ELEMENTARY PROBLEMS ON PERMUTATION & COMBINATION

NOTE : USE FUNDAMENTAL PRINCIPLE OF COUNTING & ENJOY DOING THE FOLLOWING.

Q.1 In how many ways can clean & clouded (overcast) days occur in a week assuming that an entire day is either clean or clouded. [Ans. $2^7 = 128$]

Q.2 Four visitors A, B, C & D arrive at a town which has 5 hotels. In how many ways can they disperse themselves among 5 hotels, if 4 hotels are used to accommodate them. [Ans. 5 . 4 . 3 . 2 = 120]

Q.3 If the letters of the word "VARUN" are written in all possible ways and then are arranged as in a dictionary, then the rank of the word VARUN is :
(A) 98 (B) 99 (C*) 100 (D) 101

Q.4 How many natural numbers are their from 1 to 1000 which have none of their digits repeated.
[Hint: S D = 9 ; DD = $9 \cdot 9 = 81$; TD = $9 \cdot 9 \cdot 8 = 648$] [Ans. 738]

Q.5 A man has 3 jackets, 10 shirts, and 5 pairs of slacks. If an outfit consists of a jacket, a shirt, and a pair of slacks, how many different outfits can the man make?
[Hint: $x_1 \cdot x_2 \cdot x_3 = 3 \cdot 10 \cdot 5 = 150$ outfits]

Q.6 There are 6 roads between A & B and 4 roads between B & C.

(i) In how many ways can one drive from A to C by way of B?

(ii) In how many ways can one drive from A to C and back to A, passing through B on both trips ?

(iii) In how many ways can one drive the circular trip described in (ii) without using the same road more than once. [Ans. **(i)** 24; **(ii)** 576; **(iii)** 360]

Q.7(i) How many car number plates can be made if each plate contains 2 different letters of English alphabet, followed by 3 different digits.

(ii) Solve the problem, if the first digit cannot be 0. (Do not simplify)
[Ans. **(i)** 26 . 25 . 10 . 9 . 8 = 468000 ; **(ii)** 26 . 25 . 9 . 9 . 8 = 421200]

Q.8(i) Find the number of four letter word that can be formed from the letters of the word HISTORY. (each letter to be used at most once) [Ans. 7 . 6 . 5 . 4 = 42 x 20 = 840]

(ii) How many of them contain only consonants? [Ans. 5 . 4 . 3 . 2 = 120]

(iii) How many of them begin & end in a consonant? [Ans. 5 . 4 . 4 . 4 = 400]

(iv) How many of them begin with a vowel? [Ans. 240]

(v) How many contain the letters Y? [Ans. 480]

(vi) How many begin with T & end in a vowel? [Ans. 40]

(vii) How many begin with T & also contain S? [Ans. 60]

(viii) How many contain both vowels? [Ans. 240]

Q.9 If repetitions are not permitted
(i) How many 3 digit numbers can be formed from the six digits 2, 3, 5, 6, 7 & 9? [Ans. 120]
(ii) How many of these are less than 400? [Ans. 40]
(iii) How many are even? [Ans. 40]
(iv) How many are odd ? [Ans. 80]
(v) How many are multiples of 5 ? [Ans. 20]

Q.10 In how many ways can 5 letters be mailed if there are 3 mailboxes available if each letter can be mailed in any mailbox. [243 ways]

Q.11 Every telephone number consists of 7 digits. How many telephone numbers are there which do not include any other digits but 2 , 3 , 5 & 7 ? [Ans. 4^7]

Q.12(a) In how many ways can four passengers be accommodate in three railway carriages, if each carriage can accommodate any number of passengers.
(b) In how many ways four persons can be accommodated in 3 different chairs if each person can occupy only one chair. [Ans. (a) 3^4 ; (b) 4.3.2 = 24]

Q.13 How many of the arrangements of the letter of the word "LOGARITHM" begin with a vowel and end with a consonant? [Hint : $3 \cdot 6 \cdot 7! = 9 \times 5040 = 90720$,][Ans. 90720]

Q.14 How many four digit numbers are there all whose digits are odd , if repetition of digits is allowed. [Ans. $\boxed{5}\boxed{5}\boxed{5}\boxed{5} = 5^4$]

Q.15 How many four digit numbers are there which are divisible by 2 .
[Ans. $\boxed{9}\boxed{10}\boxed{10}\boxed{5} = 4500$]

Q.16 In a telephone system four different letter P, R, S, T and the four digits 3, 5, 7, 8 are used. Find the maximum number of "telephone numbers" the system can have if each consists of a letter followed by a four-digit number in which the digit may be repeated.
[Hint: $4 \cdot 4^4 = 2^2 \cdot 2^8 = 2^{10} = 1024$] [Ans. 1024]

Q.17 Find the number of 5 lettered palindromes which can be formed using the letters from the English alphabets. [Ans. 26^3]
[Hint: A palindrome is a word or a phase that is the same whether you read it forward or backword. e.g. refer.]

Q.18 Number of ways in which 7 different colours in a rainbow can be arranged if green is always in the middle. [Ans 720]

Q.19 Two cards are drawn one at a time & without replacement from a pack of 52 cards. Determine the number of ways in which the two cards can be drawn in a definite order.
[Ans. 52 x 51 = 2652]

Q.20 It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible? [Ans. 2880]

Q.21 Numbers of words which can be formed using all the letters of the word "AKSHI", if each word begins with vowel or terminates in vowel . [Ans: $2 \cdot 24 + 2 \cdot 24 - 2 \cdot 6 = 84$]

Q.22 A letter lock consists of three rings each marked with 10 different letters. Find the number of ways in which it is possible to make an unsuccessful attempts to open the lock.
[Ans: $10^3 - 1 = 999$]

Q.23 How many 10 digit numbers can be made with odd digits so that no two consecutive digits are same. [Ans: $5 \cdot 4^9$]

Q.24 If no two books are alike, in how many ways can 2 red, 3 green, and 4 blue books be arranged on a shelf so that all the books of the same colour are together?

[Hint: $\boxed{R_1\ R_2}\ \boxed{G_1\ G_2\ G_3}\ \boxed{B_1\ B_2\ B_3\ B_4} = 3! \cdot 2! \cdot 3! \cdot 4! = 1728$] [Ans. 1728]

Q.25 How many natural numbers are there with the property that they can be expressed as the sum of the cubes of two natural numbers in two different ways.
[Ans. Infinitely many]

CLASS : XI (PQRS) **Special DPP on Permutation and Combination** **DPP. NO.-2**

Q.1 The number of arrangements which can be made using all the letters of the word LAUGH if the vowels are adjacent is :
(A) 10 (B) 24 (C*) 48 (D) 120
[Hint : $4 \times 2\,! \times 3\,! = 48$]

Q.2 The number of natural numbers from 1000 to 9999 (both inclusive) that do not have all 4 different digits is :
(A) 4048 (B*) 4464 (C) 4518 (D) 4536

$\boxed{9 \times 10^3 - 9 \cdot 9 \cdot 8 \cdot 7}$

Q.3 The number of different seven digit numbers that can be written using only three digits 1, 2 & 3 under the condition that the digit 2 occurs exactly twice in each number is :
(A*) 672 (B) 640 (C) 512 (D) none
[Hint: Two blocks for filling 2 can be selected in ${}^7C_2$ ways
and the digit 2
can be filled only in one way other 5 blocks can be filled in 2^5 ways.]
$\boxed{{}^7C_2 \cdot 2^5}$

Q.4 Out of seven consonants and four vowels, the number of words of six letters, formed by taking four consonants and two vowels is (Assume that each ordered group of letter is a word):
(A) 210 (B) 462 (C*) 151200 (D) 332640
[Hint : ${}^7C_4\ {}^4C_2\ .\ 6\,! = 151200$]

Q.5 All possible three digits even numbers which can be formed with the condition that if 5 is one of the digit, then 7 is the next digit is :
(A) 5 (B) 325 (C) 345 (D*) 365
[Hint : 5 + 8.9.5 = 365]

Q.6 For some natural N, the number of positive integral 'x' satisfying the equation ,

$$1\,! + 2\,! + 3\,! + \ldots\ldots + (x\,!) = (N)^2 \quad \text{is :}$$

(A) none (B) one (C*) two (D) infinite
[Hint: x = 1 & x = 3]

Q.7 The number of six digit numbers that can be formed from the digits 1, 2, 3, 4, 5, 6 & 7 so that digits do not repeat and the terminal digits are even is :
(A) 144 (B) 72 (C) 288 (D*) 720

[Hint: 1 . 2 . 3 . 4 . 5 . 6 . 7

T					T

${}^3C_2 \cdot 2! \cdot {}^5C_4 \cdot 4! = 6 \times 120 = 720$]

Q.8 A new flag is to be designed with six vertical strips using some or all of the colour yellow, green, blue and red. Then, the number of ways this can be done such that no two adjacent strips have the same colour is
(A*) 12×81 (B) 16×192 (C) 20×125 (D) 24×216
[Hint: 1st place can be filled in 4 ways
2nd place can be filled in 3 ways

3^{rd} place can be filled in 3 ways and |||ly 4^{th}, 5^{th} and 6^{th} each can be filled in 3 ways.
hence total ways = $4 \times 3^5 = 12 \times 81$]

Q.9 In how many ways can 5 colours be selected out of 8 different colours including red, blue, and green
(a) if blue and green are always to be included,
(b) if red is always excluded,
(c) if red and blue are always included but green excluded? [Ans: (a) 20, (b) 21, (c) 10]

Q.10 A 5 digit number divisible by 3 is to be formed using the numerals 0, 1, 2, 3, 4 & 5 without repetition. The total number of ways this can be done is :
(A) 3125 (B) 600 (C) 240 (D*) 216
[Hint: reject 0 + reject 3 $\Rightarrow$ $5! + 4 \cdot 4! = 120 + 96 = 216$]

Q.11 Number of 9 digits numbers divisible by nine using the digits from 0 to 9 if each digit is used atmost once is K . 8 ! , then K has the value equal to ______ . [Ans. 9 ! + 8 . 8 ! = 17 . 8 !]

