WARNING	Any malpractice or any attempt to commit any kind of malpractice in the Examination will DISQUALIFY THE CANDIDATE.		
PAPER – II MATHEMATICS			
Version Code	B3	Question Booklet Serial Number :	
Time : 150 Minutes		Number of Questions : 120	Maximum Marks : 480
Name of Candidate			
Roll Number			
Signature of Candidate			

INSTRUCTIONS TO THE CANDIDATE

1. **Please ensure that the VERSION CODE shown at the top of this Question Booklet is the same as that shown in the OMR Answer Sheet issued to you.** If you have received a Question Booklet with a different Version Code, please get it replaced with a Question Booklet with the same Version Code as that of the OMR Answer Sheet from the Invigilator. **THIS IS VERY IMPORTANT.**
2. Please fill in the items such as Name, Roll Number and Signature in the columns given above. Please also write Question Booklet Sl. No. given at the top of this page against item 4 in the **OMR Answer Sheet**.
3. This Question Booklet contains 120 questions. For each question, five answers are suggested and given against (A), (B), (C), (D) and (E) of which only one will be the **Most Appropriate Answer**. Mark the bubble containing the letter corresponding to the 'Most Appropriate Answer' in the OMR Answer Sheet, by using either **Blue or Black ball-point pen only.**
4. **Negative Marking**: In order to discourage wild guessing, the score will be subjected to penalization formula based on the number of right answers actually marked and the number of wrong answers marked. Each correct answer will be awarded FOUR marks. ONE mark will be deducted for each incorrect answer. More than one answer marked against a question will be deemed as incorrect answer and will be negatively marked.
5. Please read the instructions given in the OMR Answer Sheet for marking answers. Candidates are advised to strictly follow the instructions contained in the OMR Answer Sheet.

IMMEDIATELY AFTER OPENING THIS QUESTION BOOKLET, THE CANDIDATE SHOULD VERIFY WHETHER THE QUESTION BOOKLET ISSUED CONTAINS ALL THE 120 QUESTIONS IN SERIAL ORDER. IF NOT, REQUEST FOR REPLACEMENT.

DO NOT OPEN THE SEAL UNTIL THE INVIGILATOR ASKS YOU TO DO SO.

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PLEASE ENSURE THAT THIS QUESTION BOOKLET CONTAINS 120 QUESTIONS SERIALLY NUMBERED FROM 1 TO 120
PRINTED PAGES : 32

1. The equation $k \sin x + \cos 2x = 2k - 7$ has a solution if

(A) $k > 6$ (B) $2 \le k \le 6$ (C) $k < 2$ (D) $-6 \le k \le -2$ (E) $k \le -6$

2. The distance between the points $(a \cos \alpha, a \sin \alpha)$ and $(a \cos \beta, a \sin \beta)$ is

(A) $2\left|\sin\left(\frac{\alpha-\beta}{2}\right)\right|$ (B) $2\left|a \sin\left(\frac{\alpha-\beta}{2}\right)\right|$ (C) $2\left|a \cos\left(\frac{\alpha-\beta}{2}\right)\right|$

(D) $\left|a \cos\left(\frac{\alpha-\beta}{2}\right)\right|$ (E) $2\left|a\left(1-\cos(\alpha-\beta)\right)\right|$

3. The vertices of the rectangle $ABCD$ are $A(-1, 0)$, $B(2, 0)$, $C(a, b)$ and $D(-1, 4)$. Then the length of the diagonal AC is

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

4. If a straight line passes through the points $\left(\frac{-1}{2}, 1\right)$ and $(1, 2)$, then its x-intercept is

(A) -2 (B) -1 (C) 2 (D) 1 (E) 0

Space for rough work

5. The line parallel to the x-axis and passing through the point of intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a, b) \neq (0, 0)$, is

(A) above the x-axis at a distance of $\frac{3}{2}$

(B) above the x-axis at a distance of $\frac{2}{3}$

(C) below the x-axis at a distance of $\frac{2}{3}$

(D) below the x-axis at a distance of $\frac{3}{2}$

(E) below the x-axis at a distance of 3

6. The line L has intercepts a and b on the coordinate axes. Keeping the origin fixed, the coordinate axes are rotated through a fixed angle. If the line L has intercepts p and q on the rotated axes, then $\frac{1}{a^2}+\frac{1}{b^2}$ is equal to

(A) $p^2 + q^2$ (B) $p^2 - q^2$ (C) $\frac{1}{p^2}+\frac{1}{q^2}$ (D) $\frac{1}{p^2}-\frac{1}{q^2}$ (E) $\frac{1}{q^2}-\frac{1}{p^2}$

7. The equation of the perpendicular bisector of the line segment joining $A(-2, 3)$ and $B(6, -5)$ is

(A) $x - y = -1$ (B) $x - y = 3$ (C) $x + y = 3$ (D) $x + y = 1$ (E) $x + y = -1$

8. The vertices of the triangle PQR are $P\ (0, b)$, $Q\ (0, 0)$ and $R\ (a, 0)$. If the medians PM and QN of PQR are perpendicular, then

