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Book Title: Trigonometry

Chapter: Inverse Trigonometric Functions.

Topic:all topics.

Quastion & Answer

Q. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$ then (a) $x+y+z-xyz=0$ (b) $x+y+z-xyz=0$ (c) $xy+yz+zx+1=0$ (d) $xy+yz+zx-1=0$	Ans. $\because \tan^{-1}x+\tan^{-1}y+\tan^{-1}z=90°$ $\therefore$ let $\tan^{-1}x=\tan^{-1}y=\tan^{-1}z=30° \Rightarrow x=y=z=\frac{1}{\sqrt{3}}$ put values of x,y,z in options (d) will give the R.H.S.
Q. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{\pi}{2}$ then $x^2+y^2+z^2+2xyz=$ (a) 1 (b) -1 (c) 0 (d) None	Ans. $\because \sin^{-1}x+\sin^{-1}y+\sin^{-1}z=90°$ $\therefore$ Let $\sin^{-1}x=\sin^{-1}y=\sin^{-1}z=30°$ $\Rightarrow x=y=z=\sin 30°=\frac{1}{2}$ $\therefore x^2+y^2+z^2+2xyz=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+2.\frac{1}{2}.\frac{1}{2}.\frac{1}{2}=1$

Q.	If $\sin^{-1}a+\sin^{-1}b+\sin^{-1}c=\pi$ then $a\sqrt{(1-a^2)}+b\sqrt{(1-b^2)}+c\sqrt{(1-a^2)}=$ (a) 2 abc (b) abc (c) $\frac{1}{2}$abc (d) $\frac{1}{3}$abc	Ans.	$\because \sin^{-1}a+\sin^{-1}b+\sin^{-1}c=180°$ Let $\sin^{-1}a=\sin^{-1}b=\sin^{-1}c=60° \Rightarrow a=b=c=\frac{\sqrt{3}}{2}$ L.H.S. $=a\sqrt{1-a^2}+b\sqrt{1-b^2}+c\sqrt{1-c^2}=3\frac{\sqrt{3}}{2}\sqrt{1-\frac{3}{4}}=\frac{3\sqrt{3}}{4}$ put values of a, b, c in options, (a) will give R.H.S.
Q.	If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\pi$ and if $x^4+y^4+z^4+4x^2y^2z^2=k(x^2y^2+y^2z^2+z^2x^2)$ then k = (a) 1 (b) 2 (c) 4 (d) None	Ans.	$\because \sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\pi$ $\therefore$ Let $\sin^{-1}x=0^0 \Rightarrow x=0$, $\sin^{-1}y=90^0 \Rightarrow y=1$, $\sin^{-1}z=90^0 \Rightarrow z=1$ Put values of x, y, z in the given expression then we will get k = 2
Q.	If $\cos^{-1}\left(\frac{x}{2}\right)+\cos^{-1}\left(\frac{y}{3}\right)=\theta$ and if $9x^2-12xy\cos\theta+4y^2=k^2\sin^2\theta$ then k = (a) 3 (b) −3 (c) 6 (d) − 6	Ans.	Let x = 1 and y = 0, then $\theta=\cos^{-1}\left(\frac{1}{2}\right)+\cos^{-1}(0)=60°+90°=150°$ $9\times1-12\times0+4\times0=k^2\left(\frac{1}{2}\right)^2 \Rightarrow 9=\frac{k^2}{4} \Rightarrow k=\pm 6$ so (c) and (d) are correct.
Q.	If $\sin^{-1}\alpha+\sin^{-1}\beta+\sin^{-1}\gamma=\frac{3\pi}{2}$ then $1000(\alpha+\beta+\gamma)-\frac{300}{\alpha^2+\beta^2+\gamma^2}=$ (a) 0 (b) 2890 (c) 1900 (d) 2900	Ans.	$\because \sin^{-1}\alpha+\sin^{-1}\beta+\sin^{-1}\gamma=\frac{3\pi}{2}$ $\therefore$ Let $\sin^{-1}\alpha=\sin^{-1}\beta=\sin^{-1}\gamma=\frac{\pi}{2} \Rightarrow p=q=r=\sin\left(\frac{\pi}{2}\right)=1$ L.H.S. $=1000(\alpha+\beta+\gamma)-\frac{300}{\alpha^2+\beta^2+\gamma^2}$ $=1000(1+1+1)-\frac{300}{1+1+1}=3000-100=2900$
Q.	In ΔABC if $\angle A=90^0$ then $\tan^{-1}\left(\frac{c}{a+b}\right)+\tan^{-1}\left(\frac{b}{a+c}\right)=$ **[Kerela Engg. 2005]** (a) 0 (b) 1 (c) 45^0 (d) 90^0	Ans.	Consider MT - 3, $\because \angle A=90^0$ so $a=\sqrt{2}$, b = 1, c = 1 L.H.S. $=\tan^{-1}\left(\frac{1}{\sqrt{2}+1}\right)+\tan^{-1}\left(\frac{1}{\sqrt{2}+1}\right)$ $=\tan^{-1}(\sqrt{2}-1)+\tan^{-1}(\sqrt{2}-1)=22.5°+22.5°=45°$

Q. $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\left(\frac{2}{3}\right)\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\left(\frac{2}{3}\right)\right)=$ [WB JEE 2001] (a) 3 (b) -3 (c) 0 (d) 1	Ans. $\because \tan\left(\frac{\pi}{4}+\theta\right)+\tan\left(\frac{\pi}{4}-\theta\right)=2\sec 2\theta$ $\therefore \tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\left(\frac{2}{3}\right)\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\left(\frac{2}{3}\right)\right)$ $=2\sec 2\left[\frac{1}{2}\cos^{-1}\left(\frac{2}{3}\right)\right]=2\times\frac{3}{2}=3$

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all topics. | ChapterId60 | TopicId213

Quastion

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Book Title: Trigonometry
Chapter: Trigonometric functions,ratios and identities.
Topic:all

