CET – MATHEMATICS – 2014

VERSION CODE: C – 2

1. Which one of the following is not correct for the features of exponential function given by f (x) = b^x where b > 1?
   (1) For very large negative values of x, the function is very close to 0.
   (2) The domain of the function is R, the set of real numbers.
   (3) The point (1, 0) is always on the graph of the function.
   (4) The range of the function is the set of all positive real numbers.

Ans: (3)

Consider y = b^x ….. (1)

Clearly (1, 0) doesn't satisfy (1)

2. If y = (1 + x) (1 + x^2) (1 + x^4), then $\frac{dy}{dx}$ at x = 1 is
   (1) 20 (2) 28 (3) 1 (4) 0

Ans: (2)

$\frac{dy}{dx}$ = (1 + x) (1 + x^2) $4x^3$ + (1 + x^2) (1 + x^4) + (1 + x^4) (1 + x) 2x

$\left.\frac{dy}{dx}\right]_{x=1}$ = (1 + 1) (1 + 1^2) 4 + (1 + 1) (1 + 1) + (1 + 1) (1 + 1) 2

= 16 + 4 + 8 = 28

[Aliter : use logarithmic differentiation]

3. If y = $(\tan^{-1}x)^2$, then $(x^2 + 1)^2 y_2 + 2x (x^2 + 1) y_1$ is equal to
   (1) 4 (2) 0 (3) 2 (4) 1

Ans: (3)

$y^{I} = \frac{2\tan^{-1}x}{1+x^2} \Rightarrow y^{I}(1 + x^2) = 2\tan^{-1}x$

$(1 + x^2) y^{II} + y^{I}(2x) = \frac{2}{1+x^2}$

$\Rightarrow (1 + x^2)^2 y^{II} + 2x (1 + x^2) y^{I} = 2$

4. If f (x) = x^3 and g (x) = $x^3 - 4x$ in $-2 \le x \le 2$, then consider the statements:
   (a) f (x) and g (x) satisfy mean value theorem
   (b) f (x) and g (x) both satisfy Rolle's theorem
   (c) Only g (x) satisfies Rolle's theorem.
   Of these statements
   (1) (a) and (b) are correct (2) (a) alone is correct
   (3) None is correct (4) (a) and (c) are correct

Ans: (4)

f (x) and g (x) are both continuous is [-2, 2] and differentiable is (-2, 2)

$\therefore$ f (x) and g (x) satisfy Mean Value Theorem

Now f (-2) = -8, f (2) = 8 $\therefore$ f (-2) $\neq$ f (2) | g(1) = g(-2)

$\therefore$ f (x) doesn't satisfy Rolle's theorem

5. Which of the following is not a correct statement?
   (1) Mathematics is interesting (2) $\sqrt{3}$ is a prime
   (3) $\sqrt{2}$ is irrational (4) The sun is a star

Ans: (2)

$\sqrt{3}$ is prime is the false statement. Note that the question is not, "which of the following is not a statement?" in which case (1) would have been clearly the answer.

Here we have to identify a statement which is not correct. i.e., a statement whose truth value is false. Hence (2) is the answer.

6. If the function f (x) satisfies $\lim_{x \to 1} \frac{f(x)-2}{x^2-1} = \pi$, then $\lim_{x \to 1}$ f (x) =

(1) 1 (2) 2 (3) 0 (4) 3

Ans: (2)

Method of inspection

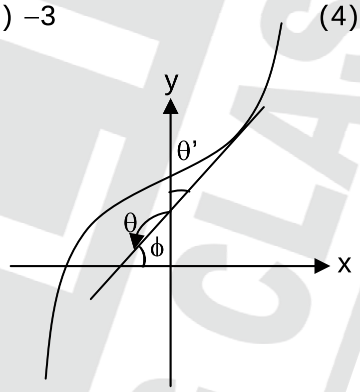
Clearly if $\lim_{x \to 1} f(x)=1$ or 0 or 3, then $\lim_{x \to 1} \frac{f(x)-2}{x^2-1}$ doesn't exist, contradicting

$\lim_{x \to 1} \frac{f(x)-2}{x^2-1} = \pi$

7. The tangent to the curve y = x^3 + 1 at (1, 2) makes an angle θ with y axis, then the value of tan θ is

(1) $-\frac{1}{3}$ (2) 3 (3) –3 (4) $\frac{1}{3}$

Ans: (1)



Clearly θ = 90 + ϕ

θ = 90 + φ

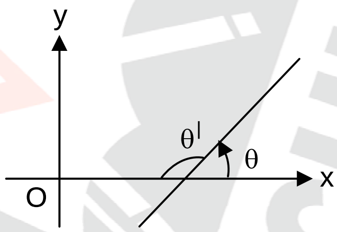
tan θ = tan (90 + ϕ)

= - cot ϕ

Now tan ϕ = $\frac{dy}{dx} = 3x^2\Big|_{(1,2)}$ = 3

$\therefore$ Required = - cot ϕ = -$\frac{1}{3}$

In the diagram above, θ is to be considered as the angle made by tangent with y axis and not θ'.[e.g When we say angle made by a line with x axis, it is the angle measured from x-axis to the line in anticlockwise direction, unless mentioned otherwise].



