{P} \times \mathrm{A}}{\mathrm{t}} \\
& =\frac{100 \times 10^{3} \times 1}{1000}=100 \mathrm{watt}
\end{aligned}
$$

23. Metric thread of 0.8 mm pitch is to be cut on a lathe. Pitch of the lead screw is 1.5 mm . If the spindle rotates at 1500 rpm , the speed of rotation of the lead screw (rpm) will be
$\qquad$ _.

Sol. (800)

$$
\begin{aligned}
\mathrm{P}_{\mathrm{t}} & =0.8 \mathrm{~mm} \text { [Petch of thread }] \\
\mathrm{N}_{\mathrm{t}} & =1500 \mathrm{rpm}[\mathrm{RPM} \text { of spindle] }] \\
\mathrm{P}_{\mathrm{s}} & =1.5 \mathrm{~mm} \\
\mathrm{~N}_{\mathrm{S}} \times \mathrm{P}_{\mathrm{S}} \times \mathrm{Z}_{\mathrm{S}} & =\mathrm{N}_{\mathrm{t}} \times \mathrm{P}_{\mathrm{t}} \times \mathrm{Z}_{\mathrm{t}} \quad\left[\mathrm{Z}_{\mathrm{s}}=\mathrm{Z}_{\mathrm{t}}=1\right] \\
\Rightarrow \mathrm{N}_{\mathrm{S}} \times 1.5 \times 1 & =1500 \times 0.8 \times 1 \\
\Rightarrow \quad \mathrm{~N}_{\mathrm{S}} & =800 \mathrm{rpm}
\end{aligned}
$$

24. Match the processes with their characteristics.

| Process | Characteristics |
| :--- | :--- |
| P: Electrical Discharge <br> Machining | 1. No residual stress |
| Q: Ultrasonic machining | 2. Machining of <br> electrically <br> conductive <br> materials |
| R: Chemical machining | 3. Machining of glass |
| S: Ion Beam Machining | 4. Nano - machining |

(a) $\mathrm{P}-2, \mathrm{Q}-3, \mathrm{R}-1, \mathrm{~S}-4$
(b) $\mathrm{P}-3, \mathrm{Q}-2, \mathrm{R}-1, \mathrm{~S}-4$
(c) $\mathrm{P}-3, \mathrm{Q}-2, \mathrm{R}-4, \mathrm{~S}-1$
(d) $\mathrm{P}-2, \mathrm{Q}-4, \mathrm{R}-3, \mathrm{~S}-1$

Sol. (a)
\(\left.$$
\begin{array}{l}\text { P EdM } \rightarrow \quad \begin{array}{l}\text { Machining of electronics } \\
\text { conductive material }\end{array}
$$ <br>

Q USM \rightarrow \quad Machining of glass\end{array}\right)\) R Chemical Machining $\rightarrow$ No reduced stress | Nano-machining |
| :---: |

25. Consider a beam with circular cross-section of diameter $d$. The ratio of the second moment of area about the neutral axis to the section modulus of the area is
(a) $\frac{\mathrm{d}}{2}$
(b) $\frac{\pi d}{2}$
(c) 3
(d) $\pi d$

Sol. (a)
Ion circular cross-section, Second moment of area of beam

$$
=\frac{\pi \mathrm{d}^{4}}{64}
$$

Section Modulus $=\frac{\pi \mathrm{d}^{3}}{32}$

$$
\therefore \quad \text { Ratio }=\frac{\mathrm{d}}{2}
$$

26. For a steady flow, the velocity field is $\vec{V}=\left(-x^{2}+3 y\right) \hat{i}+(2 x y) \hat{j}$. The magnitude of the acceleration of a particle at $(1,-1)$ is
(a) 2
(b) 1
(c) $2 \sqrt{5}$
(d) 0

Sol. (c)
Given flow filed

$$
\begin{aligned}
& \overrightarrow{\mathrm{V}}=\left(-x^{2}+3 y\right) \hat{\mathrm{i}}+(2 \mathrm{xy}) \hat{\mathrm{j}} \\
& \overrightarrow{\mathrm{~V}}=\mathrm{i} \hat{\mathrm{i}}+\mathrm{v} \hat{\mathrm{j}}
\end{aligned}
$$

So,

$$
v=2 x y
$$

$$
\mathrm{u}=-\mathrm{x}^{2}+3 \mathrm{y}
$$

For steady flow acceleration is given by

$$
\begin{aligned}
a_{x} & =u \frac{\partial u}{\partial x}+v \frac{\partial u}{\partial y} \\
a_{y} & =u \frac{\partial v}{\partial x}+v \frac{\partial v}{\partial y} \\
a_{x} & =\left(-x^{2}+3 y\right)(-2 x)+(2 x y)(3) \\
a_{x} & =2 x^{3}-6 x y+6 x y \\
a_{(1,-1) x} & =+2
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
a_{(1-1) y} & =4 \\
a_{\text {net }} & =\sqrt{a_{x}^{2}+a_{y}^{2}} \\
a_{\text {net }} & =\sqrt{4+16} \\
a_{\text {net }} & =\sqrt{20}=2 \sqrt{5} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

27. Two models, $P$ and $Q$, of a product earn profits of Rs. 100 and Rs. 80 per piece, respectively. Production times for P and Q are 5 hours and 3 hours, respectively, while the total production time available is 150 hours. For a total batch size of 40 , to maximize profit, the number of units of P to be produced is
$\qquad$
Sol. (15)
Given question can be modelised as

$$
\begin{align*}
\text { Profit, } \mathrm{Z} & =100 \mathrm{P}+80 \mathrm{Q} \\
5 \mathrm{P}+3 \mathrm{Q} & \leq 150 \text { [Time constraint] }  \tag{i}\\
\mathrm{P}+\mathrm{Q} & =40 \tag{ii}
\end{align*}
$$

Putting these equation on graph


$$
\begin{aligned}
\mathrm{Z}(0,0) & =0 \\
\mathrm{Z}(0,40) & =3200 \\
\mathrm{Z}(15,25) & =3500 \rightarrow \text { Maximum } \\
\mathrm{Z}(30,0) & =3000
\end{aligned}
$$

So desired quantity of P is 15 and Q is 25 . Note: the desired point P can be directly calculated by solving equation (i) and (ii)
28. A 10 mm deep cylindrical cup with diameter of 15 mm is drawn from a circular blank. Neglecting the variation in the sheet thickness, the diameter (upto 2 decimal points accuracy) of the blank is $\quad \mathrm{mm}$.

## Sol. (28.72 mm)



Cup dia, $\mathrm{d}=15 \mathrm{~mm}$
Cup height, $\mathrm{h}=10 \mathrm{~mm}$
We know blank diameter D

$$
\begin{aligned}
& \mathrm{D}=\sqrt{\mathrm{d}^{2}+4 \mathrm{dh}} \mathrm{~mm} \\
& \mathrm{D}=\sqrt{15^{2}+4(15 \times 10)} \mathrm{mm} \\
& \mathrm{D}=28.72 \mathrm{~mm}
\end{aligned}
$$

29. The velocity profile inside the boundary layer for flow over a flat plate is given as $\frac{u}{U_{\infty}}=\sin \left(\frac{\pi}{2} \frac{y}{\delta}\right)$, where $U_{\infty}$ is the free stream velocity and $\delta$ is the local boundary layer thickness. If $\delta^{*}$ is the local displacement thickness, the value of $\frac{\delta^{*}}{\delta}$ is
(a) $\frac{2}{\pi}$
(b) $1-\frac{2}{\pi}$
(c) $1+\frac{2}{\pi}$
(d) 0