Q.12 Number of natural numbers less than 1000 and divisible by 5 can be formed with the ten digits, each digit not occuring more than once in each number is ______ .
[Hint: single digit = 1 ; two digit = 9 + 8 = 17 ; three digit = 72 + 64 = 136 $\Rightarrow$ Total = 154]

CLASS : XI (PQRS)	Special DPP on Permutation and Combination	DPP. NO.-3

Q.1 Three men have 6 different trousers, 5 different shirts and 4 different caps . Number of different ways in which they can wear them is ______. [Ans. : ${}^6P_3 \cdot {}^5P_3 \cdot {}^4P_3$]

Q.2 The number of 9 digit numbers that can be formed by using the digits 1, 2, 3, 4 & 5 is :
(A) 9^5 (B) $9!$ (C*) 5^9 (D) ${}^9P_5$

Q.3 The number of arrangements of the letters 'a b c d' in which neither a, b nor c, d come together is:
(A) 6 (B) 12 (C) 16 (D*) none
[Hint : $4! - [3!\,2! + 3!\,2! - 2!\,2!\,2!] = 8$]

Q.4 Find the number of ways in which letters of the word VALEDICTORY be arranged so that the vowels may never be separated.
[Hint: $\boxed{\text{A E I O}}$ VLDCTRY or $8! \times 4! = 40320 \times 24 = 967680$ Ans]
['Valediction means farewell after graduation from a college. Valedictory : to take farewell]

Q.5 How many numbers between 400 and 1000 (both exclusive) can be made with the digits 2,3,4,5,6,0 if
(a) repetition of digits not allowed.
(b) repetition of digits is allowed.
[Hint: (a) $\boxed{4}\boxed{\ }\boxed{\ }$ 4/ 5/ 6 $3\times5\times4 = 60$
(b) $\boxed{\ }\boxed{\ }\boxed{\ }$ 4/ 5/ 6 $3\times6\times6 = 108 - 1 = 107$]

Q.6 If ${}^{20}P_r = 13\times {}^{20}P_{r-1}$, then the value of r is __________. [Ans : r = 8]

Q.7 The number of ways in which 5 different books can be distributed among 10 people if each person can get at most one book is :
(A) 252 (B) 10^5 (C) 5^{10} (D*) ${}^{10}C_5.5!$
[Hint: Select 5 boys in ${}^{10}C_5$ and distribute 5 books in 5! ways hence ${}^{10}C_5.\ 5!$]

Q.8 Mary typed a six-digit number, but the two 1's she typed didn't show. What appeared was 2006. Find the number of different six-digit numbers she would have typed. [Ans. ${}^6C_2 = 15$]
[Hint: × × × × × × $\Rightarrow$ ${}^6C_2 = 15$] **[13th test (29-10-2005)]**

Q.9 The 9 horizontal and 9 vertical lines on an 8 × 8 chessboard form 'r' rectangles and 's' squares. The ratio $\frac{s}{r}$ in its lowest terms is
(A) $\frac{1}{6}$ (B*) $\frac{17}{108}$ (C) $\frac{4}{27}$ (D) none
[Sol. no. of squares are

$$S = 1^2 + 2^2 + 3^2 + \ldots\ldots + 8^2 = \frac{8(9)(17)}{6} = 204$$

no. of rectangles r = ${}^9C_2 \cdot {}^9C_2 = 1296$

hence $\frac{s}{r} = \frac{204}{1296} = \frac{51}{324} = \frac{17}{108}$]

Q.10 There are 720 permutations of the digits 1, 2, 3, 4, 5, 6. Suppose these permutations are arranged from smallest to largest numerical values, beginning from 1 2 3 4 5 6 and ending with 6 5 4 3 2 1.

(a) What number falls on the 124th position?

(b) What is the position of the number 321546? [Ans. (a) 213564, (b) 267th]

[Sol. **(a)** digits 1, 2, 3, 4, 5, 6

no. of ways = 120 $\boxed{1}\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }$

no. of number = 2 $\boxed{2}\boxed{1}\boxed{3}\boxed{4}\boxed{\ }\boxed{\ }$

123rd $\boxed{2}\boxed{1}\boxed{3}\boxed{5}\boxed{4}\boxed{6}$

finally 124th is = 213564

(b) N = 321546

number of numbers beginning with 1 = 120 $\boxed{1}\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }$

number of numbers beginning with 2 = 120 $\boxed{2}\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }$

starting with 31 .... = 24 $\boxed{3}\boxed{1}\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }$

starting with 3214 .... = 2 $\boxed{3}\boxed{2}\boxed{1}\boxed{4}\boxed{\ }\boxed{\ }$

finally = 1 $\boxed{3}\boxed{2}\boxed{1}\boxed{5}\boxed{4}\boxed{6}$

hence N has 267th position]

Q.11 A student has to answer 10 out of 13 questions in an examination . The number of ways in which he can answer if he must answer atleast 3 of the first five questions is :

(A*) 276 (B) 267 (C) 80 (D) 1200

[Hint : ${}^{13}C_{10}$ – number of ways in which he can reject 3 questions from the first five

or ${}^{13}C_{10} - {}^5C_3 = 286 - 10 = 276$ or ${}^5C_3 \,.\, {}^8C_7 + {}^5C_4 \,.\, {}^8C_6 + {}^5C_5 \,.\, {}^8C_5 = 276$]

[Note that ${}^5C_3 \,.\, {}^{10}C_7$ is wrong (cases repeat]

Q.12 The number of three digit numbers having only two consecutive digits identical is :

(A) 153 (B*) 162 (C) 180 (D) 161

[Hint : $\boxed{x}\boxed{x}\boxed{\ }$ when two consecutive digits are 11, 22, etc = 9 . 9 = 81

$\boxed{\ }\boxed{0}\boxed{0}$ when two consecutive digits are 0 0 = 9

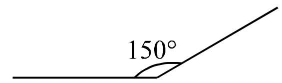
$\boxed{\ }\boxed{x}\boxed{x}$ when two consecutive digits are 11, 22, 33, ... = 9 . 8 = 72 Þ Total]

Q.1 A telegraph has x arms & each arm is capable of $(x-1)$ distinct positions, including the position of rest. The total number of signals that can be made is ______ . [Ans. $(x-1)^x - 1$]

Q.2 The interior angles of a regular polygon measure 150º each . The number of diagonals of the polygon is
(A) 35 (B) 44 (C*) 54 (D) 78
[YG '2000 Tier I]

[Hint : exterior angle = 30°

Hence number of sides = $\dfrac{360°}{30} = 12$



$\therefore$ number of diagonals = $\dfrac{12(12-3)}{2} = 54$]

[Note that sum of all exterior angles of a polygon = 2π and sum of all the interior angles of a polygon =$(2n\text{-}4)\dfrac{\pi}{2}$]

Q.3 Number of different natural numbers which are smaller than two hundred million & using only the digits 1 or 2 is :
(A*) $(3) . 2^8 - 2$ (B) $(3) . 2^8 - 1$ (C) $2 (2^9 - 1)$ (D) none
[Hint: Two hundred million = 2×10^8; $(2^1+ 2^2 +2^3+2^4+2^5+2^6+2^7+2^8) + 2^8 = 766$]

Q.4 5 Indian & 5 American couples meet at a party & shake hands . If no wife shakes hands with her own husband & no Indian wife shakes hands with a male, then the number of hand shakes that takes place in the party is :
(A) 95 (B) 110 (C*) 135 (D) 150
[Hint: ${}^{20}C_2 - (50 + 5) = 135$]

Q.5 The number of n digit numbers which consists of the digits 1 & 2 only if each digit is to be used atleast once, is equal to 510 then *n* is equal to:
(A) 7 (B) 8 (C*) 9 (D) 10
[Hint: (2 x 2 x ....2) n times-(when 1 or 2 is there at all the n places] [Ans. $2^n - 2$]

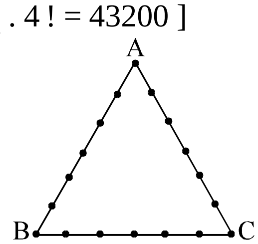
Q.6 Number of six digit numbers which have 3 digits even & 3 digits odd, if each digit is to be used atmost once is ______ . [Ans. 64800]
[Hint: alternatively, ${}^5C_3 \cdot {}^5C_3 \cdot 6!$ since all digits 0, 1, 2, ...8, 9 are equally likely at all places

$\Rightarrow$ required number = $\dfrac{{}^5C_3 \cdot {}^5C_3 \cdot 6!}{10} \cdot 9$ digits $\begin{cases} 0 \mid 2 \mid 4 \mid 6 \mid 8 \\ 1 \mid 3 \mid 5 \mid 7 \mid 9 \end{cases}$

or required number of ways = ${}^5C_3 . {}^5C_3 . 6! - {}^4C_2 . {}^5C_3 . 5 . 5!$]

Q.7 The tamer of wild animals has to bring one by one 5 lions & 4 tigers to the circus arena . The number of ways this can be done if no two tigers immediately follow each other is ______ .
[Ans. $5! \; {}^6C_4 . 4! = 43200$]

Q.8 18 points are indicated on the perimeter of a triangle ABC (see figure). How many triangles are there with vertices at these points?
(A) 331 (B) 408
(C) 710 (D*) 711



[Hint: ${}^{18}C_3 - 3\cdot {}^7C_3 = 816 - 105 = 711$] **[08-01-2005, 12th]**

Q.9 An English school and a Vernacular school are both under one superintendent . Suppose that the superintendentship, the four teachership of English and Vernacular school each, are vacant, if there be altogether 11 candidates for the appointments, 3 of whom apply exclusively for the superintendentship and 2 exclusively for the appointment in the English school, the number of ways in which the different appointments can be disposed of is :

(A) 4320 (B) 268 (C) 1080 (D*) 25920

[Sol: 11 → 3 for supdt. ; 8 Teachership → 2 E ; 6 anywhere $\Rightarrow {}^3C_1 \cdot {}^4C_2 \cdot 2! \cdot 6!$]

Q.10 A committee of 5 is to be chosen from a group of 9 people. Number of ways in which it can be formed if two particular persons either serve together or not at all and two other particular persons refuse to serve with each other, is

(A*) 41 (B) 36 (C) 47 (D) 76

[Sol. 9 → AB ; C, D ; 5 other

AB included $^7C_3 - {}^5C_1 = 30$ ($^7C_3$ denotes any 3 from CD and 5 other – no. of ways where CD nd 1 is taken from remaining)

AB excluded $^7C_5 - {}^5C_3 = 11$

41] **[18-12-2005, 12th + 13th]**

Q.11 A question paper on mathematics consists of twelve questions divided into three parts A, B and C, each containing four questions . In how many ways can an examinee answer five questions, selecting atleast one from each part .

