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	The bandwidth of output will be $B_1 + B_2$.
	So sampling rate will be $2(B_1 + B_2)$.
Q.29	The value of the integral $2\int_{0}^{\infty} \left(\frac{\sin 2\pi t}{\pi t}\right) dt$ is equal to
	(a) 0 (b) 0.5
	(d) 0.5 (c) 1 (d) 2
Ans.	(d)
	The Fourier transform of
	$\frac{2\sin(t\tau/2)}{t} \longrightarrow 2\pi \operatorname{rect}\left(\frac{\omega}{\tau}\right)$
	$rac{\sin(2\pi t)}{\pi t} \longrightarrow \operatorname{rect}\left(rac{\omega}{4\pi} ight)$
	So, $\int_{-\infty}^{\infty} \frac{\sin(2\pi t)}{\pi t} e^{-j\omega t} dt = \operatorname{rect}\left(\frac{\omega}{4\pi}\right)$
	Putting $\omega = 0$ in above equation
	$\int_{-\infty}^{\infty} \frac{\sin(2\pi t)}{\pi t} dt = 1$
	$2\int_{-\infty}^{\infty} \frac{\sin(2\pi)}{\pi t} dt = 2$
	• • End of Solution
Q.30	Let $y(x)$ be the solution of the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$ with initial
	conditions $y(0) = 0$ and $\frac{dy}{dx}\Big _{x=0} = 1$. Then the value of y (1) is
Ans.	(7.38)
	A.E. $m^2 - 4m + 4 = 0$ m = 2, 2
	$y(0) = 0 \Rightarrow \qquad \qquad y = (C_1 + C_2 x) e^{2x}$ $C_1 = 0$ $y = C_2 x e^{2x}$

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Q.31 The line integral of the vector field $F = 5xz\hat{i} + (3x^2 + 2y)j + x^2zk$ along a path from (0, 0, 0) to (1, 1, 1) parameterized by (t, t^2 , t) is _____.

Ans. (4.41)

 \Rightarrow

$$E = 5xz\overline{i} + (3x^{2} + 2y)\overline{j} + x^{2}z\overline{k}$$

$$= \int_{C} \overline{F}.\overline{d}r$$

$$= \int_{C} 5xz \, dx + (3x^{2} + 2y) dy + x^{2}z dz$$

$$x = t, \quad y = t^{2}, \quad z = t, \quad t = 0 \text{ to } 1$$

$$dx = dt$$

$$dy = 2t \, dy, \quad dz = dt$$

$$= \int_{0}^{1} 5t^{2} dt + (3t^{2} + 2t^{2}) 2t dt + t^{3} dt$$

$$= \int_{0}^{1} (5t^{2} + 11t^{3}) dt$$

$$= \left[\frac{5t^{3}}{3} + \frac{11t^{4}}{4}\right]_{0}^{1} = \frac{5}{3} + \frac{11}{4} = \frac{53}{12} = 4.41$$
End of Solution

Q.32 Let
$$P = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$
. Consider the set *S* of all vectors $\begin{pmatrix} x \\ y \end{pmatrix}$ such that $a^2 + b^2 = 1$ where $\begin{pmatrix} a \\ b \end{pmatrix} = P \begin{pmatrix} x \\ y \end{pmatrix}$. Then *S* is
(a) a circle of radius $\sqrt{10}$ (b) a circle of radius $\frac{1}{\sqrt{10}}$
(c) an ellipse with major axis along $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ (d) an ellipse with minor axis along $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$





$$P(x \le 0) = \int_{-\infty}^{0} 2e^{4x} dx$$
$$= \left[\frac{e^{4x}}{2}\right]_{-\infty}^{0} = \frac{1}{2}$$

End of Solution

The driving point input impedance seen from the source $V_{\rm S}$ of the circuit shown **Q.34** below, in Ω , is_____



(20) Ans.

