FLUID MECHANICS

GUPTA CLASSES

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Fluids

Density

Relative density or specific gravity is defined as: RD = (density of body/density of water). With rise in temperature due to thermal expansion of a given body, volume will increase while mass will remain same; so density will decrease, i.e., $\rho = \rho_0(1 - \gamma\Delta\theta)$, Where γ is temperature coefficient of cubical expansion of body and $\Delta\theta$ is change in temperature.

Pressure

If we consider a point at a depth 'h' below the surface of a liquid of density ρ, **hydrostatic pressure** 'P' is given by: $P = P_0 + h\rho g$ where P_0 represents the atmospheric pressure. The pressure difference between hydrostatic pressure and atmospheric pressure is called **gauge pressure** and will be $P - P_o = h\rho g$.

1. SI unit of pressure is N/m^2 which is also called Pascal (Pa). Other practical units of pressure are atmosphere, bar or torr (or mm of Hg). They are related with each other as follows:

   1 atmosphere (atm) = $1.01 \times 10_5\ N/m^2 = 1.01 \times 10^5$ Pa = 1.01 bar = 760 mm of Hg = 760 torr
2. Pressure always acts normal to the boundaries of the fluid because flowing ability of the fluids makes them unable to sustain a tangential force.
3. Pressure is independent of amount of liquid, shape of container or cross-sectional area considered. So if a given liquid is filled in vessels of different shapes to same height, the pressure in each vessel's base will be the same, though the volume or weight of the liquid in different vessels will be different.
4. Pressure will be same at all points in a liquid lying at the same level. This is why the height of liquid is same in vessels of different shapes containing different amounts of the same liquid at rest when they are in communication with each other.
5. In case of a fluid if an external pressure is applied to an enclosed fluid, it is transmitted undiminished to every position of the fluid and to the walls of container. This is called Pascal's law.

Archimedes' Principle and Buoyant Force

According to Archimedes' principle, when a body is immersed partly or wholly in a fluid, it is buoyed up with a force called upthrust or Buoyant force.

The magnitude of buoyant force is equal to weight of fluid displaced ($Th = V_{in}\sigma g$) by the body. It acts vertically upwards (opposite to the weight of the body) through the centre of gravity of displaced fluid (called centre of buoyancy). This is also valid for partly submerged bodies.

Due to upthrust the weight of body decreases. If W_o represents the weight of a body, W_{app} the weight in a fluid (called apparent weight) will be

$$W_{app} = W_o - Th \qquad (Th = V\sigma g)$$

Floatation

When a body of density ρ_B (which may be different from the density of material of body) and volume V is fully immersed in a liquid of density σ, following two forces act on the body:

(a) The weight of body, $W = mg = V\rho_B g$, acting downwards at centre of gravity of the body.

(b) The upthrust, $Th = V\sigma g$, acting upwards at centre of gravity of displaced liquid (centre of buoyancy).

Depending upon relative magnitudes of above two forces, following three cases are possible:

(a) If density of body is greater than that of liquid (i.e., $\rho_B > \sigma$) the body will sink.

(b) If density of body is equal to the density of liquid (i.e., $\rho_B = \sigma$) the body will float fully submerged in neutral equilibrium anywhere in the liquid.

(c) If density of body is lesser than that of liquid (i.e., $\rho_B < \sigma$) the body will move upwards and in equilibrium will float partially immersed in the liquid such that $W = V_{in}\sigma g$ or $V\rho_B g = V_{in}\sigma g$ or $V\rho_B = V_{in}\sigma$.

Important points:

(a) A body will float in a liquid only and only if $\rho_B \le \sigma$.

(b) When a body is floating, weight of body is equal to upthrust, i.e., $V\rho_B = V_{in}\sigma$

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(c) In case of floating as W = Th, the apparent weight of floating body will be zero i.e., W_{app} = W - Th = 0

Viscosity

The property of a fluid due to which it opposes the relative motion between its different layers is called viscosity and the force between the layers opposing the relative motion viscous force. A briskly stirred fluid comes to rest after a short while because of viscosity.

