FIITJEE Solutions to JEE(MAIN)-2013

XYZ

PAPER – 1 : CHEMISTRY, MATHEMATICS & PHYSICS

Test Booklet Code

Q

Read carefully the Instructions on the Back Cover of this Test Booklet.

Important Instructions:

1. Immediately fill in the particulars on this page of the Test Booklet with *Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.*
2. The test is of **3 hours duration.**
3. The Test Booklet consists of **90** questions. The maximum marks are **360.**
4. There are ***three*** parts in the question paper A, B, C consisting of **Chemistry, Mathematics** and **Physics** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for correct response.
5. *Candidates will be awarded marks as stated above in instruction No. 4 for correct response of each question.* (1/4) *(one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
6. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 5 above.

PART A –CHEMISTRY

1. An unknown alcohol is treated with the "Lucas reagent" to determine whether the alcohol is primary, secondary or tertiary. Which alcohol reacts fastest and by what mechanism:
(1) tertiary alcohol by S_N1 (2) secondary alcohol by S_N2
(3) tertiary alcohol by S_N2 (4) secondary alcohol by S_N1

Sol. **(1)**
Reaction proceeds through carbocation formation as 3^0 carbocation is highly stable, hence reaction proceeds through S_N1 with 3^0 alcohol.

2. The first ionization potential of Na is 5.1 eV. The value of electron gain enthalpy of Na^+ will be:
(1) – 5.1 eV (3) –10.2 eV
(3) + 2.55 eV (4) – 2.55 eV

Sol. **(1)**
$Na \underset{\Delta H=-5.1eV}{\overset{\Delta H=+5.1eV}{\rightleftharpoons}} Na^+ + e^-$, here the backward reaction releases same amount of energy and known as Electron gain enthalpy.

3. Stability of the species Li_2, Li_2^- and Li_2^+ increases in the order of:
(1) $Li_2^- < Li_2^+ < Li_2$ (3) $Li_2 < Li_2^- < Li_2^+$
(3) $Li_2^- < Li_2 < Li_2^+$ (4) $Li_2 < Li_2^+ < Li_2^-$

Sol. **(1)**

$Li_2(6) = \sigma 1s^2\ \overset{*}{\sigma}1s^2\ \sigma 2s^2$

B.O. = $\dfrac{4-2}{2} = 1$

$Li_2^+(5) = \sigma 1s^2\ \overset{*}{\sigma}1s^2 \sigma 2s^1$

B. O. $= \dfrac{3-2}{2} = 0.5$

$Li_2^-(7) = \sigma 1s^2\ \overset{*}{\sigma}1s^2 \sigma 2s^2\ \overset{*}{\sigma}2s^1$

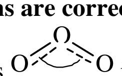
B.O. $= \dfrac{4-3}{2} = 0.5$

Li_2^+ is more stable than Li_2^- because Li_2^- has more numbers of antibonding electrons.

4. The molarity of a solution obtained by mixing 750 mL of 0.5 (M) HCl with 250 mL of 2(M)HCl will be:
(1) 1.00 M (2) 1. 75 M
(3) 0.975 M (4) 0.875 M

Sol. **(4)**
$M_1V_1 + M_2V_2 = MV$
$M = \dfrac{M_1V_1 + M_2V_2}{V} = \dfrac{0.5 \times 750 + 2 \times 250}{1000}$
M = 0. 875

5. Which of the following is the wrong statement?
(1) O_3 molecule is bent (2) Ozone is violet-black in solid state
(3) Ozone is diamagnetic gas (4) ONCl and ONO^- are not isoelectronic

Sol. **(All the options are correct statements)**
(1) Correct, as  is bent.

(2) Correct, as ozone is violet-black solid.
(3) Correct, as ozone is diamagnetic.
(4) Correct, as ONCl = 32 electrons and ONO^- = 24 electron hence are not isoelectronic.
All options are correct statements.

6. Four successive members of the first row transition elements are listed below with atomic numbers. Which one of them is expected to have the highest $E^0_{M^{3+}/M^{2+}}$ value?