Quastion & Answer

Q.	If $\sec\theta+\tan\theta = x$ then $\cos\theta =$ (a) $\frac{(e^x+e^{-x})}{2}$ (b) $\frac{2}{(e^x+e^{-x})}$ (c) $\frac{(e^x-e^{-x})}{2}$ (d) $\frac{(e^x-e^{-x})}{(e^x+e^{-x})}$	Ans.	M.F.:→ If $\boxed{\sec\theta+\tan\theta = e^x \text{ then } \sec\theta-\tan\theta = e^{-x}}$ $2\sec\theta = e^x+e^{-x}$ $\Rightarrow \sec\theta = \frac{(e^x+e^{-x})}{2} \Rightarrow \cos\theta = \frac{2}{(e^x+e^{-x})}$
Q.	$\tan 20^\circ \tan 30^\circ \tan 40^\circ \tan 80^\circ =$ (a) 3 (b) $\sqrt{3}$ (c) 1 (d) None.	Ans.	M.F.:→ $\boxed{\tan\alpha \tan(60^0-\alpha)\tan(60^0+\alpha) = \tan 3\alpha}$ $\tan 20^\circ \tan 30^\circ \tan 40^\circ \tan 80^\circ$ $= \frac{\tan 20 \tan(60^\circ-20^\circ).\tan(60^\circ+20^\circ)}{\sqrt{3}}$ $= \frac{1}{\sqrt{3}}\tan 3(20^\circ) = \frac{1}{\sqrt{3}}\tan 60^\circ = 1$
Q.	$\frac{\cos 10^\circ+\sin 10^\circ}{\cos 10^\circ-\sin 10^\circ} =$ (a) $\tan 55^\circ$ (b) $\cot 55^\circ$ (c) $-\tan 35^\circ$ (d) $-\cot 35^\circ$	Ans.	M.M.F.:→ $\boxed{\frac{\cos x+\sin x}{\cos x-\sin x} = \tan(45^\circ+x)}$ $\Rightarrow \frac{\cos 10^\circ+\sin 10^\circ}{\cos 10^\circ-\sin 10^\circ} = \tan(45^\circ+10^\circ) = \tan(55^\circ)$
Q.	$\cos 20^\circ \cos 40^\circ \cos 80^\circ =$ (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{6}$ (d) $\frac{1}{8}$	Ans.	M.M.F.:→ $\boxed{\cos\theta\cos(60^\circ-\theta).\cos(60^\circ+\theta) = \frac{\cos 3\theta}{4}}$ $\cos 20^\circ \cos 40^\circ \cos 80^\circ$ $= \cos 20^\circ.\cos(60^\circ-20^\circ)\cos(60^\circ+20^\circ)$ $= \frac{\cos(3\times 20^\circ)}{4} = \frac{\cos 60^\circ}{4} = \frac{1}{8}$

Q.	$\sin 20^\circ \sin 30^\circ \sin 40^\circ \sin 80^\circ =$ (a) $\frac{\sqrt{3}}{16}$ (b) $\frac{3}{16}$ (c) $\frac{1}{16}$ (d) None.	Ans.	M.M.F.:→ $\boxed{\sin\alpha \sin(60-\alpha) \sin(60+\alpha) = \frac{1}{4}\sin 3\alpha}$ $\sin 20^\circ \sin 30^\circ \sin 40^\circ \sin 80^\circ$ $=\frac{1}{2}\sin 20^\circ \sin(60^\circ - 20^\circ)\sin(60^\circ + 20^\circ)$ $=\frac{1}{2}\times\frac{\sin(3\times 20^0)}{4}=\frac{1}{2}\times\frac{\sin(60^0)}{4}=\frac{\sqrt{3}}{16}$
Q.	If $\sqrt{x}+\frac{1}{\sqrt{x}}=2\cos\theta$, then $x^6+x^{-6}=$ (a) $2\cos 6\theta$ (b) $2\cos 12\theta$ (c) $2\cos 3\theta$ (d) $2\sin 3\theta$.	Ans.	M.M.F.:→ $\boxed{\text{If } \sqrt{x}+\frac{1}{\sqrt{x}}=2\cos\theta \text{ then } x^n+x^{-n}=2\cos(2n\theta)}$ $\Rightarrow x^6+x^{-6}=2\cos(2\times 6\times\theta)=2\cos 12\theta$
Q.	$\sqrt{2+\sqrt{2+2\cos 4\theta}}=$ (a) $\sin\theta$ (b) $\cos\theta$ (c) $2\sin\theta$ (d) $2\cos\theta$	Ans.	Put $\theta=0^o$ in L.H.S. because $\cos 0^o=1$ and it makes calculation very easy (other values of θ also can be taken) L.H.S. $=\sqrt{2+\sqrt{2+2\times 1}}=2$ put $\theta=0^o$ in options, (d) will match with L.H.S. so (d) is the right option.
Q.	If $\tan x=\frac{b}{a}$, then $\sqrt{\frac{a+b}{a-b}}+\sqrt{\frac{a-b}{a+b}}=$ (a) $\frac{2\sin x}{\sqrt{\sin 2x}}$ (b) $\frac{2\cos x}{\sqrt{\cos 2x}}$ (c) $\frac{2\cos x}{\sqrt{\sin 2x}}$ (d) $\frac{2\sin x}{\sqrt{\cos 2x}}$	Ans.	Let $b=0$, then $\tan x=0 \Rightarrow x=0^0$ LHS $=1+1=2$ put $x=0^o$ in the options, (b) will give R.H.S
Q.	$\frac{\sec 8A-1}{\sec 4A-1}=$ (a) $\frac{\tan 2A}{\tan 8A}$ (b) $\frac{\tan 8A}{\tan 2A}$ (c) $\frac{\cot 8A}{\cot 2A}$ (d) None.	Ans.	Let $A=15^o$ then LHS $=\frac{\sec(120^\circ)-1}{\sec(60^\circ)-1}=\frac{-2-1}{2-1}=-3\sqrt{3}$ put $A=15^o$ in the options, (b) will give result.

Q.	$\tan\alpha+2\tan2\alpha+4\tan4\alpha+8\cot8\alpha=$ (a) $\tan\alpha$ (b) $\tan2\alpha$ (c) $\cot\alpha$ (d) $\cot2\alpha$	Ans.	LHS = $\tan15^\circ+2\tan30^\circ+4\tan60^\circ+8\cot120^\circ$ $=2-\sqrt{3}+2\times\frac{1}{\sqrt{3}}+4\sqrt{3}+8\left(-\frac{1}{\sqrt{3}}\right)$ $=2+\sqrt{3}=\cot(15^\circ)=\cot\alpha$
Q.	$\cos^3\theta+\cos^3(120^\circ+\theta)+\cos^3(240^\circ+\theta)=$ (a) $\frac{3}{2}$ (b) $\frac{3}{4}$ (c) 1 (d) None	Ans.	Put $\theta=0^\circ$ then $\cos^3 0+\cos^3(120^\circ)+\cos^3(240)=1+\left(-\frac{1}{2}\right)^3+\left(-\frac{1}{2}\right)^3=\frac{3}{4}$
Q.	$3(\sin x-\cos x)^4+6(\sin x+\cos x)^2+4(\sin^6x+\cos^6x)=$ (a) 11 (b) 12 (c) 13 (d) 14	Ans.	Since the value of the above expression is independent of x therefore put $x=0^\circ$ Required value $=3\times1+6\times1+4\times1=13$
Q.	If $\tan\alpha=(1+2^{-x})^{-1}$ and $\tan\beta=(1+2^{1+x})^{-1}$ then $\alpha+\beta=$ (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$	Ans.	Since $\alpha+\beta$ is independent of x $\therefore$ put $x=0$ $\therefore$ $\tan\alpha=\frac{1}{2}$ & $\tan\beta=\frac{1}{3}$ $\Rightarrow\tan(\alpha+\beta)=1\Rightarrow\alpha+\beta=\frac{\pi}{4}$
Q.	If $c\cos^3\theta+3c\cos\theta\sin^2\theta=m$ and $c\sin^3\theta+3c\cos^2\theta\sin\theta=n$ then $(m+n)^{2/3}+(m-n)^{2/3}=$ (a) $2c^{2/3}$ (b) $2c^{3/2}$ (c) $2\sqrt{c}$ (d) None	Ans.	Since Value of expression is independent of θ, then put $\theta=0^\circ$ then $c=m$, $n=0$ $\therefore$ L.H.S. $=(c)^{2/3}+(c)^{2/3}=2c^{2/3}$
Q.	$\cos2(\theta+\phi)-4\cos(\theta+\phi)\sin\theta\sin\phi+2\sin^2\phi=$ (a) $\cos2\theta$ (b) $\cos3\theta$ (c) $\sin2\theta$ (d) $\sin3\theta$	Ans.	According to options it is clear that value of the expression is independent of ϕ therefore put $\phi=0^\circ\Rightarrow$ L.H.S. $=\cos2\theta$