Sol. (b)
Given, $\quad \frac{\mathrm{U}}{\mathrm{U}_{\infty}}=\sin \left(\frac{\pi}{2} \frac{\mathrm{y}}{\delta}\right)$
Boudnary layer thickness $=\delta$
Local displacement thickness

$$
\begin{aligned}
& =\delta^{+}=\int_{0}^{\delta}\left(1-\frac{\mathrm{U}}{\mathrm{U}_{\infty}}\right) \mathrm{dy} \\
\delta^{*} & =\int_{0}^{\delta}\left[1-\sin \left(\frac{\pi \mathrm{y}}{2 \delta}\right)\right] \mathrm{dy} \\
\delta^{*} & =\left[\mathrm{y}+\frac{2 \delta}{\pi} \mathrm{x} \cos \left(\frac{\pi \mathrm{y}}{2 \delta}\right)\right]_{0}^{\delta} \\
\delta^{*} & =\left[\delta+0-0-\frac{2 \delta}{\pi}\right] \\
\delta^{*} & =\delta\left(1-\frac{2}{\pi}\right)
\end{aligned}
$$

So, $\quad \frac{\delta^{*}}{\delta}=1-\frac{2}{\pi}$
30. A parametric curve defined by $\mathrm{x}=\cos \left(\frac{\pi \mathrm{u}}{2}\right), \mathrm{y}=\sin \left(\frac{\pi \mathrm{u}}{2}\right)$ in the range $0 \leq u \leq 1$ is rotated about the X-axis by 360 degrees. Area of the surface generated is
(a) $\frac{\pi}{2}$
(b) $\pi$
(c) $2 \pi$
(d) $4 \pi$

Sol. (c)

$$
\begin{aligned}
x & =\cos \left(\frac{\pi u}{2}\right) \\
y & =\sin \left(\frac{\pi u}{2}\right) \\
x^{2}+y^{2} & =1
\end{aligned}
$$

It represents a circle in $x-y$ plane.


Given $0 \leq \mathrm{u} \leq 1$
So, $0 \leq x \leq 1, \quad 0 \leq y \leq 1$
i.e., $0 \leq \theta \leq \frac{\pi}{2}$

So, we will get as quarter circle in $x-y$ plane and by revolving it by $360^{\circ}$, we will get a hemisphere of radius unit.
Area of hemisphere $=2 \pi(1)^{2}$

$$
=2 \pi
$$

31. A horizontal bar, fixed at one end $(x=0)$, has alength of 1 m , and cross-sectional area of $100 \mathrm{~m}^{2}$. Its elastic modulus varies along its length as given by $\mathrm{E}(\mathrm{x})=100 \mathrm{e}^{-\mathrm{x}} \mathrm{GPa}$, where x is the length coordinate (in m ) along the axis of the bar. An axial tensile load of 10 kN is applied at the free end $(\mathrm{x}=1)$. The axial displacement of the free end is $\qquad$ mm .

Sol. (1.718)


Given length, $\mathrm{L}=1 \mathrm{~m}$

$$
\begin{aligned}
\mathrm{A} & =100 \mathrm{~mm}^{2} \\
\mathrm{E}(\mathrm{x}) & =100 \mathrm{e}^{-\mathrm{x}} \mathrm{GPa} \\
\mathrm{P} & =10 \mathrm{KN} \\
\text { Reaction, } \mathrm{R} & =10 \mathrm{KN}
\end{aligned}
$$

So elongation is given by

$$
\mathrm{d} \delta=\int_{0}^{\mathrm{x}} \frac{\mathrm{P}(\mathrm{x})}{\mathrm{A}(\mathrm{x}) \mathrm{E}(\mathrm{x})} \mathrm{dx}
$$

here $\mathrm{P}(\mathrm{x})=$ constant $=10 \mathrm{KN}$

$$
\begin{aligned}
& \mathrm{A}(\mathrm{x})=\text { constant }=100 \mathrm{~mm}^{2} \\
& \mathrm{E}(\mathrm{x})=100 \mathrm{e}^{-\mathrm{x}} \mathrm{GPa}
\end{aligned}
$$

$$
\mathrm{d} \delta=\frac{\mathrm{P}}{\mathrm{~A}} \int_{0}^{\mathrm{x}} \frac{1}{100 \mathrm{e}^{-\mathrm{x}}} \mathrm{dx}
$$

$$
\mathrm{d} \delta=\frac{10 \times 10^{3}\left[\mathrm{e}^{1}-\mathrm{e}^{0}\right]}{100 \times 10^{-6} \times 100 \times 10^{9}}
$$

$$
\mathrm{d} \delta=1.718 \times 10^{-3} \mathrm{~m}
$$

Axial displacement $=\mathrm{d} \delta=1.718 \mathrm{~mm}$
32. An initially stress-free massless elastic beam of length $L$ and circular cross-section with diameter $\mathrm{d}(\mathrm{d} \ll \mathrm{L})$ is held fixed between two walls as shown. The beam material has Young's modulus E and coefficient of thermal expansion $\alpha$.


If the beam is slowly and uniformly heated, the temperature rise required to cause the beam to buckle is proportional to
(a) d
(b) $\mathrm{d}^{2}$
(c) $\mathrm{d}^{3}$
(d) $\mathrm{d}^{4}$

Sol. (b)


On increasing temperature thermal stress

$$
\sigma=\mathrm{E} \alpha \Delta \mathrm{~T}
$$

Using bucklig condition buckling load

$$
\mathrm{P}=\frac{\pi^{2} \mathrm{EI}_{\mathrm{im}}}{\mathrm{~L}_{\mathrm{eff}}^{2}}
$$

Here $I_{\text {min }}$ for a circular cross-section

$$
=\frac{\pi \mathrm{d}^{4}}{64}
$$

Buckoing stress, $\sigma=\frac{\mathrm{P}}{\mathrm{A}}=\frac{\pi^{2} \mathrm{E} \cdot \pi \mathrm{d}^{4} \times 4}{\mathrm{~L}_{\text {eff }}^{2} \times 64 \times \pi \mathrm{d}^{2}}$
Equating thermal stress and buckling stress

$$
\mathrm{E} \alpha \Delta \mathrm{~T}=\frac{\pi^{2} \mathrm{Ed}^{2}}{16 \mathrm{~L}_{\mathrm{eff}}^{2}}
$$

So, $\quad \Delta \mathrm{T}$ is directly proportional to $\mathrm{d}^{2}$
33. Two cutting tools with tool life equations given below are being compared:
Tool 1 : $\mathrm{VT}^{0.1}=150$
Tool $2: \mathrm{VT}^{0,3}=300$
where $V$ is cutting speed in $m /$ minute and $T$ is tool life in minutes. The breakdown cutting speed beyond which Tool 2 will have a higher tool life is $\qquad$ $\mathrm{m} /$ minute.

## Sol. (106.07)

Given tool life equations
Tool 1, $\mathrm{VT}^{0.1}=150$
Tool 2, $\mathrm{VT}^{0.3}=300$
For break even velocity from (1)

$$
\mathrm{T}=\left(\frac{150}{\mathrm{~V}}\right)^{10}
$$

putting the above value in equation (2) we have $\mathrm{V} \times\left(\frac{150}{\mathrm{v}}\right)^{3}=300$

$$
\mathrm{V}=106.07 \mathrm{~m} / \mathrm{s}
$$

34. Two disks A and B with identical mass (m) and radius ( R ) are initially at rest. They roll down from the top of identical inclined planes without slipping. Disk A has all of its mass concentrated at the rim, while Disk B has its mass uniformly distributed. At the bottom of the plane, the ratio of velocity of the center of disk $A$ to the velocity of the center of disk B is
(a) $\sqrt{\frac{3}{4}}$
(b) $\sqrt{\frac{3}{2}}$
(c) 1
(d) $\sqrt{2}$

Regd. office : F-126, (Upper Basement), Katwaria Sarai, New Delhi-110016 Phone : 011-41013406

## Conventional Question Practice Program for ESE - 2017 Mains Exam



- Subject wise Practice, Discussion and Test
- Practice Booklets with Solution Outlines
- How to Write Answer- Test \& Counselling Session
- Discussion and Practice Session of 250-300 hrs
- Under Guidance of Mr. Kanchan Kr. Thakur