(1) Mn(Z = 25) (2) Fe(Z = 26)
(3) Co(Z = 27) (4) Cr(Z = 24)

Sol. **(3)**

$E^0_{Mn^{+3}/Mn^{+2}} = 1.57\ V$

$E^0_{Fe^{+3}/Fe^{+2}} = 0.77\ V$

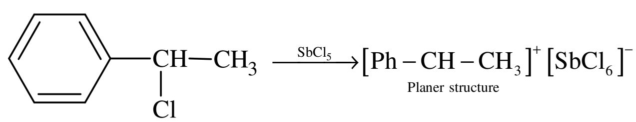
$E^0_{Co^{+3}/Co^{+2}} = 1.97\ V$

$E^0_{Cr^{+3}/Cr^{+2}} = -0.41\ V$

7. A solution of (–) –1 – chloro –1 – phenylethane is toluene racemises slowly in the presence of a small amount of $SbCl_5$, due to the formation of :

(1) carbene (2) carbocation
(3) free radical (4) carbanion

Sol. **(2)**



8. The coagulating power of electrolytes having ions Na^+, Al^{3+} and Ba^{2+} for arsenic sulphide sol increases in the order:

(1) $Na^+ < Ba^{2+} < Al^{3+}$ (2) $Ba^{2+} < Na^+ < Al^{3+}$
(3) $Al^{3+} < Na^+ < Ba^{2+}$ (4) $Al^{3+} < Ba^{2+} < Na^+$

Sol. **(1)**

As_2S_3 is an anionic sol (negative sol) hence coagulation will depend upon coagulating power of cation, which is directly proportional to the valency of cation (Hardy-Schulze rule).

9. How many litres of water must be added to 1 litre of an aqueous solution of HCl with a pH of 1 to create an aqueous solution with pH of 2?

(1) 0.9 L (2) 2.0 L
(3) 9.0 L (4) 0.1 L

Sol. **(3)**

Initial pH = 1, i.e. $[H^+] = 0.1$ mole/litre
New pH = 2, i.e. $[H^+] = 0.01$ mole/litre
In case of dilution: $M_1V_1 = M_2V_2$
$0.1 \times 1 = 0.01 \times V_2$
$V_2 = 10$ litre.
Volume of water added = 9 litre.

10. Which one of the following molecules is expected to exhibit diamagnetic behaviour?

(1) N_2 (2) O_2
(3) S_2 (4) C_2

Sol. **(1) & (4) both are correct answers.**

$N_2 \rightarrow$ Diamagnetic
$O_2 \rightarrow$ Paramagnetic
$S_2 \rightarrow$ Paramagnetic

$C_2 \rightarrow$ Diamagnetic

11. Which of the following arrangements does **not** represent the correct order of the property stated against it ?
(1) $Ni^{2+} < Co^{2+} < Fe^{2+} < Mn^{2+}$: ionic size
(2) $Co^{3+} < Fe^{3+} < Cr^{3+} < Sc^{3+}$: stability in aqueous solution
(3) $Sc < Ti < Cr < Mn$: number of oxidation states
(4) $V^{2+} < Cr^{2+} < Mn^{2+} < Fe^{2+}$: paramagnetic behaviour

Sol. **(2) & (4) both are correct answers)**
The exothermic hydration enthalpies of the given trivalent cations are:
Sc^{+3} = 3960 kJ/mole
Fe^{+3} = 4429 kJ/mole
Co^{+3} = 4653 kJ/mole
Cr^{+3} = 4563 kJ/mole
Hence Sc^{+3} is least hydrated; so least stable (not most stable)
Fe^{+2} contains 4 unpaired electrons where as Mn^{+2} contains 5 unpaired electrons hence (4) is incorrect.

12. Experimentally it was found that a metal oxide has formula $M_{0.98}O$. Metal M, is present as M^{2+} and M^{3+} in its oxide. Fraction of the metal which exists as M^{3+} would be:
(1) 4.08% (2) 6.05%
(3) 5.08% (4) 7.01%

Sol. **(1)**
Metal oxide = $M_{0.98}O$
If 'x' ions of M are in +3 state, then
$3x + (0.98 - x) \times 2 = 2$
$x = 0.04$
So the percentage of metal in +3 state would be $\frac{0.04}{0.98} \times 100 = 4.08\%$

13. A compound with molecular mass 180 is acylated with CH_3COCl to get a compound with molecular mass 390. The number of amino groups present per molecule of the former compound is:
(1) 5 (2) 4
(3) 6 (4) 2