To find impedance seen by V_s



Applying KCL at node A

$$I_{s} + 4 V_{1} = \frac{V_{A}}{3} + \frac{V_{A}}{6}$$
$$V_{A} = V_{s} - V_{1} \text{ and } V_{1} = 2I_{s}$$
$$I_{s} + 8I_{s} = \frac{V_{s} - 2I_{s}}{3} + \frac{V_{s} - 2I_{s}}{6}$$

 $V_1 = 2I_s$

So,

=

$$\Rightarrow \qquad 54 I_s = 2V_s - 4I_s + V_s - 2I_s \\ \Rightarrow \qquad 3V_s = 60I_s \\ \frac{V_s}{I_s} = 20 \Omega$$

End of Solution





C will be 3.03 mF

End of Solution

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Q.38 The single line diagram of a balanced power system is shown in the figure. The voltage magnitude at the generator internal bus is constant and 1.0 p.u. The p.u. reactances of different components in the system are also shown in the figure. The infinite bus voltage magnitude is 1.0 p.u. A three phase fault occurs at the middle of line 2. The ratio of the maximum real power that can be transferred during the pre-fault condition to the maximum real power that can be transferred under the faulted condition is _____.







(1).
$$\Rightarrow \qquad \qquad \frac{j \, 0.3 \times j \, 0.6}{j \, 1.2} = j \, 0.15$$

(2).
$$\Rightarrow \qquad \qquad \frac{j \ 0.3 \times j \ 0.6}{j \ 1.2} = j \ 0.15$$

(3).
$$\Rightarrow \qquad \qquad \frac{j \, 0.3 \times j \, 0.3}{j \, 1.2} = j \, 0.075$$

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$$\frac{P_1}{P_2} = \frac{X_2}{X_1} = \frac{j \, 1.2}{j \, 0.5} = 2.4$$

End of Solution

=

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Q.39 The open loop transfer function of a unity feedback control system is given by

$$G(s) = \frac{K(s+1)}{s(1+Ts)(1+2s)}, K > 0, T > 0.$$

The closed loop system will be stable if,

(a)
$$0 < T < \frac{4(K+1)}{K-1}$$

(b) $0 < K < \frac{4(T+2)}{T-2}$
(c) $0 < K < \frac{T+2}{T-2}$
(d) $0 < T < \frac{8(K+1)}{K-1}$

Ans.

(c)

Open loop transfer function

$$G(s) = \frac{K(s+1)}{s(1+Ts)(1+2s)}; K > 0 \text{ and } T > 0$$

For closed loop system stability, characteristic equation is 1 + G(s)H(s) = 0

$$1 + \frac{K(s+1)}{s(1+Ts)(1+2s)} \cdot 1 = 0$$

$$s(1 + Ts) (1 + 2s) + k(s + 1) = 0$$

 $2Ts^{3} + (2 + T)s^{2} + (1 + k)s + k = 0$
Using Routh's criteria



 $\begin{array}{c|ccc} s^{3} & 2T & (1+k) \\ s^{2} & (2+T) & k \\ s^{1} & \frac{(2+T)(1+k) - 2Tk}{(2+T)} & 0 \\ s^{0} & k \end{array}$

For stability, k > 0and (2 + T) (1 + k) - 2Tk > 0k(2 + T - 2T) + (2 + T) > 0or - (T - 2)k + 2(2 + T) > 0 $-k > -\frac{(2 + T)}{(T - 2)}$

or

Hence for stability,

$$0 < k < \frac{T+2}{T-2}$$

Q.40 At no load condition, a 3-phase, 50 Hz, lossless power transmission line has sending-end and receiving-end voltages of 400 kV and 420 kV respectively. Assuming the velocity of traveling wave to be the velocity of light, the length of the line, in km, is ______.

