ELASTICITY

GUPTA CLASSES

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Elasticity

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Elasticity of a material is its property by virtue of which it resists change in shape and size when deforming forces are applied on it and regain its original shape and size when deforming forces are removed.

When the body regains its original shape and size completely after the removal of deforming forces then the body is said to be perfectly elastic, e.g.. Quartz fibre.

If the body does not have any tendency to recover its original shape and size, the body is said to be perfectly plastic, e.g., plasticine.

Stress

When external forces are exerted on a body, the body gets distorted, i.e., different portions of the body move relative to each other. Due to these displacements atomic forces (restoring forces) are set up inside the body to restore the original form. The restoring force per unit area set up inside the body is called stress. This is measured by the magnitude of deforming force per unit area of the body. If F is the force applied to an area of cross-section A, then stress = (F/A).

When the stress is applied normal to a surface. This is called as **normal stress**. It produces a change in length of a wire or a change in volume of a body. The normal stress to a wire or body may be compressive or tensile according as it produces a decrease or increase in the length of a wire or volume of the body.

When the stress is applied tangential to a surface, it is called tangential or shearing stress.

Strain

The deforming forces acting on a body cause a relative displacement of its various parts and the body is then said to be strained. Either a change in length or a change, in volume or change in shape occurs. The relative change produced in the body due to the influence of external forces is called strain.

The change in length per unit length is called as **linear strain**.

The change in volume per unit volume is called as **volume strain**.

If there is a change in shape, the strain is called **shearing strain** or shear. It is measured by the angle through which a line originally perpendicular to the fixed surface is turned.

Hooke's law and moduli of elasticity

According to Hooke's law, within elastic limit, stress applied to a body is directly proportional to corresponding strain, i.e.,

$$\text{Stress} \propto \text{Strain or Stress/Strain} = \text{constant} = E$$

The constant of proportionality E is known as modulus of elasticity.

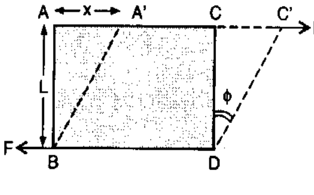
1. **Young's modulus Y:** If a rod or wire of length L and cross section area A under the action of a stretching force F applied normally to its face suffers an increase ΔL in length, then in equilibrium,

$$Y = \frac{\text{linear stress}}{\text{longitudinal strain}} = \frac{F/A}{\Delta L/L} = \frac{F/L}{A\Delta L}$$

2. **Bulk modulus B:** When a solid or fluid (liquid or gas) is subjected to change in pressure its volume changes, but the shape remains unchanged. The pressure gives the stress and the change in volume per unit volume strain. Within elastic limits, $B = \frac{\text{volume stress}}{\text{volume strain}} = \frac{\Delta P}{-\Delta V/V} = -\frac{V\Delta P}{\Delta V}$

   The reciprocal of bulk modulus is called compressibility. All the states of matter possess volume elasticity. Bulk modulus of gases is very low while that of liquids and solids is very high.

3. **Modulus of rigidity (η) :** Consider a cube of material fixed at the lower face and acted upon by a tangential force F at its upper surface having area A. The shearing stress then, will be Shearing stress =F/A



   This shearing force causes the consecutive horizontal layers of the cube to be slightly displaced or sheared relative to one another; each line such as AB or CD in the cube rotates through an angle ϕ by this shear. The

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shearing strain is defined as the angle ϕ in radians through which a line normal to a fixed surface has turned for small values of angle. Shearing stress $(\phi) = \frac{AA'}{AB} = \frac{x}{L}$ $\therefore \eta = \frac{\text{shearing stress}}{\text{shearing strain}} = \frac{F/A}{\phi} = \frac{F}{A\phi}$

Shearing is exhibited by solids only as these have definite shape.

Poisson's ratio

When longitudinal force is applied on a wire its length increases but its radius decreases. Thus two strains are produced by a single force.

(i) Longitudinal strain = $\Delta L/L$ (ii) Lateral strain = $\Delta R/R$

Poisson discovered that within limits of proportionality, the ratio of the lateral strain to the longitudinal strain is constant for a given material. This constant is called as Poisson's ratio and is represented by σ:

$$\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{-(\Delta R/R)}{(\Delta L/L)}$$

(i) σ has no units and dimensions. (ii) Theoretically, σ lies between -1 and + ½ .

(iii) Practically, no substance has been found for which σ is negative, i.e., practically σ lies between zero and + ½.