Newton found that viscous force F acting on any layer of a fluid is directly proportional to its area A and to the velocity gradient at the layer, i.e., $F = -\eta A \frac{dv}{dy}$, where η is coefficient of viscosity. The negative sign signifies that viscous force on a layer acts in a direction opposite to relative velocity of layer.

1. Viscosity of layer depends only on the nature of fluid and is independent of area considered or velocity gradient.
2. Its dimensions are $[ML^{-1}T^{-1}]$ and SI units Poiseuille (PI) while CGS units dynes-sec/cm² called Poise (P) with 1 PI = 10 Poise.
3. Viscosity of liquids is much greater (say about 100 times more) than that of gases, i.e., $\eta_L > \eta_G$
4. In case of liquids viscosity increases with density while for gases it decreases with increase in density.
5. With rise in temperature the viscosity of liquids decreases while that of gases increases.
6. With increase in pressure, the viscosity of liquids (except water) increases while that of gases is practically independent of pressure.

Stokes' law and terminal velocity

Stokes established that if a sphere of radius r moves with velocity 'v' through a fluid of viscosity η, the viscous force opposing the motion of the sphere is: F = 6πηrv.

The terminal velocity of a sphere of 'r' falling in a fluid of density σ and coefficient of viscosity η is given by: $v_T = \frac{2}{9} r^2 \left[\frac{\rho - \sigma}{\eta}\right] g$, where ρ is the density of the material of sphere.

Flow of Liquid

Streamline Flow : of a liquid is that flow in which every particle of the liquid follows exactly the path of its preceding particle and has the same velocity.

Laminar Flow : It is the flow of liquid in which liquid moves in layers of different velocities. In general laminar flow is a streamline flow.

Turbulent Flow : When a liquid moves such that motion of the particle of liquid becomes disorderly or irregular, the flow is called turbulent.

1. In case of steady flow of a liquid of viscosity η in a cylindrical tube of length 'L' and radius 'r' under a pressure difference 'P' across it, the velocity of flow at a distance y from the axis is given by: $v = \frac{P}{4\eta L}(r^2 - y^2)$, i.e., the profile of advancing liquid in a capillary is a parabola.
2. In case of steady flow of a liquid of viscosity η in a capillary tube of length 'L' and radius 'r' under a pressure difference 'P' across it, the rate of volume flow of liquid is given by: $\frac{dQ}{dt} = \frac{\pi P r^4}{8\eta L}$

Critical velocity and Reynold's number

The maximum velocity up to which fluid motion remains steady is called critical velocity.

1. Reynold through experiments established that in case of motion of fluids in thin tubes, critical velocity depends on the density (ρ), viscosity (η) of the fluid and radius of the tube (r), $v_C \propto \frac{\eta}{r\rho}$ or $v_C = N_R\left[\frac{\eta}{r\rho}\right]$, where, N_R is a dimensionless constant called Reynold's number.

Principle of continuity

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In case of steady flow of incompressible and non-viscous fluid through a tube of non-uniform cross-section, the product of the area of cross-section and the velocity of flow remains same at every point in the tube, i.e.,

$$Av = \text{constant}$$

Bernoulli's theorem

According to this theorem, in case of steady flow of incompressible and non-viscous fluid through a tube of non-uniform cross-section, the sum of the pressure, the potential energy per unit volume and the kinetic energy per unit volume is same at every point in the tube, i.e., $P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

1. **Blowing off of roofs by wind storms:** During a tornado when a high speed wind blows over a roof of straw or tin, it creates a low pressure in accordance with Bernoulli's theorem. However, pressure below the roof is still atmospheric. So due to this difference of pressure, the roof is lifted up and is then blown off by the wind,
2. **Attraction between two closely parallel moving boats or buses:** When two boats or buses move side by side in the same direction, the water (or air) in the region between them moves faster than that on the remote sides. Consequently in accordance with Bernoulli's principle, the pressure between them is reduced and hence due to pressure difference they are pulled towards each other creating the so called attraction.
3. **Magnus effect:** When a spinning ball is thrown, it deviates from its usual path in flight. This effect is called magnus effect. This effect also occurs in accordance with Bernoulli's theorem.