 $V_s = AV_R$

400 = A 420

 $k < \frac{T+2}{(T-2)}$

Ans. (294.59)

At no load,

 $A = \frac{400}{420} = 0.9524$ $A = 1 + \frac{YZ}{2} = 1 + \frac{(r + j\omega L)(g + j\omega C)}{2}$

For lossless line r = 0, g = 0

then

$$A = 1 - \frac{(\omega C) (\omega L)}{2}$$
$$\beta l = \sqrt{\omega L \omega C}$$

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End of Solution



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$$A = 0.9524 = 1 - \frac{\beta^2 l^2}{2}$$
$$\beta l = 0.3085$$
$$\beta = \frac{0.3085}{l}$$
$$\frac{V}{f} = \frac{2\pi}{\beta}$$
$$\frac{30 \times 10^5}{50} = \frac{2\pi}{\left(\frac{0.3085}{l}\right)}$$

l = 294.59 km

End of Solution

Q.41 The power consumption of an industry is 500 kVA, at 0.8 p.f. lagging. A synchronous motor is added to raise the power factor of the industry to unity. If the power intake of the motor is 100 kW, the p.f. of the motor is _

50

Ans. (0.3162)

$$\begin{array}{rcl} P_1 \ = \ 500 \, \times \, 0.8 \, = \, 400 \ \mathrm{kW} \\ Q_1 \ = \ 500 \, \times \, 0.6 \, = \, 300 \ \mathrm{kVAR} \end{array}$$

The power factor is to be raised to unity The motor has to supply 300 kVAR The motor rating is 100 kW, 300 kVAR

$$\phi_m = \tan^{-1}\left(\frac{Q}{P}\right)$$

$$\phi_m = \tan^{-1} \left(\frac{300}{100} \right) = 71.56$$

Power factor of motor = $\cos\phi_m = \cos 71.56 = 0.316$

End of Solution

Q.42 The flux linkage (λ) and current (i) relation for an electromagnetic system is $\lambda = (\sqrt{i})/g$. When i = 2A and g(air-gap length) = 10 cm, the magnitude of mechanical force on the moving part, in N, is _____



Ans. (141.4)

Stored energy in electromagnetic system

$$E = \frac{1}{2}\lambda i$$
$$E = \frac{1}{2}\frac{i\sqrt{i}}{g}$$

Restoring force in EM system

$$= -\frac{dE}{dx} = F$$

Here

x = g = air gap length

 $F = -\left(-\frac{1}{2}\cdot\frac{i^{3/2}}{g^2}\right) = \frac{1}{2}\times\frac{i^{3/2}}{g^2}$

Now

$$= \frac{1}{2} \times \frac{(2)^{3/2}}{0.1 \times 0.1} = 141.4 \text{ N}$$

End of Solution

Q.43 The starting line current of a 415 V, 3-phase, delta connected induction motor is 120 A, when the rated voltage is applied to its stator winding. The starting line current at a reduced voltage of 110 V, in ampere, is _____.

Ans. (31.807)

415 V, 3-phase, Δ connected induction motor $(I_{\rm st})_{\rm line}$ = 120 A at rated voltage. at, V = 110 V, i.e. reduced voltage

 $I_{\rm st} = x(I_{\rm st})_{\rm rated}$

where,

$$x = \frac{V_{reduced}}{V_{rated}}$$
$$x = \frac{110}{415}$$
$$st)_{at \ 110 \ V} = \left(\frac{110}{415}\right) \times 120$$
$$= 31.807 \ A$$

End of Solution

(I

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Q.45 A full-bridge converter supplying an RLE load is shown in figure. The firing angle of the bridge converter is 120°. The supply voltage $v_m(t) = 200 \pi \sin(100\pi t) V$, $R = 20 \Omega$, E = 800 V. The inductor L is large enough to make the output current I_L a smooth dc current. Switches are lossless. The real power feedback to the source, in kW, is_____.