10
Subject wise
Tests

Branches
Complete Course Conventional Test + Classroom Program only for CE and ME

Conventional Test Series
CE, ME, EE, ECE

## Admissions Open

Sol. (a)


Given mass of both disks $=m$
Radius of both disks $=\mathrm{R}$
Initially both have same potential energy finally they will also have same energy.

$$
\begin{equation*}
\text { So, } \quad \frac{1}{2} \mathrm{I}_{\mathrm{A}} \mathrm{w}_{\mathrm{A}}^{2}=\frac{1}{2} \mathrm{I}_{\mathrm{B}} \mathrm{w}_{\mathrm{B}}^{2} \tag{1}
\end{equation*}
$$

Where $I_{A}$ and $I_{B}$ are moment of inertia about point of contact.
So,

$$
\begin{aligned}
& \mathrm{I}_{\mathrm{A}}=2 \mathrm{mR}^{2} \\
& \mathrm{I}_{\mathrm{B}}=\frac{3}{2} \mathrm{mR}^{2}
\end{aligned}
$$

So from (1)

$$
\begin{aligned}
& \frac{\mathrm{w}_{\mathrm{A}}}{\mathrm{w}_{\mathrm{B}}} & =\sqrt{\frac{\mathrm{I}_{\mathrm{B}}}{\mathrm{I}_{\mathrm{A}}}} \\
\therefore & \frac{\mathrm{w}_{\mathrm{A}}}{\mathrm{w}_{\mathrm{B}}} & =\frac{\mathrm{V}_{\mathrm{A}}}{\mathrm{~V}_{\mathrm{B}}}=\sqrt{\frac{3}{4}}
\end{aligned}
$$

35. For the vector $\vec{V}=2 y z \hat{i}+3 x z \hat{j}+4 x y \hat{k}$, the value of $\nabla \cdot(\nabla \times \vec{V})$ is

Sol. (0)
Given,

$$
\overrightarrow{\mathrm{V}}=2 y z \hat{i}+3 x z \hat{j}+4 x y \hat{k}
$$

$$
\begin{aligned}
\nabla \times \overrightarrow{\mathrm{V}} & =\left|\begin{array}{ccc}
\mathrm{i} & \mathrm{j} & \mathrm{k} \\
\frac{\partial}{\partial \mathrm{x}} & \frac{\partial}{\partial \mathrm{y}} & \frac{\partial}{\partial \mathrm{z}} \\
2 \mathrm{yz} & 3 \mathrm{xz} & 4 \mathrm{xy}
\end{array}\right| \\
& =\mathrm{x} \hat{\mathrm{i}}-2 \mathrm{y} \hat{j}+\mathrm{z} \hat{\mathrm{k}} \\
\nabla \cdot(\nabla \times \overrightarrow{\mathrm{V}}) & =\frac{\partial \mathrm{x}}{\partial \mathrm{x}}+\frac{\partial}{\partial \mathrm{y}}(-2 \mathrm{y})+\frac{\partial}{\partial z}(\mathrm{z}) \\
& =1-2+1
\end{aligned}
$$

$$
\nabla \cdot(\nabla \times \overrightarrow{\mathrm{V}})=0
$$

Alternatively :
Divergence of a curl is always zero.
36. A rectangular region in a solid is in a state of plane strain. The ( $x, y$ ) coordinates of the corners of the underformed rectangle are given by $\mathrm{P}(0,0), \mathrm{Q}(4,0), \mathrm{R}(4,3) \mathrm{S}(0.3)$. The rectangle is subjected to uniform strains, $\varepsilon_{\mathrm{xx}}=0.001, \varepsilon_{\mathrm{yy}}=0.002, \gamma_{\mathrm{xy}}=0.003$. The deformed length of the elongated diagonal, upto three decimal places. is $\qquad$ units.

Sol. (5.014 mm)

$$
\begin{aligned}
& \cos _{1}=\frac{4}{5} \\
& \sin \theta_{1}=\frac{3}{5} \\
& \varepsilon_{\mathrm{xx}}=0.001 \\
& \gamma_{\mathrm{xy}}=0.002 \\
& \frac{\Delta \mathrm{PR}}{\mathrm{PR}}=\varepsilon_{1}(\mathrm{along} \mathrm{PR}) \\
& \Rightarrow \varepsilon_{\mathrm{xx}} \cos ^{2} \theta_{1}+\varepsilon_{\mathrm{yy}} \sin \theta_{1}+\gamma_{\mathrm{xy}} \sin \theta_{1} \cos \theta_{1} \\
& \Rightarrow \quad \frac{\Delta \mathrm{PR}}{\mathrm{PR}}=\frac{7}{2500} \mathrm{~mm} \\
& \Delta \mathrm{PR}=0.014 \mathrm{~mm}
\end{aligned}
$$

Length of elongated diagonal $=P R+\Delta \mathrm{PR}$

$$
=5.014 \mathrm{~mm}
$$

37. $P(0,3), Q(0.5 .4)$, and $R(1,5)$ are three points on the curve defined by $f(x)$. Numerical integration is carried out using both Trapezoidal rule and Simpson's rule within limits $x=0$ and $x=1$ for the curve. The difference between the two results will be
(a) 0
(b) 0.25
(c) 0.5
(d) 1

Sol. (a)


From $\beta$ trapezoidal rule,
$\int_{a}^{h} f(x) d x=\frac{h}{2}\left[\left(y_{0}+y_{n}\right)+2\left(y_{1}+y_{2}+\ldots\right)\right]$
$=\frac{1}{2} \times(3+4) \times 0.5+\frac{1}{2} \times(4+5) \times 0.5$
$=4$
From simpson 1/3rd router
$\int_{a}^{b} f(x) d x=\frac{h}{3}\left[\left(y_{0}+y_{n}\right)+4\left(y_{1}+y_{3}+\ldots\right)+2\left(y_{2}+y_{a}+..\right)\right]$
$=\frac{0.5}{3} \times[(3+5)+4 \times 4]$
$=4$
Difference between result $=4-4=0$
38. Air contains $79 \%$ to $\mathrm{N}_{2}$ and $21 \% \mathrm{O}_{2}$ on a molar basis. Methane $\left(\mathrm{CH}_{4}\right)$ is burned with $50 \%$ excess air than required stoichiometrically. Assuming complete combustion of methane, the molar percentage of $\mathrm{N}_{2}$ in the products is $\qquad$ .

Sol. (73.821)
$\mathrm{CH}_{4}+1.5 \times 2\left(\mathrm{O}_{2}+3.76 \mathrm{~N}_{2}\right) \rightarrow \mathrm{CO}_{2}+2 \mathrm{H}_{2} \mathrm{O}$
$+\mathrm{O}_{2}+3 \times 3.76 \mathrm{~N}_{2}$
$\therefore \%$ of $\mathrm{N}_{2}$ is product

$$
\begin{aligned}
& =\frac{3 \times 3.76}{3 \times 3.76+1+2+1} \times 100 \\
& =73.821 \%
\end{aligned}
$$

39. Moist air is treated as an ideal gas mixture of water vapor and dry air (molecular weight of air = 28.84 and molecular weight of water $=18$ ). At a location, the total pressure is 100 kPa , the temperature is $30^{\circ} \mathrm{C}$ and the relative humidity is $55 \%$. Given that the saturation pressure of water at $30^{\circ} \mathrm{C}$ is 4246 Pa , the mass of water vapor per kg of dry air is - grams.