Velocity of efflux

If a liquid is filled in a vessel up to height H and a small hole is made at a depth h below the free surface of the liquid, then

1. Velocity of liquid coming out from the hole is given by: $v = \sqrt{2gh}$
2. The time taken by the liquid to reach the ground-level $t = \sqrt{\frac{2(H-h)}{g}}$
3. Range of liquid flowing out from the orifice is $R = 2\sqrt{h(H-h)}$

Intermolecular forces (Cohesive and adhesive forces)

The force of attraction between molecules of same substance is called cohesive force while the force of attraction between the molecules of different substances is called adhesive force.

Surface tension

The property of the surface of a liquid by virtue of which it tends to contract and occupy the minimum possible surface area (as it is in a state of tension) is called surface tension.

1. The surface tension of a liquid is defined as the force per unit length in the plane of the liquid surface, acting at right angles on either side of an imaginary line drawn in that surface.
2. With a rise in temperature, surface tension of a liquid decreases till it becomes zero at the boiling point or the critical temperature whichever is lower. There is also an exception to it. Surface tension of molten cadmium and copper increases with increase in temperature.
3. With the presence of impurity in a liquid, surface tension of the liquid decreases if the impurity is sparingly soluble in the liquid (e.g., soap, phenol, camphor, sodium in water) and surface tension of liquid increases when the impurity is highly soluble in .the liquid (e.g., common salt or sugar added to water).

Some important points concerning surface tension

1. Free surface of a liquid always remains in a state of tension and tends to have minimum possible surface area due to property of surface tension. If the area of liquid surface is increased, work will be done. This work is stored as potential energy in the surface. The amount of this energy per unit area of the surface under isothermal condition is called free surface energy density.
2. As work done by external force in increasing the total surface area of film by dA is W = T dA, hence free surface energy density = (W/dA) = T.
3. Work done in forming a drop or bubble inside a liquid, $W = T \times 4\pi r^2$.

Liquid drop which consists of only one free surface
Air bubble in a liquid which also consists of only one free surface
Liquid bubble in air which consists of two free surfaces

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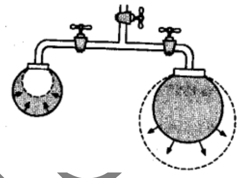
4. Work done In forming a bubble in air, $W = T \times (2 \times 4\pi r^2) = 8\pi r^2 T$.
5. Work done in increasing the surface of a soap bubble of radius r_1 into a bubble of radius r_2 is : $W = 8\pi(r_2^2 - r_1^2)\, T$
6. Work done in spraying a liquid drop of radius R into N equal droplets $W = T(4\pi R^2)[N^{1/3}-1]$
7. If a ring of negligible thickness and radius r floats on liquid surface, then the force of surface tension is: $F = T \times 2 \times (2\pi r)$ because the length of the ring in contact with the liquid surface is $2 \times (2\pi r)$.

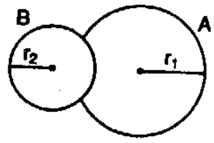
Excess pressure



The pressure on the concave side of the liquid surface is always greater than the pressure on the convex side. The difference of pressure is called as excess pressure.

In case of liquid drop in air or an air bubble in a liquid the excess pressure, $\Delta P = (2T/R)$. In case of a spherical film like a soap bubble in air, the excess pressure, $\Delta P = (4T/R)$ where R is the radius of the spherical surface.

1. When two bubbles of different sizes are in communication with each other, air passes from smaller one to larger one and larger one grows at the expense of smaller one. This happens due to pressure inside the smaller bubble being higher than that inside the larger bubble.
2. If two spherical soap bubbles of radii r_1 and r_2 coalesce in vacuum to form a bigger bubble of radius R, then there is no change in temperature and surface energy. This implies that surface area remains unchanged. i.e., $4\pi r_1^2 + 4\pi r_2^2 = 4\pi R^2$ or $R = (r_1^2 + r_1^2)^{1/2}$
3. If two soap bubbles of different radii r_1 and r_2 $(r_1 > r_2)$ coalesce to form a single double bubble having a common surface, then the radius of curvature of the interface is given by: $r = r_1 r_2/(r_1 - r_2)$ and the interface will be concave towards smaller bubble and convex towards larger bubble.