(A*) 624 (B) 208 (C) 2304 (D) none

[Hint: $3\,({}^4C_2 \cdot {}^4C_2 \cdot {}^4C_1) + 3\,({}^4C_1 \cdot {}^4C_1 \cdot {}^4C_3) = 432 + 194 = 624$]

Alternative : $^{12}C_5 - 3$ [no of ways in which he does not select any question from any one section]

$^{12}C_5 - 3 \cdot {}^8C_5$]

Q.12 If *m* denotes the number of 5 digit numbers if each successive digits are in their descending order of magnitude and *n* is the corresponding figure. When the digits and in their ascending order of magnitude then $(m - n)$ has the value

(A) $^{10}C_4$ (B*) $^9C_5$ (C) $^{10}C_3$ (D) $^9C_3$

CLASS : XI (PQRS) **Special DPP on Permutation and Combination** **DPP. NO.-5**

Q.1 There are m points on a straight line AB & n points on the line AC none of them being the point A. Triangles are formed with these points as vertices, when
(i) A is excluded (ii) A is included. The ratio of number of triangles in the two cases is:
(A*) $\frac{m+n-2}{m+n}$ (B) $\frac{m+n-2}{m+n-1}$ (C) $\frac{m+n-2}{m+n+2}$ (D) $\frac{m\,(n-1)}{(m+1)\,(n+1)}$

[Hint: $\frac{m\,.\,{}^nC_2 + n\,.\,{}^mC_2}{m\,.\,{}^nC_2 + n\,.\,{}^mC_2 + mn}$]

Q.2 Number of ways in which 9 different prizes be given to 5 students if one particular boy receives 4 prizes and the rest of the students can get any numbers of prizes, is :
(A*) ${}^9C_4\,.\,2^{10}$ (B) ${}^9C_5\,.\,5^4$ (C) $4\,.\,4^5$ (D) none
[Hint : 4 prizes to be given to the particular boys can be selected in ${}^9C_4$ ways Remaining 5 prizes to the 4 students can be given 4^5 ways $\Rightarrow$ total ways ${}^9C_4\,.\,4^5 = {}^9C_4\,.\,2^{10}$]

Q.3 In a certain algebraical exercise book there are 4 examples on arithmetical progressions, 5 examples on permutation – combination and 6 examples on binomial theorem . Number of ways a teacher can select for his pupils atleast one but not more than 2 examples from each of these sets, is ______ .
[Hint: $({}^4C_1 + {}^4C_2)\,({}^5C_1 + {}^5C_2)\,({}^6C_1 + {}^6C_2)$] [Ans. : 3150]
[Alternatively : add one dummy exercies in each and compute ${}^5C_2 \cdot {}^6C_2 \cdot {}^7C_2$]

Q.4 The kindergarten teacher has 25 kids in her class . She takes 5 of them at a time, to zoological garden as often as she can, without taking the same 5 kids more than once . Find the number of visits, the teacher makes to the garden and also the number of of visits every kid makes. [Ans. ${}^{25}C_5$; ${}^{24}C_4$]

Q.5 There are n persons and m monkeys $(m > n)$. Number of ways in which each person may become the owner of one monkey is
(A) n^m (B) m^n (C*) ${}^mP_n$ (D) mn
[Hint: n monkeys out of m can be selected in ${}^mC_n$ and distributed in n! $\Rightarrow$ ${}^mC_n \cdot n! = {}^mP_n$]

Q.6 Seven different coins are to be divided amongst three persons . If no two of the persons receive the same number of coins but each receives atleast one coin & none is left over, then the number of ways in which the division may be made is :
(A) 420 (B*) 630 (C) 710 (D) none
[Hint: 1, 2, 4 groups] [Ans: $\frac{7\,!}{1!\;2!\;4!}$ x 3 !]

Q.7 Let there be 9 fixed points on the circumference of a circle . Each of these points is joined to every one of the remaining 8 points by a straight line and the points are so positioned on the circumference that atmost 2 straight lines meet in any interior point of the circle . The number of such interior intersection points is :
(A*) 126 (B) 351 (C) 756 (D) none of these
[Hint: Any interior intersection point corresponds to 4 of the fixed points , namely the 4 end points of the intersecting segments . Conversely, any 4 labled points determine a quadrilateral, the diagonals of which intersect once within the circle . Thus the answer is **A**]

Q.8 The number of 5 digit numbers such that the sum of their digits is even is :

(A) 50000 (B*) 45000 (C) 60000 (D) none

[Hint: $\frac{9\times10^4}{2}$]

Q.9 A forecast is to be made of the results of five cricket matches, each of which can be win, a draw or a loss for Indian team. Find

(i) the number of different possible forecasts

(ii) the number of forecasts containing 0, 1, 2, 3, 4 and 5 errors respectively

[Ans: $3^5 = 243$; 1, 10, 40, 80, 80, 32]

[Hint: $N(0) = 1$; $N(1) = 2.{}^5C_4$; $N(2)\ 2^2.\ {}^5C_3$; $N(3)\ 2^3.\ {}^5C_2$; $N(4) = 2^4.\ {}^5C_1$; $N(5) = 2^5$]

Q.10 The number of ways in which 8 non-identical apples can be distributed among 3 boys such that every boy should get atleast 1 apple & atmost 4 apples is $K \cdot {}^7P_3$ where K has the value equal to :

(A) 88 (B) 66 (C) 44 (D*) 22

[Hint: (4 3 1) ; (3 3 2) ; (4 2 2)]

Q.11 A women has 11 close friends. Find the number of ways in which she can invite 5 of them to dinner, if two particular of them are not on speaking terms & will not attend together.

[Ans. ${}^{11}C_5 - {}^9C_3$ or $3 \cdot {}^9C_4$] [Ans. : 378]

[□□□□: $11 \begin{cases} \text{A B} \\ \text{9 other} \end{cases} \xrightarrow[\text{selected}]{\text{5 to be}} \Rightarrow {}^9C_3 + {}^9C_4 + {}^9C_4$]

Q.12 A rack has 5 different pairs of shoes . The number of ways in which 4 shoes can be chosen from it so that there will be no complete pair is :

(A) 1920 (B) 200 (C) 110 (D*) 80

[Hint: ${}^5C_4 \cdot 2^4$ or $\frac{10\ .\ 8\ .\ 6\ .\ 4}{4\ !} = 80$]

CLASS : XI (PQRS) — ***Special DPP on Permutation and Combination*** — **DPP. NO.-6**

Q.1 Number of different ways in which 8 different books can be distributed among 3 students, if each student receives atleast 2 books is ______. [Ans. 2940]

[Hint: 8 books can be distributed in a group of (2, 2, 4) or (2, 3, 3). Number of groups are $\left(\frac{8!}{2!\,2!\,4!\,2!}+\frac{8!}{2!\,3!\,3!\,2!}\right)$ and can be distributed in 3 ! ways]

Q.2$_{203/1}$ In how many different ways a grandfather along with two of his grandsons and four grand daughters can be seated in a line for a photograph so that he is always in the middle and the two grandsons are never adjacent to each other. [Ans. 528]

[Sol. Total number of ways they can sit = 6! $\times\times\times \boxed{GF} \times\times\times$
no. of ways when the two grandsons are always adjacent = $4 \cdot 2! \cdot 4! = 192$
where 4 denotes the no. of adjacent positions
(2!) no. of ways in which two sons can be seated
and 4! no. of ways in which the daughter can be seated in the remaining places.
$\therefore$ required no. of ways = 720 – 192 = 528 Ans]

Q.3 There are 10 seats in a double decker bus, 6 in the lower deck and 4 on the upper deck. Ten passengers board the bus, of them 3 refuse to go to the upper deck and 2 insist on going up. The number of ways in which the passengers can be accommodated is _____. (Assume all seats to be duly numbered)
[Ans. ${}^4C_2$. 2! ${}^6C_3$. 3! 5! or 172800]

Q.4 Find the number of permutations of the word "AUROBIND" in which vowels appear in an alphabetical order. [Ans. ${}^8C_4 \cdot 4\,!$]

[Hint : A, I, O, U $\rightarrow$ treat them alike. Now find the arrangement of 8 letters in which 4 alike and 4 different = $\frac{8!}{4!}$]

Q.5 The greatest possible number of points of intersection of 9 different straight lines & 9 different circles in a plane is:
(A) 117 (B) 153 (C*) 270 (D) none

[Hint : ${}^9C_2 \cdot 1 + {}^9C_1 \cdot {}^9C_1 \cdot 2 + {}^9C_2 \cdot 2$ = Ans. 270]

Q.6 An old man while dialing a 7 digit telephone number remembers that the first four digits consists of one 1's, one 2's and two 3's. He also remembers that the fifth digit is either a 4 or 5 while has no memorising of the sixth digit, he remembers that the seventh digit is 9 minus the sixth digit. Maximum number of distinct trials he has to try to make sure that he dials the correct telephone number, is
(A) 360 (B*) 240 (C) 216 (D) none

[Hint: $\left(\frac{4!}{2!}\right)\left(\begin{matrix}2\text{ ways}\\ \text{for fifth place}\end{matrix}\right)\left(\begin{matrix}10\text{ ways}\\ 6^{th}\text{ place}\end{matrix}\right)\left(\begin{matrix}1\text{ way}\\ 7^{th}\text{ place}\end{matrix}\right)$ $\underbrace{\times\times\times\times}_{1\,2\,3\,3}\times\times\underbrace{\times}_{7^{th}}$
$x_7 = 9 - x_6$
x_6 can take 0 to 9
= 240]

Q.7 If as many more words as possible be formed out of the letters of the word "DOGMATIC" then the number of words in which the relative order of vowels and consonants remain unchanged is ______ .