Sol. **(1)**

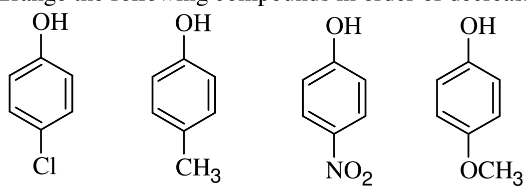
$$R-NH_2 + CH_3-\overset{\overset{\large O}{\|}}{C}-Cl \longrightarrow R-NH-\overset{\overset{\large O}{\|}}{C}-CH_3 + HCl$$

Each $CH_3-\overset{\overset{O}{\|}}{C}$ addition increases the molecular wt. by 42.
Total increase in m.wt. = 390 – 180 = 210
Then number of NH_2 groups $= \frac{210}{42} = 5$

14. Given
$E^0_{Cr^{3+}/Cr} = -0.74$ V; $E^0_{MnO_4^-/Mn^{2+}} = 1.51$ V
$E^0_{Cr_2O_7^{2-}/Cr^{3+}} = 1.33$ V; $E^0_{Cl/Cl^-} = 1.36$ V
Based on the data given above, strongest oxidising agent will be:
(1) Cr^{3+} (2) Mn^{2+}
(3) MnO_4^- (4) Cl^-

Sol. **(3)**
As per data mentioned
MnO_4^- is strongest oxidising agent as it has maximum SRP value.

15. Arrange the following compounds in order of decreasing acidity:



(I) (II) (III) (IV)

(1) I > II > III > IV
(2) III > I > II > IV
(3) IV > III > I > II
(4) II > IV > I > III

Sol. **(2)**

Correct order of acidic strength is III > I > II > IV

16. The rate of a reaction doubles when its temperature changes from 300K to 310K. Activation energy of such a reaction will be:
($R = 8.314\ JK^{-1}\ mol^{-1}$ and log 2 = 0.301)

(1) $48.6\ kJ\ mol^{-1}$
(2) $58.5\ kJ\ mol^{-1}$
(3) $60.5\ kJ\ mol^{-1}$
(4) $53.6\ kJ\ mol^{-1}$

Sol. **(4)**

As per Arrhenius equation:

$$\ln\frac{k_2}{k_1} = -\frac{E_a}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

$$2.303\log 2 = -\frac{E_a}{8.314}\left(\frac{1}{310} - \frac{1}{300}\right)$$

$\Rightarrow E_a = 53.6$ kJ/mole

17. Synthesis of each molecule of glucose in photosynthesis involves:

(1) 10 molecules of ATP
(2) 8 molecules of ATP
(3) 6 molecules of ATP
(4) 18 molecules of ATP

Sol. **(4)**

$$12H_2O + 12NADP + 18ADP \xrightarrow{\text{Light reaction}} 6O_2 + 18ATP + 12NADPH$$

$$6CO_2 + 12NADPH + 18ATP \xrightarrow{\text{Dark reaction}} C_6H_{12}O_6 + 12NADP + 18ADP + 6H_2O$$

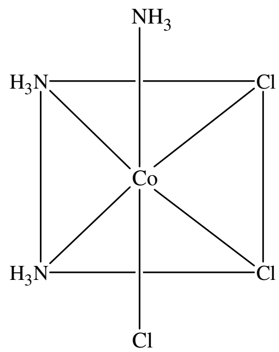
Net reaction: $6CO_2 + 6H_2O \longrightarrow C_6H_{12}O_6 + 6O_2$

18. Which of the following complex species is not expected to exhibit optical isomerism?

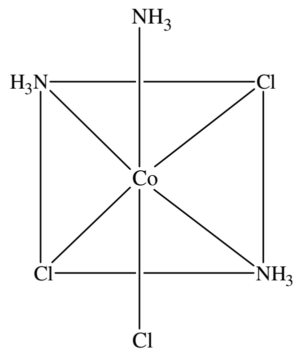
(1) $\left[Co(en)_2Cl_2\right]^+$
(2) $\left[Co(NH_3)_3Cl_3\right]$
(3) $\left[Co(en)(NH_3)_2Cl_2\right]^+$
(4) $\left[Co(en)_3\right]^{3+}$

Sol. **(2)**

$[Co(NH_3)_3Cl_3]$ exists in two forms (facial and meridonial)



facial



meridonial

Both of these forms are achiral. Hence, $[Co(NH_3)_3Cl_3]$ does not show optical isomerism.