4. When a bigger drop splits into smaller drops, energy is required to break it but when smaller drops coalesce to form bigger drop energy is released.

Angle of contact

When the free surface of a liquid comes in contact with a solid, it becomes curved near the place of contact. The angle between the tangent to the liquid surface and the tangent to the solid surface at the point of contact (inside the liquid) is known as angle of contact.

1. The angle of contact is different for different pairs of solids and liquids. For mercury and glass, the angle of contact is 135°. For ordinary water and glass, the angle of contact is nearly 8°.
2. Angle of contact increases on increasing the temperature.
3. Angle of contact decreases on adding soluble impurity in a liquid.
4. Angle of contact does not depend upon the inclination of the tube.

Capillarity

The phenomenon of rise or fall of liquids in a capillary tube is known as capillarity. The rise or fall of liquid in a capillary (= h) is given by; $h = \frac{2T\cos\theta}{rdg}$, where r is the radius of capillary tube; g, the acceleration due to gravity; d, the density of liquid and θ, the angle of contact. In case of pure water and clean glass, angle of contact θ = 0°.

1. Phenomenon of capillarity depends on the nature of liquid and solid both, i.e., surface tension (T), angle of contact (θ) and density of fluid (d). If θ > 90°, i.e., meniscus is convex, h will be negative, i.e., the liquid will fall in the capillary tube as in case of mercury in a glass capillary. If θ = 90°, i.e., meniscus is plane, h = 0, so no phenomenon of capillarity. If θ < 90°, i.e., meniscus is concave towards air, h will be + ive, i.e., the liquid will rise in the capillary.
2. The vertical height 'h' of a liquid column in capillaries of different shapes and sizes will be same if the radius of meniscus remains the same and also the vertical height of the liquid in a capillary does not change when it is inclined to the vertical.
3. In case of capillary of insufficient length, i.e., L < h, the liquid will neither overflow from the upper end like a fountain nor will trickle along the vertical sides of the tube but after reaching the upper end the radius of the meniscus will increase without changing its nature such that: *hr = Lr'*.

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Fluid Mechanics Assignment

1. A piece of ice has a stone in it and floats in a vessel containing water. When the ice melts, the level of water in the vessel would
   (a) remain unchanged
   (b) fall
   (c) rise
   (d) may rise or fall depending on the size of Stone

2. A piston of cross-sectional area 100 cm^2 is used in a hydraulic press to exert a force of 10^7 dynes on the water. The cross-sectional area of the other piston which supports a truck of mass 2000 kg is
   (a) 9.8×10^2 cm^2 (b) 9.8×10^3 cm^2
   (c) 1.96×10^3 cm^2 (d) 1.96×10^4 cm^2

3. A piece of wood of relative density 0.36 floats in oil of relative density 0.90. The fraction of volume of wood above the surface of oil is
   (a) 0.3 (b) 0.4 (c) 0.6 (d) 0.8

4. A bird, sitting on the floor of an airtight box which is being carried by a boy, starts flying. The boy feels that the box now
   (a) is heavier
   (b) is lighter
   (c) shows no change in weight
   (d) is lighter in the beginning and heavier later.

5. Two solids A and B float in water. A float with half its volume immersed and B floats with 2/3rd of its volume immersed. The densities of A and B are in the ratio
   (a) 2 : 3 (b) 4 : 3 (c) 3 : 4 (d) 3 : 2

6. Two pieces of different metals, when completely immersed in water, experience equal upthrust. Then
   (a) both pieces have equal weights in air
   (b) both pieces have the same density
   (c) both pieces have equal volumes
   (d) both are immersed to the same depth.