Q.	$\tan^{-1}\left(\frac{c_2x-y}{c_1y+x}\right)+\tan^{-1}\left(\frac{c_2-c_1}{1+c_2c_1}\right)+\ldots\ldots+\tan^{-1}\left(\frac{1}{c_n}\right)=$ (a) $\tan^{-1}\left(\frac{y}{x}\right)$ (b) $\tan^{-1}(yx)$ (c) $\tan^{-1}\left(\frac{x}{y}\right)$ (d) $\tan^{-1}(x-y)$	Ans.	$\because$ The value of expression is independent of $c_0, c_1, c_2 \ldots\ldots\ldots c_n$ $\therefore$ Let $c_1=c_2=c_3=\ldots\ldots\ldots\ldots=c_n=0$ $\therefore$ L.H.S. $=\tan^{-1}\left(-\frac{y}{x}\right)+0+\ldots\ldots\ldots\ldots+\tan^{-1}(\infty)$ $=-\tan^{-1}\left(\frac{y}{x}\right)+\frac{\pi}{2}$ $=\frac{\pi}{2}-\tan^{-1}\left(\frac{y}{x}\right)=\cot^{-1}\left(\frac{y}{x}\right)=\tan^{-1}\left(\frac{x}{y}\right)$
Q.	If $\cos\alpha+\sec\alpha=2$ then $\cos^8\alpha+\sec^8\alpha=$ (a) 2 (b) 4 (c) 8 (d) None	Ans.	$\because \cos\alpha+\sec\alpha=2$ $\therefore$ Let $\alpha=0°$ $\therefore \cos^8\alpha+\sec^8\alpha=2$
Q.	If $2\sec 2\alpha=\tan\beta+\cot\beta$ then $\alpha+\beta=$ (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) π (d) 2π	Ans.	If $2\sec 2\alpha=\tan\beta+\cot\beta$ then $\alpha+\beta=$ (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) π (d) 2π
Q.	If $\frac{3\pi}{4}<\alpha<\pi$ then $\sqrt{\text{cosec}^2\alpha+2\cot\alpha}=$ (a) $1+\cot\alpha$ (b) $1-\cot\alpha$ (c) $-1-\cot\alpha$ (d) $-1+\cot\alpha$	Ans.	$\because \frac{3\pi}{4}<\alpha<\pi$, so let $\alpha=150°$, L.H.S $=\sqrt{\text{cosec}^2 150°+2\cot 150°}$ $=\sqrt{4-2\sqrt{3}}=\sqrt{(\sqrt{3}-1)^2}=\sqrt{3}-1$ put value of α in the options, (c) in right option.
Q.	If $\text{cosec}\theta=\frac{p+q}{p-q}$ then $\cot\left(\frac{\pi}{4}+\frac{\theta}{2}\right)=$ (a) $\sqrt{(p/q)}$ (b) $\sqrt{(q/p)}$ (c) $\sqrt{pq}$ (d) pq.	Ans.	If we put $\theta=30°$ $\therefore$ $2=\frac{p+q}{p-q}\Rightarrow p=3q$ L.H.S $=\cot(45°+15°)=\cot 60°=\frac{1}{\sqrt{3}}$ put the value of θ in options, (b) will give $\frac{1}{\sqrt{3}}$.

Q.	If $0 \le \alpha_1, \alpha_2, \alpha_3, \ldots\ldots\alpha_n \le \frac{\pi}{2}$ and if $\tan\alpha_1 \tan\alpha_2 \tan\alpha_3 \ldots\ldots\tan\alpha_n = 1$ then $\cos\alpha_1 \cos\alpha_2 \cos\alpha_3 \ldots\ldots\ldots\ldots\cos\alpha_n =$ (a) $2^{n/2}$ (b) $2^{-n/2}$ (c) $2^{n/4}$ (d) $2^{-n/4}$	Ans.	Let $\alpha_1 = \alpha_2 = \alpha_3 = \ldots\ldots = \alpha_n = 45^0$ (Satisfying given conditions) Required value $= \cos 45^0 \times \cos 45^0 \ldots\ldots$ n terms $= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \ldots$ n terms $= \frac{1}{(\sqrt{2})^n} = 2^{-n/2}$
Q.	If $t_1 = (\tan\theta)^{\tan\theta}, t_2 = (\tan\theta)^{\cot\theta}, t_3 = (\cot\theta)^{\tan\theta}$ and $t_4 = (\cot\theta)^{\cot\theta}$ and if $\theta \in \left(0, \frac{\pi}{4}\right)$ then (a) $t_1 > t_2 > t_3 > t_4$ (b) $t_4 > t_3 > t_1 > t_2$ (c) $t_2 > t_3 > t_1 > t_4$ (d) $t_3 > t_1 > t_2 > t_4$	Ans.	$\because \theta \in \left(0, \frac{\pi}{4}\right) \therefore$ let $\theta = \frac{\pi}{6} = 30^o$, $\tan 30^o = \frac{1}{\sqrt{3}} = 0.57$ & $\cot 30^o = \sqrt{3} = 1.732$ so $t_1 = (0.57)^{0.57}$, $t_3 = (1.732)^{0.57}$ $t_2 = (0.57)^{1.732}$, $t_4 = (1.732)^{1.732}$ clearly $t_4 > t_3 > t_1 > t_2$
Q.	If $a_1 = \left(\tan\frac{\pi}{8}\right)^{\tan\frac{\pi}{8}}$, $a_2 = \left(\tan\frac{\pi}{8}\right)^{\cot\frac{\pi}{8}}$, $a_3 = \left(\cot\frac{\pi}{8}\right)^{\cot\frac{\pi}{8}}$ and $a_4 = \left(\cot\frac{\pi}{8}\right)^{\tan\frac{\pi}{8}}$ then which of the following statement(s) is / are true: - (a) $a_3 > a_1 > a_{21} > a_4$ (b) $a_3 > a_4 > a_1 > a_2$ (c) $a_4 > a_3 > a_2 > a_1$ (d) $a_3 > a_4 > a_2 > a_1$	Ans.	$0 < \frac{\pi}{8} < \frac{\pi}{4} \Rightarrow \cot\frac{\pi}{8} > 1 > \tan\frac{\pi}{8} \Rightarrow a_3 > a_4$, Again, $\cot\frac{\pi}{8} > 1 > \tan\frac{\pi}{8} > 0 \Rightarrow a_1 < a_3$ so $a_2 < a_1 < a_4 < a_3$.
Q.	If $\alpha \in \left(0, \frac{\pi}{2}\right)$ then $\sqrt{x^2+x} + \frac{\tan^2\alpha}{\sqrt{x^2+x}}$ is always greater than or equal to [IIT - JEE 2004] (a) $2\tan\alpha$ (b) 1 (c) 2 (d) $\sec^2\alpha$	Ans.	Let $a = \sqrt{x^2+x}$ and $b = \frac{\tan^2\alpha}{\sqrt{x^2+x}}$ we know that $A \ge G$ so $\sqrt{x^2+x} + \frac{\tan^2\alpha}{\sqrt{x^2+x}} \ge 2\tan\alpha$