## Sol. (14.872)

$\mathrm{P}=$ total pressure $=100 \mathrm{kPa}$

$$
\mathrm{T}=30^{\circ} \mathrm{C}
$$

Relative humidity $\phi=55 \%$

$$
\mathrm{P}_{\mathrm{VS}}=4246 \mathrm{~Pa}
$$

We know
Relative humidity, $\phi=\frac{\mathrm{P}_{\mathrm{V}}}{\mathrm{P}_{\mathrm{VS}}}$
where $P_{V}=$ Vapour pressure
$P_{V S}=$ Vapour pressure at saturated
So, $\quad 0.55=\frac{\mathrm{P}_{\mathrm{V}}}{4246}$

$$
\mathrm{P}_{\mathrm{V}}=2335.3 \mathrm{~Pa}
$$

So, mass of water vapour per kg of dry air is called specific humidity and given by

$$
\begin{aligned}
& \mathrm{w}=\frac{0.622 \mathrm{P}_{\mathrm{v}}}{\mathrm{P}-\mathrm{P}_{\mathrm{v}}} \\
& \omega=\frac{0.622 \mathrm{P}_{\mathrm{V}} \times 2335.3}{\left[\left(100 \times 10^{3}\right)-2335.3\right]} \\
& \omega=14.872 \text { gm per kg of dry air }
\end{aligned}
$$

Regd. office : F-126, (Upper Basement), Katwaria Sarai, New Delhi-110016 • Phone : 011-41013406
40. A thin uniform rigid bnar of length $L$ and mass $M$ is hinged at point $O$, located at a distance of $\frac{L}{3}$ from one of its ends. The bar is further supported using springs, each of stiffness k, located at the two ends. A particle of mass $m=\frac{M}{4}$ is fixed at one end of the bar, as shown in the figure. For small rotations of the bar about O , the natural frequencyof the system is

(a) $\sqrt{\frac{5 \mathrm{~K}}{\mathrm{M}}}$
(b) $\sqrt{\frac{5 \mathrm{~K}}{2 \mathrm{M}}}$
(c) $\sqrt{\frac{3 \mathrm{~K}}{2 \mathrm{M}}}$
(d)


Sol. (b)


Mass moment of inertia about 0 ,
$\mathrm{I}=\frac{\mathrm{M} l^{2}}{12}+\mathrm{M}\left(\frac{l}{2}-\frac{l}{3}\right)^{2}+\mathrm{m} \times\left(\frac{2 l}{3}\right)^{2}$
$=\frac{\mathrm{M} l^{2}}{12}+\frac{\mathrm{M} l^{2}}{36}+\frac{4 \mathrm{~m} l^{2}}{9}$
$=\frac{\mathrm{M} l^{2}}{9}+\frac{4 \mathrm{M} l^{2}}{4 \times 9}$
$=\frac{2 \mathrm{M} l^{2}}{9}$
Balancing torque about 0 ,
$\mathrm{I} \alpha=\mathrm{K} \times \frac{2 \mathrm{~L}}{3} \times\left(\frac{2 \mathrm{~L}}{3} \theta\right)+\mathrm{K} \times \frac{\mathrm{L}}{3} \times\left(\frac{\mathrm{L}}{3} \theta\right)$
$\Rightarrow \frac{2 \mathrm{M} l^{2}}{9} \frac{\mathrm{~d}^{2} \theta}{\mathrm{dt}}=\frac{5 \mathrm{~K}}{2 \mathrm{M}}=\omega_{\mathrm{n}}^{2} \theta$
$\therefore \quad \omega_{\mathrm{n}}=\sqrt{\frac{5 \mathrm{~K}}{2 M}}$
41. For an inline slider-crank mechanism, the lengths of the crank and connecting rod are 3 m and 4 m , respectively. At the instant when the connecting rod is perpendicular to the crank, if the velocity of the slider is $1 \mathrm{~m} / \mathrm{s}$, the magnitude of angular velocity (upto 3 decimal points accuracy) of the crank is $\qquad$ radian/s.

Sol. (0.266)


$$
\begin{aligned}
& \mathrm{V}_{\text {connecting rod }}=1 \cos \theta=\frac{4}{5} \mathrm{~m} / \mathrm{s} \\
& \mathrm{~V}_{\text {connecting rod }}=\omega_{\text {crank }} \times \mathrm{r}
\end{aligned}
$$

$$
\Rightarrow \quad \frac{4}{5}=\omega_{\mathrm{crank}} \times 3
$$

$$
\Rightarrow \quad \omega_{\text {crank }}=\frac{4}{15}=0.266 \mathrm{rad} / \mathrm{s}
$$

## Alternate :



## 14 | GATE-2017 Mechanical Engineering Session-1 Question and Details Solution

Applying Kennedy's theorem at $\mathrm{I}_{24}$,

$$
\begin{aligned}
\omega_{2} \times\left(\mathrm{I}_{24} \mathrm{I}_{12}\right) & =\mathrm{V}_{\mathrm{A}}=\mathrm{V}_{\mathrm{B}}=1 \\
\Rightarrow \omega_{2} \times\left(\mathrm{I}_{24} \mathrm{I}_{12}\right) & =1 \\
\Rightarrow \quad \omega_{2} & =\frac{1}{\mathrm{I}_{24} \mathrm{I}_{12}}=\frac{1}{\mathrm{OB} \tan \theta} \\
& =\frac{1}{5 \times \frac{3}{4}}=\frac{4}{15}=0.266 \mathrm{rad} / \mathrm{s}
\end{aligned}
$$

42. Consider steady flow of an incompressible fluid through two long and straight pipes of diameters $\mathrm{d}_{1}$ and $\mathrm{d}_{2}$ arranged in series. Both pipes are of equal length and the flow is turbulent in both pipes. The friction factor for turbulent flow though pipes is of the form, $\mathrm{f}=\mathrm{K}(\mathrm{Re})^{-\mathrm{n}}$, where K and $\pi$ are known positive constants and Re is the Reynolds number. Neglecting minor losses, the ratio of the frictional pressure drop in pipe 1 to that in pipe $2\left(\frac{\Delta \mathrm{P}_{1}}{\Delta \mathrm{P}_{2}}\right)$, is given by
(a) $\left(\frac{\mathrm{d}_{2}}{\mathrm{~d}_{1}}\right)^{(5-\mathrm{n})}$
(b) $\left(\frac{\mathrm{d}_{2}}{\mathrm{~d}_{1}}\right)^{5}$
(c) $\left(\frac{d_{2}}{d_{1}}\right)^{(2}$

(d) $\left(\frac{\mathrm{d}_{2}}{\mathrm{~d}_{1}}\right)^{(5+\mathrm{n})}$

Sol. (a)

$$
\begin{aligned}
\frac{\Delta \mathrm{P}_{1}}{\Delta \mathrm{P}_{2}} & =\frac{\rho \mathrm{gh}_{\mathrm{f} 1}}{\rho \mathrm{gh}_{\mathrm{f} 2}}=\frac{\mathrm{h}_{\mathrm{f} 1}}{\mathrm{~h}_{\mathrm{f} 2}} \\
& =\frac{\frac{\mathrm{f}_{1} / \mathrm{V}_{1}^{2}}{2 \mathrm{gd}_{1}}}{\frac{\mathrm{f}_{2} V_{2}^{2}}{2 \mathrm{gd}_{2}}} \\
& =\frac{\frac{\mathrm{f}_{1} \mathrm{Q}^{2}}{\mathrm{~d}_{1}^{5}}}{\frac{\mathrm{f}_{2} \mathrm{Q}^{2}}{\mathrm{~d}_{2}^{5}}}
\end{aligned}
$$


43. One kg of an ideal gas (gas constant, $\mathrm{R}=$ $400 \mathrm{~J} / \mathrm{kg} . \mathrm{K}$ : specific heat at constant volume, $\left.\mathrm{c}_{\mathrm{v}}=1000 \mathrm{~J} / \mathrm{kg} . \mathrm{K}\right)$ at 1 bar , and 300 K is contained in a sealed rigid cylinder. During an adiabatic process, 100 kJ of work is done on the system by a stirrer. The increase in entropy of the system is $\qquad$ J/K.

