Motion in One & Two Dimension

GUPTA CLASSES

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Motion in One & Two Dimension

Distance and Displacement

1. The magnitude of displacement is equal to minimum possible distance between two positions. In general, magnitude of displacement is not equal to distance. However, it can be so if the motion is along a straight line without change in direction. i.e. Distance ≥ | Displacement |
2. For a moving particle distance can never decrease with time while displacement can. Decrease in displacement with time means that body is moving towards the initial position.
3. For a moving particle distance can never be negative or zero while displacement can be. (Zero displacement means that the body after motion has come back to initial position.).

Average Speed and Velocity

1. For a moving body average speed can never be negative or zero while average velocity can be. In general, average speed is not equal to magnitude of average velocity. However, it can be so if the motion is along a straight line without change in direction.
2. If a particle travels equal distances at speeds v_1, v_2, v_3,…..etc respectively, then the average speed (v_{av}) is given by $\frac{n}{v_{av}} = \frac{1}{v_1} + \frac{1}{v_2} + \ldots\ldots\ldots \frac{1}{v_n}$.
3. If a particle moves a distance at speed v_1 and comes back with speed v_2 the average velocity is zero while average speed $v_{av} = \frac{2v_1v_2}{v_1 + v_2}$
4. If a particle travels at speeds v_1, v_2, v_3…….etc., for equal time intervals, then average speed is given by $v_{av} = \frac{v_1 + v_2 + \ldots\ldots v_n}{n}$

Instantaneous Speed and Velocity

Instantaneous velocity or simply velocity is defined as rate of change of particle's position with time. Instantaneous velocity is given by $\vec{v}_{inst} = \frac{d\vec{r}}{dt}$

Instantaneous speed is the magnitude of instantaneous velocity.

Average and Instantaneous Acceleration

If the velocity of a particle changes (either in magnitude or direction or both) with time the motion is said to be accelerated or non-uniform. We define the acceleration as the rate of change of velocity with respect to time e.g. $\text{Average acceleration}\,(\vec{a}_{av}) = \frac{\text{change in velocity}\,(\Delta\vec{v})}{\text{time interval}\,(\Delta t)} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$

and $\text{Instantaneous acceleration}\,(\vec{a}_{inst}) = \frac{d\vec{v}}{dt}$

Uniform Motion

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The motion is uniform if velocity of an object remains constant throughout the motion. Since the velocity is a vector quantity. It means both the magnitude and direction should be same.

Displacement Time Graph

It is also known as position time graph. In this graph one axis represent the time and another axis represent the position or displacement of object.

1. The slope of the line in displacement time graph represents velocity.
2. No line in displacement time graph can be perpendicular to time axis because it will represent infinite velocity.

Velocity Time Graph

1. The slope of the line in velocity time graph represents acceleration.
2. If the velocity-time graph is a straight line parallel to time axis, then the acceleration of the body is zero.
3. The total displacement of the body in the given time is equal to the area which velocity time graph encloses with time axis. The area is to be added with sign.
4. The total distance travelled by the body in the given time is equal to the area which velocity time graph encloses with time axis. The area is to be added without sign.

Equations of uniformly accelerated motion

(a) Velocity – Time Relation ⇒ $v = u + at$ …………….. (1)

(b) Displacement – Time Relation ⇒ $s = ut + ½ at^2$ …………….. (2)

(c) Velocity – Displacement Relation ⇒ $v^2 = u^2 + 2as$ …………….. (3)

Regarding these equations it is worth noting that:

1. These equations can be applied only and only when acceleration is constant. When acceleration is not constant, integration of acceleration will give velocity and integration of velocity will give displacement.
2. If the velocity and acceleration are collinear, we conventionally take the direction of motion to be positive and so variables in the direction of motion are taken to be positive while opposite to it negative.
4. The velocity and acceleration of a body may not necessarily be in the same direction.
5. If a body is starting from rest and is moving with a uniform acceleration, the distance traveled by the body in successive seconds is in the ratio of 1 : 3 : 5 : 7.......