(A) $b^2 = 2a^2$ (B) $b = a^2$ (C) $a^2 = 2b^2$ (D) $a = b$ (E) $a = -b$

Space for rough work

9. The slope of the straight line which does not intersect x-axis is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\sqrt{3}$ (D) 1 (E) 0

10. The length of the tangent drawn from any point on the circle $x^2 + y^2 + 2fy + \lambda = 0$ to the circle $x^2 + y^2 + 2fy + \mu = 0$, where $\mu > \lambda > 0$, is

(A) $\sqrt{\mu - \lambda}$ (B) $\sqrt{\mu + \lambda}$ (C) $\sqrt{\mu^2 - \lambda^2}$ (D) $\mu + \lambda$ (E) $\mu - \lambda$

11. The sum of the minimum distance and the maximum distance from the point (4, – 3) to the circle $x^2 + y^2 + 4x - 10y - 7 = 0$ is

(A) 20 (B) 12 (C) 10 (D) 16 (E) 22

12. The equation of one of the diameters of the circle $x^2 - y^2 - 6x + 2y = 0$ is

(A) $x - 3y = 0$ (B) $x + 3y = 0$ (C) $3x + y = 0$ (D) $3x - y = 0$ (E) $x + 2y = 0$

Space for rough work

13. The parametric equations of the circle $x^2+y^2+x+\sqrt{3}y=0$ are

(A) $x=1+\cos\theta,\ y=\frac{\sqrt{3}}{2}+\sin\theta$ (B) $x=-\frac{1}{2}+\cos\theta,\ y=-\frac{\sqrt{3}}{2}+\sin\theta$

(C) $x=\frac{1}{2}+\cos\theta,\ y=-\frac{\sqrt{3}}{2}+\sin\theta$ (D) $x=\frac{1}{2}+\frac{1}{2}\cos\theta,\ y=\frac{\sqrt{3}}{2}+\frac{1}{2}\sin\theta$

(E) $x=\cos\theta-1,\ y=-\frac{\sqrt{3}}{2}+\sin\theta$

14. An equilateral triangle is inscribed in the parabola $y^2 = 4x$. If a vertex of the triangle is at the vertex of the parabola, then the length of side of the triangle is

(A) $\sqrt{3}$ (B) $8\sqrt{3}$ (C) $4\sqrt{3}$ (D) $3\sqrt{3}$ (E) $2\sqrt{3}$

15. The equation of the latus rectum of the conic $y^2=\frac{5}{2}x$ is

(A) $8x-5=0$ (B) $8x+5=0$ (C) $5x+8=0$ (D) $x-5=0$ (E) $x-8=0$

16. For each point (x, y) on an ellipse, the sum of the distances from (x, y) to the points (2, 0) and (–2, 0) is 8. Then the positive value of x so that $(x, 3)$ lies on the ellipse is

(A) 2 (B) $2\sqrt{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) 4 (E) 0

17. The focus of the parabola $y^2+6x-2y+13=0$ is at the point

(A) $\left(\frac{7}{2},1\right)$ (B) $\left(\frac{-1}{2},1\right)$ (C) $\left(-2,\frac{1}{2}\right)$ (D) $\left(-\frac{7}{2},1\right)$ (E) $\left(-\frac{1}{2},-1\right)$

Space for rough work

18. The distance between the vertex of the parabola $y = x^2 - 4x + 3$ and the centre of the circle $x^2 = 9 - (y-3)^2$ is

(A) $2\sqrt{3}$ (B) $3\sqrt{2}$ (C) $2\sqrt{2}$ (D) $\sqrt{2}$ (E) $2\sqrt{5}$

19. If $\vec{a}$ is perpendicular to $\vec{b}$, then the vector $\vec{a} \times \{\vec{a} \times \{\vec{a} \times (\vec{a} \times \vec{b})\}\}$ is equal to

(A) $|\vec{a}|^2 \vec{b}$ (B) $|\vec{a}|\vec{b}$ (C) $|\vec{a}|^3 \vec{b}$ (D) $|\vec{a}|^4 \vec{b}$ (E) 0

20. If the vector $8\hat{i} + a\hat{j}$ of magnitude 10 is in the direction of the vector $4\hat{i} - 3\hat{j}$, then the value of a is equal to

(A) 6 (B) 3 (C) –3 (D) 5 (E) –6

21. If $\vec{a} = 2\hat{i} - 7\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$ and $\vec{a} \cdot m\vec{b} = 120$, then the value of m is equal to

(A) 5 (B) –24 (C) –5 (D) 120 (E) 24

22. If the angle between $\vec{a}$ and $\vec{c}$ is 25°, the angle between $\vec{b}$ and $\vec{c}$ is 65° and $\vec{a} + \vec{b} = \vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(A) 40° (B) 115° (C) 25° (D) 65° (E) 90°

Space for rough work

23. The position vector of the centroid of the triangle ABC is $2\hat{i}+4\hat{j}+2\hat{k}$. If the position vector of the vertex A is $2\hat{i}+6\hat{j}+4\hat{k}$, then the position vector of midpoint of BC is

(A) $2\hat{i}+3\hat{j}+\hat{k}$ (B) $2\hat{i}+3\hat{j}-\hat{k}$ (C) $2\hat{i}-3\hat{j}-\hat{k}$

