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QUESTION BANK

Career Point,

Guwahati Centre, G.S. Road, Bora Service Bye-Lane, Guwahati

Ph | 0361-2466191 | 98643-42927 | 94351-01613

MATHEMATICS

APPLICATION OF DERIVATIVE

Straight Objective
Section-I

choice questions, each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct. Mark your response in OMR sheet against the question number of that question. +3 marks will be given for correct answer and -1 mark for wrong answer.

1. The angle at which the curve y= Ke^{Kx} intersects the y-axis is :
   (a) $tan^{-1}k^2$ (b) $tcot^{-1}(k^2)$
   (c) $sec^{-1}\sqrt{1+k^4}$ (d) none

2. The angle between the tangent lines to the graph of the function f(x)$\int_2^x (2t-5)$dt at the points where the graph cuts the x-axis is
   (a)$\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

3. The difference between the greatest and the least values of the function f(x) = sin2x –x on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
   (a) π (b) 0
   (c) $\frac{\sqrt{3}}{2}+\frac{\pi}{3}$ (d)$-\frac{\sqrt{3}}{2}+\frac{2\pi}{3}$

4. In which of the following functions Rolle's theorem is applicable?
   (a) $f(x) = \begin{cases} x, & 0, \le x < 1 \\ 0, & x = 1 \end{cases}$ on [0, 1]
   (b) $f(x) = \begin{cases} \frac{\sin x}{x}, & -\pi, \le x < 0 \\ 0, & x = 0 \end{cases}$ on [$-\pi$, 0]
   (c) $f(x) = \frac{x^2-x-6}{x-1}$ on [-2, 3]
   (d) $f(x) =$
   $\begin{cases} \frac{x^2 - 2x^2 - 5x + 6}{x-1}, & if\ x \neq 1, on[-2,3] \\ -6, & if\ x = 1 \end{cases}$

5. Equation of the line through the point (1/2. 2)and tangent to the parabola y =$\frac{-x^2}{2}$ + $2\ and$ secant to the curve y=$\sqrt{4+x^2}$ is:
   (a)2x + 2y -5 =0 (b) 2x + 2y – 3 =0
   (c) y -2 =0 (d) None

6. The least value of 'a' for which the equation,$\frac{4}{sinx}+\frac{4}{1-sinx}=$ a has atleast one solution on the interval (0, $\pi/2$) is:
   (a) 3 (b) 5 (c) 7 (d) 9

7. For all a, b ϵ R the function f(x) =3x^4 – 4x^3 + 6x^2 + ax + b has
   (a) no extremum (b) exactly one extremum
   (c) exactly two extremum
   (d) three extremum

8. Let $f(x) = \begin{cases} x^{3/5}, & if\ x \le 1 \\ -(x-2)^3, & if\ x > 1 \end{cases}$

then the number of critical points on the graph of the function is

(a) 1 (b) 2 (c) 3 (d) 4

9. Number of roots of the function $f(x)=\frac{1}{(x+1)^3}$ -3x + sinx is

(a) 0 (b) 1 (c) 2 (d) more than 2

10. If f(x) $=\int_x^{x^2}(t-1)dt, 1 \le x \le 2$, then global maximum value of f(x) is

(a) 1 (b) 2 (c) 4 (d) 5

11. Given f'(1) = 1 and $\frac{d}{dx}$(f(2x)) = f'(x)∀x> 0. If f' (x) is differentiable then there exists a number cϵ(2, 4) such that f'' (c) equals

(a) – ¼ (b) $-1/8$

(c) 1/ 4 (d) 1/ 8

12. Consider the function

$$f(x) = \begin{bmatrix} 2 + x^3, & if\ x \le 1 \\ 3x, & if\ x > 1 \end{bmatrix}$$,then

(a) f is continuous on $[-1, 2]$ but is not differentiable on $(-1, 2)$

(b)Mean value theorem is not applicable for the function on $[-1, 2]$

(c)Mean value theorem is applicable on $[-1, 2]$ and the value of c =1

(d) Mean value theorem is applicable on $[-1, 2]$ and the value of c is $\pm\frac{\sqrt{5}}{3}$

13. Number of critical points of the function, f(x) $=\frac{2}{3}\sqrt{x^3}-\frac{x}{2}+\int_1^x\left(\frac{1}{2}+\frac{1}{2}\cos 2t-\sqrt{t}\right)$dt Which lie in the interval $[-2\pi, 2\pi]$ is:

(a) 2 (b) 4 (c) 6 (d) 8

14. Let f(x) = x+ $\sqrt{x}$ on [1, 4]. The mean value theorem says that there must be some number 'c' between 1 and 4 so that f'(c) is equal to the average slope of f(x) on [1, 4] the number 'c' must be

(a) $\frac{5}{2}$ (b) $\frac{9}{4}$ (c) $\frac{11}{4}$ (d) 3

15. Two sides of a triangle are to have lengths 'a' cm & 'b' cm. If the triangle is to have the maximum area then the length of the median from the vertex containing the sides 'a' and 'b' is

(a)$\frac{1}{2}\sqrt{a^2+b^2}$ (b) $\frac{2a+b}{3}$

(c)$\sqrt{\frac{a^2+b^2}{2}}$ (d) $\frac{a+2b}{3}$

FUNCTIONS

Multiple correct
Section-II

Multiple choice questions, each question has four choices (A), (B), (C) and (D), out of which MULTIPLE (ONE OR MORE) is correct. Mark your response in OMR sheet against the question number of that question. +4 marks will be given for correct answer and -2 marks for each wrong answer.

16. Which of the following functions are periodic?

(a) f(x) =sgn (e^{-x})

(b)$f(x) = \begin{cases} 1\ if\ x\ is\ a\ rational\ number \\ if\ x\ is\ an\ irrational\ number \end{cases}$

(c)f(x)=$\sqrt{\frac{8}{1+cosx}+\frac{8}{1-cosx}}$

(d) f(x) =$\left[x+\frac{1}{2}\right]$+$\left[x-\frac{1}{2}\right]$ +2[-x] where [.] denotes greatest integer function)

17. f(x) =$\sin\left(2\left(\sqrt{[a]}\right)x\right)$, where [.] denote the greatest integer function, has fundamental period π for
(a) a =$\frac{3}{2}$ (b) a =$\frac{5}{4}$
(c) a =$\frac{2}{3}$ (d) a =$\frac{4}{5}$

18. Let f(x) = $[x]^2$ + [x + 1) – 3, where [x] = the greatest integer $\leq x$. Then
(a) f(x) is a many –one and into function
(b)f(x)=0 for infinite number of values of x
(c)f(x) =0 for only two real values
(d) none of these

19. If f:R→R, f(x) =$e^{-|x|}-e^{x}$ is a given function, then which of the following are correct:
(a) f is many –one into function
(b) f is many one onto function
(c)range of f is [0,∞]
(d) range of f is (-∞, 0]

20. If f(x) = sin { [x+5] + {x-{x-{x}}}} for x $\epsilon\left(0,\frac{\pi}{4}\right)$ is invertible, where {.} and [.] represent fractional part and greatest integer functions respectively, then f^{-1}(x) is
(a) $\sin^{-1}x$ (b) $\frac{\pi}{2}\sin^{-1}x$
(c) $\sin^{-1}\{x\}$ (d) $cos^{-1}\{x\}$

21. Range of f(x) =$log_{\sqrt[3]{10}}$ ($\sqrt{5}$ (2 sinx + cosx)+ 5) is
(a)[0, 1] (b) [0, 3]
(c)$\left(-\infty,\frac{1}{3}\right)$ (d) none of these

MONOTONICITY

22. Let f(x) be a non-constant twice differentiable function defined on$(-\infty,\infty)$ such that f(x) = f(1-x) and $f'\left(\frac{1}{4}\right)$=0. Then
(a) f''(x) vanishes at least twice on [0, 1]
(b) f'$\left(\frac{1}{2}\right)=0$
(c) $\int_{-1/2}^{1/2} f\left(x+\frac{1}{2}\right)$sin x dx =0
(d) $\int_0^{1/2} f(t)e^{sin\pi t}dt = \int_{1/2}^{1} f(1-t)\, e^{sin\pi t}$dt

23. Let g'(x) > 0 and f'(x) < 0 ,∀ $x\epsilon R$, then
(a) (f(x+1)) > g(f(x−1))
(b) f(g(−1)) > f(g(x+ 1))
(c)g(f(x+1)) < g (f(x−1))
(d) g(g(x+ 1)) < g(g(x−1))

24. If f(x) = $x^3 - x^2$ + 100x + 1001, then
(a) f(2000) > f(2001)
(b) f$\left(\frac{1}{1999}\right) > f\left(\frac{1}{2000}\right)$

(c) f(x+ 1) > f(x −1)

(d) f(3x −5) > f(3x)

Comprehension
Section-III

This section contains 2 paragraphs; each has 3 multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct. Mark your response in OMR sheet against the question number of that question. +4 marks will be given for correct answer and -1 mark for each wrong answer.