Here we consider θ as the angle made by the line with +ve x-axis and not $\theta^{|}$]

8. If the function f (x) defined by f (x) = $\frac{x^{100}}{100} + \frac{x^{99}}{99} + ... + \frac{x^2}{2} + x + 1$, then f' (0) =

(1) 100 f' (0) (2) 100 (3) 1 (4) –1

Ans: (3)

$f^{|}(x) = 1 + x + x^2 + ... + x^{99}$

$f^{|}(0) = 1$

9. If f (x) = f (π + e – x) and $\int_e^{\pi} f(x)\,dx = \frac{2}{e+\pi}$, then $\int_e^{\pi} x f(x)$ dx is equal to

(1) π - e (2) $\frac{\pi+e}{2}$ (3) 1 (4) $\frac{\pi-e}{2}$

Ans: (3)

I = $\int_e^{\pi} xf(x)\,dx = \int_e^{\pi}(e+\pi-x)\,f(e+\pi-x)\,dx$

$= \int_e^{\pi}(e+\pi-x)\,f(x)\,dx = \int_e^{\pi}(e+\pi)\,f(x) - I$

$2I = (e+\pi)\frac{2}{e+\pi} \Rightarrow I = 1$

10. If linear function f (x) and g (x) satisfy
$\int\left[(3x-1)\cos x + (1-2x)\sin x\right]$ dx = f (x) cos x + g (x) sin x + C, then

(1) f (x) = 3 (x – 1) (2) f (x) = 3x – 5 (3) g (x) = 3 (x – 1) (4) g (x) = 3 + x

Ans: (3)

$\int[(3x-1)\cos x + (1-2x)\sin]\,dx = f(x)\cos x + g(x)\sin x + C$

= $(3x-1)\sin x - \int \sin x.3dx + (1-2n)(-\cos x) + \int \cos x(-2)\,dx$

= (3x – 1) sin x + 3 cos x – cos x + 2x cos x – 2 sin x + C

= (3x – 1 – 2) sin x + (2 + 2x) cos x + C

= 3 (x – 1) sin x + 2 (x + 1) cos x + C

$\therefore$ f (x) = 2 (x + 1), g (x) = 3 (x – 1)

11. The value of the integral $\int_{-\pi/4}^{\pi/4} \log\left(\sec\theta - \tan\theta\right)d\theta$ is

(1) 0 (2) $\frac{\pi}{4}$ (3) π (4) $\frac{\pi}{2}$

Ans: (1)

The value of $\int_{-\pi/4}^{\pi/4} \log(\sec\theta - \tan\theta)\,d\theta = 0$

$\therefore$ log (sec θ - tan θ) is an odd function

$\because$ if f (θ) = log (sec θ - tan θ)

then, f (-θ) = log [sec θ + tan θ]

= -log (sec θ - tan θ) = -f(θ)

12. $\int \frac{\sin 2x}{\sin^2 x + 2\cos^2 x}$ dx =

(1) –log (1 + $\sin^2$x) + C (2) log (1 + $\cos^2$ x) + C

(3) –log (1 + $\cos^2$ x) + C d) log (1 + $\tan^2$ x) + C

Ans: (3)

$\int \frac{\sin 2x}{\sin^2 x + 2\cos^2 x}$ = $\int \frac{\sin 2x}{1+\cos^2 x}dx$ $\because$ $\sin^2$ x = 1 – $\cos^2$ x

put $\cos^2$x = t , 1 + $\cos^2$ x = t

-2 cos x sin x dx = dt

or sin 2x dx = -dt

= $-\int \frac{dt}{t}$ = - log t + c = - log (1 + $\cos^2$ x) + C

13. Let S be the set of all real numbers. A relation R has been defined on S by aRb $\Leftrightarrow$ $|a - b| \le 1$, then R is
(1) symmetric and transitive but not reflexive
(2) reflexive and transitive but not symmetric
(3) reflexive and symmetric but not transitive
(4) an equivalence relation

Ans: (3)

$a \; R \; b \Leftrightarrow |a - b| \le 1,$

$a \; R \; a = |a - a| = 0 \le 1 \; \therefore$ R is reflexive

if $a \; R \; b \Rightarrow |a - b| \le 1$

then $b \; R \; a \Rightarrow |b - a| \le 1, \Rightarrow |a - b| \le 1$, which is true. $\therefore$ R is symmetric

But R is not transitive, $\because$ take a = 1, b = 2

Then $|a - b| = |1 - 2| = 1 = 1$

Let b = 2 and c = 3 $|b - c| = |2 - 3| = 1$

But $a \not R c \;\; \because |a - c| = |1 - 3| = 2 > 1$

14. For any two real numbers, an operation $*$ defined by $a * b = 1 + ab$ is
(1) neither commutative nor associative
(2) commutative but not associative
(3) both commutative and associative
(4) associative but not commutative

Ans: (2)

$a * b = 1 + ab$

Now $a * b = 1 + ab$

$b * a = 1 + ba = 1 + ab = a * b$

$\therefore$ * is commutative

$(a * b) * c = (1 + ab) * c = 1 + (1 + ab) c \;\; = 1 + c + abc$

but $a * (b * c) = a * (1 + bc) = 1 + a (1 + bc) \;\; = 1 + a + abc$

$\therefore (a * b) * c \ne a * (b * c)$

$\therefore$ * is not associate

15. Let $f : N \to N$ defined by $f(n) = \begin{cases} \frac{n+1}{2} & if\ n\ is odd \\ \frac{n}{2} & if\ n\ is\ even \end{cases}$ then f is

(1) onto but not one-one
(2) one-one and onto
(3) neither one-one nor onto
(3) one-one but not onto

Ans: (1)

$f : f : N \to N$

$$f(n) = \begin{cases} \frac{n+1}{2} & if \quad n\ is\ odd \\ \frac{n}{2} & if \quad n\ is\ even \end{cases}$$

Now for n = 1, $f(1) = \frac{1+1}{2} = 1$

And if n = 2, $f(2) = \frac{2}{2} = 1$

$\therefore f(1) = f(2)$, But $1 \ne 2$.