Q.	If $(1+\tan 1^\circ)(1+\tan 2^\circ)\ldots\ldots\ldots\ldots(1+\tan 45^\circ)=2^n$ then the value of n is (a) 22 (b) 23 (c) 24 (d) 21	Ans.	M.M.F. $\Rightarrow$ if $A+B=45^0$ then $(1+\tan A)(1+\tan B)=2$ $(1+\tan 1^\circ)(1+\tan 2^\circ)\ldots\ldots(1+\tan 44^\circ)(1+\tan 45^\circ)=2^n$ $(1+\tan 1^\circ)(1+\tan 44^\circ)(1+\tan 2^\circ)(1+\tan 43^\circ)\ldots 2=2^n$ $22\times 2=2^n \Rightarrow n=23$
Q.	The value of $\frac{\sin 5\theta}{\sin\theta}$ is :- (a) $16\cos^4\theta-12\cos^2\theta+1$ (b) $16\cos^4\theta+12\cos^2\theta+1$ (c) $16\cos^4\theta-12\cos^2\theta-1$ (d) $16\cos^4\theta+12\cos^2\theta-1$	Ans.	Put $\theta=60^0$ then L.H.S. $=\frac{\sin 5\theta}{\sin\theta}=\frac{\sin 300^0}{\sin 60^0}=\frac{-\sin 60^0}{\sin 60^0}=-1$ put $\theta=60^0$ in options then only (a) will give (-1).
Q.	If $\frac{\sin^4 A}{a}+\frac{\cos^4 A}{b}=\frac{1}{a+b}$ then $\frac{\sin^8 A}{a^3}+\frac{\cos^8 A}{b^3}=$ (a) $\frac{1}{(a+b)^2}$ (b) $\frac{1}{(a+b)^3}$ (c) $\frac{1}{(a+b)^4}$ (d) None	Ans.	Put $A=B=45^\circ$ then $\frac{1}{4}\left[\frac{a+b}{ab}\right]=\frac{1}{(a+b)}$ $\Rightarrow (a+b)^2-4ab=0 \Rightarrow (a-b)^2=0 \Rightarrow a=b$ L.H.S $=\frac{\left(\frac{1}{\sqrt{2}}\right)^8}{1}+\frac{\left(\frac{1}{\sqrt{2}}\right)^8}{1}=\frac{1}{8}$ Now putting a=b=1 in options only (b) is right.
Q.	If $\frac{\cos^4 x}{3}+\frac{\sin^4 x}{2}=\frac{1}{5}$ then (IITJEE 2009) (a) $\tan^2 x=\frac{2}{3}$ (b) $\frac{\cos^8 x}{27}+\frac{\sin^8 x}{8}=\frac{1}{125}$ (c) $\tan^2 x=\frac{1}{3}$ (d) $\frac{\cos^8 x}{27}+\frac{\sin^8 x}{8}=\frac{2}{125}$	Ans.	$\frac{(1-\sin^2 x)^2}{3}+\frac{\sin^4 x}{2}=\frac{1}{5}\Rightarrow 3\sin^4 x+2(1-\sin^2 x)^2=\frac{6}{5}$ $\Rightarrow 25\sin^4 x-20\sin^2 x+4=0\Rightarrow \sin^2 x=\frac{2}{5}$ & $\cos^2 x=\frac{3}{5}$ $\therefore \tan^2 x=\frac{2}{3}$ and $\frac{\cos^8 x}{27}+\frac{\sin^8 x}{8}=\frac{1}{125}$
Q.	If $\tan 2\theta\tan\theta=1$, then the general value of θ is : [KCET 2003] (a) $\left(n+\frac{1}{2}\right)\frac{\pi}{3}$ (b) $\left(n+\frac{1}{2}\right)\pi$ (c) $\left(2n\pm\frac{1}{2}\right)\frac{\pi}{3}$ (d) none	Ans.	$\because \theta=30^\circ$ satisfy the given equation, so (a) is right.

Book Title: Trigonometry
Chapter: Trigonometric Equations
Topic:General solution.

Quastion & Answer

If $\cos 2\theta = (\sqrt{2}+1)\left(\cos\theta - \frac{1}{\sqrt{2}}\right)$, then G.S. is :

(a) $2n\pi + \frac{\pi}{4}$ (b) $2n\pi \pm \frac{\pi}{4}$ (c) $2n\pi - \frac{\pi}{4}$ (d) None

Q.

Ans.

$\because$ $\theta = 45^\circ$ satisfy the given equation and we know that $\cos(-\theta) = \cos\theta$

$\therefore$ (b) is the right option.

Q.

If $\left(\frac{\sin\theta}{\sin\phi}\right)^2 = \frac{\tan\theta}{\tan\phi} = 3$, then G.S. for θ & ϕ are :

(a) $\theta = n\pi \pm \frac{\pi}{3}, \phi = n\pi \pm \frac{\pi}{6}$ (b) $\theta = n\pi - \frac{\pi}{3}, \phi = n\pi - \frac{\pi}{6}$

(c) $\theta = n\pi \pm \frac{\pi}{2}, \phi = n\pi \pm \frac{\pi}{3}$ (d) None of these

Ans.

$\theta = \pm\frac{\pi}{3}$ & $\phi = \pm\frac{\pi}{6}$ satisfy given equations.

So option (a) is right.

Q.

If θ and ϕ are acute angle such that

$\sin\theta = \frac{1}{2}$ and $\cos\theta = \frac{1}{3}$, then $(\theta + \phi)$ lies in :

(a) $\left(\frac{\pi}{3}, \frac{\pi}{6}\right]$ (b) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ (c) $\left(\frac{2\pi}{3}, \frac{5\pi}{3}\right)$ (d) $\left(\frac{5\pi}{6}, \pi\right)$

Ans.

Since $\sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$

and $\cos\phi = \frac{1}{3} = 0.33 \Rightarrow \phi \in (60^\circ \quad 90^\circ)$

therefore $\frac{\pi}{2} < (\theta + \phi) < \frac{2\pi}{3}$ so (b) is correct.