Some important points concerning the moduli of elasticity (Y, B and η)

1. The value of moduli of elasticity is independent of the magnitude of the stress and strain. It depends only on the nature of the material of the body. Greater the value of modulus of elasticity, more elastic is the material.
2. The moduli of elasticity Y and η exist only for solids as liquids and gases cannot be deformed along one dimension only and also cannot sustain shear strain. However, B exists for all states of matter, viz; solid, liquid or gas.
3. Gases being most compressible are least elastic while solids are most, i.e., the bulk modulus of gases is very low while that for liquids and solids is very high, i.e., $E_{\text{solid}} > E_{\text{liquid}} > E_{\text{gas}}$
4. Gases have two bulk moduli, namely. Isothermal elasticity E_θ and adiabatic elasticity E_ϕ. It can be easily proved that at a given pressure P, E_θ = P and $E_\phi = \gamma P$

   So that $\frac{E_\phi}{E_\theta} = \gamma = \frac{C_p}{C_v} > 1$ i.e. $E_\phi > E_\theta$
5. With rise in temperature, the distance between atoms increases and so the elastic restoring force will decrease. This in turn decreases the elasticity, i.e., with rise in temperature, Y, B and η decrease.

Elongation in a wire by its own weight

In case of elongation by its own weight, F (= Mg) will act at centre of gravity of the wire, so that length of wire which is stretched is (L/2). $\therefore$ $\Delta L = \frac{Mg(L/2)}{AY} = \frac{MgL}{2AY} = \frac{\rho g L^2}{2Y}$

Thermal stress

If a rod is fixed between two supports, due to change in temperature its length will change and so it will exert a normal stress (compressive, if temperature increases and tensile, if temperature decreases) on the supports. This stress is called thermal stress. Thermal stress = $Y\alpha\, d\theta$

Work done in stretching a wire

In stretching a wire, work is done against internal restoring forces. This work is stored in the body as elastic potential energy or strain energy.

Total work done in increasing the length by ΔL, $W = \frac{1}{2}\frac{YA}{L}(\Delta L)^2$

Work done per unit volume or energy density $u = \frac{1}{2}Y\left(\frac{\Delta L}{L}\right)^2 = \frac{1}{2}Y(\text{strain})^2 = \frac{1}{2} \times \text{stress} \times \text{strain}$

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Bending of Beam

In case of bending of a beam of length L, breadth b and thickness d, by a load Mg at the middle, depression δ is given by : $\delta = \dfrac{MsL^3}{4bd^3Y}$ or $\dfrac{MgL^3}{3YI}$ (I = moment of inertia)

Twisting of cylinder

In case of twisting of a cylinder (or wire) of length L and radius r, elastic restoring couple per unit twist is given by: $C = \pi\eta r^4/2L$

Elastic after-effect

When a wire is loaded continuously then a state is reached when the wire does not return to its original state immediately after the removal of force. This temporary absence of elastic property in the wire is called elastic after-effect.

Effect of hammering and rolling on elasticity

Operations like hammering, rolling, etc., which help in breaking up the crystal grains into smaller units, result in an increase of elastic property of substances.

Effect of change of temperature on elasticity

A rise of temperature generally decreases the elastic properties of materials.

Elasticity & Surface Tension Assignment

1. Two wires of the same material have diameters in the ratio 2 : 1 and lengths in the ratio 1 : 2. If they are stretched by the same force, their elongations will be in the ratio

   (a) 8 : 1 (b) 1 : 8 (c) 2 : 1 (d) 1 : 4

2. The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied

   (a) length = 50 cm, diameter = 0.5 mm

   (b) length = 100 cm, diameter = 1 mm

   (c) length = 200 cm, diameter = 2 mm

   (d) length = 300 cm, diameter = 3 mm.

3. The extension produced in a wire by the application of a load is 3.0 mm. The extension in a wire of the same material and length but half the radius, by the same load, is

   (a) 0.75 mm (b) 1.5 mm (c) 6.0mm (d) 12.0 mm.

4. Young's modulus of steel is 2×10^{11} N/m^2. A steel wire has a length of 1 m and area of cross section 1 mm^2. The work required to increase its length by 1 mm is

   (a) 0.1 J (b) 1 J (c) 10 J (d) 100 J.

5. A substance breaks down by a stress of 10^6N/m^2. If the density of the material of the wire is 3×10^3 kg/m^3, then the length of the wire of that substance which will break under its own weight when suspended vertically is

   (a) 3.4m (b) 34 m

   (c) 340 m (d) none of these.