Sol. (287.68)
Given

$$
\begin{aligned}
\mathrm{m} & =1 \mathrm{Kg} \\
\mathrm{R} & =400 \mathrm{~J} / \mathrm{Kg} \mathrm{~K} \\
\mathrm{C}_{\mathrm{V}} & =1000 \mathrm{~J} / \mathrm{KgK} \\
\mathrm{~T}_{1} & =300 \mathrm{~K} \\
\mathrm{~W} & =100 \mathrm{KJ}
\end{aligned}
$$

Rigid cylinder, adiabatic process
Applying first law of thermodynamics

$$
\mathrm{dQ}=\mathrm{dU}+\mathrm{dW}
$$

$[\because \mathrm{d} Q=0$ adiabatic and $\mathrm{dU}=\mathrm{MC}_{\mathrm{V}} \mathrm{dT}$ for constant volume]

$$
\begin{aligned}
\mathrm{mC}_{\mathrm{V}} \mathrm{dT} & =\mathrm{dW} \\
\mathrm{dT} & =\frac{100 \times 10^{3}}{1 \times 1000} \\
\mathrm{dT} & =100 \\
\mathrm{~T}_{2} & =\mathrm{T}_{1}+\mathrm{dT}=400 \mathrm{~K}
\end{aligned}
$$

For ideal gas

# TM IES MASTER <br> IES MASTER <br> Institute for Engineers (IES/GATE/PSUs) 

## ESE-2017 Conventional Test Schedule, Mechanical Engineering



## Subject Code Details

| Thermodynamic | TH-1 |  | TH-2 |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Thermodynamic systems and processes; Zeroth, First and Second Laws of Thermodynamics. properties of pure substance. |  | Entropy, Irreversibility and availability; Real and Ideal gases; compressibility factor; Gas mixtures. |  |
| Heat Transfer | HT-1 |  | HT-2 |  |
|  | Steady and unsteady heat conduction, Fins, Radiative heat transfer. |  | Free and forced convection, boiling and condensation, Heat exchanger. |  |
| IC Engines | ICE-1 |  | ICE-2 |  |
|  | SI and Cl Engines, Engine Systems and Components, Fuels. |  | Performance characteristics and testing of IC Engines; Emissions and Emission Control. Otto, Diesel and Dual Cycles. |  |
| Refrigeration Air Conditioning | RAC-1 |  | RAC-2 |  |
|  | Vapour compression refrigeration, Refrigerants, Compressors, Other types of refrigeration systems like Vapour Absorption, Vapour jet, thermo electric and Vortex tube refrigeration and Heat pump. |  | Psychometric properties and processes, Comfort chart, Comfort and industrial air conditioning, Load calculations and Condensers, Evaporators and Expansion devices. |  |
| Fluid Mechanics and Machinery | FMM-1 | FMM-2 |  | FMM-3 |
|  | Basic Concepts and Properties of Fluids, Manometry, Fluid Statics, Buoyancy, Equations of Motion such as velocity potential, Stream Function. | Bernoulli's equation and applications, Viscous flow of incompressible fluids, Laminar and Turbulent flows, Flow through pipes and head losses in pipes. |  | Reciprocating and Centrifugal pumps, Hydraulic Turbines and other hydraulic machines. |
| Power Plant Engineering | PPE-1 | PPE-2 |  | PPE-3 |
|  | Steam and Gas Turbines, Rankine and Brayton cycles with regeneration and reheat. | Fuels and their properties, Flue gas analysis, Theory of Jet Propulsion Pulse jet and Ram Jet Engines, Reciprocating and Rotary Compressors. |  | Boilers, power plant components like condensers, air ejectors, Electrostatic precipitators and cooling towers. |
| Renewable Sources of Energy | RSE-1 |  | RSE-2 |  |
|  | Solar Radiation, Solar Thermal Energy collection - <br> Flat Plate andfocusing collectors their materials and performance. Solar Thermal Energy Storage, Applications - heating, cooling and Power Generation. |  | Solar Photovoltaic Conversion; Harnessing of Wind Energy, Bio-mass and Tidal Energy - Methods and Applications, Working principles of Fuel Cells. |  |
| Engineering Mechanics (SoM) | Mech-1 | Mech-2 |  | Mech-3 |
|  | Analysis of System of Forces, Friction, Centroid and Centre of Gravity, Dynamics. | Stresses and Strains-Compound Stresses and Strains, Bending Moment and Shear Force Diagrams. |  | Theory of Bending Stresses-Slope and deflection-Torsion, Thin and thick Cylinders, Spheres. |
| Engineering Materials | MS-1 |  | MS-2 |  |
|  | Basic Crystallography, Alloys and Phase diagrams, Heat Treatment. |  | Ferrous and Non Ferrous Metals, Non metallic materials, Basics of Nano-materials, Mechanical Properties and Testing, Corrosion prevention and control. |  |
|  | ToM-1 | ToM-2 |  | ToM-3 |
| Mechanisms and Machines | Mechanisms, Kinematic Analysis, Velocity and Acceleration. CAMs with uniform acceleration, cycloidal motion, oscillatingfollowers; Effect of Gyroscopiccouple on automobiles, ships and aircrafts. Governors. | Vibrations -Free and forced vibration of undamped and damped SDOF systems, Transmissibility Ratio, Vibration Isolation, Critical Speed of Shafts. |  | Geometry of tooth profiles, Law of gearing, Interference, Helical, Spiral and Worm Gears, Gear Trains- Simple, compound and Epicyclic. Slider crank mechanisms, Balancing. |
| Design of Machine Elements | MD-1 |  | MD-2 |  |
|  | Design for static and dynamic loading; failure theories; fatigue strength and the S-N diagram; principles of the design of machine elements such as riveted, welded and bolted joints. |  | Shafts, Spur gears, rolling and sliding contact bearings, Brakes and clutches, flywheels. |  |
| Manufacturing, Industrial and Maintenance Engineering | PROD-1 | IE-1 |  | RE-1 |
|  | Metal casting-Metal forming, Metal Joining, computer Integrated manufacturing, FMS. | Production planning and Control, Inventory control |  | Failure concepts and characteristicsReliability, Failure analysis, Machine Vibration, Data acquisition, Fault Detection, Vibration Monitoring. |
|  | PROD-2 | IE-2 |  | RE-2 |
|  | Machining and machine tool operations, Limits, fits and tolerances, Metrology and inspection. | Operations research - CPM-PERT |  | Field Balancing of Rotors, Noise Monitoring, Wear and Debris Analysis, Signature Analysis, NDT Techniques in Condition Monitoring. |
|  | MR-1 |  | MR-2 |  |
| Mechatronics and Robotics | Microprocessors and Micro controllers: Architecture, programming, I/O,Computer interfacing, Programmable logic controller. Sensors and actuators, Piezoelectric accelerometer, Hall effect sensor, Optical Encoder, Resolver, Inductosyn, Pneumatic and Hydraulic actuators, stepper motor, Control Systems- Mathematical modeling of Physicalsystems, control signals, controllability and observability |  | Robotics, Robot Classification, Robot Specification, notation; Direct and Inverse Kinematics; Homogeneous Coordinates and Arm Equation of four Axis SCARA Robot. |  |

$$
\mathrm{S}_{2}-\mathrm{S}_{1}=\mathrm{mC}_{\mathrm{V}} \ln \frac{\mathrm{~T}_{2}}{\mathrm{~T}_{1}}+\mathrm{R} \ln \frac{\mathrm{~V}_{2}}{\mathrm{~V}_{1}}
$$

$\left[\because \mathrm{V}_{2}=\mathrm{V}_{1}\right.$ rigid cylinder $]$

$$
\begin{aligned}
\mathrm{S}_{2}-\mathrm{S}_{1} & =\mathrm{m} \times 1000 \times \ln \left(\frac{400}{300}\right)+0 \\
(\Delta \mathrm{~S})_{\text {system }} & =\mathrm{S}_{2}-\mathrm{S}_{1}=287.68 \mathrm{~J} / \mathrm{K}
\end{aligned}
$$

44. A sprue in a sand mould has a top diameter of 20 mm and height of 200 mm . The velocity of the molten metal at entry of the sprue is $0.5 \mathrm{~m} / \mathrm{s}$. Assume acceleration due to gravity as $9.8 \mathrm{~m} / \mathrm{s}^{2}$ and neglect all losses. If the mould is well ventilated the velocity (upto 3 decimal points accuracy) of the molten metal at the bottom of the sprue is $\qquad$ $\mathrm{m} / \mathrm{s}$.