Ans. (6)

$$V_o = 2 \frac{V_m}{\pi} \cos \alpha = 2 \frac{200\pi}{\pi} \cos 120^\circ$$
$$V_o = -200 \text{ V}$$
$$|V_o| = 200 \text{ V}$$

Power balance equation, $EI_o = I_o^2 R + V_o I_o$

800
$$I_o = I_o^2(20) + 200I_o$$

 $I_o = 30 \text{ A}$
 $I_o = I_{or}$



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Power fed to source =
$$V_o I_o$$

= 200 × 30 = 6 kW

End of Solution

Q.46 A three-phase Voltage Source Inverter (VSI) as shown in the figure is feeding a delta connected resistive load of 30 Ω /phase. If it is fed from a 600 V battery, with 180° conduction of solid-state devices, the power consumed by the load, in kW, is_____.



Ans. (24)

$$V_{L} = V_{ph} = \sqrt{\frac{2}{3}} V_{s}$$
$$V_{ph} = \sqrt{\frac{2}{3}} \times 600$$
$$P = 3 \frac{V_{ph}^{2}}{R} = \frac{3 \times \frac{2}{3} \times 600^{2}}{30} = 24 \text{ kW}$$

• • • End of Solution

Q.47 A DC-DC boost converter, as shown in the figure below, is used to boost 360V to 400 V, at a power of 4 kW. All devices are ideal. Considering continuous inductor current, the rms current in the solid state switch (*S*), in ampere, is ______.





Q.48 A single-phase bi-directional voltage source converter (VSC) is shown in the figure below. All devices are ideal. It is used to charge a battery at 400 V with power of 5 kW from a source $V_s = 220$ V (rms), 50 Hz sinusoidal AC mains at unity p.f. If its AC side interfacing inductor is 5 mH and the switches are operated at 20 kHz, then the phase shift (δ) between AC mains voltage (V_s) and fundamental AC rms VSC voltage (VC1), in degree, is _____.



Ans. (9.21)



 $P = V_s I_s \text{ p.f.}$ $J_s x_s = 220 \times I_s \times 1$ $I_s = 22.72 \text{ A}$ $\tan \delta = \frac{I_s X_s}{V_s}$ $\delta = \tan^{-1} \left(\frac{22.72 \times 2\pi \times 50 \times 5 \times 10^{-3}}{220}\right)$

 $\delta = 9.21^{\circ}$

End of Solution

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Q.49	Consider a linear time invariant system $x = Ax$, with initial conditions $x(0)$ at
	$t = 0$. Suppose α and β are eigenvectors of (2×2) matrix A corresponding to
	distinct eigenvalues λ_1 and λ_2 respectively. Then the response $x(t)$ of the system
	due to initial condition $x(0) = \alpha$ is
	(a) $\alpha e^{\lambda_1 t}$ (b) $e^{\lambda_2 t_\alpha} \beta$ (c) $e^{\lambda_2 t_\alpha} \alpha$ (d) $e^{\lambda_1 t_\alpha} + e^{\lambda_2 t \beta}$
	(c) $e^{\lambda_2 t_{\alpha}} \alpha$ (d) $e^{\lambda_1 t_{\alpha}} + e^{\lambda_2 t_{\beta}}$
Ans.	(a)
	x = Ax
	Eigen values are λ_1 and λ_2
	We can write,
	$\begin{bmatrix} e^{\lambda_1 t} & 0 \end{bmatrix}$
	$\phi(t) = \begin{vmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{vmatrix}$
	Response due to initial conditions,
	$x(t) = \phi(t) \cdot x(0)$
	$\begin{bmatrix} e^{\lambda_1 t} & 0 \end{bmatrix} \begin{bmatrix} \alpha \end{bmatrix}$
	$x(t) = \begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix} \begin{bmatrix} \alpha \\ 0 \end{bmatrix}$
	$= \alpha e^{\lambda_1 t}$
	• • End of Solution
Q.50	A second -order real system has the following properties :
	(a) the damping ratio $\zeta = 0.5$ and undamped natural frequency $\omega_n = 10$ rad/s,
	(a) the damping ratio $\zeta = 0.5$ and undamped natural nequency $\omega_n = 10$ radis, (b) the steady state value of the output, to a unit step input, is 1.02.
	The transfer function of the system is
	(a) $\frac{1.02}{s^2 + 5s + 100}$ (b) $\frac{1.02}{s^2 + 10s + 100}$
	$s^2 + 5s + 100$ $s^2 + 10s + 100$
	100 102
	(c) $\frac{100}{s^2 + 10s + 100}$ (d) $\frac{102}{s^2 + 5s + 100}$
Ans.	(b) $\xi = 0.5$
	Undamped natural frequency $\omega_n = 10$ rad/sec
	Steady state output to a unit step input
	$C_{\rm ss} = 1.02$
	Hence steady state error = $1.02 - 1.00$
	$e_{ss} = 0.02$