$\therefore f(x)$ is not one-one

f (x) = $\frac{n+1}{2}$ if n is odd

if y = $\frac{n+1}{2}$ then n = 2y – 1, $\forall$ y

also, f (x) = $\frac{n}{2}$ if n is even i.e., y = $\frac{n}{2}$ or n = 2y $\forall$ y

$\therefore$ f (x) is onto

16. Suppose f (x) = $(x + 1)^2$ for x ≥ –1. If g (x) is a function whose graph is the reflection of the graph of f (x) in the line y = x, then g (x) =

(1) $\frac{1}{(x+1)^2}$ x > –1 (2) $-\sqrt{x}$ - 1 (3) $\sqrt{x}$ + 1 (4) $\sqrt{x}$ – 1

Ans: (4)

f (x) = $(x + 1)^2$ for x ≥ -1 g (x) is the reflection of f (x) in the line y = x, then g (x) is the inverse of f (x)

let y = $(x + 1)^2$

$\Rightarrow \sqrt{y}$ = x + 1

x = $\sqrt{y}-1$

i.e., f^{-1} (y) = $\sqrt{y}-1$

or g (x) = $\sqrt{x}-1$

17. The domain of the function f (x) = $\sqrt{cos x}$ is

(1) $\left[\frac{3\pi}{2}, 2\pi\right]$ (2) $\left[0, \frac{\pi}{2}\right]$ (3) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (4) $\left[0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right]$

Ans: (*)

f (x) = $\sqrt{\cos x}$ $\Rightarrow$ cos x ≥ 0

0 ≤ cos x ≤ 1 x is in I quad or IV quad

i.e., x varies from 0 to $\frac{\pi}{2}$ (in I quadrant)

also from $\frac{3\pi}{2}$ to 2π, cos x

is ≥ 0

$\therefore$ In = $\left[0,\frac{\pi}{2}\right]\cup\left[\frac{3\pi}{2},2\pi\right]$, cos x > 0

However, $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ also is the domain of the function. Infact $\left[\frac{3\pi}{2},2\pi\right]$ and $\left[0,\frac{\pi}{2}\right]$ are also domains since cos x > 0 when x belongs to either of these two intervals.

18. In a class of 60 students, 25 students play cricket and 20 students play tennis and 10 students play both the games, then the number of students who play neither is

(1) 45 (2) 0 (3) 25 (4) 35

Ans: (3)

n ($\cup$) = 60 n (C) = 25 n (T) = 20

n (C $\cap$ T) = 10 then n $(C \cap T)^{l}$ = ?

n (C $\cup$ T) = n (C) + n (T) – n (C $\cap$ T) = 25 + 20 – 10 = 35

$\therefore$ n $(C \cap T)^{l}$ = n (n) – n (C $\cup$ T)

= 60 – 35 = 25

19. Given $0 \le x \le \frac{1}{2}$ then the value of $tan\left[sin^{-1}\left\{\frac{x}{\sqrt{2}}+\frac{\sqrt{1-x^2}}{\sqrt{2}}\right\}-sin^{-1}x\right]$ is

(1) 1 (2) $\sqrt{3}$ (3) –1 (4) $\frac{1}{\sqrt{3}}$

Ans: (1)

$0 \le x \le \frac{1}{2}$

tan $\left[\sin^{-1}\left\{\frac{x}{\sqrt{2}}+\frac{\sqrt{1-x^2}}{\sqrt{2}}\right\}-\sin^{-1}x\right]$ is

= tan $\left[\sin\left\{\frac{x+\sqrt{1-x^2}}{\sqrt{2}}\right\}-\sin^{-1}x\right]$

put $\sin^{-1} x = \theta$ or $x = \sin\theta$

$\therefore$ given = tan $\left[\sin^{-1}\left\{\frac{\sin\theta+\cos\theta}{\sqrt{2}}\right\}-\theta\right]$ = tan $\left[\sin^{-1}\left[\sin\left(\theta+\frac{\pi}{4}\right)-\theta\right]\right]$

tan $\left[\theta+\frac{\pi}{4}-\theta\right]=\tan\frac{\pi}{4}=1$

20. The value of $\sin(2\sin^{-1} 0.8)$ is equal to

(1) 0.48 (2) $\sin 1.2^o$ (3) $\sin 1.6^o$ (4) 0.96

Ans: (4)

The value of $\sin(2\sin^{-1}(0.8))$ is

Let $\sin^{-1} 0.8 = \theta \Rightarrow \sin\theta = 0.8$

$\therefore \cos\theta = \sqrt{1-\sin^2\theta} = 0.6$

given exp = $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2 \times 0.8 \times 0.6$