7. The pressure in a water tap at the base of a building is 3×10^6 dynes/cm^2 and on its top it is 1.6×10^6 dynes/cm^2. The height of the building is approximately
   (a) 7 m (b) 14 m (c) 70 m (d) 140 m

8. The weight of a body in air is 100 N. How much will it weigh in water, if it displaces 400 cc of water?
   (a) 90 N (b) 94 N (c) 98 N (d) none of these.

9. A solid weighs 5 N in air, 4 N in water and 4.5 N in some other liquid. The specific gravity of the liquid is
   (a) 0.5 (b) 1.5 (c) 0.9 (d) None of these

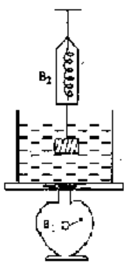
10. A solid weighs 30 N when dipped in water and weighs 40 N in air. The density of the solid is
    (a) 4 kg/m^3 (b) 4000 kg/m^3
    (c) 4/3 kg/m^3 (d) 4000/3 kg/m^3

11. The density of a block of wood that floats in water with 10% of its volume outside water is (density of water = 10 kg/m^3)
    (a) 10 kg/m^3 (b) 90 kg/m^3
    (c) 900 kg/m^3 (d) none of these

12. The specific gravity of ice is 0.9. The area of the smallest slab of ice of height 0.5 m, floating in fresh water that will just support a 100 kg man is
    (a) 1.5m^2 (b) 2.0 m^2 (c) 3.0m^2 (d) 4.0 m^2

13. A vessel contains oil (density 0.8 g/cm^3) over mercury (density 13.6 g/cm^3). A homogeneous sphere floats with half its volume immersed in mercury and the other half in oil. The density of the material of the sphere in g/cm^3 is
    (a) 3.3 (b) 6.4 (c) 7.2 (d) 12.8

14. A beaker containing water is kept on a spring balance B_1. The weight of beaker and water is 5 kg. A piece of iron (specific gravity 7.5) weighing 1.5 kg is hung from a spring balance B_2. If the iron piece is lowered in water till it is fully immersed but does not touch the bottom of the beaker, the readings of B_1 and B_2 will be respectively



    (a) 5.0 kg, 1.3 kg (b) 5.2 kg, 1.5kg
    (c) 5.2 kg, 1.3kg (d) 5.0 kg, 1.5 kg

15. A body is floating in a liquid. The upthrust on the body is
    (a) equal to the weight of the liquid displaced

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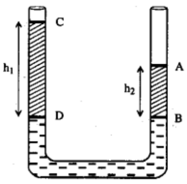
(b) zero

(c) less than the weight of the liquid displaced

(d) equal to the weight of the body minus the weight of the liquid displaced.

16. In a U tube experiment, a column AB of water is balanced by a column CD of paraffin, as shown in the figure. The relative density of paraffin is



(a) h_2/h_1 (b) h_1/h_2

(c) $(h_1-h_2)/h_1$ (d) $h_2/(h_1+h_2)$

17. An open vessel containing water is given a constant acceleration a in the horizontal direction. Then the free surface of water gets sloped with the horizontal at an angle given by

(a) $\tan^{-1}\frac{a}{g}$ (b) $\tan^{-1}\frac{g}{a}$

(c) $\tan^{-1}\frac{a}{\sqrt{a^2+g^2}}$ (d) $\tan^{-1}\frac{g}{\sqrt{a^2+g^2}}$

18. A wooden block of mass 8 kg is tied to a string attached to the bottom of a water tank and is completely immersed in water in equilibrium. If the relative density of wood is 0.8, then the tension in the string is (g = 10 m/s^2 ; density of water g = 10^3 kg/m^3)

(a) 20 N (b) 80 N (c) 100 N (d) 120 N

19. A ball floats on the surface of water in a container exposed to the atmosphere. If the container is now covered and the air is pumped out, then

(a) the ball will remain at its former depth

(b) the ball will rise a little

(c) the ball will sink a little

(d) the ball will sink completely.

20. When a fluid passes through the constricted part of a pipe, its

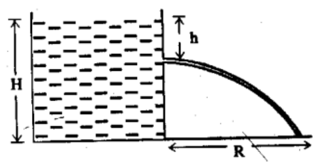
(a) velocity and pressure decrease

(b) velocity and pressure increase

(c) velocity decreases and pressure increases

(d) velocity increases and pressure decreases.