Sol. (2.042)


Applying bernaulli's equation between (1) and (2).

$$
\begin{aligned}
\mathrm{V}_{1} & =0.5 \mathrm{~m} / \mathrm{s} \\
\mathrm{~h}_{1} & =200 \mathrm{~mm} \\
\mathrm{~h}_{2} & =0 \\
\mathrm{P}_{1} & =\mathrm{P}_{2}=\mathrm{Patm} \\
\frac{\mathrm{P}_{1}}{\rho \mathrm{~g}}+\frac{\mathrm{V}_{1}^{2}}{2 \mathrm{~g}}+\mathrm{h}_{1} & =\frac{\mathrm{P}_{2}}{\rho \mathrm{~g}}+\frac{\mathrm{V}_{2}^{2}}{2 \mathrm{~g}}+\mathrm{h}_{2} \\
\mathrm{~V}_{2} & =2.042 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

45. A block of length 200 mm is machined by a slab milling cutter 34 mm in diameter. The depth of cut and table feed are set at 2 mm and $18 \mathrm{~mm} /$ minute, respectively. Considering the approach and the over travel of the cutter to be same, the minimum estimated machining time per pass in $\qquad$ minutes.

Sol. (12)

$$
\begin{aligned}
\text { Approach } & =\text { over travel } \\
& =\sqrt{\mathrm{d}(\mathrm{D}-\mathrm{d})} \\
& =\sqrt{2 \times(34-2)} \\
& =8 \mathrm{~mm}
\end{aligned}
$$

Estimated machine time per pass
$=\frac{\text { Block length }+ \text { Approach }+ \text { Over travel }}{\text { table feed }}$
$=\frac{200+8+8}{18}$ minute
$=12$ minute
46. A point mass of 100 kg is dropped onto a massless elastic bar (cross-sectional area $=$ $100 \mathrm{~mm}^{2}$, length $=1 \mathrm{~m}$, Young's modulus $=$ 100 GPa from a height H of 10 mm as shown (figure is not to scale). If $g=10 \mathrm{~m} / \mathrm{s}^{2}$, the maximum compression of the elastic bar is __mm.


Sol. ( $\mathbf{1 . 5 1 7} \mathbf{~ m m}$ )


$$
\operatorname{mg}(\mathrm{h}+\mathrm{x})=\frac{1}{2} \mathrm{~K}_{\mathrm{bar}} \mathrm{x}^{2}
$$

[By energy conserved]

$$
\begin{aligned}
\mathrm{K}_{\text {bar }} & =\frac{E A}{L} \\
& =\frac{100 \times 10^{9} \times 100 \times 10^{-6}}{1} \mathrm{~N} / \mathrm{m} \\
& =10^{7} \mathrm{~N} / \mathrm{m}
\end{aligned}
$$

Solving quadratic,

$$
\mathrm{x}=1.317 \mathrm{~mm}
$$

47. Following data refers to the jobs (P, Q, R, S) which have arrived at a machine for scheduling. The shortest possible average flow time is $\qquad$ days.

| Job | Processing Time <br> (days) |
| :---: | :---: |
| P | 15 |
| Q | 9 |
| R | 22 |
| S | 12 |

Sol. (31)
According to shortest possible time sequencing the job sequence will be
$\mathrm{Q} \rightarrow \mathrm{S} \rightarrow \mathrm{P} \rightarrow \mathrm{R}$

Job Proces sing Job flow time

| Q | 9 | 9 |
| :---: | :---: | :---: |
| S | 12 | 21 |
| P | 15 | 36 |
| R | 22 | 58 |

Total job flow time $=124$
Average job flow time $=\frac{\text { Total job flow time }}{\text { no of jobs }}$

$$
\begin{aligned}
& =\frac{124}{4} \\
& =31 \text { days }
\end{aligned}
$$

48. Two black surfaces, $A B$ and $B C$, of lengths 5 m and 6 m , respectively, are oriented as shown. BOth surfaces extend infinitely into the third dimension. Given that view factor $\mathrm{F}_{12}=0.5, \mathrm{~T}_{1}=800 \mathrm{~K} . \mathrm{T}_{2}=600 \mathrm{~K}, \mathrm{~T}_{\text {surrounding }}$ $=300 \mathrm{~K}$ and Stefan Boltzmann constant, $\sigma=5.64 \times 10^{-8} \mathrm{~W} /\left(\mathrm{m}^{2} \mathrm{~K}^{4}\right)$, the heat transfer rate from Surface 2 to the surrounding environment is $\qquad$ kW .

## Sol. (14.696)

$$
\begin{aligned}
& \text { Surf.2 } \\
& \mathrm{AB}=5 \mathrm{~m} \\
& \mathrm{BC}=6 \mathrm{~m} \\
& \mathrm{~F}_{12}=0.5 \\
& \mathrm{~A}_{1} \mathrm{~F}_{12}=\mathrm{A}_{2} \mathrm{~F}_{21} \text { [Reciprocity relation][ } \\
& \Rightarrow(2 \times 6) \times 0.5=(\mathrm{L} \times 5) \times \mathrm{F}_{21} \\
& \Rightarrow \quad F_{21}=0.6 \\
& \mathrm{~F}_{21}+\mathrm{F}_{22}+\mathrm{F}_{23}=1 \\
& \Rightarrow 0.6+0+\mathrm{F}_{23}=1 \\
& \Rightarrow \quad \mathrm{~F}_{23}=0.4
\end{aligned}
$$

Therefore transfer rate from surface to surrounding

$$
\dot{\mathrm{q}}_{1-2}=\mathrm{F}_{23} \sigma \mathrm{~A}_{2} \mathrm{~T}_{2}^{4}
$$

$$
\begin{gathered}
=0.4 \times\left(5.67 \times 10^{-8}\right) \times(5 \times 1) \times 6000^{4} \mathrm{~h} \\
=14.696 \mathrm{KW}
\end{gathered}
$$

49. Heat is generated uniformly in a long solid cylindrical rod (diameter $=10 \mathrm{~mm}$ ) at the rate of $4 \times 10^{7} \mathrm{~W} / \mathrm{m}^{3}$. the thermal conductivity of the rod material is $25 \mathrm{~W} / \mathrm{mK}$. Under steady state conditions, the temperature difference between the centre and the surface of the rod is $\qquad$ ${ }^{\circ} \mathrm{C}$.

Sol. (10)
Given
Cylindrical rod dia $=10 \mathrm{~mm}$
Rate of heat generation $\dot{\mathrm{q}}_{\mathrm{g}}=4 \times 10^{7} \mathrm{~W} / \mathrm{m}^{3}$
Thermal conductivity, $\mathrm{K}=25 \mathrm{~W} / \mathrm{mK}$
Temperature distribution in a cylindrical rod with uniform heat generation under steady state is given by

$$
\mathrm{T}_{0}-\mathrm{T}_{\infty}=\frac{\dot{\mathrm{q}}_{\mathrm{g}} \mathrm{R}^{2}}{4 \mathrm{~K}}\left(1-\left(\frac{\mathrm{r}}{\mathrm{R}}\right)^{2}\right)
$$

[ $\mathrm{T}_{0} \rightarrow$ Centre temperature]

for $\mathrm{T}=\mathrm{T}_{0}=\mathrm{T}_{\text {centre }}$ means $\mathrm{r}=0$
So, $T_{0}-T_{\infty}=\frac{\dot{\mathrm{q}}_{\mathrm{g}} \mathrm{R}^{2}}{4 \mathrm{k}}$

$$
\begin{aligned}
\mathrm{T}_{0}-\mathrm{T}_{\text {wall }} & =\frac{4 \times 10^{7} \times(0.005)^{2}}{4 \times 25} \\
\mathrm{~T}_{\text {centre }}-\mathrm{T}_{\text {wall }} & =10
\end{aligned}
$$

50. In an epicyclic gear train, shown in the figure, the outer ring gear is fixed, while the sun gear rotates counterclockwise at 100 rpm .