$= 1.6 \times 0.6$

$= 0.96$

21. If A is 3 x 4 matrix and B is a matrix such that A'B and BA' are both defined, then B is of the type

(1) 4 x 4 (2) 3 x 4 (3) 4 x 3 (4) 3 x 3

Ans: (2)

22. The symmetric part of the matrix $A=\begin{pmatrix}1 & 2 & 4\\ 6 & 8 & 2\\ 2 & -2 & 7\end{pmatrix}$ is

(1) $\begin{pmatrix}0 & -2 & -1\\ -2 & 0 & -2\\ -1 & -2 & 0\end{pmatrix}$ (2) $\begin{pmatrix}1 & 4 & 3\\ 2 & 8 & 0\\ 3 & 0 & 7\end{pmatrix}$ (3) $\begin{pmatrix}0 & -2 & 1\\ 2 & 0 & 2\\ -1 & 2 & 0\end{pmatrix}$ (4) $\begin{pmatrix}1 & 4 & 3\\ 4 & 8 & 0\\ 3 & 0 & 7\end{pmatrix}$

Ans: (4)

Symmetric part of A = $\frac{1}{2}(A+A^{|})$

$= \frac{1}{2}\left\{\begin{pmatrix}1 & 2 & 4\\ 6 & 8 & 2\\ 2 & -2 & 7\end{pmatrix}+\begin{pmatrix}1 & 6 & 2\\ 2 & 8 & -2\\ 4 & 2 & 7\end{pmatrix}\right\} = \begin{pmatrix}1 & 4 & 3\\ 4 & 8 & 0\\ 3 & 0 & 7\end{pmatrix}$

23. If A is a matrix of order 3, such that A (adj A) = 10 I, then |adj A| =

(1) 1 (2) 10 (3) 100 (4) 10 I

Ans: (3)

We know A . Adj A = |A| I

Clearly |A| = 10

$|\text{Adj } A| = |A|^{3-1} = |A|^2 = 10^2 = 100$

24. Consider the following statements:

(a) If any two rows or columns of a determinant are identical, then the value of the determinant is zero

(b) If the corresponding rows and columns of a determinant are interchanged, then the value of determinant does not change.

(c) If any two rows (or columns) of a determinant are interchanged, then the value of the determinant changes in sign.

Which of these are correct?

(1) (a) and (c) (2) (a) and (b) (3) (a), (b) and (c) (4) (b) and (c)

Ans: (3)

Since each option is correct options (1), (2), (3), (4) are all correct answers.

25. The inverse of the matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is

(1) $\frac{1}{24}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ (2) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ (3) $\frac{1}{24}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (4) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$

Ans: (4)

If A = $\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$, A^{-1} = $\begin{pmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{pmatrix}$

When a ≠ 0, b ≠ 0, c ≠ 0

26. If a, b and c are in A.P., then the value of $\begin{vmatrix} x+2 & x+3 & x+a \\ x+4 & x+5 & x+b \\ x+6 & x+7 & x+c \end{vmatrix}$ is

(1) 0 (2) x – (a + b + c) (3) a + b + c (4) $9x^2 + a + b + c$

Ans: (1)

$R_1^1 = R_1 - R_2, R_2^1 = R_2 - R_3$

$$\begin{vmatrix} -2 & -2 & a-b \\ -2 & -2 & b-c \\ x+6 & x+7 & x+c \end{vmatrix}$$

= $R_1 \equiv R_2$ since a – b = b – c

(∵ a, b, c are in AP ⇒ b – a = c – b)

27. The local minimum value of the function $f^{|}$ given by f (x) = 3 + |x|, x $\in$ R is

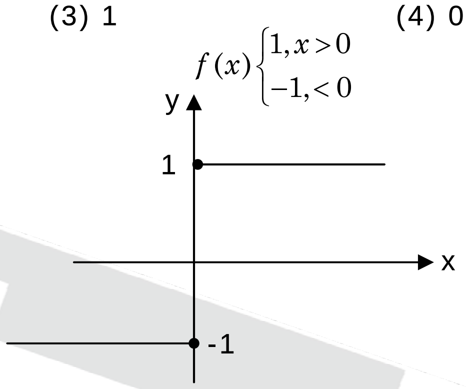
(1) -1 (2) 3 (3) 1 (4) 0

Ans: (1 & 3) or (2)

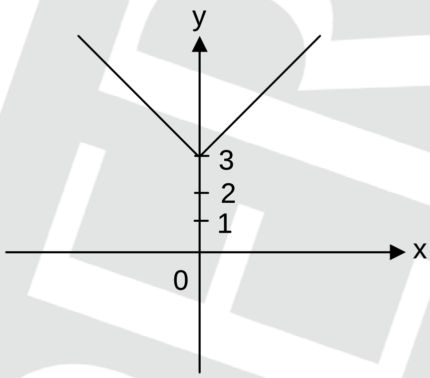
Clearly when x > 0, local minimum = 1

When x < 0, local minimum = -1

$\therefore$ option (1) and (3) both are true



However, if the question was given as “The local minimum value of the function **f** given by f (x) = 3 + |x|, x $\in$ R is”, then (3) is correct answer. It is clear from the graph of the function y = |x| + 3. Eventhough it is not differentiable at x = 3, it still has a relative minimum.



28. A stone is dropped into a quiet lake and waves move in circles at the speed of 5 cm/sec. At that instant, when the radius of circular wave is 8 cm, how fast is the enclosed area increasing?