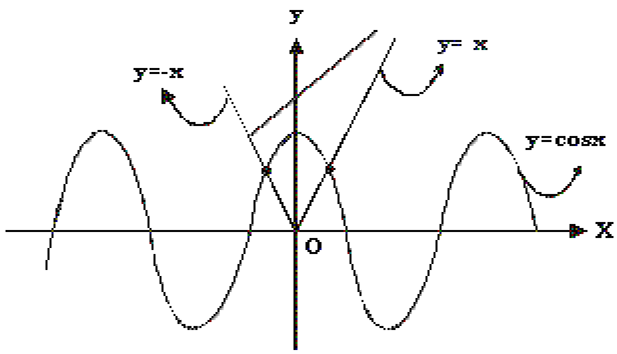
Q.

The number of real solution of $|x| = \cos x$ is

[IIT - (Screening) 2006]

(a)1 (b) 2 (c) 3 (d)0

Ans.



Concept : - the point of intersection of two curver gives their solution no. of solution is 2.

Q.

If $\tan 2\theta \tan\theta = 1$, then the general value of θ is :

[KCET 2003]

(a) $\left(n+\frac{1}{2}\right)\frac{\pi}{3}$ (b) $\left(n+\frac{1}{2}\right)\pi$ (c) $\left(2n\pm\frac{1}{2}\right)\frac{\pi}{3}$ (d) none

Ans.

$\because$ $\theta = 30°$ satisfy the given equation, so (a) is right.

Q.

If θ and ϕ are acute angle and $\sin\theta = \frac{1}{2}$ & $\cos\theta = \frac{1}{3}$ then $(\theta + \phi)$ lies in

[IITJEE 2004]

(a) $\left(\frac{\pi}{3},\frac{\pi}{6}\right]$ (b) $\left(\frac{\pi}{2},\frac{2\pi}{3}\right)$ (c) $\left(\frac{2\pi}{3},\frac{5\pi}{3}\right)$ (d) $\left(\frac{5\pi}{6},\pi\right)$

Ans.

Since $\sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$

and $\cos\phi = \frac{1}{3} = 0.33 \Rightarrow \phi \in (60° \ \ 90°)$

$\frac{\pi}{2} < (\theta + \phi) < \frac{2\pi}{3}$ therefore option (b) is correct.

Q.

If $\sin\theta = \cos\phi$ then possible values of $\frac{1}{\pi}\left(\theta \pm \phi - \frac{\pi}{2}\right)$ are:

(IITJEE2006)

(a) 0, 1 (b) 0, 2 (c) 1, 3 (d) 2, 3

Ans.

$\sin\theta = \cos\phi \Rightarrow \cos\left(\frac{\pi}{2} - \theta\right) = \cos\phi \Rightarrow \frac{\pi}{2} - \theta = 2n\pi \pm \phi$

$\Rightarrow -2n\pi = \theta \pm \phi - \frac{\pi}{2} \Rightarrow \frac{1}{\pi}\left(\theta \pm \phi - \frac{\pi}{2}\right) = -2n \ \ (n \in Z)$

from given values possible values of n are 0 & 2

Q.

The number of integral values of k for which $5\cos x + 9\sin x = 2k + 1$ has a solutions is:

(a) 7 (b) 8 (c) 10 (d) 12

Ans.

Solution the equation $a\cos x + b\sin x = c$ is possible only when $-1 \le \frac{c}{\sqrt{a^2+b^2}} \le 1$

$\therefore -1 \le \frac{2k+1}{\sqrt{106}} \le 1 \Rightarrow \sqrt{106} \le 2k+1 \le \sqrt{106}$

$\Rightarrow -10 \le 2k+1 \le 10 \Rightarrow -11 \le 2k \le 9 \Rightarrow -5.5 \le k \le 4.5$

$\Rightarrow k = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4$

Q.

The number of integral value of k for which $7\cos x + 5\sin x = 2k + 1$ has a solution is

[IIT - JEE 2002]

(a) 4 (b) 8 (c) 10 (d) 12

Ans.

$7\cos x + 5\sin x$ lies in $\left[-\sqrt{74}, \sqrt{74}\right]$

$\Rightarrow -\sqrt{74} < 2k+1 < \sqrt{74} \Rightarrow k = 0, \pm 1, \pm 2, \pm 3, -4$

so number of solutions are 8.

Q.

If $\tan(\cot x) = \cot(\tan x)$, then $\sin 2x$ may have which of the followings value.

(a) $\frac{1}{\pi}$ (b) $\frac{2}{\pi}$ (c) $\frac{1}{3\pi}$ (d) $\frac{4}{3\pi}$

Ans.

$$\tan(\cot x) = \tan\left(\frac{\pi}{2} - \tan x\right)$$

$$\Rightarrow \cot x + \tan x = n\pi + \frac{\pi}{2} = (2n+1)\frac{\pi}{2}$$

$$\Rightarrow \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x} = (2n+1)\frac{\pi}{2} \Rightarrow \frac{2}{\sin 2x} = (2n+1)\frac{\pi}{2}$$

$$\Rightarrow \sin 2x = \frac{4}{(2n+1)\pi} \neq 0 \Rightarrow \sin 2x = \frac{4}{3\pi}(\text{for } n = 1)$$

Q.

The number of solutions of $\tan\left(x + \frac{\pi}{6}\right) = 2\tan x$ is / are :

(a) 0 (b) 1 (c) 2 (d) 3

Ans.

Let $k = \tan x$ $\tan\left(x + \frac{\pi}{6}\right) = 2\tan x \Rightarrow \dfrac{k + \frac{1}{\sqrt{3}}}{1 - \frac{k}{\sqrt{3}}} = 2k$

$\Rightarrow 2k^2 - \sqrt{3}k + 1 = 0$, $D < 0$ so no solution.

Q.

The values of θ, $\theta \in [0\ 2\pi]$ satisfying $\sin\theta\sqrt{8\cos^2\theta} = 1$ are in A.P. then common difference is :

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{8}$ (c) $\frac{3\pi}{8}$ (d) $\frac{5\pi}{8}$

Ans.

Q. The value of $\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x \sin x^2}{\sin x^2 + \sin(\ln 6 - x^2)} dx$ is

(a) $\frac{1}{4}\ln\left(\frac{3}{2}\right)$ (b) $\frac{1}{2}\ln\left(\frac{3}{2}\right)$ (c) $\ln\left(\frac{3}{2}\right)$ (d) $\frac{1}{6}\ln\left(\frac{3}{2}\right)$

ST. Let $x^2 = t \Rightarrow 2x dx = dt$

$$\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \left(\frac{x \sin x^2}{\sin x^2 + \sin(\ln 6 - x^2)}\right) dx = \frac{1}{2}\int_{\ln 2}^{\ln 3} \frac{\sin t\, dt}{\sin t + \sin(\ln 6 - t)} \left(\because \int_a^b \frac{f(x)}{f(x) + f(a + b - x)} = \frac{b - a}{2}\right)$$

$$= \frac{1}{2}\frac{(\ln 3 - \ln 2)}{2} = \frac{1}{4}\ln\left(\frac{3}{2}\right)$$

Q.