[Hint : $\boxed{0}\boxed{\ }\boxed{\ }\boxed{\ }\boxed{A}\boxed{\ }\boxed{I}\boxed{\ }$ 3! × 5! – 1 = 719]

Q.8 Number of ways in which 7 people can occupy six seats, 3 seats on each side in a first class railway compartment if two specified persons are to be always included and occupy adjacent seats on the same side, is $(5\,!) \cdot k$ then k has the value equal to :
(A) 2 (B) 4 (C*) 8 (D) none

[Hint: including the two specified people, 4 others can be selected in ${}^5C_4$ ways . The two adjacent seats can be taken in 4 ways and the two specified people can be arranged in 2 ! ways, remaining 4 people can be arranged in 4 ! ways .

$\Rightarrow \quad 5C_4 . 4 . 2! \ 4! = 5! \ 8 = 8 . 5! \ \Rightarrow$ C]

Q.9 Number of ways in which 9 different toys be distributed among 4 children belonging to different age groups in such a way that distribution among the 3 elder children is even and the youngest one is to receive one toy more, is :

(A) $\frac{(5!)^2}{8}$ (B) $\frac{9!}{2}$ (C*) $\frac{9!}{3!(2!)^3}$ (D) none

[Hint : distribution 2, 2, 2 and 3 to the youngest . Now 3 toys for the youngest can be selected in ${}^9C_3$ ways, remaining 6 toys can be divided into three equal groups in

$\frac{6!}{(2!)^3 . 3!}$ way and can be distributed in 3 ! ways $\Rightarrow$ ${}^9C_3 . \frac{6!}{(2!)^3} = \frac{9!}{3!(2!)^3}$]

Q.10 In an election three districts are to be canvassed by 2, 3 & 5 men respectively . If 10 men volunteer, the number of ways they can be alloted to the different districts is :

(A*) $\frac{10!}{2!\ 3!\ 5!}$ (B) $\frac{10!}{2!\ 5!}$ (C) $\frac{10!}{(2!)^2\ 5!}$ (D) $\frac{10!}{(2!)^2\ 3!\ 5!}$

[Hint : number of groups of 2, 3, 5 = $\frac{10!}{2!\ 3!\ 5!}$ & can be deputed only in one way]

Q.11 Let P_n denotes the number of ways in which three people can be selected out of 'n' people sitting in a row, if no two of them are consecutive. If , $P_{n+1} - P_n = 15$ then the value of 'n' is :

(A) 7 (B*) 8 (C) 9 (D) 10

[Hint : $P_n = {}^{n-2}C_3$; $P_{n+1} = {}^{n-1}C_3$;

Hence ${}^{n-1}C_3 - {}^{n-2}C_3 = 15$

${}^{n-2}C_3 + {}^{n-2}C_2 - {}^{n-2}C_3 = 15$

or ${}^{n-2}C_2 = 15 \Rightarrow n = 8 \Rightarrow$ C]

Q.12 A cricket team consisting of eleven players is to be selected from two sets consisting of six and eight players respectively. In how many ways can the selection be made, on the supposition that the set set of six shall contribute not fewer than four players.

[Hint: ${}^6C_4 . {}^8C_7 + {}^6C_5 . {}^8C_6 + {}^6C_6 . {}^8C_5 = 344$]

Q.13 An organisation has 25 members, 4 of whom are doctors. In how many ways can a committee of 3 members be selected so as to included at least 1 doctor. [Ans. ${}^{25}C_3 - {}^{21}C_3 = 970$ ways]

Q.14 A has 3 maps and B has 9 maps. Determine the number of ways in which they can exchange their maps if each keeps his initial number of maps.

[Hint: ${}^{12}C_3 - 1 = 219$]

Q.15 Number of three digit number with atleast one 3 and at least one 2 is

(A) 58 (B) 56 (C) 54 (D*) 52

[Hint: When exactly one 2, exactly one 3 and
1 other non zero digit = $7 \times 3! = 42$
one 2, one 3 and one 0 = 4
two 2's and one 3 = 3
two 3's and one 4 = 3

Total = 52]

[12th & 13th (25-9-2005)]

CLASS : XI (PQRS) — ***Special DPP on Permutation and Combination*** — *DPP. NO.-7*

Q.1 Total number of ways in which 6 '+' & 4 '–' signs can be arranged in a line such that no 2 '–' signs occur together is _______ . [Ans. 35][IIT'88, 2]

Q.2 There are 10 red balls of different shades & 9 green balls of identical shades. Then the number of arranging them in a row so that no two green balls are together is
(A) $(10\,!)\,.\,{}^{11}P_9$ (B*) $(10\,!)\,.\,{}^{11}C_9$ (C) $10\,!$ (D) $10\,!\ 9\,!$

Q.3 Number of ways in which n distinct objects can be kept into two identical boxes so that no box remains empty, is ______ .

[Hint: Consider the boxes to be different for a moment. T_1 can be kept in either of the boxes in 2 ways, silmilarly for all other things $\Rightarrow$ Total ways = 2^n
but this includes when all the things are in B_1 or B_2 $\Rightarrow$ number of ways = $2^n - 2$
Since the boxes are identical $\Rightarrow$ $\dfrac{2^n - 2}{2} = 2^{n-1} - 1$]

Q.4 A shelf contains 20 different books of which 4 are in single volume and the others form sets of 8, 5 and 3 volumes respectively. Number of ways in which the books may be arranged on the shelf, if the volumes of each set are together and in their due order is
(A) $\dfrac{20!}{8!\,5!\,3!}$ (B) 7! (C*) 8! (D) 7 . 8!

[Hint: Volume of each set may be in due order in two ways, either from left to right or from right to left. Now we have
$D_1\ D_2\ D_3\ D_4\ ,\ \boxed{V_1\ V_2\ V_3\ ....\ V_8\ ,}\ \ \boxed{P_1\ P_2\ P_3\ P_4\ P_5\ ,}\ \ \boxed{E_1\ E_2\ E_3\ ,} = 7! \times 2!\ .\ 2!\ .\ 2! = 8!$]

Q.5 If all the letters of the word "QUEUE" are arranged in all possible manner as they are in a dictionary, then the rank of the word QUEUE is :
(A) 15^{th} (B) 16^{th} (C*) 17^{th} (D) 18^{th}

Q.6 There are 12 different marbles to be divided between two children in the ratio 1 : 2 . The number of ways it can be done is :
(A*) 990 (B) 495 (C) 600 (D) none

[Hint: $\dfrac{12!}{4!\ 8!} \times 2\,! =$ Ans. : 990]

Q.7 All the five digits number in which each successive digit exceeds its predecessor are arranged in the increasing order of their magnitude. The 97^{th} number in the list does not contain the digit
(A) 4 (B*) 5 (C) 7 (D) 8

[Sol. All the possible number are ${}^9C_5$ (none containing the digit 0) = 126
Total starting with 1 = ${}^8C_4$ = 70 $\boxed{1}\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }$
(using 2, 3, 4, 5, 6, 7, 8, 9)
Total starting with 23 = ${}^6C_3$ = 20 $\boxed{2}\boxed{3}\boxed{\ }\boxed{\ }\boxed{\ }$
(4, 5, 6, 7, 8, 9)
Total starting with 245 = ${}^4C_2$ = 6 $\boxed{2}\boxed{4}\boxed{5}\boxed{\ }\boxed{\ }$
(6, 7, 8, 9)
97^{th} number = $\boxed{2}\boxed{4}\boxed{6}\boxed{7}\boxed{8}$]

Q.8 The number of combination of 16 things , 8 of which are alike and the rest different, taken 8 at a time is ______ . [Ans. 256]

[Hint: A A A A A A A A $D_1\ D_2$ .... D_8 $\Rightarrow$ ${}^8C_0 + {}^8C_1 + {}^8C_2 + ... + {}^8C_8 = 256$]

Q.9 The number of different ways in which five 'dashes' and eight 'dots' can be arranged, using only seven of these 13 'dashes' & 'dots' is :
(A) 1287 (B) 119 (C*) 120 (D) 1235520

Q.10 In a certain college at the B.Sc. examination, 3 candidates obtained first class honours in each of the following subjects: Physics, Chemistry and Maths, no candidates obtaining honours in more than one subject; Number of ways in which 9 scholarships of different value be awarded to the 9 candidates if due regard is to be paid only to the places obtained by candidates in any one subject is __________.