19. A piston filled with 0.04 mol of an ideal gas expands reversibly from 50.0 mL to 375 mL at a constant temperature of 37.0^0C. As it does so, it absorbs 208J of heat. The values of q and w for the process will be: (R = 8.314 J/mol K) ($\ell n\ 7.5 = 2.01$)

(1) q = – 208 J, w = – 208 J (2) q = – 208 J, w = + 208 J
(3) q = + 208 J, w = + 208 J (4) q = + 208 J, w = – 208 J

Sol. **(4)**
Process is isothermal reversible expansion, hence $\Delta U = 0$.
$\therefore$ q = –W
As q = +208 J
Hence W = –208 J

20. A gaseous hydrocarbon gives upon combustion 0.72 g of water and 3.08 g of CO_2. The empirical formula of the hydrocarbon is:

(1) C_3H_4 (2) C_6H_5
(3) C_7H_8 (4) C_2H_4

Sol. **(3)**

$$C_xH_y + \left(x + \frac{y}{4}\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$$

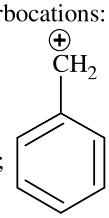
Weight (g)	3.08 g	0.72 g
moles	0.07	0.04

$$\frac{x}{y/2} = \frac{0.07}{0.04}$$

$$\Rightarrow \frac{x}{y} = \frac{7}{8}$$

21. The order of stability of the following carbocations:

$H_2C{=}CH{-}\overset{\oplus}{C}H_2$; $H_3C{-}CH_2{-}\overset{\oplus}{C}H_2$;



I II III

is:

(1) II > III > I (2) I > II > III
(3) III > I > II (4) III > II > I

Sol. **(3)**
Order of stability is III > I > II.

(Stability ∝ extent of delocalization)

22. Which of the following represents the correct order of increasing first ionization enthalpy for Ca, Ba, S, Se and Ar?
(1) S < Se < Ca < Ba < Ar
(2) Ba < Ca < Se < S < Ar
(3) Ca < Ba < S < Se < Ar
(4) Ca < S < Ba < Se < Ar

Sol. **(2)**
Increasing order of first ionization enthalpy is
Ba < Ca < Se < S < Ar

23. For gaseous state, if most probable speed is denoted by C*, average speed by $\overline{C}$ and mean square speed by C, then for a large number of molecules the ratios of these speeds are:
(1) $C^*:\overline{C}:C = 1.128:1.225:1$
(2) $C^*:\overline{C}:C = 1:1.128:1.225$
(3) $C^*:\overline{C}:C = 1:1.125:1.128$
(4) $C^*:\overline{C}:C = 1.225:1.128:1$

Sol. **(2)**
$$C^* = \sqrt{\frac{2RT}{M}},\ \overline{C} = \sqrt{\frac{8RT}{\pi M}},\ C = \sqrt{\frac{3RT}{M}}$$

24. The gas leaked from a storage tank of the Union Carbide plant in Bhopal gas tragedy was:
(1) Methylamine
(2) Ammonia
(3) Phosgene
(4) Methylisocyanate

Sol. **(4)**
It was methyl isocyanate (CH_3NCO)

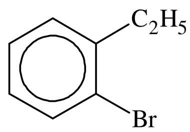
25. Consider the following reaction:
$$xMnO_4^- + yC_2O_4^{2-} + zH^+ \longrightarrow xMn^{2+} + 2yCO_2 + \frac{z}{2}H_2O$$
The values of x, y and z in the reaction are, respectively:
(1) 2, 5 and 8
(2) 2, 5 and 16
(3) 5, 2 and 8
(4) 5, 2 and 16

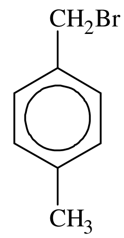
Sol. **(2)**
$$2MnO_4^- + 5C_2O_4^{-2} + 16H^+ \longrightarrow 2Mn^{+2} + 10CO_2 + 8H_2O$$
x = 2, y = 5, z = 16