(1) $6\pi cm^2/s$ (2) $8\pi cm^2/s$ (3) $\frac{8}{3} cm^2/s$ (4) $80\pi cm^2/s$

Ans: (4)

Given $\frac{dr}{dt}$ = 5 cm/sec

$A = \pi r^2 \Rightarrow \frac{dA}{dt} = \pi.2r\frac{dr}{dt} = 2\pi.(8).\ 5 = 80\pi cm^2/sec$

29. A gardener is digging a plot of land. As he gets tired, he works more slowly. After ‘t’ minutes he is digging at a rate of $\frac{2}{\sqrt{t}}$ square metres per minute. How long will it take him to dig an area of 40 square metres?

(1) 100 minutes (2) 10 minutes (3) 30 minutes (4) 40 minutes

Ans: (1)

Given, $\frac{dA}{dt} = \frac{2}{\sqrt{t}} \Rightarrow \int dA = \int \frac{2}{\sqrt{t}}$ dt

$\Rightarrow A = 2.2\sqrt{t} + C$

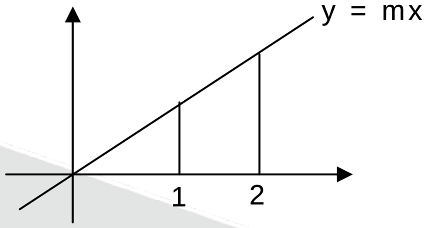
Where t = 0, C = 0

$\therefore 4\sqrt{t} = 40 \Rightarrow t = 100$

30. The area of the region bounded by the lines y = mx, x = 1, x = 2, and x axis is 6 sq. units, then 'm' is

(1) 3 (2) 1 (3) 2 (4) 4

Ans: (4)



Area = $\int_1^2 mx\,dx$ = 6

$m\left.\frac{x^2}{2}\right|_1^2 = 6 \Rightarrow$ m $(2^2 - 1^2)$ = 12

$\Rightarrow$ 3m = 12 $\Rightarrow$ m = 4

31. Area of the region bounded by two parabolas y = x^2 and x = y^2 is

(1) $\frac{1}{4}$ (2) $\frac{1}{3}$ (3) 4 (4) 3

Ans: (2)

y^2 = 4ax, x^2 = 4by is $\frac{4a.4b}{3}$

Required = $\frac{1.1}{3} = \frac{1}{3}$

32. The order and degree of the differential equation y = x$\frac{dy}{dx} + \frac{2}{\frac{dy}{dx}}$ is

(1) 1, 2 (2) 1, 3 (3) 2, 1 (4) 1, 1

Ans: (1)

$\frac{dy}{dx}y = x\left(\frac{dy}{dx}\right)^2 + 2$

order = 1, degree = 2

33. The general solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = 3x$ is

(1) $y = x - \frac{c}{x}$ (2) $y = x + \frac{c}{x}$ (3) $y = x^2 - \frac{c}{x}$ (4) $y = x^2 + \frac{c}{x}$

Ans: (4 & 3)

$x\frac{dy}{dx} + y = 3x^2$

$\frac{dy}{dx}(xy) = 3x^2$

$d(xy) = 3x^2dx$ -----(*)

$on\,\text{int}\,egrate,$

$xy = x^3 + C$

$y = x^2 + \frac{C}{x}$

Infact, after integrating (*) one may also write

$xy = x^3 - C$

$\Rightarrow y = x^2 - \frac{C}{x}$

34. The distance of the point P (a, b, c) from the x-axis is

(1) $\sqrt{a^2+b^2}$ (2) $\sqrt{b^2+c^2}$ (3) a (4) $\sqrt{a^2+c^2}$

Ans: (2)

P (a, b, c), A (a, 0, 0)

Distance = $\sqrt{0^2+b^2+c^2} = \sqrt{b^2+c^2}$

35. Equation of the plane perpendicular to the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and passing through the point (2, 3, 4) is

(1) 2x + 3y + z = 17 (2) x + 2y + 3z = 9

(3) 3x + 2y + z = 16 (4) x + 2y + 3z = 20

Ans: (4)

Since plane is perpendicular to $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$

Direction ratio of normal to the plane is 1, 2, 3.

$\therefore$ Eq is 1x + 2y + 3z + d = 0.

Passes through the point (2, 3, 4) $\therefore$ 2 + 6 + 12 + d = 0

d = -20

$\therefore$ Eq is x + 2y + 3z = 20

36. The line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ is parallel to the plane

(1) 2x + 3y + 4z = 0 (2) 3x + 4y + 5z = 7

(3) 2x + y – 2z = 0 (4) x + y + z = 2

Ans: (3)

D.R of line = 3, 4, 5

Line and plane are parallel

$\therefore$ normal to plane and line are perpendicular

$\therefore$ $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\therefore$ For plane 2x + y – 2z = 0

3 (2) + 4(1) – 2(5) = 0

$\therefore$2x + y – 2z = 0

37. The angle between two diagonals of a cube is

(1) $\cos^{-1}\left(\frac{1}{3}\right)$ (2) 30° (3) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (4) 45°

Ans: (1)

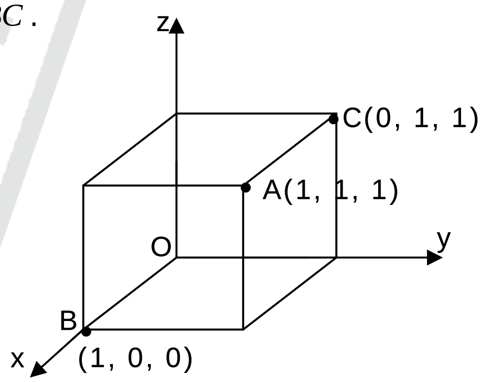
Consider a diagonal with each side 1. Now BC and OA are diagonals.