Trignometrical Equations

General Solution of T. E. :–

General Solution – A Set of values of θ which satisfy the given trigonometric equation is known as General Solution of that equations.

$$\text{Sin}\,\theta = \frac{1}{2} \Rightarrow \theta = 30^0, 150^0, \ldots\ldots$$

If is not that only Sin $30^0 = 1/2$, there are so many angles whose sine is $1/2$

$\therefore$ **General Solution given us a set of values of angles.**

$\because$ **Sin θ = Sin α**

$$\theta = n\pi + (-1)^n . \alpha$$

For different integer values of n we will got different values of θ.

Master Concept (Short Technique)

Any equation wheter it is algebric or Trigonometric. The solutions of the equations are their roots.

Ex.: $\tan\theta = \frac{1}{\sqrt{3}}$

Roots are $\theta \Rightarrow 30^0,$

The angle of the current option will satisfy the given equation. Therefore putting n = 0, 1, ... in the option and putting the angle thus obtainded in the equation the angle that satisfy the given equation will be the right answer.

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Book Title: Trigonometry
Chapter: Properties and Solutions of Triangles.
Topic:all topics.

Quastion & Answer

With usual notations, in a triangle ABC,

$$\frac{b^2 - c^2}{a \sec C} + \frac{c^2 - a^2}{b \sec C} + \frac{a^2 - b^2}{c \sec C} =$$

(a) 1 (b) 0 (c) abc (d) None of these

Q. Ans.

Let the triangle is an equilateral, therefore
$a = b = c = 1$ and $A = B = C = 60^\circ$
after putting the values of a, b, c,
and A, B, C, the L.H.S. = 0.

Q.

In ΔABC, if $A + C = 2B$ then $\dfrac{a + c}{\sqrt{a^2 - ac + c^2}} =$

(a) $2\cos\left(\frac{A - C}{2}\right)$ (b) $\sin\left(\frac{A + C}{2}\right)$

(c) $\sin\left(\frac{A}{2}\right)$ (d) None

Ans.

$\because$ The data of MT–1 , A = 60°, B = 60° and C = 60°
satisfy the given condition, therefore,
the triangle is an equilater also, let a = b = c = 1,

$$\text{L.H.S.} = \frac{a+c}{\sqrt{a^2 - ac + c^2}} = \frac{2}{1} = 2$$

put the values of A and C in the options,
(a) will give the R.H.S

Q.

In ΔABC if $\sin^3 A + \sin^3 B + \sin^3 C = 3\sin A \sin B \sin C$

then $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} =$

(a) 0
(b) $(a + b + c)$
(c) $(a + b + c)(ab + bc + ca)$
(d) None of these

Ans.

Let $A = B = C = 60^0$ so

$\sin^3 60^0 + \sin^3 60^0 + \sin^3 60^0 = 3\sin 60^0 . \sin 60^0 . \sin 60^0$

$3\sin^3 60^0 = 3\sin^3 60^0$ (Satisfy the given condition)

therefore, the triangle is an equilateral so

take $a = b = c = 1 \therefore \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$

Q.

In triangle ABC if $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$ and if a = 2 then area of triangle will be:

(a) 1 (b) 2 (c) $\sqrt{3}/2$ (d) $\sqrt{3}$

Ans.

The data of MT–1 satisfies the given condition.
so the triangle is An equilateral

$$\text{Area } \Delta = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times (2)^2 = \sqrt{3}$$

Q.

In a Δ ABC the sides a, b and c are in A.P. then $\left(\tan\frac{A}{2} + \tan\frac{C}{2}\right) : \cot\frac{B}{2}$ will be:

(a) 3 : 2 (b) 1 : 2 (c) 3 : 4 (d) None of these

Ans.

Let the triangle be an equilateral.
therefore, A = B = C = 60°

$$\left(\tan\frac{A}{2} + \tan\frac{C}{2}\right) : \cot\frac{B}{2} = (\tan 30^\circ + \tan 30^\circ) : \cot 30^\circ$$

$$= 2\tan 30^\circ : \cot 30^\circ = \frac{2}{\sqrt{3}} : \sqrt{3} = \frac{2}{3}$$

Q.

If the length of the sides of the triangle ABC satisfy,

$2(bc^2 + ca^2 + ab^2) = b^2c + c^2a + a^2b + 3abc$,

then triangle :

(a) Is a rightangled. (b) Is Isosceles

(c) Is an equilateral (d) None of these

Ans.

$\because$ Data of MT-1 :- a = b = c = 1, satisfy given condition

$\therefore$ the triangle is an equilateral.

Q.

In ΔABC $\dfrac{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}{4b^2c^2} =$

(a) $\sin^2 A$ (b) $\cos^2 A$ (c) $\tan^2 A$ (d) None of these

Ans.

Let the triangle be an equilateral.

According to MT–1, a = b = c = 1 and A = B = C = 60°

so L.H.S. $= \dfrac{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}{4b^2c^2} = \dfrac{3}{4}$

after putting the value of A in the options,

(a) will give R.H.S.

Q.

In triangle ABC , $\cos A + \cos B + \cos C$ =

(EAMCET 2000)

(a) $1 + \dfrac{r}{R}$ (b) $1 - \dfrac{r}{R}$ (c) $1 + \dfrac{R}{r}$ (d) $1 - \dfrac{R}{r}$

Ans.

Let the triangle is an equilateral. so, according to MT-1

$A = B = C = 60^\circ$ and $r = \frac{1}{2\sqrt{3}}$, $R = \frac{1}{\sqrt{3}}$

$\text{L.H.S.} = 3\cos 60^\circ = 3 \times \frac{1}{2} = \frac{3}{2}$

put values of r and R in options, then (a) will match with L.H.S.

Q.

In an equilateral triangle the in-radius (r) and the circumradius (R) are connected by:

(a) $r = \frac{4}{R}$ (b) $r = \frac{R}{2}$ (c) $r = \frac{R}{3}$ (d) None.

Ans.

According to data of MT –1, $r = \frac{1}{2\sqrt{3}}$ $R = \frac{1}{\sqrt{3}}$.

therefore, $r = \frac{R}{2}$ therefore option (b) is right.

Q.

In ΔABC , $\frac{1}{a}\cos^2\left(\frac{A}{2}\right) + \frac{1}{b}\cos^2\left(\frac{B}{2}\right) + \frac{1}{c}\cos^2\left(\frac{C}{2}\right) =$

(a) s (b) $\frac{s}{abc}$ (c) $\frac{s^2}{abc}$ (d) none

Ans.

Let the triangle is an equilateral triangle,
so $a = b = c = 1$ & $A = B = C = 60^0$

L.H.S. $= 3\cos^2 30^o = 3 \times \frac{3}{4} = \frac{9}{4}$

put values of a, b, c and s in the options,
then (c) will give the required result.