Motion under Gravity

(a) Velocity – Time Relation ⇒ $v = u \pm gt$ …………….. (1)

(b) Displacement – Time Relation ⇒ $h = ut \pm ½ gt^2$ …………….. (2)

(c) Velocity – Displacement Relation ⇒ $v^2 = u^2 \pm 2gh$ …………….. (3)

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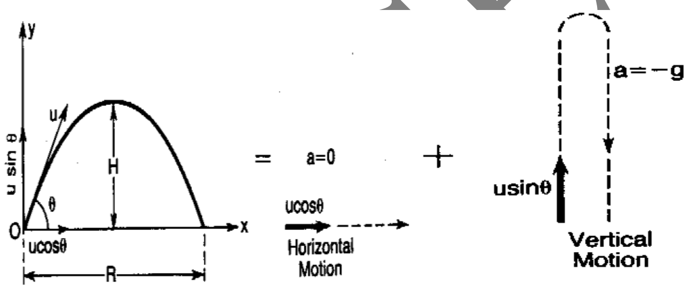
1. We conventionally take the direction of motion to be positive and so variables in the direction of motion are taken to be positive while opposite to it negative.
2. The acceleration is constant, i.e. g = 9.8 m/s^2 and directed vertically downwards.
3. In case of motion under gravity time taken to go up is equal to the time taken to fall down through the same distance.
4. In case of motion under gravity, the speed with which a body is projected up is equal to the speed with which it comes back to the point of projection.

Projectile Motion (on a Horizontal Plane)



If an object is given an initial velocity in any direction and then allowed to travel freely under gravity, the motion is called projectile motion. In case of projectile motion it is assumed that acceleration due to gravity is constant and the effect of air resistance is negligible. Projectile motion is a two-dimensional motion and can be regarded as simultaneous superposition of two motions one horizontal with velocity **ucosθ** and acceleration = 0 and the other vertical with velocity **usinθ** and acceleration = –g

Trajectory

It is the path followed by the projectile, i.e., a curve joining positions of the same particle in motion at different instants of time. Equation of trajectory of projectile is given by

$$y = (\tan\theta)x - \frac{g}{2u^2\cos^2\theta}x^2$$

This is an equation of the type, which represents a parabola; so *the path of a projectile is a parabola*.

Time of Flight (T)

The total time taken by the projectile to go up and come down to the same level from where it was projected is called time of flight and it is given by $T = \frac{2u\sin\theta}{g}$

Horizontal Range (R)

It is the horizontal distance travelled by a body during the time of flight and is given by $R = \frac{u^2\sin 2\theta}{g}$

Maximum Height (H)

It is the maximum height from the point of projection a projectile can reachand is given by $H = \frac{u^2\sin^2\theta}{2g}$

Regarding projectile motion it is worth noting that;

1. In case of projectile motion, the horizontal component of velocity (u cos θ), acceleration (g) and mechanical energy remains constant while, speed, velocity, vertical component of velocity (u sin θ), momentum, kinetic energy and potential energy all change. Velocity and KE are maximum at the point of projection while minimum (but not zero) at the highest point.
2. For two angle of projection θ and (90 – θ) with same speed the range is same though time of flight, maximum height and trajectories are different.
3. A projectile will have maximum range when it is projected at an angle of 45° to the horizontal and the maximum range will be (u^2/g)
4. When the range is maximum, the height H reached by the projectile is (R_{max} /4)

Motion in One Dimension Assignment

1. When a body is accelerated
   1. Its velocity always changes
   2. Its speed always changes
   3. Its direction always changes
   4. Its speed always changes
2. Tripling the speed of vehicle multiplies the minimum distance needed for stopping it by

   1. 3 2. 6 3. 9 4. 9/2
3. A stone, dropped from the top of a tower, travels 25 m in the last second of its journey. The height of the tower is (g = 10 m/s²)

   1. 45 m 2. 90 m 3. 72 m 4. 135 m
4. A particle starting from rest has a velocity that increases linearly with time v = kt. Then the distance covered by it in first 3 s is

   1. 3k/2 2. 3k 3. 6k 4. 9k/2
5. A stone falls freely from rest. The distance covered by it in the last second is equal to the distanced covered by it in the first three seconds. The time taken by the stone to reach the ground is