Angle between diagonals = Angle between $\overrightarrow{OA}$ and $\overrightarrow{BC}$.

$\overrightarrow{OA}$ = (1, 1, 1) – (0, 0, 0) = (1, 1,1)

$\overrightarrow{BC}$ = (0, 1, 1) – (1, 0, 0) = (-1, 1,1)

Now cos θ = $\frac{1(-1)+1(1)+(1)(1)}{\sqrt{1^2+1^2+1^2}\sqrt{(-1)^2+1^2+1^2}} = \frac{1}{3}$



38. Lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-K}$ and $\frac{x-1}{K}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar if

(1) K = 2 (2) K = 0 (3) K = 3 (4) K = -1

Ans: (2)

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

$$\begin{vmatrix} 1-2 & 4-3 & 5-4 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix}=0$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix}=0$$

$\therefore$ **k = 0**

39. A and B are two events such that P (A) $\neq$ 0, P (B/A) if

(i) A is a subset of B

(ii) A $\cap$ B = Φ are respectively

(1) 1, 1 (2) 0 and 1 (3) 0, 0 (4) 1, 0

Ans: (4)

P(B | A) = $\frac{P(A\cap B)}{P(A)}$

$\therefore$ P(B | A) = 0

$\therefore$ Since P(A) $\neq$ 0, P(A) = 1 (Inspection)

$\therefore$ P(A) = 1, P(A $\cap B)$ =0 $\because A\cap B=\phi$

40. Two dice are thrown simultaneously. The probability of obtaining a total score of 5 is

(1) $\frac{1}{9}$ (2) $\frac{1}{18}$ (3) $\frac{1}{36}$ (4) $\frac{1}{12}$

Ans: (1)

O (s) = 36

E = {(1, 4), (4, 1), (2,3) , (3, 2)}

$\therefore$ P(E) = 4/36 = 1/9

41. If the events A and B are independent if P ($A^{|}$) = $\frac{2}{3}$ and P ($B^{|}$) = $\frac{2}{7}$, then P (A $\cap$ B) is equal to

(1) $\frac{4}{21}$ (2) $\frac{5}{21}$ (3) $\frac{1}{21}$ (4) $\frac{3}{21}$

Ans: (2)

P (A $\cap B)$ = P(A) . P(B) (independent events)

= [1- P($A^{|}$)] [1- P($B^{|}$)]

= [1 – 2/3] [1 – 2/7]

= $\frac{1}{3}.\frac{5}{7}=\frac{5}{21}\,\frac{1}{3}.\frac{5}{7}=\frac{5}{21}$

42. A box contains 100 bulbs, out of which 10 are defective. A sample of 5 bulbs is drawn. The probability that none is defective is

(1) $\frac{9}{10}$ (2) $\left(\frac{1}{10}\right)^5$ (3) $\left(\frac{9}{10}\right)^5$ (4) $\left(\frac{1}{2}\right)^5$

Ans: (3)

$p=\frac{10}{100}=0.1 \quad q=0.9 \quad n=5$

$\therefore p(x)={}^5C_x(0.1)^x(0.9)^{5-x}$

$p(0)={}^5C_0(0.1)^0(0.9)^5=\left(\frac{9}{10}\right)^5$

43. The area of the parallelogram whose adjacent sides are $\hat{i}+\hat{k}$ and $2\hat{i}+\hat{j}+\hat{k}$ is

(1) 3 (2) $\sqrt{2}$ (3) 4 (4) $\sqrt{3}$

Ans: (4)

Area = $\left|\vec{a}\times\vec{b}\right|$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 0 & 1\\ 2 & 1 & 1\end{vmatrix}=-\hat{i}+\hat{j}+\hat{k}$$

$\therefore$ area = $\sqrt{1+1+1}=\sqrt{3}$ $\sqrt{1+1+1}=\sqrt{3}$

44. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at an angle $\frac{\pi}{3}$, then the value of $\left|\vec{a}+\vec{b}\right|$ is

(1) equal to (2) greater than 1 (3) equal to 0 (4) less than 1

Ans: (2)

$|\vec{a}|=|\vec{b}|=1, \theta=\pi/3$

$|\vec{a}+\vec{b}|^2=|\vec{a}|^2+|\vec{b}|^2+2(|\vec{a}|.|\vec{b}|\cos\theta)$

$=1+1+2.1.1.1/2=3$

$=|\vec{a}+\vec{b}|=\sqrt{3}$

$\therefore|\vec{a}+\vec{b}|>1$

45. The value of $\left[\vec{a}-\vec{b}\ \ \vec{b}-\vec{c}\ \ \vec{c}-\vec{a}\right]$ is equal to

(1) 0 (2) 1 (3) $2\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ (4) 2

Ans: (1)