(D) $-2\hat{i}-3\hat{j}-\hat{k}$ (E) $2\hat{i}-3\hat{j}+\hat{k}$

24. The projection of the vector $2\hat{i}+a\hat{j}-\hat{k}$ on the vector $\hat{i}-2\hat{j}+\hat{k}$ is $\frac{-5}{\sqrt{6}}$. Then the value of a is equal to

(A) 1 (B) 2 (C) –2 (D) 3 (E) –3

25. A unit vector in the XOY- plane that makes an angle 30° with the vector $\hat{i}+\hat{j}$ and makes an angle 60° with $\hat{i}-\hat{j}$ is

(A) $\frac{1}{4}\left[\left(\sqrt{6}+\sqrt{2}\right)\hat{i}-\left(\sqrt{6}-\sqrt{2}\right)\hat{j}\right]$ (B) $\frac{1}{2}\left[\left(\sqrt{6}-\sqrt{2}\right)\hat{i}+\left(\sqrt{6}+\sqrt{2}\right)\hat{j}\right]$

(C) $\frac{1}{4}\left[\left(\sqrt{6}-\sqrt{2}\right)\hat{i}+\left(\sqrt{6}+\sqrt{2}\right)\hat{j}\right]$ (D) $\frac{1}{3}\left[\left(\sqrt{6}+\sqrt{2}\right)\hat{i}+\left(\sqrt{2}-\sqrt{6}\right)\hat{j}\right]$

(E) $\frac{1}{4}\left[\left(\sqrt{6}+\sqrt{2}\right)\hat{i}+\left(\sqrt{6}-\sqrt{2}\right)\hat{j}\right]$

26. The angle between the line $\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(2\hat{i}+3\hat{j}+4\hat{k})$ and the plane $\vec{r}\cdot(\hat{i}+2\hat{j}-2\hat{k})=3$ is

(A) 0° (B) 60° (C) 30° (D) 90° (E) 45°

27. The lines $\vec{r}=\hat{i}+\hat{j}-\hat{k}+\lambda(3\hat{i}-\hat{j})$ and $\vec{r}=4\hat{i}-\hat{k}+\mu(2\hat{i}+3\hat{k})$ intersect at the point

(A) (0, 0, 0) (B) (0, 0, 1) (C) (0, –4, –1) (D) (4, 0, –1) (E) (4, 1, –1)

Space for rough work

28. An equation of the plane through the points (1, 0, 0) and (0, 2, 0) and at a distance $\frac{6}{7}$ units from the origin is

(A) $6x+3y+z-6=0$ (B) $6x+3y+2z-6=0$

(C) $6x+3y+z+6=0$ (D) $6x+3y+2z+6=0$

(E) $6x+2y+3z+6=0$

29. The projection of a line segment on the axes are 9, 12 and 8. Then the length of the line segment is

(A) 15 (B) 16 (C) 17 (D) 18 (E) 21

30. The straight line passing through the point (1, 0, –2) and perpendicular to the plane $x-2y+5z-7=0$ is

(A) $\frac{x-1}{1}=\frac{y}{0}=\frac{z-5}{-2}$ (B) $\frac{x-1}{5}=\frac{y}{-2}=\frac{z+2}{1}$ (C) $\frac{x-5}{-2}=\frac{y-1}{-5}=\frac{z}{1}$

(D) $\frac{x-1}{-1}=\frac{y}{-2}=\frac{z-2}{5}$ (E) $\frac{x-1}{1}=\frac{y}{-2}=\frac{z+2}{5}$

31. The equation of the plane passing through (1, 2, 3) and parallel to $3x-2y+4z=5$ is

(A) $3x-2y+4z=11$ (B) $3x-2y+4z=0$ (C) $3x-2y+4z=10$

(D) $3(x-1)-2(y-2)+4(z-3)=5$ (E) $3(x-1)-2(y-2)+4(z-3)=11$

Space for rough work

32. If the straight lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{0}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar, then the value of k is

(A) -3 (B) 0 (C) 1 (D) -2 (E) 6

33. The line $\frac{x-x_1}{0}=\frac{y-y_1}{1}=\frac{z-z_1}{2}$ is

(A) perpendicular to the x-axis
(B) perpendicular to the yz-plane
(C) parallel to the y-axis
(D) parallel to the xz-plane
(E) perpendicular to the z-axis

34. The A.M. of 9 terms is 15. If one more term is added to this series, then the A.M. becomes 16. The value of the added term is

(A) 30 (B) 27 (C) 25 (D) 23 (E) 20

35. If the average of the numbers $1, 2, 3, \cdots, 98, 99, x$ is $100x$, then the value of x is

(A) $\frac{51}{100}$ (B) $\frac{50}{99}$ (C) $\frac{1}{2}$ (D) $\frac{51}{99}$ (E) $\frac{50}{101}$

36. If the median of $\frac{x}{5}, x, \frac{x}{4}, \frac{x}{2}, \frac{x}{3}$ $(x>0)$ is 8, then the value of x is

(A) 24 (B) 32 (C) 8 (D) 16 (E) 40

Space for rough work

37. If the standard deviation of 3, 8, 6, 10, 12, 9, 11, 10, 12, 7 is 2.71, then the standard deviation of 30, 80, 60, 100, 120, 90, 110, 100, 120, 70 is