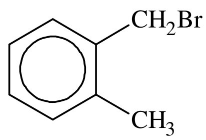
26. Which of the following exists as covalent crystals in the solid state?
(1) Silicon
(2) Sulphur
(3) Phosphorous
(4) Iodine

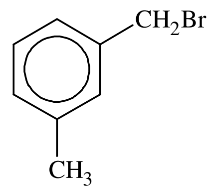
Sol. **(1)**
Silicon (Si) – covalent solid
Sulphur (S_8) – molecular solid
Phosphorous (P_4) – Molecular solid
Iodine (I_2) – Molecular solid

27. Compound (A), C_8H_9Br, gives a white precipitate when warmed with alcoholic $AgNO_3$. Oxidation of (A) gives a acid (B), $C_8H_6O_4$. (B) easily forms anhydride on heating. Identify the compound (A).

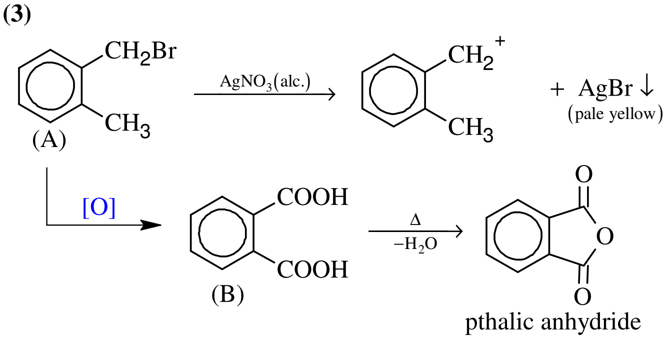
(1) 

(2) 

(3) 

(4) 

Sol. **(3)**



28. Energy of an electron is given by $E = -2.178 \times 10^{-18} J\left(\frac{Z^2}{n^2}\right)$. Wavelength of light required to excite an electron in an hydrogen atom from level n = 1 to n = 2 will be
($h = 6.62 \times 10^{-34}$ Js and $c = 3.0 \times 10^{8}\ ms^{-1}$)

(1) 2.816×10^{-7} m
(2) 6.500×10^{-7} m
(3) 8.500×10^{-7} m
(4) 1.214×10^{-7} m

Sol. **(4)**

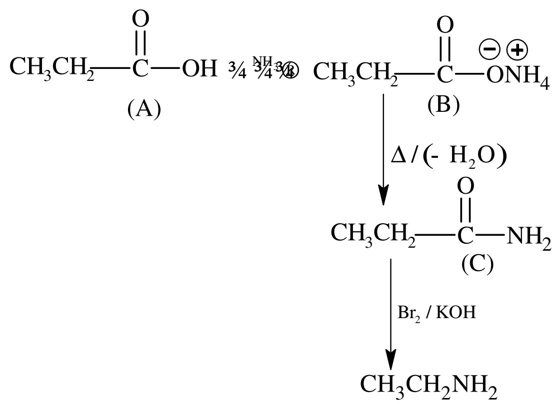
$$E = \frac{hc}{\lambda} = 2.178 \times 10^{-18} \times Z^2 \left[\frac{1}{1^2} - \frac{1}{2^2}\right]$$

$$\Rightarrow \lambda = 1.214 \times 10^{-7}\ m$$

29. An organic compound A upon reacting with NH_3 gives B. On heating B gives C. C in presence of KOH reacts with Br_2 to give $CH_3CH_2NH_2$. A is

(1) $CH_3CH_2CH_2COOH$
(2) $H_3C—\underset{\displaystyle CH_3}{\underset{|}{CH}}—COOH$
(3) CH_3CH_2COOH
(4) CH_3COOH

Sol. **(3)**



30. In which of the following pairs of molecules/ions, both the species are not likely to exist?

(1) H_2^-, He_2^{2-}
(2) H_2^{2+}, He_2
(3) H_2^-, He_2^{2+}
(4) H_2^+, He_2^{2-}

Sol. **(2)**

Bond order of H_2^{2+} and He_2 is zero, thus their existence is not possible.