$\left[\vec{a}-\vec{b}\ \ \vec{b}-\vec{a}\ \ \vec{c}-\vec{a}\right]$

$=\left(\vec{a}-\vec{b}\right)\left[\left(\vec{b}-\vec{c}\right)\times\left(\vec{c}-\vec{a}\right)\right]$

$=\left(\vec{a}-\vec{b}\right)\left[\vec{b}\times\vec{c}-\vec{b}\times\vec{a}-\vec{c}\times\vec{c}+\vec{c}\times\vec{a}\right]$

$= \left(\vec{a} - \vec{b}\right)\left[\vec{b} \times \vec{c} - \vec{b} \times \vec{a} + \vec{c} \times \vec{a}\right]$

$\because$ $\vec{c} \times \vec{c}$ = 0

$\vec{a}.\left(\vec{b} \times \vec{c}\right) - \vec{a}.\left(\vec{b} \times \vec{a}\right) + \vec{a}.\left(\vec{c} \times \vec{a}\right) - \vec{b}.\left(\vec{b} \times \vec{c}\right) + \vec{b}.\left(\vec{b} \times \vec{a}\right) - \vec{b}.\left(\vec{c} \times \vec{a}\right)$

= $\left[\vec{a}\,\vec{b}\,\vec{c}\right]$ - 0 + 0 – 0 + 0 - $\left[\vec{a}\,\vec{b}\right]$

= 0

46. If x + y ≤ 2, x ≥ 0, y ≥ 0 the point at which maximum value of 3x + 2y attained will be

(1) (0, 2) (2) (0, 0) (3) (2, 0) (4) $\left(\frac{1}{2}, \frac{1}{2}\right)$

Ans: (3)

Corner points are (0, 0), (2, 0) (0, 2)

Max. of 2x + 3y is 6 at (2, 0)

47. If sin θ = sin α, then

(1) $\frac{\theta+\alpha}{2}$ is any multiple of $\frac{\pi}{2}$ and $\frac{\theta-\alpha}{2}$ is any odd multiple of π.

(2) $\frac{\theta+\alpha}{2}$ is any odd multiple of $\frac{\pi}{2}$ and $\frac{\theta-\alpha}{2}$ is any multiple of π.

(3) $\frac{\theta+\alpha}{2}$ is any multiple of $\frac{\pi}{2}$ and $\frac{\theta-\alpha}{2}$ is any even multiple of π.

(4) $\frac{\theta+\alpha}{2}$ is any even multiple of $\frac{\pi}{2}$ and $\frac{\theta-\alpha}{2}$ is any odd multiple of π.

Ans: (*)

sin θ = sin α

sin θ - sin α = 0

$2\cos\left(\frac{\theta+\alpha}{2}\right)\sin\left(\frac{\theta-\alpha}{2}\right) = 0$

It is not necessary that $\frac{\theta+\alpha}{2}$ is odd multiple of $\frac{\pi}{2}$ and $\frac{\theta-\alpha}{2}$ is any multiple of π should be simultaneously hold good for the above equation to be true. Hence the correct answer should be

$\Rightarrow \frac{\theta+\alpha}{2} = odd\ multiple\ of\ \pi/2\ or\ \frac{\theta-\alpha}{2} = any\ multiple\ of\ \pi$

48. If tan x = $\frac{3}{4}$, π < x < $\frac{3\pi}{2}$, then the value of cos $\frac{x}{2}$ is

(1) - $\frac{1}{\sqrt{10}}$ (2) $\frac{3}{\sqrt{10}}$ (3) $\frac{1}{\sqrt{10}}$ (4) - $\frac{3}{\sqrt{10}}$

Ans: (1)

Tan x = 3/4

$\therefore$ cos x = - 4/5

1+ cos x = 2 $\cos^2$ (x/2)

1 – 4/5 = 2 $\cos^2$ (x/2)

$\frac{1}{10} = \cos^2(x/2)$ $\therefore$ cos (x/2) = - $\frac{1}{\sqrt{10}}$ $\frac{1}{\sqrt{10}}$

49. In a triangle ABC, a [b cos C – c cos B] =

(1) 0 (2) a^2 (3) $b^2 - c^2$ (4) b^2

Ans: (3)

a[b cos C – c cos B]

(b cos C + c cos B) (b cos C - c cos B)

$b^2 \cos^2 C - c^2 \cos^2 B$

$b^2 (1 - \sin^2 C) - c^2 (1 - \sin^2 B)$

$b^2 \left(1-\frac{c^2}{4R^2}\right)-c^2\left(1-\frac{b^2}{4R^2}\right)$

$= b^2 - \frac{b^2c^2}{4R^2}-c^2+\frac{c^2b^2}{4R^2}\left(1-\frac{c^2}{4R^2}\right)-c^2\left(1-\frac{b^2}{4R^2}\right)$

$= b^2 - c^2$

50. If α and β are two different complex numbers with $|\beta| = 1$, then $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$ is equal to

(1) $\frac{1}{2}$ (2) 0 (3) – 1 (4) 1

Ans: (1)

Take, $\alpha = 0$, $\beta = 1$

Then $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|=\left|\frac{1-0}{1-0}\right|=1$

51. The set A = {x :|2x + 3| < 7} is equal to the set

(1) D = {x : 0 < x + 5 < 7} (2) B = {x : - 3 < x < 7}

(3) E = {x : - 7 < x < 7} (4) C = {x : - 13 < 2x < 4}

Ans: (1)

$|2x + 3| < 7 \rightarrow -7 < 2x + 3 < 7$

$-10 < 2x < 4$ $-5 < x < 2$ $0 < x + 5 < 7$

52. How many 5 digit telephone numbers can be constructed using the digits 0 to 9, if each number starts with 67 and no digit appears more than once?