[Ans: 1680]

[Hint : 3 candidates in P ; 3 in C and 3 in M . Now 9 scholarships can be divided into three groups in $\frac{9\,!}{(3\,!)^3\,.\,3\,!}$ ways and distributed to P, C, M in $\frac{9\,!}{(3\,!)^3}$ ways]

Q.11 There are n identical red balls & m identical green balls . The number of different linear arrangements consisting of "n red balls but not necessarily all the green balls" is ${}^{x}C_y$ then
(A) $x = m + n\,,\ y = m$ (B*) $x = m + n + 1\,,\ y = m$
(C) $x = m + n + 1\,,\ y = m + 1$ (D) $x = m + n\,,\ y = n$

[Hint : put one more red ball & find the arrangement of n + 1 red and m green balls = ${}^{m+n+1}C_m$]

Direction for Q.12 & Q.13
In how many ways the letters of the word "COMBINATORICS" can be arranged if

Q.12 All the vowels are always grouped together to form a contiguous block.

Q.13 All vowels and all consonants are alphabetically ordered. [Ans.**Q.12** $\frac{(9!)(5!)}{(2!)^3}$; **Q.13** $\frac{(13!)}{(8!)(5!)}$]

[Sol. COMBINATORICS
C's = 2 A, M, B, N, T, R, S
O's = 2 Total 13 letter
I's = 2 Vowel : O O I I A
$\boxed{\text{O O I I A}}$ C C M B N T R S

$\frac{9!}{2!}\cdot\frac{5!}{2!2!} = \frac{(9!)(5!)}{(2!)^3}$ **Ans.(12)**

8 places for consonant's can be selected in ${}^{13}C_8$ these 8 places
$\underbrace{\times\times\times\times\times\times\times\times\times\times\times\times\times}_{13\text{ places}}$
can be filled by consonants in 1 way and remaining 5 places by vowels in one way. hence total ways = ${}^{13}C_5$.]

Q.14 How many different arrangements are possible with the factor of the term $a^2b^4c^5$ written at full length.

[Ans. $\frac{11!}{2!\cdot 4!\cdot 5!} = 6930$]

Q.15$_{209/1}$ Find the number of 4 digit numbers starting with 1 and having exactly two identical digits.

[Ans. 432]

[Sol. **Case-I :** When the two identical digits are both unity as shown.

| 1 | x | y | 1 |

any one place out of 3 block for unity can be taken in 3 ways and the remaining two blocks can be filled in $9 \cdot 8$ ways.

Total ways in this case = $3 \cdot 9 \cdot 8 = 216$

Case-II : When the two identical digit are other than unity.

| 1 | x | x | y | ; | 1 | x | y | x | ; | 1 | y | x | x |

two x's can be taken in 9 ways and filled in three ways and y can be taken in 8 ways.

Total ways in this case = $9 \cdot 3 \cdot 8 = 216$

Total of both case = 432 Ans.]

CLASS : XI (PQRS) **Special DPP on Permutation and Combination** **DPP. NO.-8**

Q.1 Number of different words that can be formed using all the letters of the word "DEEPMALA" if two vowels are together and the other two are also together but separated from the first two is :
(A) 960 (B) 1200 (C) 2160 (D*) 1440

[Hint : | D | P | M | L | can be arranged in 4 ! ways & the two gaps out of 5 gaps can be selected in ${}^5C_2$ ways.
(A A or E E) or A E or A E can be placed in $\frac{4!}{2!2!} = 6$ ways .
$\Rightarrow \quad 4\,!\;.\;{}^5C_2\;.\;\frac{4!}{2!2!} = 1440$]

Q.2 The number of ways in which 10 boys can take positions about a round table if two particular boys must not be seated side by side is :
(A) 10 (9) ! (B) 9 (8) ! (C*) 7 (8) ! (D) none

Q.3 In a unique hockey series between India & Pakistan, they decide to play on till a team wins 5 matches . The number of ways in which the series can be won by India, if no match ends in a draw is :
(A*) 126 (B) 252 (C) 225 (D) none

[Hint: India wins exactly in 5 matches $\Rightarrow$ looses in none $\Rightarrow$ ${}^5C_0$ ways
India wins exactly in 6 matches $\Rightarrow$ wins the 6^{th} and looses anyone in the 1^{st} five
$\Rightarrow$ ${}^5C_1$ ways and so on . $\Rightarrow$ ${}^5C_0 + {}^5C_1 + {}^6C_2 + {}^7C_3 + {}^8C_4 = 126$
alternatively Total number of ways in which Series can be won by India or Pakistan = ${}^{10}C_5$ $\Rightarrow$
required number of ways = $\frac{{}^{10}C_5}{2} = 126$]

Q.4 Number of ways in which n things of which r alike & the rest different can be arranged in a circle distinguishing between clockwise and anticlockwise arrangement, is :
(A) $\frac{(n+r-1)!}{r!}$ (B) $\frac{(n-1)!}{r}$ (C) $\frac{(n-1)!}{(r-1)!}$ (D*) $\frac{(n-1)!}{r!}$

[Hint: $x \,.\, r! = (n-1)! \Rightarrow x = \frac{(n-1)!}{r!}$]

Q.5 The number of ways of arranging 2m white & 2n red counters in straight line so that each arrangement is symmetrical with respect to a central mark is ______ .
(assume that all counters are alike except for the colour) [Ans. $\frac{(m+n)!}{n!\;m!}$]

Q.6 A gentleman invites a party of m + n (m ≠ n) friends to a dinner & places m at one table T_1 and n at another table T_2 , the table being round . If not all people shall have the same neighbour in any two arrangement, then the number of ways in which he can arrange the guests, is
(A*) $\frac{(m+n)!}{4\;mn}$ (B) $\frac{1}{2}\;\frac{(m+n)!}{mn}$ (C) $2\;\frac{(m+n)!}{mn}$ (D) none

[Hint: $\frac{(m+n)!}{m!\;n!}\;\frac{(m-1)!}{2}.\frac{(n-1)!}{2} \Rightarrow$ (A)]

Q.7 Delegates from 9 countries includes countries A, B, C, D are to be seated in a row . The number of possible seating arrangements, when the delegates of the countries A and B are to be seated next to each other and the delegates of the countries C and D are not to be seated next to each other is :
(A) 10080 (B) 5040 (C) 3360 (D*) 60480

[Hint : | AB | E | F | G | H | I $\Rightarrow$ ${}^7C_2$ 2 ! 6 ! 2 ! = 60480]

Q.8 There are 12 guests at a dinner party . Supposing that the master and mistress of the house have fixed seats opposite one another, and that there are two specified guests who must always, be placed next to one another ; the number of ways in which the company can be placed, is:

(A*) 20 . 10 ! (B) 22 . 10 ! (C) 44 . 10 ! (D) none

[Hint: 6 places on either sides $\Rightarrow G_1G_2$ will have 5 places each on either side and can be seated in 2 ways $\Rightarrow 10 \times 2! \times 10!$ Ans]

Q.9 Let P_n denotes the number of ways of selecting 3 people out of 'n' sitting in a row, if no two of them are consecutive and Q_n is the corresponding figure when they are in a circle . If $P_n - Q_n = 6$, then 'n' is equal to :

(A) 8 (B) 9 (C*) 10 (D) 12

[Hint: $P_n = {}^{n-2}C_3$; $Q_n = {}^{n}C_3 - [\, n + n\,(n-4)\,]$ or $Q_n = \dfrac{{}^{n}C_1 \cdot {}^{n-4}C_2}{3}$

$P_n - Q_n = 6 \quad \Rightarrow \quad n = 10$]

Q.10$_{207/1}$ Define a 'good word' as a sequence of letters that consists only of the letters A, B and C and in which A never immidiately followed by B, B is never immediately followed by C, and C is never immediately followed by A. If the number of n-letter good words are 384, find the value of n. [Ans. n = 8]

[Sol. There are 3 choices for the first letter and two choices for each subsequent letters.
Hence using fundamental principle
number of good words = $3 \cdot 2^{n-1} = 384$; $2^{n-1} = 128 = 2^7$; n = 8 Ans.]

Q.11 Six married couple are sitting in a room. Find the number of ways in which 4 people can be selected so that

(a) they do not form a couple (b) they form exactly one couple
(c) they form at least one couple (d) they form atmost one couple

[Ans. 240, 240, 255, 480]

[Hint : ${}^{12}C_4 = {}^{6}C_2 + {}^{6}C_1 \cdot {}^{5}C_2 \cdot 2^4 + {}^{6}C_4 \cdot 2^4$]

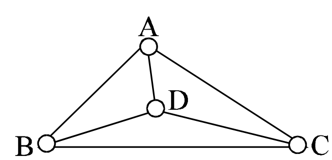
Q.12 Find the number of different permutations of the letters of the word " BOMBAY" taken four at a time. How would the result be affect if the name is changed to "MUMBAI". Also find the number of combinations of the letters taken 3 at a time in both the cases. [Ans: 192 ; no change ; 14]

Q.13 Fifty college teachers are surveyed as to their possession of colour TV, VCR and tape recorder. Of them, 22 own colour TV, 15 own VCR and 14 own tape recorders. Nine of these college teachers own exactly two items out of colour TV, VCR and tape recorders ; and, one college teacher owns all three. how many of the 50 college teachers own none of three, colour TV, VCR or tape recorder?

(A) 4 (B) 9 (C*) 10 (D) 11

[Hint: By drawing a Venn diagram , we see that the number of teachers with some possession
= [22 + 15 + 14 – 9 – 2(1)] = 40 .
$\therefore$ the number of people havig no possesion = 50 – 40 = 10]

Q.14 A road network as shown in the figure connect four cities. In how many ways can you start from any city (say A) and come back to it without travelling on the same road more than once ?