Passage I

Let f and g are two functions such that f(x) & g(x) are continuous in [a, b] and differentiable in (a, b) Then at least one c ϵ(a, b) such that f' (c) $\frac{f(b)-f(a)}{b-a}$

(i) if f(a) = f(b), then f'(c)=0 (RMVT)

(ii) If f(a) ≠f(b) and a≠b, (LMVT)

(iii) If g'(x) $\neq$ 0,then$\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}$

(Cauchy theorem)

25. The set of values of k, for which equation x^3 -3x + k = 0 has two distinct roots in (0, 1) is

(a) (1, 4) (b) (0, ∞)

(c)(0, 1) (d)ϕ

26. Which of the following is true ?

(a) $|tan^{-1}x - tan^{-1}y| \leq |x-y| \forall x, y \epsilon R$

(b) $|tan^{-1}x - tan^{-1}y| \geq |x-y| \forall x, y \epsilon R$

(c) $|sinx - siny| \geq |x-y| \forall x, y \epsilon R$

(d)none of these

27. Let 0 < $\alpha < 0 < \beta < \frac{\pi}{2}$, then$\frac{sin\alpha - sin\beta}{cos\alpha - cos\beta}$ is equal to

(a) tanθ (b) –tanθ

(c) cotθ (d) $-cot\theta$

Passage II

Consider the function f(x) = max (x^2, $(1-x)^2$, 2x(1-x)}, where $0 \leq x \leq 1$

28. The interval in which f(x) is increasing is

(a)$\left(\frac{1}{3}, \frac{2}{3}\right)$ (b) $\left(\frac{1}{3}, \frac{1}{2}\right)$

(c) $\left(\frac{1}{3}, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \frac{2}{3}\right)$

(d) $\left(\frac{1}{3}, \frac{1}{2}\right) \cup \left(\frac{2}{3}, 1\right)$

29. The interval in which f(x) is decreasing is

(a)$\left(\frac{1}{3}, \frac{2}{3}\right)$ (b) $\left(\frac{1}{3}, \frac{1}{2}\right)$

(c) $\left(0, \frac{1}{3}\right) \cup \left(\frac{1}{2}, \frac{2}{3}\right)$

(d) $\left(0, \frac{1}{2}\right) \cup \left(\frac{2}{3}, 1\right)$

30. Let RMVT is applicable for f(x) on (a, b) then a+ b +c is (where c is point such that f'(c) =0)

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{2}$

Passage III

Consider the function f(x)=$\frac{x^2}{x^2-1}$

31. The interval in which f is increasing is

(a) (-1, 1) (b) $(-\infty, -1) \cup (-1, 0)$

(c) $(-\infty, \infty) - \{-1, 1\}$

(d) (0, 1) $\cup(1, \infty)$

32. If f is defined from R – {-1, 1} → R *then f is*

(a) injective but not surjective

(b)surjective but not injective

(c)injective as well as surjective

(d)neither injective nor surjective.

33. F has

(a) local maxima but no local minima

(b) local minima but no local maxima

(c) both local maxima and local minima

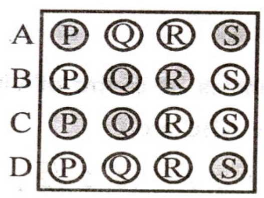
(d) neither local maxima nor local minima

MAXIMA & MINIMA

Matrix

Section-IV

Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in Column-I have to be matched with statements (P, Q, R, S) in Column-II. The answers to these questions have to be appropriately bubbled as illustrated in the following example. If the correct matches are A-P, A-S, B-Q, B-R, C-P, C-Q and D-S, then the correctly bubbled 4 x 4 matrix should be as follows:



Mark your response in OMR sheet against the question number of that question in section-IV. +6 marks will be given for correct answer and No negative marks for wrong answer. However, +1 mark will be given for a correctly marked answer in any row.