(1) 335 (2) 336 (3) 338 (4) 337

Ans: (2)

3 digits from 0, 1, 2, 3, 4, 5, 8, 9 (arrangement of 8 digits taking 3 at a time

${}^8P_3 = 720 = 8 \times 7 \times 6 = 42 \times 8 = 336$

53. If 21^{st} and 22^{nd} terms in the expansion of $(1 + x)^{44}$ are equal, then x is equal to

(1) $\frac{8}{7}$ (2) $\frac{21}{22}$ (3) $\frac{7}{8}$ (4) $\frac{23}{24}$

Ans: (3)

$${}^{44}C_{20}x^{20} = {}^{44}C_{21}x^{21} \Rightarrow x = \frac{{}^{44}C_{20}}{{}^{44}C_{21}}$$

$$\frac{(44-21)!21!}{(44-20)!20!}=\frac{23!}{24!}.\frac{21!}{20!}=\frac{21}{24}=\frac{7}{8}$$

54. Consider an infinite geometric series with first term 'a' and common ratio 'r'. If the sum is 4 and the second term is $\frac{3}{4}$, then

(1) a = 2, r = $\frac{3}{8}$ (2) a = $\frac{4}{7}$, r = $\frac{3}{7}$ (3) a = $\frac{3}{2}$, r = $\frac{1}{2}$ (4) a = 3, r = $\frac{1}{4}$

Ans: (4)

$4 = \frac{a}{1-r} \Rightarrow 4$

$\Rightarrow a = 4 - 44$

$\Rightarrow 4r = 4 - a$

check with options

55. A straight lien passes through the points (5, 0) and (0, 3). The length of perpendicular form the point (4, 4) on the line is

(1) $\frac{15}{\sqrt{34}}$ (2) $\frac{\sqrt{17}}{2}$ (3) $\frac{17}{2}$ (4) $\sqrt{\frac{17}{2}}$

Ans: (4)

$y - 0 = \left(\frac{3-0}{-5}\right)(x-5)$

$-5y = 3x - 15$

$d = \left|\frac{3(4)+5(4)-15}{\sqrt{3^2+5^2}}\right| = \frac{17}{\sqrt{34}} = \sqrt{\frac{17}{2}}$

56. Equation of circle with centre (-a, -b) and radius $\sqrt{a^2 - b^2}$ is

(1) $x^2 + y^2 + 2ax + 2by + 2b^2 = 0$ (2) $x^2 + y^2 - 2ax - 2by - 2b^2 = 0$

(3) $x^2 + y^2 - 2ax - 2by + 2b^2 = 0$ (4) $x^2 + y^2 - 2ax + 2by + 2a^2 = 0$

Ans: (1)

Only (1) has centre (a, b)

57. The area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of Latus rectum is

(1) 20 sq. units (2) 18 sq. units (3) 17 sq. units (4) 19 sq. units

Ans: (1)

$x^2 = 12y \Rightarrow 4a = 12 \Rightarrow a = 3$

area of triangle = $\frac{1}{2}(base)(height)$

$\frac{1}{2}(4|a|)(|a|) = \frac{1}{2}(12)(3) = 18$

58. If the coefficient of variation and standard deviation are 60 and 21 respectively, the arithmetic mean of distribution is

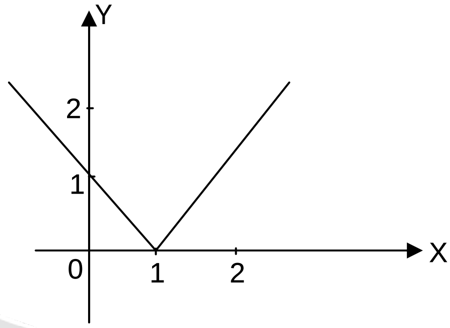
(1) 60 (2) 30 (3) 35 (4) 21

Ans: (3)

coefficient of variation = $\frac{\sigma}{\bar{x}}100$

$60 = \frac{21}{x}.100 \Rightarrow \bar{x} = 35$

59. The function represented by the following graph is



(1) Continuous but not differentiable at x = 1
(2) Differentiable but not continuous at x = 1
(3) Continuous and differentiable at x = 1
(4) Neither continuous nor differentiable at x = 1

Ans: (1)

$\because f(x) = |x-1|$

60. If f (x) = $\begin{cases} \dfrac{3\sin \pi x}{5x} & x \neq 0 \\ 2K & x = 0 \end{cases}$ is continuous at x = 0, then the value of K is

(1) $\dfrac{\pi}{10}$ (2) $\dfrac{3\pi}{10}$ (3) $\dfrac{3\pi}{2}$ (4) $\dfrac{3\pi}{5}$

Ans: (2)

$$\lim_{x\to 0}\left(\frac{3\sin \pi x}{5x}\right) = 2k$$

$$\pi\frac{3}{5}\lim_{x\to 0}\frac{\sin \pi x}{x\pi} = 2k$$

$$\frac{3\pi}{5} = 2k$$

$$k = 3\pi/10$$