Let the number of teeth on the sun, planet and outer gears to be 50,25 , and 100 , respectively. The ratio of magnitudes of angular velocity of the planet gear to the angular velocity of the carrier arm is $\qquad$ _.


Sol. (3)


|  | Sun(S) | Planet(P) | Outer ring |
| :---: | :---: | :---: | :---: |
| Wihtout (orpm) <br> arm | x | $-\mathrm{x} \times \frac{50}{25}=-2 \mathrm{x}$ | $-\mathrm{x} \times \frac{50}{25} \times \frac{25}{100}=\frac{-\mathrm{x}}{2}$ |
| With arm (y rpm) | $\mathrm{x}+\mathrm{y}=100$ | $-2 \mathrm{x}+\mathrm{y}$ | $-\frac{\mathrm{x}}{2}+\mathrm{y}=0$ |

$$
\begin{align*}
x+y & =100  \tag{1}\\
-\frac{x}{2}+y & =0 \tag{2}
\end{align*}
$$

Eqn. (1) and (2), we get

$$
\begin{aligned}
\frac{3 \mathrm{x}}{2} & =100 \\
\Rightarrow \quad \mathrm{x} & =\frac{200}{3} \\
\mathrm{y} & =\frac{100}{3}
\end{aligned}
$$

$\omega_{\mathrm{p}},($ Angular vel. of plant gear $)=-2 \mathrm{x}+\mathrm{y}$

$$
=\frac{-400}{3}+\frac{100}{3}=-100
$$

$$
\frac{\left|\omega_{\mathrm{p}}\right|}{\left|\omega_{\text {arm }}\right|}=\frac{|-100|}{\left|\frac{100}{3}\right|}=3
$$

51. The pressure ratio across a gas turbine (for air, specific heat of constant pressure, $\mathrm{c}_{\mathrm{p}}=$ $1040 \mathrm{~J} / \mathrm{kg} . \mathrm{K}$ and ratio of specific heats, $\gamma=1.4$ is 10 . If the inelt temperature to the turbine is 1200 K and the isentropic efficiency is 09 , the gas temperature at turbine exit is $\qquad$ K.

Sol. (679.38)


Given,

$$
\begin{aligned}
\frac{\mathrm{P}_{1}}{\mathrm{P}_{2}} & =10 \\
\mathrm{C}_{\mathrm{P}} & =1040 \mathrm{~J} / \mathrm{kg} \\
\mathrm{Y} & =1.4 \\
\mathrm{~T}_{1} & =1200 \mathrm{~K} \\
\eta_{\text {isentropic }} & =0.9
\end{aligned}
$$

for process 1-2

$$
\frac{\mathrm{T}_{2}}{\mathrm{~T}_{1}}=\left(\frac{\mathrm{P}_{2}}{\mathrm{P}_{1}}\right)^{\frac{\gamma-1}{\gamma}}
$$

$$
\begin{aligned}
\therefore \quad \mathrm{T}_{2} & =1200\left(\frac{1}{10}\right)^{0.4 / 1.4} \\
\mathrm{~T}_{2} & =621.54 \mathrm{~K}
\end{aligned}
$$

Now, we know

$$
\eta_{\text {isentropic }}=\frac{\mathrm{T}_{1}-\mathrm{T}_{2}^{\prime}}{\mathrm{T}_{1}-\mathrm{T}_{2}}
$$

$$
\begin{aligned}
0.9 & =\frac{1200-\mathrm{T}_{2}^{\prime}}{1200-621.54} \\
\mathrm{~T}_{2}^{\prime} & =679.38 \mathrm{~K}
\end{aligned}
$$

52. Consider the matrix $\mathrm{P}=\left[\begin{array}{ccc}\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}}\end{array}\right]$

Which one of the following statements about P is INCORRECT?
(a) Determinant of P is equal to 1.
(b) P is orthogonal
(c) Inverse of P is equal to its transpose.
(d) All eigenvalues of P are real numbers

Sol. (d)
(i)

$$
\mathrm{P}=\left[\begin{array}{ccc}
\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\
0 & 1 & 0 \\
-\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}}
\end{array}\right]
$$

$$
|\mathrm{P}|=1
$$

(ii) $\quad \mathrm{P}^{\mathrm{T}}=\left[\begin{array}{ccc}\frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}}\end{array}\right]$
P. $\mathrm{P}^{\mathrm{T}}=\left[\begin{array}{ccc}\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}}\end{array}\right]\left[\begin{array}{ccc}\frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}}\end{array}\right]$

$$
=\left[\begin{array}{lll}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right]=\mathrm{I}
$$

Hence P is orthogonal as $\mathrm{P} \cdot \mathrm{P}^{\mathrm{T}}=\mathrm{I}$
(iii) $\quad \mathrm{P}^{-1}=\left[\begin{array}{ccc}\frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}}\end{array}\right]=\mathrm{P}^{\mathrm{T}}$

Hence (iv) is wrong.
53. A machine element has an ultimate strength $\left(\sigma_{\mathrm{u}}\right)$ of $600 \mathrm{~N} / \mathrm{mm}^{2}$, and endurnace limit $\left(\sigma_{\mathrm{en}}\right)$ of $250 \mathrm{~N} / \mathrm{mm}^{2}$. The fatigue curve for the element on a log-log plot is shown below. If the element is to be designed for a finite life of 10000 cycles, the maximum amplitude of a completely reversed operating stress is
$\qquad$


Sol. (386.19 MPa)


Coordinates of points are :
$\mathrm{A} \rightarrow \mathrm{A} \rightarrow\left(\log \left(0.8 \sigma_{\mathrm{u}}\right), 3\right)$
$B \rightarrow(\log S, 4)$
$\mathrm{C} \rightarrow\left(\log \sigma_{\mathrm{en}}, 6\right)$

Equating slope of ine-segment A-B-C

$$
\begin{aligned}
& \frac{\log \left(0.8 \sigma_{u}\right)-\log S}{3-4}=\frac{\log \left(0.8 \sigma_{u}\right)-\log \left(\sigma_{\mathrm{en}}\right)}{3-6} \\
& \Rightarrow \log S=\log \left(0.8 \sigma_{u}\right)-\frac{\log \left(0.8 \sigma_{u}\right)-\log \left(\sigma_{\mathrm{n}}\right)}{3} \\
& \Rightarrow \quad \mathrm{~S}=386.34
\end{aligned}
$$

54. Assume that the surface roughness profile is triangular as shown schematically in the figure. If the peak to valley height is $20 \mu \mathrm{~m}$, the central line average surface roughness

(a) 5
(b) 6.67
(c) 10
(d) 20

Sol. (a)
Average surface roughness, $\mathrm{R}_{\mathrm{a}}=\mathrm{Z}_{1}+\mathrm{Z}_{2}-$ $+\frac{Z_{n}}{n}$

$$
\begin{aligned}
& =\frac{\mathrm{h}}{4} \\
& =\frac{20}{4} \\
& =5 \mathrm{~mm}
\end{aligned}
$$

55. Circular arc on a part profile is being machined on a vertical CNC milling machine, CNC part program using metric units with absolute dimensins is listed below:

N60 G01 X 30 Y 55 Z-5 F50
N70 G02 X 50 Y 35 R 20
N80 G01 Z 5
The coordinates of the centre of the circular arc are:
(a) $(30,55)$
(b) $(50,55)$
(c) $(50,35)$
(d) $(30,35)$

Sol. (d)


Two possible centre are $(30,35) \rightarrow$ For $\mathrm{R} \rightarrow+$ ve $\rightarrow(50,55) \rightarrow$ for $\mathrm{R} \rightarrow-\mathrm{ve}$.