GATE-2016 Exam Solutions MADE Page EAS India's Best Institute for IES. GATE & PSUs **Electrical Engineering** (Seesion-8, Set-2) 35 $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$ $s^2 + 2 \times 0.5 \times 10 \text{ s} + 100 = 0$ $s^2 + 10s + 100 = 0$ From options, if we take option (b) then $C_{ss} = \lim_{s \to 0} s.C(s) = \lim_{s \to 0} s \times \frac{1}{s} \times \frac{102}{s^2 + 10s + 100}$ $C_{ss} = 1.02$ Hence option (b) is correct answer. End of Solution Q.51 Three single-phase transformers are connected to form a delta-star three-phase transformer of 110 kV/11 kV. The transformer supplies at 11 kV a load of 8 MW at 0.8 p.f. lagging to a nearby plant. Neglect the transformer losses. The ratio of phase currents in delta side to star side is (b) $10\sqrt{3}:1$ (a) $1:10\sqrt{3}$ (d) $\sqrt{3}:10$ (c) 1 : 10 Ans. (a) At 11 kV, load is 8 MW, 0.8 Pf lagging $\frac{\left(V_{ph}\right)_{\Delta}}{\left(V_{ph}\right)_{\chi}} = \frac{\left(I_{ph}\right)_{Y}}{\left(I_{ph}\right)_{\Delta}}$ \Rightarrow $(I_{ph})_{\Delta} = (I_{ph})_{\gamma} \times \frac{(V_{ph})_{\gamma}}{(V_{ph})_{\gamma}}$ \Rightarrow $\frac{(I_{\rm ph})_{\Delta}}{(V_{\rm ph})_{\chi}} = \frac{11/\sqrt{3}}{110} = 1:10\sqrt{3}$ End of Solution Q.52 The gain at the breakaway point of the root locus of a unity feedback system with open loop transfer function $G(s) = \frac{Ks}{(s-1)(s-4)}$ is (a) 1 (b) 2 (c) 5 (d) 9

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Ans. (a)

(

DLTF
$$\Rightarrow$$
 $G(s) = \frac{\text{Ks}}{(s-1)(s-4)}$

Now, characteristics equation 1 + G(s)H(s) = 0

$$\frac{\mathrm{Ks}}{(\mathrm{s}-1)(\mathrm{s}-4)} + 1 = 0$$

$$\Rightarrow \qquad Ks + (s^2 - 5s + 4) = 0$$

$$K = -\frac{\left(s^2 - 5s + 4\right)}{s} = [s - 5 + 4/s]$$

For break away point: $\frac{dK}{ds} = 0$

$$\frac{\mathrm{dK}}{\mathrm{ds}} = -\left[1 - 0 - \frac{4}{\mathrm{s}^2}\right] = 0$$

we get $s = \pm 2$

Therefore valid break away point is

s = 2, Now gain at s = 2 is

 $\Rightarrow K = \frac{\text{Product of distances from all the poles to break away point}}{\text{Product of distance from all the zeros to break away point}}$

Gain, K =
$$\frac{1 \times 2}{2} = 1$$

• • • End of Solution