Q.

In any ΔABC, $a \cot A + b \cot B + c \cot C =$

(a) $r + R$ (b) $r - R$ (c) $2(r + R)$ (d) $2(r - R)$

Ans. Q.

In a ΔABC, If r&R are in- radius and circum-radius of the triangle then $2(r + R) =$

[AIEEE2005]

(a) $a + b$ (b) $b + c$ (c) $c + a$ (d) $a + b + c$

Ans.

Let the triangle be a right angled isoscales,
According to MT–3, $a = 1$, $b = 1$, $c = \sqrt{2}$

and $r = 1 - \frac{1}{\sqrt{2}}$, $R = \frac{1}{\sqrt{2}}$

L.H.S. $= 2\ (r + R) = 2\left(1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = 2$

put values of a, b & c in options,(a) will give R.H.S.

Q.

For a triangle ABC which of the following is true :–

(a) $a\cos A + b\cos B + c\cos C = R\sin A\sin B\sin C$

(b) $a\cos A + b\cos B + c\cos C = 2R\sin A\sin B\sin C$

(c) $a\cos A + b\cos B + c\cos C = 4R\sin A\sin B\sin C$

(d) $a\cos A + b\cos B + c\cos C = 8R\sin A\sin B\sin C$

Ans.

Let the triangle is an equilateral ,therefore

According to MT-1 : – $a = b = c = 1$, $A = B = C = 60^\circ$

and $R = \dfrac{1}{\sqrt{3}}$

put these values in the options, then (c) is satisfied

So, it is the right answer.

Q.

The sum of the radii of the incircle and circumcircle of a regular polygone is:

[**AIEEE 2003**]

(a) $a\cot\dfrac{\pi}{2n}$ (b) $a\cot\dfrac{\pi}{n}$ (c) $\dfrac{a}{2}\cot\dfrac{\pi}{2n}$ (d) none of these

Ans.

Let the polygone is an equilateral triangle then $n = 3$

therefore, According to MT–1, $r = \dfrac{1}{2\sqrt{3}}$ & $R = \dfrac{1}{\sqrt{3}}$

$$\therefore \quad \text{L.H.S.} = r + R = \frac{1}{2\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{1+2}{2\sqrt{3}} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$

put values of a & n in options, then (a) will give R.H.S.

Q.

The distance between circumcentre and incentre in a Δ ABC is :

(a) $\sqrt{R^2+2Rr}$ (b) $\sqrt{R^2+Rr}$ (c) $\sqrt{R^2-2Rr}$ (d) $\sqrt{2R^2+r}$

Ans.

Let triangle be an equilateral, then we know that in equilateral triangle circumcentre and incentre coincide, therefore, the distance between these two centres is zero.

After putting the data of MT–1, $R=\dfrac{1}{\sqrt{3}}$ & $r=\dfrac{1}{2\sqrt{3}}$

Option (c) will give zero so it is the right option.

Q.

In a ΔABC If $c^2=a^2+b^2$, then $4s(s-a)(s-b)(s-c)=$

(a) s^4 (b) b^2c^2 (c) c^2a^2 (d) a^2b^2

Ans.

$\because c^2=a^2+b^2$, therefore the triangle is a Right angled and c is the hypoteneous.

L.H.S. $=4s(s-a)(s-b)(s-c)$

$$=4\Delta^2=4\times\left(\frac{1}{2}\times a\times b\right)^2=4\times\frac{a^2b^2}{4}=a^2b^2$$

Therefore, option (d) is the right option.

Q.

In a ΔABC if $\sin A + \sin B + \sin C = 1 + \sqrt{2}$ and $\cos A + \cos B + \cos C = \sqrt{2}$, then the triangle is :

(a) Equilateral. (b) Isosceles.
(c) Right angled. (d) Right angle isosceles.

Ans.

$\because$ The data of MT–3, $A = 45°$, $B = 45°$ and $C = 90°$ satisfies the given condition, therefore, it is right angle isoscales

Q.

In any ΔABC, $(a-b)^2 \cos^2\left(\frac{C}{2}\right) + (a+b)^2 \sin^2\left(\frac{C}{2}\right) =$

(a) a^2 (b) b^2 (c) c^2 (d) None

Ans.

Let the triangle be right angled isoscales.
so, according to data of MT–3, $a = 1, b = 1, c = \sqrt{2}$ and $A = 45°$, $B = 45°$, $C = 90°$

$$\text{L.H.S.} = (a-b)^2 \cos^2\left(\frac{C}{2}\right) + (a+b)^2 \sin^2\left(\frac{C}{2}\right)$$

$$= 0 + 4 \times \frac{1}{2} = 2 = c^2 \quad (\text{since } c = \sqrt{2})$$

Q.

In ΔABC if Δ is the area of the triangle then $a^2 \sin 2C + c^2 \sin 2A =$

(a) Δ (b) 2Δ (c) 3Δ (d) 4Δ

Ans.

Let triangle be an equilateral so according to MT–1 ,

$a = b = c = 1,\ A = B = C = 60°, \Delta = \frac{\sqrt{3}}{4}(1)^2 = \frac{\sqrt{3}}{4} \Rightarrow \sqrt{3} = 4\Delta$

$LHS = a^2\sin 2C + c^2\sin 2A = 1\times\sin 120° + 1\times\sin 120°$

$= 2\sin 120° = 2\times\frac{\sqrt{3}}{2} = \sqrt{3} = 4\Delta.$

Q.

In ΔABC if $a = 5,\ b = 13,\ c=12$ then $\tan\left(\frac{B}{4}\right) =$

(a) 2 (b) $\sqrt{2}$ (c) $\sqrt{2}+1$ (d) $\sqrt{2}-1$

Ans.

$\because a = 5, b = 13, c=12$, so triangle is a right angled

and $\angle B = 90^0$ so $\tan\left(\frac{B}{4}\right) = \tan(22.5°) = \sqrt{2}-1.$

Book Title: Trigonometry
Chapter: Heights and Distances
Topic:Master figures.

Quastion & Answer

A triangular park is enclosed on two sides byfence and on the third side by a straight river bank. The two sides having fence are of same length x. the maximum area enclosed by the park is

[AIEEE 2006]

(a) $\sqrt{x^3/8}$ (b) $\frac{1}{2}x^2$ (c) πx^2 (d) $\frac{3}{2}x^2$

Q. Ans.

Concept : we know that if two sides of a triangle are given then area of this triangle will be maximum when the included angle between these sides will be 90^0, so maximum area of this triangular park $= \frac{1}{2} \times x \times x = \frac{x^2}{2}$

Q.

AB is a vertical pole with B at ground level and A at the top. A man finds that angle of elevation of point A from a certain point C on ground is 60^0 He moves away from the pole along line BC to a point D such that CD = 7m. From D angle of elevation of the point A is 45^0. Then height of the pole is

[AIEEE 2008]

(a) $\frac{7\sqrt{3}(\sqrt{3}+1)}{2}$ (b) $\frac{7\sqrt{3}(\sqrt{3}-1)}{2}$

(c) $\frac{7\sqrt{3}}{2(\sqrt{3}+1)}$ (d) $\frac{7\sqrt{3}}{2(\sqrt{3}-1)}$

Ans.