21. Water stands at a height H in a large tank whose sides are vertical. A hole is made in one of the walls of the tank at a depth h below the surface of water. The distance R from the foot of the wall where the emerging stream of water strikes the floor is



(a) $\sqrt{h(H-h)}$ (b) $\sqrt{hH}$

(c) $2\sqrt{h(H-h)}$ (d) $2\sqrt{hH}$

22. Two large tanks a and b, open at the top, contain different liquids. A small hole is made in the side of each tank at the same depth h below the liquid surface, but the hole in a has twice the area of the hole in b. The ratio of the densities of the liquids in a and b so that the mass flux is the same for each hole should be

(a) 2 (b) 0.5 (c) 4 (d) 0.25

23. Water is flowing through a tube of non-uniform cross-section. If the radii of the tube at the entrance and the exit are in the ratio 3 : 2, then the ratio of the velocities of flow of water at the entrance and the exit is

(a) 9 : 4 (b) 4 : 9 (c) 8 : 27 (d) 27 : 8.

24. It temperature rises the coefficient of viscosity of a liquid

(a) decreases

(b) increases

(c) remains unchanged

(d) increases for some liquids and decreases for others.

25. Under a pressure head the rate of orderly volume flow of a liquid through a capillary tube is Q. If the length of the capillary tube is doubled and the diameter of the bore is halved, the rate of flow would become

(a) Q/32 (b) Q/8 (c) Q/4 (d) 8 Q

26. A steel ball of radius 2 mm acquires a terminal velocity of 20 cm/s in a liquid. The terminal velocity of another steel ball of radius 1 mm in the same liquid will be

(a) 5 cm/s (b) 10 cm/s (c) 40 cm/s (d) 80 cm/s

27. The rate of steady volume flow of water through a capillary tube of length l and radius r, under a pressure difference p, is V. What is the rate of steady flow through a parallel combination of this tube with another tube of the same length and half the radius if the same pressure difference p is maintained across combination ?

(a) V/16 (b) V/17 (c) 16V/17 (d) 17V/16

28. Two capillary tubes of the same length and radii r_1 and r_2 are fitted horizontally side by side to the bottom of a vessel containing water. The radius of a single tube that can replace the two tubes such that the rate of steady flow through this tube equals the combined rate of flow through the two tubes, is

(a) $r_1 + r_2$ (b) $(r_1r_2)^{1/2}$

(c) $(r_1^2 + r_2^2)^{1/2}$ (d) $(r_1^4 + r_2^4)^{1/4}$

29. A small sphere is dropped into a medium of infinite extent. As the sphere falls, the net force acting on it

(a) remains constant throughout

(b) decreases for some time and then becomes constant

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(c) increases for some time and then becomes constant

(d) decreases for some time and then becomes zero.

30. Two drops of equal size are falling through air with a constant speed of 10 cm/s. If the drops coalesce, the new constant speed will be

(a) 20 cm/s (b) $10\sqrt{2}$ cm/s

(c) $10 \times 2^{1/3}$ cm/s (d) $10 \times 2^{2/3}$ cm/s

31. There is a 1 mm thick layer of glycerine between a flat plate of area 100 cm^2 and a big plate. If the coefficient of viscosity of glycerine is 1.0 kg/(m-s), then the force required to move the plate with a velocity of 7 cm/s is

(a) 0.35 N (b) 0.7N (c) 1.05N (d) 1.4N

32. The terminal velocity of a small sized spherical body of radius r falling in a viscous liquid is proportional to

(a) $1/r^2$ (b) $1/r$ (c) r (d) r^2

ANSWERS

1b ,2d ,3c ,4c ,5c ,6c ,7b ,8d ,9a ,10b ,11c ,12b ,13c ,14c ,15a ,16a ,17a ,18a ,19c ,20d ,21c ,22b ,23b ,24a ,25a ,26a ,27d ,28d ,29d ,30d ,31b ,32d

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