(A) 2.71 (B) 27.1 (C) $(2.71)\sqrt{10}$ (D) $(2.71)\sqrt{2}$ (E) 0.271

38. The domain of the function $\cos^{-1}(\log_2(x^2+5x+8))$ is

(A) [2, 3] (B) [– 2, 2] (C) [3, 1] (D) (– 2, – 2) (E) [– 3, – 2]

39. $\lim_{x\to 0} \frac{(1+2x)^{10}-1}{x}$ is equal to

(A) 5 (B) 10 (C) 15 (D) 20 (E) 0

40. The range of the function $f(x) = \log_e(3x^2+4)$ is equal to

(A) $[\log_e 2, \infty)$ (B) $[\log_e 3, \infty)$ (C) $[2\log_e 3, \infty)$ (D) $[0, \infty)$ (E) $[2\log_e 2, \infty)$

41. $\lim_{x\to 2} \frac{x^{100}-2^{100}}{x^{77}-2^{77}}$ is equal to

(A) $\frac{100}{77}$ (B) $\frac{100}{77}(2^{22})$ (C) $\frac{100}{77}(2^{21})$ (D) $\frac{100}{77}(2^{23})$ (E) $\frac{100}{77}(2^{24})$

Space for rough work

42. $\lim_{k\to\infty}\left(\frac{1^3+2^3+3^3+\cdots+k^3}{k^4}\right)$ is equal to

(A) 0 (B) 2 (C) $\frac{1}{3}$ (D) ∞ (E) $\frac{1}{4}$

43. If $y=\sin^2\cot^{-1}\sqrt{\frac{1+x}{1-x}}$, then $\frac{dy}{dx}$ is equal to

(A) $2\sin 2x$ (B) $\sin 2x$ (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$ (E) $\cos 2x$

44. If $x=\sin^{-1}(3t-4t^3)$ and $y=\cos^{-1}(\sqrt{1-t^2})$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{3}$ (D) $\frac{2}{5}$ (E) $\frac{1}{5}$

45. If $y=(x+1)(x+2)(x+3)(x+4)(x+5)$, then the value of $\frac{dy}{dx}$ at $x=0$ is equal to

(A) 374 (B) 742 (C) 472 (D) 247 (E) 274

Space for rough work

46. If $y = \cot^{-1}\left(\tan\frac{x}{2}\right)$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{1}{2}$ (B) 0 (C) $\frac{x}{2}$ (D) $-\frac{1}{2}$ (E) $-\frac{x}{2}$

47. If $y = (\sin^{-1} x)^2$, then $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx}$ is equal to

(A) 0 (B) -1 (C) -2 (D) 1 (E) 2

48. If $x^y y^x = 16$, then $\frac{dy}{dx}$ at (2, 2) is

(A) 1 (B) 2 (C) -1 (D) -2 (E) 0

49. If $2y = \sin^{-1}(x+5y)$, then $\frac{dx}{dy}$ is equal to

(A) $\cos 2y - 5$ (B) $2\cos y + 5$ (C) $\cos 2y + 5$ (D) $2\cos 2y + 5$ (E) $2\cos 2y - 5$

50. The total revenue in rupees received from the sale of x units of a product is given by $R(x) = 13x^2 + 26x + 15$. Then the marginal revenue in rupees, when $x = 15$ is

(A) 116 (B) 126 (C) 136 (D) 416 (E) 146

Space for rough work

51. The function $f(x)=(x(x-2))^2$ is increasing in the set

(A) $(-\infty,0)\cup(2,\infty)$ (B) $(-\infty,1)$ (C) $(0,1)\cup(2,\infty)$

(D) $(1,2)$ (E) $(0,2)$

52. If a tangent to the curve $y=2+\sqrt{4x+1}$ has slope $\frac{2}{5}$ at a point, then the point is

(A) (0, 2) (B) $\left(\frac{3}{4},4\right)$ (C) (2, 5) (D) (7, 6) (E) (6, 7)

53. The equation of the line parallel to x-axis and tangent to the curve $y=\frac{1}{x^2+2x+5}$ is

(A) $y=\frac{1}{4}$ (B) $y=4$ (C) $y=\frac{1}{2}$ (D) $y=0$ (E) $y=2$

54. The equation of the tangent to the curve $x=\frac{t-1}{t+1}$, $y=\frac{t+1}{t-1}$ at $t=2$ is

(A) $x+9y-6=0$ (B) $9x-y-6=0$ (C) $9x+y+6=0$

(D) $x+9y+6=0$ (E) $9x+y-6=0$

55. The point on the hyperbola $3x^2-4y^2=72$ which is nearest to the line $3x+2y+1=0$ is

(A) $(-6,3)$ (B) $(6,3)$ (C) $(-6,-3)$ (D) $(6,-3)$ (E) $(\sqrt{24},0)$

Space for rough work

56. The value of x in the interval [4, 9] at which the function $f(x)=\sqrt{x}$ satisfies the mean value theorem is

(A) $\frac{13}{4}$ (B) $\frac{17}{4}$ (C) $\frac{21}{4}$ (D) $\frac{23}{4}$ (E) $\frac{25}{4}$