(A) 8 (B) 9 (C*) 12 (D) 16

[Hint: 3 . 2. 1 = 6 + 6 (anticlockwise) = 12 Ans]

Q.15 There are (p + q) different books on different topics in Mathematics. ($p \ne q$)
If L = The number of ways in which these books are distributed between two students X and Y such that X get p books and Y gets q books.

M = The number of ways in which these books are distributed between two students X and Y such that one of them gets p books and another gets q books.

N = The number of ways in which these books are divided into two groups of p books and q books then,

(A) L = M = N (B) L = 2M = 2N (C*) 2L = M = 2N (D) L = M = 2N

CLASS : XI (PQRS) **Special DPP on Permutation and Combination** **DPP. NO.-9**

Q.1 On a Railway route from Kota to Bina there are 12 stations.A booking clerk is to be deputed for each of these stations out of 12 candidates of whom five are Marathis, four are Oriyas and the rest are Bengalis. The number of ways of deputing the persons on these stations so that no two Bengali's serve on two consecutive stations, is ______________.
(Persons of the same religieon are not to be distinguished)

[Hint:
B B B
X | X | X | X | X | X | X | X | X |

Select 3 gaps = ${}^{10}C_3 = 120 \Rightarrow$ number of arrangements = $120 \times \dfrac{9!}{5!.4!} = 120 \times 126 = 15120$ Ans]

Q.2 Let m denote the number of ways in which 4 different books are distributed among 10 persons, each receiving none or one only and let n denote the number of ways of distribution if the books are all alike. Then :
(A) m = 4n (B) n = 4m (C*) m = 24n (D) none

[Hint : $m = {}^{10}C_4 . 4\,!$ and $n = {}^{10}C_4$]

Q.3 The number of ways in which we can arrange n ladies & n gentlemen at a round table so that 2 ladies or 2 gentlemen may not sit next to one another is :
(A) (n – 1) ! (n – 2) ! (B*) (n !) (n – 1) !
(C) (n + 1) ! (n) ! (D) none

[Hint: arrange them alternately on the circle]

Q.4 The number of ways in which 10 identical apples can be distributed among 6 children so that each child receives atleast one apple is :
(A*) 126 (B) 252 (C) 378 (D) none of these

Q.5 The number of all possible selections of one or more questions from 10 given questions, each equestion having an alternative is :
(A) 3^{10} (B) $2^{10} - 1$ (C*) $3^{10} - 1$ (D) 2^{10}

[Hint: 1^{st} question can be selected in three ways and so on]

Q.6 The number of ways in which 14 men be partitioned into 6 committees where two of the committees contain 3 men & the others contain 2 men each is :

(A) $\dfrac{14\,!}{(3!)^2\ (2!)^4}$ (B) $\dfrac{14\,!}{(3!)^2\ (2!)^5}$ (C) $\dfrac{14\,!}{4!\,(3!)^2 . (2!)^4}$ (D*) $\dfrac{14\,!}{(2!)^5 . (3!)^2 . 4!}$

Q.7 The number of divisors of the number 21600 is _____ and the sum of these divisors is ______.
[Ans. 72, 78120]

Q.8 10 IIT & 2 PET students sit in a row. The number of ways in which exactly 3 IIT students sit between 2 PET student is ______ . [Ans. 8 . 2 ! 10 ! = 16 . 10 ! or ${}^{10}C_3 . 3\,!\,2\,! . 8\,!$]

[Hint: 10 IIT students $T_1, T_2, ... T_{10}$ can be arranged in 10 ! ways . Now the number of ways in which two PET student can be placed will be equal to the number of ways in which 3 consecutive IIT students can be taken i.e. in 8 ways and can be arranged in two ways $\Rightarrow$ (10 !) (8 !) (2 !) .

Alternatively 3 IIT student can be selected in ${}^{10}C_3$ ways . Now each selection of 3 IIT and 2 PET students in $P_1\,T_1\,T_2\,T_3\,P_2$ can be arranged in (2 !) (3 !) ways . Call this box X. Now this X and the remaining & IIT students can be arranged in 8 ! ways
⇒ Total ways ${}^{10}C_3$ (!) (3 !) (8 !)]

Q.9 The number of ways of choosing a committee of 2 women & 3 men from 5 women & 6 men, if Mr. A refuses to serve on the committee if Mr. B is a member & Mr. B can only serve, if Miss C is the member of the committee, is :
(A) 60 (B) 84 (C*) 124 (D) none

[Hint : 5 W < Miss C, 4 W ; 6 M < Mr. A, Mr. B, 4 M ; Committee < 2 W, 3 M

(i) Miss C is taken
(a) B included Þ A excluded Þ ${}^4C_1 \cdot {}^4C_2 = 24$
(b) B excluded Þ ${}^4C_1 \cdot {}^5C_3 = 40$

(ii) Miss C is not taken
Þ B does not comes ; ${}^4C_2 \cdot {}^5C_3 = 60$ Þ Total = 124
Alt. Total - [A, B, C present + A,B present & C absent + B present & A, C absent]
<u>Alternatively :</u> **Case 1** : Mr. 'B' is present Þ 'A' is excluded & 'C' included
Hence number of ways = ${}^4C_2 \cdot {}^4C_1$ = 24
Case 2 : Mr. 'B' is absent Þ no constraint
Hence number of ways = ${}^5C_3 \cdot {}^5C_2$ = 100 **Total = 124**]

Q.10 Six persons A, B, C, D, E and F are to be seated at a circular table . The number of ways this can be done if A must have either B or C on his right and B must have either C or D on his right is :
(A) 36 (B) 12 (C) 24 (D*) 18
[Hint : when A has B or C to his right we have AB or AC
when B has C or D to his right we have BC or BD
Thus Þ we must have ABC or ABD or AC and BD
for $\boxed{\text{A B C}}$ D, E, F on a circle number of ways = 3 ! = 6
for $\boxed{\text{A B D}}$ C, E, F on a circle number of ways = 3 ! = 6
for $\boxed{\text{A C}}$, $\boxed{\text{B D}}$ E, F the number of ways = 3 ! = 6 Þ Total = 18]

Q.11 There are 2 identical white balls, 3 identical red balls and 4 green balls of different shades. The number of ways in which they can be arranged in a row so that atleast one ball is separated from the balls of the same colour, is :
(A*) 6 (7 ! – 4 !) (B) 7 (6 ! – 4 !) (C) 8 ! – 5 ! (D) none

[Hint : $\frac{9\,!}{2\,!\;3\,!}$ – number of ways when balls of the same colour are together

$= \frac{9\,!}{2\,!\;3\,!} - 3\,!\;4\,! = 6\,(7\,! - 4\,!)$]

Q.12 Sameer has to make a telephone call to his friend Harish, Unfortunately he does not remember the 7 digit phone number. But he remembers that the first three digits are 635 or 674, the number is odd and there is exactly one 9 in the number. The maximum number of trials that Sameer has to make to be successful is
(A) 10,000 (B*) 3402 (C) 3200 (D) 5000

[Sol. There are 2 ways of filling the first 3 digits, either 635 or 674. Of the remaining 4 digits, one has to be 9 and the last has to be odd. If the last digit is 9 then there are 9 ways of filling each of the remaining 3 digits. thus the total number of phone numbers that can be formed are $2 \times 9^3 = 1458$.

If the last digit is not 9, then there are only 4 ways of filling the last digit. (one of 1, 3, 5 and 7). The 9 could occur in any of the 3 remaining places and the remaining 2 places can be filled in 9^2 ways. Thus the total number of such numbers is : $2 \times 4 \times 3 \times 9^2 = 1944 \Rightarrow 1944 + 1458 = 3402$ Ans]

Q.13 Six people are going to sit in a row on a bench. A and B are adjacent. C does not want to sit adjacent to D. E and F can sit anywhere. Number of ways in which these six people can be seated, is
(A) 200 (B*) 144 (C) 120 (D) 56

[Hint: $\boxed{AB}$; C and D separated ; E and F any where **[11-12-2005, 11th (PQRS)]**
$\boxed{AB}$ and E, F can be seated in 3! 2!
no. of gaps are 4 | $\boxed{AB}$ | E | F |
C D can be seated in ${}^4C_2 \cdot 2!$
Total ways $3! \cdot 2! \cdot {}^4C_2 \cdot 2! = 144$ Ans.]

Q.14 Boxes numbered 1, 2, 3, 4 and 5 are kept in a row, and they are necessarily to be filled with either a red or a blue ball, such that no two adjacent boxes can be filled with blue balls. Then how many different arrangements are possible, given that the balls of a given colour are exactly identical in all respects?
(A) 8 (B) 10 (C*) 13 (D) 22

[Hint: Justify with a tree diagram or
alternatively :
0 B $\Rightarrow$ R R R R R (1) = 1
1 B $\Rightarrow$ | R | R | R | R | $({}^5C_1) = 5$
2 B $\Rightarrow$ | R | R | R | $({}^4C_2) = 6$
3 B $\Rightarrow$ | R | R | $({}^3C_3) = 1$

Total = 13]

Q.15 There are 6 boxes numbered 1, 2, ... 6. Each box is to be filled up either with a red or a green ball in such a way that at least 1 box contains a green ball and the boxes containing green balls are consecutive. The total number of ways in which this can be done, is
(A*) 21 (B) 33 (C) 60 (D) 6

[Hint □ □ □ □ □ □ **[11-12-2005, PQRS]**
B_1 B_2 B_3 B_4 B_5 B_6
all six green 1
5 green 2
4 green 3
3 green 4
2 green 5
1 green 6]

Q.16 Find the number of 10 digit numbers using the digits 0, 1, 2, ... 9 without repetition. How many of these are divisible by 4.