## GENERAL APTITUDE

1. A right-angled cone (with base radius 5 cm and height 12 cm ), as shown in the figure below, is rolled on the ground keeping the point $P$ fixed until the point $Q$ (at the base of the cone, as shown) touches the ground again


By what angle (in radians) about $P$ does the cone travel?
(a) $\frac{5 \pi}{12}$
(b) $\frac{5 \pi}{24}$
(c) $\frac{24 \pi}{5}$
(d) $\frac{10 \pi}{13}$

Sol. (d)


While rotating Q the whole cone will also rotate in a circle of radius which will be equal to its and slant height.
So rotating $Q$ it will cover $2 \pi R$ distance in horizontal circle.

So angle made will be $\frac{2 \pi \mathrm{R}}{2 \pi l} \times 2 \pi$ radians

$$
\begin{aligned}
& =\frac{5}{13} \times 2 \pi \\
Q & =\frac{10 \pi}{13}
\end{aligned}
$$

2. As the two speakers became increasingly agitated, the debate became $\qquad$ -.
(a) lukewarm
(b) poetic
(c) forgiving
(d) heated

Sol. (d)
Lukewarm $\rightarrow$ milld; other poetic and for giving is not suitable here.
3. In a company with 100 employees, 45 earn Rs. 20,000 per month 25 earn Rs. $30,000,20$ earn Rs. 40,000, 8 earn Rs. 60,000, and 2 earn Rs. 150,000. The median of the salaries is
(a) Rs. 20,000
(b) Rs. 30,000
(c) Rs. 32,300
(d) Rs. 40,000

Sol. (b)
Medium is the middle term of the data arranged in increasing under if no of terms are odd, if is even then median will be the average of two middle terms.
So for above question, arranging data

$$
\text { Median }=\frac{30000+30000}{2}=30000
$$

4. He was one of my best $\qquad$ and I felt his loss
$\qquad$ _.
(a) friend, keenly
(b) friends, keen
(c) friend, keener
(d) friends, keenly

Sol. (d)
5. P, Q, and R talk about S' 5 car collection P states that $S$ has at least 3 cars. $Q$ believes that $S$ has been than 3 cars $R$ indicates that to his knowledge, S has at least one car. Only one of $P, Q$ and $R$ is right. The number of cars owned by S is
(a) 0
(b) 1
(c) 3
(d) Cannot be determined

Sol. (a)


As per given condition no of car according to

$$
\begin{aligned}
& \mathrm{P} \geq 3 \\
& \mathrm{Q}<3 \\
& \mathrm{R} \geq 1
\end{aligned}
$$

and only one is correct.
So only Q cars is satisfying the given condition.
6. What is the sum of the missing digits in the subtraction problem below?

$$
\begin{array}{r}
5---7 \\
\frac{-48 \_89}{1111}
\end{array}
$$

(a) 8
(b) 10
(c) 11
(d) Cannot be determined

Sol. (a, b)

$$
\begin{array}{r}
5----- \\
-\quad 48 \_89 \\
\hline 01111 \\
\hline
\end{array}
$$

By hit and trial we find that the missing digit in lower number an be either 8 or 9 .

If it is 8
$\Rightarrow$ Sum of digits $=8+0+0+0+0=8$
If it is 9
$\Rightarrow$ Sum of digits $=9+0+1+0+0=10$
7. "Here, throughout the early 1820 s, Stuart contained to fight his losing battle to allow his sepoys to wear their caste-marks and their own choice of facial hair on parade, being again repromanded by the commander-inchied. His retort that 'A stronger instance than this of European prejudice with relation to this country has never come under my observations' had no effect on his superiors."
According to this paragraph, which of the statements below is most accurte?
(a) Stuart's commander-in-chief was moved by this demonstration of his prejudice
(b) The Europeans were accommodaing of the sepoys' desire to wear their castemarks.
(c) Stuart's losing battle refers to his inability to succeed in enabling sepoys to wear cast-marks.
(d) The commander-in-chief was exempt from the European prejudice that dicatated how the sepoys were to dress

## Sol. (c)

8. Let $S_{1}$ be the plane figure consisting of the points ( $\mathrm{x}, \mathrm{y}$ ) given by the inequalities $\mid \mathrm{x}$ $1 \mid \leq 2$ and $|y+2| \leq 3$. Let $S_{2}$ be the plane figure given by the inequalities $x-y \geq-2, y \geq 1$, and $x \leq 3$. Let $S$ be the union of $S_{1}$ and $S_{2}$. The area of $S$ is
(a) 26
(b) 28
(c) 32
(d) 34
OUR TOP RESULTS IN ESE-2016 IES MASTER

JATIN KUMAR RACHIT JAIN ADARSH R. SRIVASTAV

SHIVAM DWIVEDI
AMRIT ANAND AVDHESH MEENA

| AIR | AIR | AIR |
| :---: | :---: | :---: |
| $\begin{aligned} & 10 \\ & C E \end{aligned}$ | $\begin{aligned} & 11 \\ & C E \end{aligned}$ | $\begin{aligned} & 12 \\ & \text { CE } \end{aligned}$ |


BHARAT BHUSHAN DIXIT

MOHAMMAD IDUL AHMED CHIRAG SRIVASTAV



Sol. (c)


\[

\]

Intersection point of $x-y=-2$ and $x=3$

$$
\begin{aligned}
3-y & =-2 \\
y & =3+2=5
\end{aligned}
$$

Point is $(3,5)$
Area of $\mathrm{S}=$ Area of $\mathrm{S}_{1}+$ Area of $\mathrm{S}_{2}$

$$
\begin{aligned}
& =(6 \times 4)+\frac{1}{2} \times 4 \times 4 \\
& =24+8=32
\end{aligned}
$$

9. Two very famous sportsmen Mark and Steve happened to be brothers, and played for country K. Mark teased James, an opponent from country E, "There is no way you are good enought to play for your country." James replied, "Maybe not, but at least I am the best player in my own family."
Which one of the following can be inferred from this conversation?
(a) Mark was known to play better than James
(b) Steve was known to play better than Mark
(c) James and Steve were good friends
(d) James played better than Steve

Sol. (b)
The statement by James, "May be not, but at least I am the best player in my own family" suggests that mark is not best play in is family so stene is known to play better than mark.
10. The growth of bacteria (lactobacillus) in milk leads to curd formation A minimum bacterial population density of 0.8 (in suitable units) is needed to form curd. In the graph below, the population density of lactobacillus in 1 litre of milk is plotted as a function of time at two different temperatures, $25^{\circ} \mathrm{C}$ and $37^{\circ} \mathrm{C}$


Consider the following statements based on the data shown above
i. The growth in bacterial population stops earlier at $37^{\circ} \mathrm{C}$ as compared to $25^{\circ} \mathrm{C}$
ii. The time taken for curd formation at $25^{\circ} \mathrm{C}$ is twice the time taken at $37^{\circ} \mathrm{C}$
Which one of the following options is correct?
(a) Only i
(b) Only ii
(c) Both i and ii
(d) Neither i nor ii

Sol. (a)
(i) the growth in bacterial population stops almost 140 s in $37^{\circ} \mathrm{C}$ as compared to 180 s in $25^{\circ} \mathrm{C}$.
(ii) time taken for curd formation at $25^{\circ} \mathrm{C}$ is approximately 90 s while it is 130 s in $37^{\circ} \mathrm{C}$ which is not double.