6. If a rubber ball is taken down to a 100 m deep lake, its volume decreases by 0.1%. If g = 10 m/s^2 then the bulk modulus of elasticity for rubber, in N/m^2, is

   (a) 10^8 (b) 10^9 (c) 10^{11} (d) 10^{10}

7. A wire can support a load W without breaking. It is cut into two equal parts. The maximum load that each part can support is

   (a) W/4 (b) W/2 (c) W (d) 2W

8. If the work done in stretching a wire by 1 mm is 2J, the work necessary for stretching another wire of the same material but double the radius and half the length by 1 mm is

   (a) 16 J (b) 8 J (c) 4J (d) (1/4)J

9. Two rods A and B of the same material and length have radii r_1 and r_2, respectively. When they are rigidly fixed at one end and twisted by the same couple applied at the other end, the ratio of their angle of twist is

   (a) r_1^2/r_2^2 (b) r_1^3/r_2^4

   (c) r_2^4/r_1^4 (d) r_1^4/r_2^4

10. An aluminium wire has a breaking strain 0.2 %. Young's modulus of aluminium is 7.0×10^9 N/m^2.

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If the wire can support a maximum load of 10^4 N, its area of cross section is

(a) 7.1 x 10^{-3} m^2 (b) 7.1 x 10^{-4} m^2

(c) 3.5 x 10^{-3} m^2 (d) 3.5 x 10^{-4} m^2

11. Which of the following is most elastic?

(a) Rubber (b) Wet clay

(c) Plastic (d) Steel

12. The compressibility of water is 4 x 10^{-5} per unit atmospheric pressure. The decrease in volume of 100 cm^3 of water under a pressure of 100 atmosphere will be

(a) 0.4 cm^3 (b) 4 x 10^{-5} cm^3

(c) 0.025 cm^3 (d) 0.004 cm^3

13. A copper wire and a steel wire of the same diameter and length are connected end to end and a force is applied which stretches their combined length by 1 cm, Then the two wires have

(a) the same stress and strain.

(b) the same stress but different strains.

(c) the same strain but different stresses.

(d) different stresses and strains.

14. A soap bubble (surface tension 30 dynes/cm) has radius 2 cm. The work done in doubling the radius is

(a) 4525 ergs (b) 2262 ergs

(c) 1130 ergs (d) 9050 ergs.

15. Water rises to a height of 2 cm in a capillary tube held vertically. When the tube is tilted 60° from the vertical, the length of the water column in the tube will be

(a) 1.0 cm (b) 2.0 cm (c) 3.0 cm (d) 4.0 cm

16. A long capillary tube with both ends open is filled with water and set in a vertical position. The radius of the capillary is 0.1 cm and surface tension of water is 70 dynes/cm. The length of the water column remaining in the capillary tube will be approximately

(a) 1.5 cm (b) 2.9 cm (c) 3.6 cm (d) 5.8 cm

17. The excess pressure inside one soap bubble is three times that inside a second. The volumes of the two bubbles are in the ratio

(a) l : 9 (b) 9 : 1 (c) 1 : 27 (d) 27 : 1

18. In a capillary tube water rises to a height of 5 cm. If the area of Cross-section of the tube were one-fourth, water would have risen to a height

(a) 2.5 cm (b) 5 cm (c) 7.5 cm (d) 10 cm

19. Two spherical soap bubbles of radii r_1 and r_2 in vacuum coalesce under isothermal conditions. The resulting bubble has a radius equal to

(a) $\frac{r_1 + r_2}{2}$ (b) $\frac{r_1 r_2}{r_1 + r_2}$

(c) $\sqrt{r_1 r_2}$ (d) $\sqrt{r_1^2 + r_2^2}$

20. A mercury drop of radius 1 cm is sprayed into 10^6 drop of equal size. If the surface tension of mercury is 35 x 10^{-3} N/m. the energy expended is

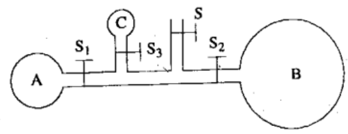
(a) 4.35 x 10^{-3} J (b) 8.7 x 10^{-3} J

(c) 4.35 x 10^{-2} J (d) 8.7 x 10^{-2} J

21. A long cylindrical glass vessel has a small hole of radius r at its bottom. The depth to which the vessel can be lowered vertically in a deep water bath (surface tension T, density d) without any water entering inside is

(a) T/rdg (b) 2T/rdg (c) 3T/rdg (d) 4T/rdg

22. The given figure shows three soap bubbles A, B and C prepared by blowing the capillary tube having stop cocks S, S_1, S_2 and S_3. If S is closed and S_1, S_2, S_3 remain open then



(a) A and C will both start collapsing with volume of B increasing

(b) B will start collapsing with volumes of A and C increasing

(c) Volumes of A, B and C will become equal

(d) C will start collapsing with volumes of A and B increasing.

23. A capillary tube is immersed in water and the mass of water that rises in the capillary tube is M, If another tube of double the radius is immersed then the mass of water that will rise in the capillary is

(a) M (b) 2M (c) 4 M (d) M/2

24. The force necessary to pull a circular plate of radius 5 cm from the surface of water (surface tension 75 dynes/cm) is

(a) 30 dynes (b) 60 dynes

(c) 750 dynes (d) 750π dynes

ANSWERS:

1b ,2a ,3d ,4a ,5b ,6b ,7c ,8a ,9c ,10b ,11d ,12a ,13b ,14d ,15d ,16b ,17c ,18d ,19d ,20a ,21b ,22a ,23b ,24d

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