57. $\int \frac{dx}{(x+1)\sqrt{x}}$ is equal to

(A) $\tan^{-1}\sqrt{x}+C$ (B) $2\tan^{-1}x+C$ (C) $2\tan^{-1}\left(\sqrt{x}\right)+C$

(D) $\tan^{-1}\left(x^{\frac{3}{2}}\right)+C$ (E) $2\tan^{-1}\left(x^{\frac{3}{2}}\right)+C$

58. $\int \frac{\log x}{x^2}dx$ is equal to

(A) $\frac{\log x}{x}+\frac{1}{x^2}+C$ (B) $-\frac{\log x}{x}+\frac{2}{x}+C$ (C) $-\frac{\log x}{x}-\frac{1}{2x}+C$

(D) $x\log x+\frac{1}{x^2}+C$ (E) $-\frac{\log x}{x}-\frac{1}{x}+C$

59. If $\int \frac{f(x)}{\log\cos x}dx=-\log(\log\cos x)+C$, then $f(x)$ is equal to

(A) $\tan x$ (B) $-\sin x$ (C) $-\cos x$ (D) $-\tan x$ (E) $\sin x$

Space for rough work

60. $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}\,dx$ is equal to

(A) $x-\sin^{-1}x+C$

(B) $x-\sqrt{1-x^2}\sin^{-1}x+C$

(C) $x+\sin^{-1}x+C$

(D) $x+\sqrt{1-x^2}\sin^{-1}x+C$

(E) $x\sin^{-1}x+\sqrt{1-x^2}+C$

61. $\int\frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}\,dx$ is equal to

(A) $\frac{3}{2}x+\frac{35}{36}\log\left|9e^{2x}-4\right|+C$

(B) $\frac{3}{2}x-\frac{35}{36}\log\left|9e^{2x}-4\right|+C$

(C) $-\frac{3}{2}x+\frac{35}{36}\log\left|9e^{2x}-4\right|+C$

(D) $-\frac{5}{2}x+\frac{35}{36}\log\left|9e^{2x}-4\right|+C$

(E) $\frac{5}{2}x+\frac{35}{36}\log\left|9e^{2x}-4\right|+C$

62. $\int\sqrt{\frac{1-x}{1+x}}\,dx$ is equal to

(A) $\sin^{-1}x+\sqrt{1-x^2}+C$

(B) $\sin^{-1}x-2\sqrt{1-x^2}+C$

(C) $2\sin^{-1}x-\sqrt{1-x^2}+C$

(D) $\sin^{-1}x-\sqrt{1-x^2}+C$

(E) $-\cos^{-1}x-\sqrt{1-x^2}+C$

Space for rough work

63. $\int \frac{dx}{1+\tan x}$ is equal to

(A) $\frac{1}{2}+\frac{1}{2}\log|\cos x + \sin x| + C$

(B) $\frac{x}{2}+\frac{1}{2}\log|\cos x - \sin x| + C$

(C) $\frac{1}{2}+\frac{1}{2}\log|\cos x - \sin x| + C$

(D) $\frac{x}{2}+\frac{1}{2}\log|\cos x + \sin x| + C$

(E) $\frac{1}{2}+\frac{1}{2}\log|\cos x + \sin x| + C$

64. If $\int_a^0 \frac{x^2-1}{1-x}dx = -\frac{1}{2}$, then the value of a is equal to

(A) -1 (B) 1 (C) 2 (D) -2 (E) $-\frac{1}{2}$

65. The value of the integral $\int_0^1 x(1-x)^5\,dx$ is equal to

(A) $\frac{1}{6}$ (B) $\frac{1}{7}$ (C) $\frac{6}{7}$ (D) $\frac{5}{6}$ (E) $\frac{1}{42}$

Space for rough work

66. If $[x]$ denotes the greatest integer less than or equal to x, then the value of $\int_0^2 \left(|x-2| + [x] \right) dx$ is equal to

(A) 2 (B) 3 (C) 1 (D) 4 (E) $\frac{3}{2}$

67. $\int_0^1 x\, e^{-5x}\, dx$ is equal to

(A) $\frac{1}{25} - \frac{6e^{-5}}{25}$ (B) $\frac{1}{25} + \frac{6e^{-5}}{25}$ (C) $-\frac{1}{25} - \frac{6e^{-5}}{25}$

(D) $\frac{1}{25} - \frac{1}{5}e^{-5}$ (E) $\frac{1}{25} + \frac{1}{5}e^{-5}$

68. The area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$ is (in square units)

(A) 1 (B) 2 (C) 0 (D) 4 (E) 2π

69 The differential equation representing the family of curves $y^2 = 2c\,(x + \sqrt{c}\,)$, where c is a positive parameter, is of