[Sol. Digit 0, 1, 2,........8, 9

For a number to be divisible by 4 the number formed by last two digits must be divisible by 4 and can be
04, 08, 12, ..., 96 ; Total of such numbers = 24
Out of these 44 and 88 are to be rejected. (as repetition is not allowed)
Hence accepted number of cases = 22
Out of these number of cases with '0' always include 04, 08, 20, 40, 60, 80 (six)
no. of such numbers with such one of these as last two digits $= 6 \cdot 8!$ ...(1)
e.g. [× × × × × × × × 04]
no. of other numbers $= 16 \cdot 7 \cdot 7! = 14 \cdot 8!$ ....(2)
e.g. [× × × × × × × × 16]
$\therefore$ Total number $= 6 \cdot 8! + 14 \cdot 8! = (20) \cdot 8!$ **Ans.**]

Q.1 The combinatorial coefficient C(n, r) can not be equal to the
(A) number of possible subsets of r members from a set of n distinct members.
(B) number of possible binary messages of length n with exactly r 1's.
(C*) number of non decreasing 2-D paths from the lattice point (0, 0) to (r, n).
(D) number of ways of selecting r things out of n different things when a particular thing is always included plus the number of ways of selecting 'r' things out of n, when a particular thing is always excluded.

[Hint: In C it should be (r, n + r) ; (D) ${}^{n}C_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$] **[18-12-2005, 12th & 13th]**

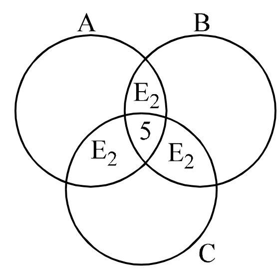
Q.2 Delegates of the five of the member countries of SAARC decide to hold a round table conference. There are 5 Indians, 4 Bangladeshis, 4 Pakistanis , 3 Sri Lankans and 3 Nepales. In how many ways can they be seated ? In how many ways can they be seated, if those of the same nationality sit together ?
[Hint: South Asian Association for Regional Cooperation] [Ans: 18! ; $(3!)^2\,(4!)^3\,(5!)$]

Q.3 Given 11 points, of which 5 lie on one circle, other than these 5, no 4 lie on one circle . Then the maximum number of circles that can be drawn so that each contains atleast three of the given points is :
(A) 216 (B*) 156 (C) 172 (D) none

[Hint : 11 5 (One circle)
6 (no. 4 lies on the same circle)
$\Rightarrow$ ${}^{5}C_2 \,.\, {}^{6}C_1 + {}^{6}C_2\ {}^{5}C_1 + {}^{6}C_3 + 1 = 156$ alternatively ${}^{11}C_3 - {}^{5}C_3 + 1$]

Q.4 One hundred management students who read at least one of the three business magazines are surveyed to study the readership pattern. It is found that 80 read Business India, 50 read Business world, and 30 read Business Today. Five students read all the three magazines. How many read exactly two magazines?
(A*) 50 (B) 10 (C) 95 (D) 25

[Hint: $n(A \cup B \cup C) = 100$
$n(A) = 80$; $n(B) = 50$; $n(C) = 30$



now $n(A \cup B \cup C) = \sum n(A) - \sum n(A \cap B) + n(A \cap B \cap C)$

$$100 = 80 + 50 + 30 - \sum n(A \cap B) + 5$$

$$\therefore \quad \sum n(A \cap B) = 65$$

now $n(E_2) = \sum n(A \cap B) - 3n(A \cap B \cap C) = 65 - 15 = 50$]

Q.5 The number of ways of arranging the letters AAAAA, BBB, CCC, D, EE & F in a row if the letter C are separated from one another is :
(A*) ${}^{13}C_3 \cdot \dfrac{12!}{5!\,3!\,2!}$ (B) $\dfrac{13!}{5!\,3!\,3!\,2!}$ (C) $\dfrac{14!}{3!\,3!\,2!}$ (D) none

Q.6 The maximum number of different permutations of 4 letters of the word "EARTHQUAKE" is :
(A) 2910 (B) 2550 (C*) 2190 (D) 2091

Q.7 How many ways are there to seat *n* married couples ($n \ge 3$) around a table such that men and women alternate and each women is not adjacent to her husband. [Ans. $n!(n-1)! - 2(n-1)!$]

[Hint: No. of ways of seating n couples in an alternating manner
i.e. one women one man and so on...... = $(n-1)!\, n!$
No. of ways couples can be seated together always is $(n-1)! \times 2$
$= n!\,(n-1)! - 2(n-1)!$ Ans.]

Q.8 Two classrooms A and B having capacity of 25 and (n–25) seats respectively. A_n denotes the number of possible seating arrangements of room 'A', when 'n' students are to be seated in these rooms, starting from room 'A' which is to be filled up full to its capacity.
If $A_n - A_{n-1} = 25!\,({}^{49}C_{25})$ then 'n' equals
(A*) 50 (B) 48 (C) 49 (D) 51

[Hint: ${}^{n}C_{25}.\, 25! - {}^{n-1}C_{25}\, .\, 25! = 25!\; {}^{49}C_{25} \Rightarrow {}^{n-1}C_{24} = {}^{49}C_{24} \Rightarrow n-1 = 49 \Rightarrow n = 50 \Rightarrow$ (A)]

Q.9 12 normal dice are thrown once. The number of ways in which each of the values 2, 3, 4, 5 and 6 occurs exactly twice is :
[1,1, 2,2, 3,3, 4,4, 5,5, 6,6 can come in any order]
(A) $\frac{(12)!}{6}$ (B) $\frac{(12)!}{2^6\,.\,6!}$ (C*) $\frac{(12)!}{2^6}$ (D) none

Q.10$_{186/1}$ 10 identical balls are to be distributed in 5 different boxes kept in a row and labled A, B, C, D and E. Find the number of ways in which the balls can be distributed in the boxes if no two adjacent boxes remain empty. [Ans. 771 ways]

[Sol. Case-1 : When no box remains empty
it is equivalently distributing
10 coins in 5 beggar

$$\underbrace{0\ 0\ 0\ 0\ 0}_{5}\ \underbrace{\varnothing\ \varnothing\ \varnothing\ \varnothing}_{4} = {}^{9}C_4 = 126$$

Case-2: Exactly one is empty

$${}^{5}C_1\ \underbrace{0\ 0\ 0\ 0\ 0\ 0}_{6}\ \varnothing\ \varnothing\ \varnothing = 5 \cdot {}^{9}C_3 = 420$$

Case-3: Exactly two remains empty

$$({}^{5}C_2 - \text{two adjacent})\ {}^{9}C_2 \quad \underbrace{0\ 0\ 0\ 0\ 0\ 0\ 0}_{7}\ \varnothing\ \varnothing$$

$(10-4) \times {}^{9}C_2$
$6 \times 36 = 216$

Case-4 : Exactly three empty. There is only 1 way to select 3 no. two adjacent

$$\text{Hence } 1 \cdot {}^{9}C_1 = 9 \qquad \underbrace{0\ 0\ 0\ 0\ 0\ 0\ 0\ 0}_{8}\ \varnothing$$

Total = 771 ways]

Q.11 The streets of a city are arranged like the lines of a chess board . There are m streets running North to South & 'n' streets running East to West . The number of ways in which a man can travel from NW to SE corner going the shortest possible distance is :
(A) $\sqrt{m^2+n^2}$ (B) $\sqrt{(m-1)^2\,.\,(n-1)^2}$ (C) $\frac{(m+n)!}{m!\,.\,n!}$ (D*) $\frac{(m+n-2)!}{(m-1)!\,.\,(n-1)!}$

[Hint: (m – 1) paths of one kind & (n – 1) paths of other kind, taken all at a time]

Q.12 The sum of all numbers greater than 1000 formed by using digits 1, 3, 5, 7 no digit being repeated in any number is :

(A) 72215 (B) 83911 (C*) 106656 (D) 114712

[Hint: $\frac{24}{2}[1357 + 7531]$]

Q.13 The number of times the digit 3 will be written when listing the integers from 1 to 1000 is :

(A*) 300 (B) 269 (C) 271 (D) 302

[Hint: $\boxed{}\boxed{}\boxed{}$ A three digit block from 000 to 999 mean 1000 numbers, each number constituting 3 digits. Hence, total digit which we have to write is 3000 . Since the total number of digits is 10 (0 to 9) no digit is filled preferentiely.

$\Rightarrow$ number of times we write 3 = $\frac{3000}{10}$ = 300.

Alternatively : any one block can be selected in $^3C_1$ ways and the digit 3 can be filled in it. Now the remaining two blocks can be filled in $9 \times 9 = 81$ ways (excluding the digit 3)

Total ways it can be done = $^3C_1 . 9 . 9 = 243$

Similarly any two blocks can be selected in $^3C_2$ ways and the digit 3 can be put in both of them. Remaining one block can be filled in 9 ways .

Total ways this can be done = $^3C_2 . 2 . 9$

When all the blocks are taken we have $^3C_3 . 3$

Thus the total = $(^3C_1 . 9^2)\, 1 + (^3C_2 . 9)\, 2 + {}^3C_3 . 3 = 300$]

Q.14 The number of ways in which the number 108900 can be resolved as a product of two factors is ______ .

[Ans. 41]

[Hint : $108900 = 2^2 . 3^2 . 5^2 . 11^2 = \frac{3 \times 3 \times 3 \times 3 + 1}{2} = 41$]

Q.15 The number of non negative integral solution of the inequation $x + y + z + w \le 7$ is ____ .