   1. 4 s 2. 5 s 3. 8 s 4. 10 s
6. A stone is dropped from the top of a 30 m high cliff. At the same instant another stone is projected vertically upwards from the ground with a speed of 30 m/s. The two stones will cross each other after a time (g = 10 m/s²)

   1. 1 s 2. 2 s 3. 3 s 4. 4 s
7. From the top of a building, 16 m high, water drops are falling at equal intervals of time such that when the first drop reaches the ground, the fifth drop just starts. The distances between the successive drops, in metres, at that instant is

   1. 8, 4, 2, 1 2. 7, 5, 3, 1
   3. 7.5, 5, 2.5, 1 4. None of the above
8. The initial velocity of a particle is u and the acceleration at time *t* is *at* 'a' being a constant. Then the velocity *v* at time *t* is given by

   1. v = u 2. v = u + at
   3. v = u + at² 4. v = u + ½ at²
9. A bullet fired into a fixed wooden block loses half its velocity after penetrating 60 cm. It comes to rest after penetrating a further distance of

   1 . 10 cm 2. 20 cm 3. 40 cm 4. 60 cm
10. The displacement x of a body varies with time as $x = -t^2/3 + 16t + 3$ Where x is metres and t is in seconds. The time taken by body to come to rest is

    1. 12 s 2. 24 s 3. 30 s 4. 36 s
11. 3A body starts from rest and moves with a constant acceleration. The ratio of the distance covered in nth second to the distance covered in n seconds is

    1. $1/n - 2/n^2$ 2. $1/ n + 2/n^2$
    3. $2/n - 1/n^2$ 4. $2/n + 1/n^2$
12. A person throws balls upward one after the other with the same speed at an interval of 1 second. The next ball is thrown when the velocity of the previous ball is zero. The balls rise to a height (g = 10 m/s²)

    1. 5 m 2. 10 m 3. 150 m 4. 20 m
13. If x denotes displacement in time t and x = acos t, then acceleration is

    1. a cos t 2. – a cos t

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3. a sin t 4. – asin t

14. A conveyer belt is moving horizontally at a speed of 4m/sec. A box of mass 20 kg is gently laid on it. It takes 0.1 second for the box to come to rest on the belt. The distance moved by the box on the conveyer belt is

1. 0 m 2. 0.2m 3. 0.4m 4. 0.8m

15. A bus is moving with a velocity 10m/s on a straight road. A scooterist wishes to overtake the bus in 100 s. If the bus is at of 1 km from the scooterist, with what velocity should the scooterist chase the bus

1. 50 m/s 2. 40 m/s 3. 30 m/s 4. 20 m/s

16. A particle moving with constant acceleration covers distance of 30 m in 3^{rd} second. It covers a distance of 50 m in the 5^{th} second. What is the acceleration of the particle?

1. 3 m/s 2. 5 m/s² 3. 8 m/s² 4. 10 m/s²

17. A train accelerates from rest for time t_1 at a constant rate α and then it retards at the constant rate β for time t_2 and come to rest. The ratio of t_1/t_2 is equal to

1. α/β 2. β / α 3. α^2 / β^2 4. β^2 / α^2

18. A parachutist drops freely from an aeroplane for 10 s before the parachute opens out. Then he descends with a net retardation of 2.5 m/s². If he bails out of the plane at a height of 2495 m and g = 10 m/s², his velocity on reaching the ground will be

1. 5 m/s 2. 10 m/s 3. 15 m/s 4. 20 m/s

19. A body freely falling from a vertical height of 10 m pierced through a distance of 1 m in sand. It faced an average retardation in sand amounting

1. g 2. 10 g 3. 100 g 4. 1000g

20. A stone falls from the top of tower in 8 s. How much time it will take to cover the first quarter of the distance starting from top?

1. 4 s 2. 2 s 3. 1 s 4. 8 s

Answers: 1-a, 2-c, 3-a, 4-d, 5-b , 6-a , 7-b , 8-d, 9-b, 10-b, 11-c, 12-a, 13-b, 14-b, 15-d, 16-d, 17-b, 18-a, 19-b, 20-a,

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