According to MF − 3, h = ?, $\alpha = 45^0$, $\beta = 60^0$ and x = 7

$$\therefore \ h = \frac{7}{1-\frac{1}{\sqrt{3}}} = \frac{7\sqrt{3}}{(\sqrt{3}-1)} = \frac{7\sqrt{3}(\sqrt{3}+1)}{2}$$

Q.

A man from the top of a 100 m high tower sees a car moving towards the tower above an angle of depression is 30° .After some time this angle becomes 60° . Then the distance travelled by the car during this time in meter is

[IITJEE 2001]

(a) $100\sqrt{3}$ (b) $\frac{200\sqrt{3}}{3}$ (c) $\frac{100\sqrt{3}}{3}$ (d) none

Ans.

According to MF − 3 , h = 100, $\alpha = 30^\circ$, $\beta = 60^\circ$ and $x = ?$

$$\therefore 100 = \frac{x}{\sqrt{3} - \frac{1}{\sqrt{3}}} = \frac{x\sqrt{3}}{2} \Rightarrow x = \frac{200}{\sqrt{3}} = \frac{200\sqrt{3}}{3}$$

Q.

The angle of elevation of a stationary cloud from a point 2500 m above a lake is 15° and the angle of depression of it reflection in the lake is 45°. The height of cloud above lake level in meter is:

(a) $2500\sqrt{3}$ (b) 2500 (c) $500\sqrt{3}$ (d) none

Ans.

According to MF-4: d=2500, $A = 15^\circ$, $B = 45^\circ$, $D = ?$

$$D = \frac{d\sin(A+B)}{\sin(B-A)} = \frac{2500\sin(15^\circ + 45^\circ)}{\sin(45^\circ - 15^\circ)}$$

$$= \frac{2500 \cdot \frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2500\sqrt{3} \text{ mts.}$$

Q.

An aeroplane flying at a height of 300 meters above the ground passes vertically above another plane at an instant when the angle of elevation of the two planes from the same point on the ground are 60^0 and 45^0 respectively. Then the height of the lower plane from the ground is

(a) $100\sqrt{3}$ m (b) $\frac{100}{\sqrt{3}}$ m

(c) 50 m (d) None

Ans.

According to MT – 1

$A + B = 60^0$, $A = 45^0$

$x + y = 300$, $x = ?$

$$y = x\left[\frac{\tan(A+B) - \tan A}{\tan A}\right]$$

$$300 - x = x\left[\frac{\sqrt{3} - 1}{1}\right]$$

$$300 - x = \sqrt{3}x - x$$

$$x = 100\sqrt{3}$$

Q.

A person observes the angle of elevation of a building as 30^0. The person proceeds towards the buiding with a speed of $25(\sqrt{3}-1)$ m / hour. After two hours, he observes that the angle of elevation becomes 45^0. Then height of the building is :

(a) $50(\sqrt{3}-1)$ m (b) $50(\sqrt{3}+1)$ m
(c) 50 m (d) 100 m

Ans.

According to MF - 2

$A = 30^0, \quad B = 45^0$

$\therefore$ distance = speed x time

$y = 25(\sqrt{3}-1) \cdot 2$

$= 50(\sqrt{3}-1)$

$$\therefore = \frac{y}{\cot A - \cot B}$$

$$= \frac{50(\sqrt{3}-1)}{\cot 30^0 - \cot 45^0}$$

$$= \frac{50(\sqrt{3}-1)}{\sqrt{3}-1}$$

= 50 mts.

Q.

At a point A, the angle of elevation of a tower is such that its tangent is $\frac{5}{12}$ on walking 240 meters nearer the tower the tangent of the angle of elevation is $\frac{3}{4}$. The height of the tower is :

(a) 225 meters (b) 200 meters

(c) 230 meters (d) None of these

Ans.

According to MF - 2

$\tan A = \frac{5}{12}, \tan B = \frac{3}{4}$

$y = 240\,m$

$\therefore y = h(\cot A - \cot B)$

$240 = h\left(\frac{12}{5} - \frac{4}{3}\right)$

$240 = h\left(\frac{16}{15}\right)$

$h = 225$

$\therefore$ (a) is correct option

Q.

The angles of elevation of the top of a tower at the top and the foot of a pole of height 10 meters are 30^0 and 60^0 respectively. The height of the tower is:

(a) 10 meters (b) 15 meters
(c) 20 meters (d) None of these

Ans.

According to MF - 2

$A = 30^0$, $B = 45^0$

$\therefore$ distance = speed x time

$y = 25(\sqrt{3} - 1) \cdot 2$

$= 50(\sqrt{3} - 1)$

$$\therefore h = \frac{y}{\cot A - \cot B}$$

$$= \frac{50(\sqrt{3} - 1)}{\cot 30^0 - \cot 45^0}$$

$$= \frac{50(\sqrt{3} - 1)}{\sqrt{3} - 1}$$

$= 50$ mts.

Q.

The angles of elevation of the top of a tower (A) from the top (B) and bottom(D) at a building of height a are 30° and 45° respectively. If the tower and the building stand at the same level then the height of the tower is

(a) $a\sqrt{3}$ **(b)** $\frac{a\sqrt{3}}{\sqrt{3}-1}$

(c) $\frac{a(3+\sqrt{3})}{2}$ **(d)** $a(\sqrt{3}-1)$

Ans.

According to MF - 3

$A = 30^0$

$B = 45^0, \ y = a, \ x = ?$

$$\therefore \ x = \frac{y\cot A}{\cot A - \cot B} = \frac{a.\cot 30^0}{\cot 30^0 - \cot 45^0}$$

$$= \frac{\sqrt{3}a}{\sqrt{3}-1} x \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{a(3+\sqrt{3}}{2}$$

Q.

The angular elevation of a tower CD at a point A due south of it is 60° and at a point B due west of A, the elevationis 30° if AB = 3 km, then the height o: the tower is :

(a) $2\sqrt{3}$ km **(b) $2\sqrt{6}$ km**

(c) $\frac{3\sqrt{3}}{2}$ km **(d) $\frac{3\sqrt{6}}{4}$ km**

Ans.

According to MF - 5

$h = ?$

$A = 60^0, B = 30^0$

$d = 3$ km.

$$\therefore h = \frac{d}{\sqrt{\cot^2 B - \cot^2 A}}$$

$$h = \frac{3}{\sqrt{\cot^2 30^0 - \cot^2 30^0}}$$

$$= \frac{3}{\sqrt{3 - \frac{1}{3}}}$$

$$= \frac{3\sqrt{3}}{2\sqrt{2}} . \frac{\sqrt{3}}{\sqrt{2}} = \frac{3\sqrt{6}}{4} \text{km}$$

(d) is right option.

Q. Ans.