34. **Column I** **Column II**

(a) Number of points which are local extrema of (p)1

$$f(x) = \begin{cases} (2+x)^3 \;\; ; & -3 < x \le x \le -1 \\ x^{2/3}, & -1 < x < 2 \end{cases}$$

(b) If a+b=1; a>0, b > 0, then the minimum value of (q) 2

$\sqrt{\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)}$ is

(c)The maximum value attained by y = $10 - |x - 10|$,- 1$\le x \le 3$, is (r) 3

(d) If P(t^2, 2t), tϵ [0, 2] is an arbitrary point on parabola y^2 =4x. Q is foot of perpendicular from focus S on the tangent at P, then maximum area of triangle PQS is (s) 4

(t) 5

35. **Column –I** **Column II**

(a) The minimum value of $\frac{x^2+2x+4}{x+2}$ is (p) 0

(b) Let A and B be 3× 3 matrices of real numbers, where A is symmetric, B is skew –symmetric and (A+B) A-B) = (A-B) (A +B). If $(AB)^t = (-1)^k$ AB, where $(AB)^t$ is the transpose of the matrix AB, then the possible values of k are (q) 1

(c) Let a = $\log_3 \log_3 2$. An integer k satisfying $1 < 2^{(-k+3^{-a})} < 2$, must be less than (r)2

(d) If $\sin\theta = cos\phi$, then the possible values of $\frac{1}{\pi}(\theta \pm \phi - \frac{\pi}{2})$ are (s) 3

(t)4

36. **Column I** **Column II**

(a)If smallest positive integral value of x for which $x^2 - x - \sin^{-1}(\sin 2) < 0$ is λ, then 3+ λ is equal to (p) 4

(b) Number of solution of 2[x] = x + 2{x} is (where [.], {.} are greatest integer and least integer functions respectively) (q) 1

(c) If $x^2 + y^2 = 1$ and maximum value of x + y is $\frac{\sqrt{2}\lambda}{3}$, then λ is equal to (r) 2

(d)$f\left(x + \frac{1}{2}\right) + f\left(x, -\frac{1}{2}\right)$= f (x) for all $x \epsilon R$, then period of f(x) is (s) 0

(t) 3

37. **Column I** **Column II**

(a) If function f(x) is defined in [-2, 2], then domain of f$(|x| + 1) is$ (p) (- ∞, -4)

(b) Range of the function f(x) =$\frac{sin^{-1}x + cos^{-1} tan^{-1}x}{\pi}$ is (q) [- 1, 1]

(c) Range of the function f(x) =3 $|\sin x|$-4$|cos|x$ is (r) [-4, 3]

(d) Range of f(x) = $(\sin^{-1}x)$ sin x is (s)$\left[0, \frac{\pi}{2}\sin 1\right]$

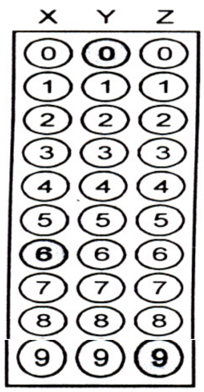
(t) $\left[\frac{1}{4}, \frac{3}{4}\right]$

Integer Type

Section-V

Integer Answer type

This section contains 3 questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the OMR have to be darkened. For example, if the correct answers to question number X, Y and Z (say) are 6, 0, 9 respectively, then the correct darkening of bubbles will look like the following



38. The smallest value of k, for which both the equation x^2 -8kx + 16 (k^2 –k + 1) =0 are real , distinct and have values at least 4, is
39. Let f: R→R be a continuous function which satisfies f(x)=$\int_0^x f(t)$dt. Then the value of f(In 5) is
40. Let p(x) be a polynomial degree 4 having extremum at x = 1, 2 and $\lim_{x\to 0}\left(1+\frac{P(x)}{x^2}\right)$ =2. Then the value of p(2) is

ANSWER KEY

Q.NO	1	2	3	4	5	6	7	8	9	10
ANS	B	D	A	D	A	D	B	C	C	C
Q.NO	11	12	13	14	15	16	17	18	19	20
ANS	B	D	B	B	A	A,B,C,D	A	A,B	A,D	A,B,C
Q.NO	21	22	23	24	25	26	27	28	29	30
ANS	D	A,B,C	B,C	B,C	D	A	D	D	C	D
Q.NO	31	32	33	34		35		36		
ANS	B	D	A	A-q, B-r	C-r,D-t	A-r, B-q,s	C-r,s, D-p,r	A-p, B-t	C-t,	D-t
Q.NO	37				38	39	40			
ANS	A-q	B-t	C-r	D-s	2	0	0			