(A) order 1, degree 2 (B) order 1, degree 3 (C) order 2, degree 3

(D) order 2, degree 2 (E) order 1, degree 1

Space for rough work

70. An integrating factor of the differential equation $(1+x^2)\frac{dy}{dx}+xy=x$ is

(A) $\frac{x}{1+x^2}$ (B) $\frac{1}{2}\log(1+x^2)$ (C) $\sqrt{1+x^2}$

(D) x (E) $\frac{1}{1+x^2}$

71. The solution of the differential equation $x\frac{dy}{dx}+y=\frac{1}{x^2}$ at $(1, 2)$ is

(A) $x^2y+1=3x$ (B) $x^2y+1=0$ (C) $xy+1=3x$

(D) $x^2(y+1)=3x$ (E) $x^2y=3x+1$

72. The general solution of the differential equation $\frac{dy}{dx}=e^y\left(e^x+e^{-x}+2x\right)$ is

(A) $e^{-y}=e^x-e^{-x}+x^2+C$ (B) $e^{-y}=e^{-x}-e^x-x^2+C$

(C) $e^{-y}=-e^{-x}-e^x-x^2+C$ (D) $e^y=e^{-x}+e^x+x^2+C$

(E) $e^y=e^{-x}+e^x+C$

Space for rough work

73. If the function $f:[1,\infty)\to[1,\infty)$ is defined by $f(x)=2^{x(x-1)}$, then $f^{-1}(x)$ is

(A) $\left(\frac{1}{2}\right)^{x(x-1)}$ (B) $\frac{1}{2}\left(1-\sqrt{1+4\log_2 x}\right)$ (C) $\frac{1}{2}\sqrt{1+4\log_2 x}$

(D) $\frac{1}{2}\left[1+\sqrt{1+4\log_2 x}\right]$ (E) not defined

74. If $n(A)=8$ and $n(A\cap B)=2$, then $n((A\cap B)'\cap A)$ is equal to

(A) 2 (B) 4 (C) 6 (D) 8 (E) 10

75. If $f(x)=\sin x+\cos x,\ x\in(-\infty,\infty)$ and $g(x)=x^2,\ x\in(-\infty,\infty)$, then $(fog)(x)$ is equal to

(A) 1 (B) 0 (C) $\sin^2(x)+\cos(x^2)$

(D) $\sin(x^2)+\cos^2(x)$ (E) $\sin(x^2)+\cos(x^2)$

Space for rough work

76. If the set A contains 5 elements, then the number of elements in the power set $P(A)$ is equal to

(A) 32 (B) 25 (C) 16 (D) 8 (E) 10

77. The domain of the function $f(x) = \dfrac{1}{\sqrt{9-x^2}}$ is

(A) $-3 \le x \le 3$ (B) $-3 < x < 3$ (C) $-9 \le x \le 9$

(D) $-9 < x < 9$ (E) $-\infty < x < \infty$

78. The period of the function $f(x) = |\sin 2x| + |\cos 8x|$ is

(A) 2π (B) π (C) $\dfrac{2\pi}{3}$ (D) $\dfrac{\pi}{2}$ (E) $\dfrac{\pi}{4}$

79. The value of $i - i^2 + i^3 - i^4 + \cdots - i^{100}$ is equal to

(A) i (B) $-i$ (C) $1-i$ (D) $1+i$ (E) 0

80. If the imaginary part of $\dfrac{2+i}{ai-1}$ is zero, where a is a real number, then the value of a is equal to

(A) $\dfrac{1}{2}$ (B) 2 (C) $-\dfrac{1}{2}$ (D) -2 (E) $\dfrac{3}{2}$

Space for rough work

81. The argument of the complex number $\left(\frac{i}{2}-\frac{2}{i}\right)$ is equal to

(A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$ (C) $\frac{\pi}{12}$ (D) $\frac{\pi}{2}$ (E) $\frac{3\pi}{2}$

82. Let $z_1=3+4i$ and $z_2=-1+2i$. Then $|z_1+z_2|^2-2\left(|z_1|^2+|z_2|^2\right)$ is equal to

(A) $|z_1-z_2|^2$ (B) $-|z_1-z_2|^2$ (C) $|z_1|^2+|z_2|^2$

(D) $|z_1|^2-|z_2|^2$ (E) $|z_1|^2+|z_2|^2-2|z_1||z_2|$

83. If z_1 and z_2 are two non-zero complex numbers such that $|z_1+z_2|=|z_1|+|z_2|$, then $\arg\left(\frac{z_1}{z_2}\right)$ is equal to

(A) 0 (B) $-\pi$ (C) $-\frac{\pi}{2}$ (D) $\frac{\pi}{2}$ (E) π

84. If the equation $x^2-(2+m)x+(m^2-4m+4)=0$ in x has equal roots, then the values of m are

(A) $\frac{2}{3}, 1$ (B) $\frac{2}{3}, 6$ (C) 0, 1 (D) 0, 2 (E) $\frac{2}{3}, 0$

Space for rough work

85. The number of integral values of b, for which the equation $x^2+bx-16=0$ has integral roots, is

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

86. If $(1+i)$ is a root of the equation $x^2-x+(1-i)=0$, then the other root is

(A) $1-i$ (B) i (C) $-i$ (D) $2i$ (E) $-2i$

87. If the roots of the quadratic equation $3x^2+2x+a^2-a=0$ in x are of opposite signs, then a lies in the interval

(A) $(-\infty,-2)$ (B) $(-\infty,0)$ (C) $(-1,0)$ (D) $(0,1)$ (E) $(1,3)$

88. The number of real roots of the equation $|x|^2-3|x|+2=0$ is

(A) 1 (B) 2 (C) 3 (D) 6 (E) 4

89. Let a, b, c be positive real numbers. If $\frac{x^2-bx}{ax-c}=\frac{m-1}{m+1}$ has two roots which are numerically equal but opposite in sign, then the value of m is