[Hint : find $x + y + z + w + t = 7 = {}^{11}C_4 = 330$] [Ans. 330]

Q.16 On the normal chess board as shown, I_1 & I_2 are two insects which starts moving towards each other . Each insect moving with the same constant speed . Insect I_1 can move only to the right or upward along the lines while the insect I_2 can move only to the left or downward along the lines of the chess board . Prove that the total number of ways the two insects can meet at same point during their trip is equal to

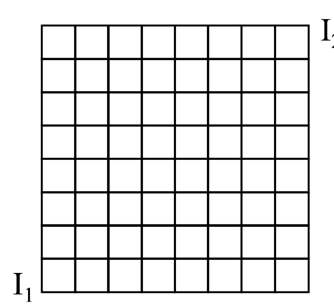
$$\binom{9}{8}\binom{10}{7}\binom{11}{6}\binom{12}{5}\binom{13}{4}\binom{14}{3}\binom{15}{2}\binom{16}{1}$$

OR

$$2^8\binom{1}{1}\binom{3}{2}\binom{5}{3}\binom{7}{4}\binom{9}{5}\binom{11}{6}\binom{13}{7}\binom{15}{8}$$

OR

$$\binom{2}{1}\binom{6}{2}\binom{10}{3}\binom{14}{4}\binom{18}{5}\binom{22}{6}\binom{26}{7}\binom{30}{8}$$



[Hint: $(^8C_0 . {}^8C_0) + (^8C_1 . {}^8C_1) + ... + (^8C_8 . {}^8C_8) = {}^{16}C_8) = 12870$]

Q.17 How many numbers gretater than 1000 can be formed from the digits 112340 taken 4 at a time.

[Ans: 159]

Q.18 Distinct 3 digit numbers are formed using only the digits 1, 2, 3 and 4 with each digit used at most once in each number thus formed. The sum of all possible numbers so formed is

(A*) 6660 (B) 3330 (C) 2220 (D) none

[Hint: all possible = 24

$6(1 + 2 + 3 + 4)(1 + 10 + 10^2) = 6 \cdot 10 \cdot 111 = 6660$

reject 1 or 2 or 3 or 4] **[12 & 13th test (09-10-2005)]**

Q.19 5 balls are to be placed in 3 boxes . Each box can hold all the 5 balls . Number of ways in which the balls can be placed so that no box remains empty, if :

	Column I	**Column II**	
(A)	balls are identical but boxes are different	(P)	2
(B)	balls are different but boxes are identical	(Q)	25
(C)	balls as well as boxes are identical	(R)	50
(D)	balls as well as boxes are identical but boxesare kept in a row	(S)	6

You may note that two or more entries of column I can match with only only entry of column II.

[Ans. A, D $\rightarrow$ S ; B $\rightarrow$ Q ; C $\rightarrow$ P]

Q.20 In maths paper there is a question on "Match the column" in which column A contains 6 entries & each entry of column A corresponds to exactly one of the 6 entries given in column B written randomly. 2 marks are awarded for each correct matching & 1 mark is deducted from each incorrect matching . A student having no subjective knowledge decides to match all the 6 entries randomly . The number of ways in which he can answer, to get atleast 25 % marks in this question is ______ .

[Hint . ${}^6C_6 + {}^6C_4 . 1 + {}^6C_3 . 2$ = atleast 3 correct = 56 ways]

Q.21 If $N = 2^{p-1} . (2^p - 1)$, where $2^p - 1$ is a prime, then the sum of the divisors of N expressed in terms of N is equal to ______ . [Ans. 2 N]

[Hint: Let $2^p - 1 = q$ (prime) $\Rightarrow N = 2^{p-1} \times q$

$\Rightarrow$ Sum $= (2^0 + 2^1 + 2^2 + ... + 2^{p-1})(q^0 + q^1)$

$$= \left(\frac{2^p - 1}{2 - 1}\right)(1 + 2^p - 1) = 2^p(2^p - 1) = 2N \;]$$

Q.22 171/1 Tom has 15 ping-pong balls each uniquely numbered from 1 to 15. He also has a red box, a blue box, and a green box.

(a) How many ways can Tom place the 15 distinct balls into the three boxes so that no box is empty?

(b) Suppose now that Tom has placed 5 ping-pong balls in each box. How many ways can he choose 5 balls from the three boxes so that he chooses at least one from each box?

[Sol.(a) $3^{15} - [{}^3C_2 + {}^3C_1(2^{15} - 2) = 3^{15} - 3 \cdot 2^{15} + 3$ **Ans.**

(b) The 5 balls can be chosen either as 1, 1, 3 (1 from a box, 1 from another box, 3 from remaining box) or as 1, 2, 2. There are 3 ways to select as 1, 1, 3 (take the 3 balls from red or 3 from blue or 3 from green). There are 3 ways to select as 1, 2, 2. Thus, recalling that the balls are uniquely numbered, the answer is

$3 \cdot {}^5C_1 \cdot {}^5C_2 \cdot {}^5C_2 + 3 \cdot {}^5C_1 \cdot {}^5C_1 \cdot {}^5C_3$ $B_1 \; B_2 .... B_{15}$

R	B	G
1	2	2
1	1	3

= 1500 + 750

= 2250]

Q.23 Number of ways in which 12 identical coins can be distributed in 6 different purses, if not more than 3 & not less than 1 coin goes in each purse is ______ .

[Hint : $\underbrace{000000}_{\text{remaining}}$ (i) 2 coins in each of 3 purses = ${}^6C_3$ (selecting 3 purses from 6 different purses = 20.

(ii) 2 coins in one + 1 coin in 4 purses = ${}^6C_1 \, . \, {}^5C_4$ = 20

(iii) 2 coins in each of two purses + 1 coin in each of two purses = ${}^6C_2 \, . \, {}^4C_2$ = 90

(iv) 1 coin in each of 6 purses = ${}^6C_6$ = 1 or co-efficient of x^{12} in $(x + x^2 + x^3)^6 = 141$

Alternatively : co-efficient of x^{12} in $(x + x^2 + x^3)^6 = 141$]

Q.24 A drawer is fitted with n compartments and each compartment contains n counter, no two of which marked alike. Number of combinations which can be made with these counters if no two out of the same compartment enter into any combination, is ______ . [Ans. : $(n + 1)^n - 1$]

Q.25 Sum of all the numbers that can be formed using all the digits 2, 3, 3, 4, 4, 4 is :

(A*) 22222200 (B) 11111100 (C) 55555500 (D) 20333280

[Hint: $4 \times 30\,[x] + 3 \times 20\,[x] + 2 \times 10\,[x]$ where $[x] = 1 + 10 + 10^2 + 10^3 + 10^4 + 10^5$

Total number = $\dfrac{6!}{3!\,2!} = 60$;

4 remains at all the places 30 times

|||ly 3 remains at all the places 20 times.

2 remains at all the places 10 times.]

Q.26 The number of ways in which we can choose 6 chocolates out of 8 different brands available in the market is

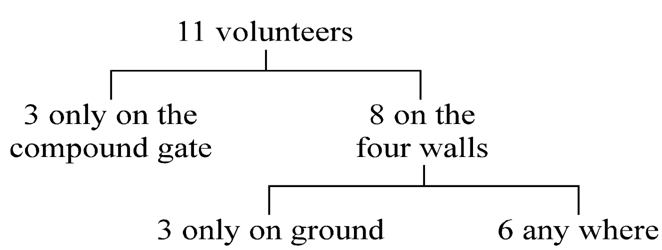
(A*) ${}^{13}C_6$ (B) ${}^{13}C_8$ (C) 8^6 (D) none

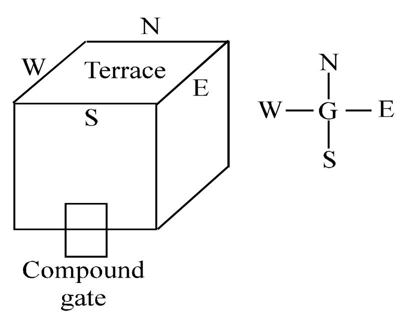
[Hint : consider 8 different brands to be beggar and compute the distribution of 6 identical things among 8 people each receiving none, one or more. Alternatively find co-efficient of x^6 in $(1 + x + x^2 + \ldots.. \infty)^8$]

Q.27 During the times of riots, residents of a building decide to guard their building from ground and terrace level ; one man posted at each of the four different sides and one watching the compound gate . If out of 11 volunteers, 2 suffer from acrophobia and other 3 wish to watch only the compound gate then the number of ways in which the watch teams which can be posted is ______ .

[Ans. : 25920]

[Hint :





one person for the compound gate can be taken in ${}^3C_1$ ways and cna be arranged only in one way 2 positions on the ground can be selected in ${}^4C_2$ ways and two people can be arranged in 3 ! ways 6 persons for the remaining positions can be filled in 6 ! ways.

Hence total = ${}^3C_1 \, . \, {}^4C_2 \, . \, 2\,! \, . \, 6\,!$ = 25920]

Q.28 An ice cream parlour has ice creams in eight different varieties . Number of ways of choosing 3 ice creams taking atleast two ice creams of the same variety, is :

(A) 56 (B*) 64 (C) 100 (D) none

(Assume that ice creams of the same variety are identical & available in unlimited supply)

[Ans. : ${}^{10}C_3 - {}^{8}C_3 = 120 - 56 = 64$]

Q.29 Number of cyphers at the end of ${}^{2002}C_{1001}$ is

(A) 0 (B*) 1 (C) 2 (D) 200

[Hint: $${}^{2002}C_{1001} = \frac{(2002)!}{(1001)!(1001)!}$$

no. of zeros in (2002)! are

$400 + 80 + 16 + 3 = 499$

no. of zeroes in $(1001\,!)^2 = 2(200 + 40 + 8 + 1) = 498$

Hence no. of zeroes is $\frac{(2002)!}{(1001!)^2} = 1$]

Q.30 There are 12 books on Algebra and Calculus in our library, the books of the same subject being different. If the number of selections each of which consists of 3 books on each topic is greatest then the number of books of Algebra and Calculus in the library are respectively:

(A) 3 and 9 (B) 4 and 8 (C) 5 and 7 (D*) 6 and 6