(A) c (B) $\frac{1}{c}$ (C) $\frac{a+b}{a-b}$ (D) 1 (E) $\frac{a-b}{a+b}$

Space for rough work

90. If the 9^{th} term of an A.P. is zero, then the ratio of 29^{th} term to 19^{th} term is

(A) 1 : 2 (B) 1 : 3 (C) 2 : 1 (D) 3 : 1 (E) 9 : 1

91. Let $s_1, s_2, \ldots, s_{101}$ be consecutive terms of an A.P. If $\frac{1}{s_1 s_2} + \frac{1}{s_2 s_3} + \cdots + \frac{1}{s_{100} s_{101}} = \frac{1}{6}$ and $s_1 + s_{101} = 50$, then $|s_1 - s_{101}|$ is equal to

(A) 10 (B) 20 (C) 30 (D) 40 (E) 50

92. If $a_1, a_2, a_3, \ldots, a_n$ are in A.P. and $a_1 = 0$, then the value of $\left(\frac{a_3}{a_2} + \frac{a_4}{a_3} + \cdots + \frac{a_n}{a_{n-1}}\right) - a_2\left(\frac{1}{a_2} + \frac{1}{a_3} + \cdots + \frac{1}{a_{n-2}}\right)$ is equal to

(A) $(n-2) + \frac{1}{(n-2)}$ (B) $\frac{1}{n-2}$ (C) $(n-2)$

(D) $n-1$ (E) $n+2$

93. The value of $1^2 - 2^2 + 3^2 - 4^2 + \cdots + 11^2$ is equal to

(A) 55 (B) 66 (C) 77 (D) 88 (E) 99

Space for rough work

94. Let S_n denote the sum of first n terms of an A.P. and $S_{2n} = 3S_n$. If $S_{3n} = k\,S_n$, then the value of k is equal to

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

95. The first four terms of an A.P. are $a,\ 9,\ 3a-b,\ 3a+b$. The 2011^{th} term of the A.P. is

(A) 2015 (B) 4025 (C) 5030 (D) 6035 (E) 8045

96. If $(n+2)! = 2550 \times n!$, then the value of n is equal to

(A) 48 (B) 49 (C) 50 (D) 51 (E) 52

97. If ${}^{n}C_{r-1} = 28$, ${}^{n}C_{r} = 56$ and ${}^{n}C_{r+1} = 70$, then the value of r is equal to

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

98. The number of integers greater than 6000 that can be formed with 3, 5, 6, 7 and 8, where no digit is repeated, is

(A) 120 (B) 192 (C) 216 (D) 72 (E) 202

Space for rough work

99. The sum of the coefficients in the expansion of $\left(x^2 - \frac{1}{3}\right)^{199} \times \left(x^3 + \frac{1}{2}\right)^{200}$ is

(A) $\frac{1}{3}$ (B) $-\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{2}$ (E) 0

100. If $(1+ax)^n = 1 + 6x + \frac{27}{2}x^2 + \cdots + a^n x^n$, then the values of a and n are respectively

(A) 2, 3 (B) 3, 2 (C) $\frac{3}{2}$, 4 (D) 1, 6 (E) $\frac{3}{2}$, 6

101. If $(1-x)^n = c_0 - c_1 x + c_2 x^2 - c_3 x^3 + \cdots + (-1)^n c_n x^n$, then

$\frac{c_0}{2} - \frac{c_1}{3} + \frac{c_2}{4} - \frac{c_3}{5} + \cdots + (-1)^n \frac{c_n}{n+2} =$

(A) $\frac{1}{n(n+1)}$ (B) $\frac{1}{(n+1)(n+2)}$ (C) $\frac{1}{(n+1)(n+3)}$

(D) $\frac{1}{n(n+3)}$ (E) $\frac{1}{(n+2)(n+3)}$

Space for rough work

102. If $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ a & 2 & b\end{bmatrix}$ is a matrix satisfying $AA^T = 9I_3$, then the values of a and b are respectively

(A) 1, 2 (B) −1, 2 (C) −1, −2 (D) 2, 1 (E) −2, −1

103. If $A=\begin{bmatrix}2 & 1\\ 0 & x\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}\frac{1}{2} & \frac{1}{6}\\ 0 & \frac{1}{x}\end{bmatrix}$, then the value of x is equal to

(A) −3 (B) 3 (C) −2 (D) 6 (E) −6

104. If $A=\begin{pmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{pmatrix}$, then A^{10} is equal to

(A) $\begin{pmatrix}\cos^{10}\alpha & \sin^{10}\alpha\\ -\sin^{10}\alpha & \cos^{10}\alpha\end{pmatrix}$

(B) $\begin{pmatrix}\cos^{10}\alpha & -\sin^{10}\alpha\\ \sin^{10}\alpha & \cos^{10}\alpha\end{pmatrix}$

(C) $\begin{pmatrix}\cos^{10}\alpha & \sin^{10}\alpha\\ -\sin^{10}\alpha & -\cos^{10}\alpha\end{pmatrix}$

(D) $\begin{pmatrix}\cos 10\alpha & \sin 10\alpha\\ -\sin 10\alpha & \cos 10\alpha\end{pmatrix}$

(E) $\begin{pmatrix}\cos 10\alpha & -\sin 10\alpha\\ -\sin 10\alpha & -\cos 10\alpha\end{pmatrix}$

Space for rough work

105. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$ and I is the unit matrix of order 3, then $A^2 + 2A^4 + 4A^6$ is equal to

(A) $7A^8$ (B) $7A^7$ (C) $8I$ (D) $6I$ (E) I

106. If $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$ and A^2 is the unit matrix, then the value of $x^3 + x - 2$ is equal to

(A) -8 (B) -2 (C) 0 (D) 1 (E) 8

107. If $(b+c)(y+z) - ax = b - c$, $(c+a)(z+x) - by = c - a$ and $(a+b)(x+y) - cz = a - b$, where $a+b+c \neq 0$, then x is equal to

(A) $\frac{c+b}{a+b+c}$ (B) $\frac{c-b}{a+b+c}$ (C) $\frac{a-b}{a+b+c}$ (D) $\frac{a+b}{a+b+c}$ (E) $\frac{b-a}{a+b+c}$

108. If $|2x-3| < |x+5|$, then x lies in the interval

(A) $(-3, 5)$ (B) $(5, 9)$ (C) $\left(\frac{-2}{3}, 8\right)$ (D) $\left(-8, \frac{2}{3}\right)$ (E) $\left(-5, \frac{2}{3}\right)$

Space for rough work

109. The solution set of $\frac{x+3}{x-2} \leq 2$ is

(A) $(-\infty, \infty)$ (B) $(-\infty, 2] \cup [7, \infty)$ (C) $(-\infty, 2) \cup [7, \infty)$

(D) $[7, \infty)$ (E) $(-\infty, 2)$

110. Let p : roses are red and q : The sun is a star. Then the verbal translation of $(\sim p) \vee q$ is

(A) Roses are not red and the sun is not a star

(B) It is not true that roses are red or the sun is not a star

(C) It is not true that roses are red and the sun is not a star

(D) Roses are not red or the sun is a star

(E) It is not true that roses are red and the sun is a star

111. The statement $p \rightarrow (\sim q)$ is equivalent to

(A) $q \rightarrow p$ (B) $\sim q \vee \sim p$ (C) $p \wedge \sim q$ (D) $\sim q \rightarrow p$ (E) $\sim p \vee q$

112. The negation of $(p \vee \sim q) \wedge q$ is

(A) $(\sim p \vee q) \wedge \sim q$ (B) $(p \wedge \sim q) \vee q$ (C) $(\sim p \wedge q) \vee \sim q$

(D) $(p \wedge \sim q) \vee \sim q$ (E) $(\sim p \vee \sim q) \wedge \sim q$

Space for rough work

113. The value of $\cos 20° + \cos 100° + \cos 140°$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\sqrt{3}$ (D) 0 (E) 1

114. If $\frac{-\pi}{2} < \theta < \frac{\pi}{2}$ and $\theta \neq \pm\frac{\pi}{4}$, then the value of $\cot\left(\frac{\pi}{4}+\theta\right)\cot\left(\frac{\pi}{4}-\theta\right)$ is

(A) 0 (B) −1 (C) 1 (D) − 2 (E) 2

115. If $\sin\theta = 3\sin(\theta+2\alpha)$, then the value of $\tan(\theta+\alpha)+2\tan\alpha$ is

(A) 3 (B) 2 (C) − 1 (D) 0 (E) 1

116. If $\alpha, \beta, \gamma \in [0, \pi]$ and if α, β, γ are in A.P. then $\frac{\sin\alpha - \sin\gamma}{\cos\gamma - \cos\alpha}$ is equal to

(A) $\sin\beta$ (B) $\cos\beta$ (C) $\cot\beta$ (D) $2\cos\beta$ (E) $\mathrm{cosec}\,\beta$

Space for rough work

117. If $2\sin^{-1}x - \cos^{-1}x = \frac{\pi}{2}$, then x is equal to

(A) $\frac{1}{\sqrt{2}}$ (B) $-\frac{1}{\sqrt{2}}$ (C) $\frac{-\sqrt{3}}{2}$ (D) $\frac{\sqrt{3}}{2}$ (E) $\frac{1}{2}$

118. The value of $\frac{1}{8}(3 - 4\cos 2\theta + \cos 4\theta)$ is

(A) $\cos 4\theta$ (B) $\sin 4\theta$ (C) $\sin^4\theta$ (D) $\cos^4\theta$ (E) $\sin^4(\theta/2)$

119. If $8\cos 2\theta + 8\sec 2\theta = 65$, $0 < \theta < \frac{\pi}{2}$, then the value of $4\cos 4\theta$ is equal to

(A) $\frac{-33}{8}$ (B) $\frac{-31}{8}$ (C) $\frac{-31}{32}$ (D) $\frac{-33}{32}$ (E) $\frac{-31}{4}$

120. The value of $\tan^{-1}(2) + \tan^{-1}(3)$ is equal to

(A) $\frac{3\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\tan^{-1}(6)$ (E) $\tan^{-1}(5)$

Space for rough work