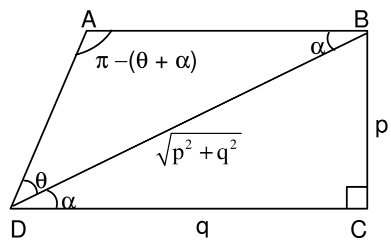
PART B – MATHEMATICS

31. The circle passing through (1, –2) and touching the axis of x at (3, 0) also passes through the point
(1) (2, –5) (2) (5, –2)
(3) (–2, 5) (4) (–5, 2)

Sol. **(2)**
$(x - 3)^2 + y^2 + \lambda y = 0$
The circle passes through (1, – 2)
$\Rightarrow 4 + 4 - 2\lambda = 0 \Rightarrow \lambda = 4$
$(x - 3)^2 + y^2 + 4y = 0 \Rightarrow$ Clearly (5, – 2) satisfies.

32. ABCD is a trapezium such that AB and CD are parallel and BC⊥CD. If $\angle ADB = \theta$, BC = p and CD = q, then AB is equal to
(1) $\dfrac{p^2 + q^2 \cos\theta}{p\cos\theta + q\sin\theta}$ (2) $\dfrac{p^2 + q^2}{p^2\cos\theta + q^2\sin\theta}$
(3) $\dfrac{(p^2 + q^2)\sin\theta}{(p\cos\theta + q\sin\theta)^2}$ (4) $\dfrac{(p^2 + q^2)\sin\theta}{p\cos\theta + q\sin\theta}$

Sol. **(4)**



Using sine rule in triangle ABD
$$\frac{AB}{\sin\theta} = \frac{BD}{\sin(\theta + \alpha)}$$
$$\Rightarrow AB = \frac{\sqrt{p^2 + q^2}\,\sin\theta}{\sin\theta\cos\alpha + \cos\theta\sin\alpha} = \frac{\sqrt{p^2 + q^2}\,\sin\theta}{\dfrac{\sin\theta \cdot q}{\sqrt{p^2 + q^2}} + \dfrac{\cos\theta \cdot p}{\sqrt{p^2 + q^2}}}$$
$$\Rightarrow AB = \frac{(p^2 + q^2)\sin\theta}{(p\cos\theta + q\sin\theta)}.$$

33. Given : A circle, $2x^2 + 2y^2 = 5$ and a parabola, $y^2 = 4\sqrt{5}\,x$.
Statement – I : An equation of a common tangent to these curves is $y = x + \sqrt{5}$.
Statement – II : If the line, $y = mx + \dfrac{\sqrt{5}}{m}$ $(m \neq 0)$ is their common tangent, then m satisfies $m^4 - 3m^2 + 2 = 0$.
(1) Statement - I is True; Statement -II is true; Statement-II is **not** a correct explanation for Statement-I
(2) Statement -I is True; Statement -II is False.
(3) Statement -I is False; Statement -II is True
(4) Statement -I is True; Statement -II is True; Statement-II is a **correct** explanation for Statement-I

Sol. **(1)**
Let the tangent to the parabola be $y = mx + \dfrac{\sqrt{5}}{m}$ $(m \neq 0)$.
Now, its distance from the centre of the circle must be equal to the radius of the circle.
So, $\left|\dfrac{\sqrt{5}}{m}\right| = \dfrac{\sqrt{5}}{\sqrt{2}}\sqrt{1 + m^2} \Rightarrow (1 + m^2)\,m^2 = 2 \Rightarrow m^4 + m^2 - 2 = 0.$
$\Rightarrow (m^2 - 1)(m^2 + 2) = 0 \Rightarrow m = \pm 1$
So, the common tangents are $y = x + \sqrt{5}$ and $y = -x - \sqrt{5}$.

34. A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected rays is

(1) $\sqrt{3}y = x - \sqrt{3}$ (2) $y = \sqrt{3}x - \sqrt{3}$
(3) $\sqrt{3}y = x - 1$ (4) $y = x + \sqrt{3}$

Sol. **(1)**
Slope of the incident ray is $-\frac{1}{\sqrt{3}}$.
So, the slope of the reflected ray must be $\frac{1}{\sqrt{3}}$.
The point of incidence is $\left(\sqrt{3},\ 0\right)$. So, the equation of reflected ray is $y = \frac{1}{\sqrt{3}}\left(x - \sqrt{3}\right)$.

35. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given ?
(1) median (2) mode
(3) variance (4) mean

Sol. **(3)**
Variance is not changed by the change of origin.
Alternate Solution:
$$\sigma = \sqrt{\frac{\sum\left|x - \bar{x}\right|^2}{n}} \text{ for } y = x + 10 \Rightarrow \bar{y} = \bar{x} + 10$$
$$\sigma_1 = \sqrt{\frac{\sum\left|y + 10 - \bar{y} - 10\right|^2}{n}} = \sqrt{\frac{\sum\left|y - \bar{y}\right|^2}{n}} = \sigma.$$

36. If x, y, z are in A.P. and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then
(1) $2x = 3y = 6z$ (2) $6x = 3y = 2z$
(3) $6x = 4y = 3z$ (4) $x = y = z$

Sol. **(4)**
If x, y, z are in A.P.
$2y = x + z$
and $\tan^{-1}x$, $\tan^{-1}y$, $\tan^{-1}z$ are in A.P.
$2\tan^{-1}y = \tan^{-1}x + \tan^{-1}z \Rightarrow x = y = z$.
Note: If y = 0, then none of the options is appropriate.

37. If $\int f(x)\,dx = \Psi(x)$, then $\int x^5 f\left(x^3\right)dx$ is equal to
(1) $\frac{1}{3}x^3\Psi\left(x^3\right) - 3\int x^3\Psi\left(x^3\right)dx + C$ (2) $\frac{1}{3}x^3\Psi\left(x^3\right) - \int x^2\Psi\left(x^3\right)dx + C$
(3) $\frac{1}{3}\left[x^3\Psi\left(x^3\right) - \int x^3\Psi\left(x^3\right)dx\right] + C$ (4) $\frac{1}{3}\left[x^3\Psi\left(x^3\right) - \int x^2\Psi\left(x^3\right)dx\right] + C$

Sol. **(2)**
$\int f(x)\,dx = \psi(x)$
Let $x^3 = t$
$3x^2dx = dt$
then $\int x^5 f\left(x^3\right)dx = \frac{1}{3}\int tf(t)\,dt$
$$= \frac{1}{3}\left[t\int f(t)dt - \int\left\{1\cdot\int f(t)\,dt\right\}dt\right] = \frac{1}{3}x^3\psi\left(x^3\right) - \int x^2\psi\left(x^3\right)dx + C.$$

38. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$, and having centre at (0, 3) is

(1) $x^2+y^2-6y+7=0$ (2) $x^2+y^2-6y-5=0$

(3) $x^2+y^2-6y+5=0$ (4) $x^2+y^2-6y-7=0$

Sol. **(4)**

foci ≡ (± ae, 0)

We have $a^2e^2 = a^2 - b^2 = 7$

Equation of circle $(x-0)^2 + (y-3)^2 = \left(\sqrt{7}-0\right)^2+\left(0-3\right)^2$

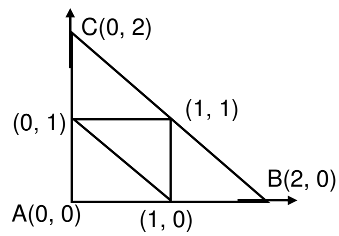
$\Rightarrow x^2 + y^2 - 6y - 7 = 0.$

39. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1) (1, 1) and (1, 0) is

(1) $2-\sqrt{2}$ (2) $1+\sqrt{2}$

(3) $1-\sqrt{2}$ (4) $2+\sqrt{2}$

Sol. **(1)**



x-coordinate = $\frac{ax_1+bx_2+cx_3}{a+b+c}$

$= \frac{2\times2+2\sqrt{2}\times0+2\times0}{2+2+2\sqrt{2}}$

$= \frac{4}{4+2\sqrt{2}} = \frac{2}{2+\sqrt{2}} = 2-\sqrt{2}.$

Alternate Solution:

x-coordinate = r = (s – a) tan A/2

$= \left(\frac{4+2\sqrt{2}}{2}-2\sqrt{2}\right)\tan\frac{\pi}{4} = 2-\sqrt{2}.$

40. The intercepts on x-axis made by tangents to the curve, $y=\int_0^x |t|\,dt$, $x \in R$, which are parallel to the line y = 2x, are equal to

(1) ± 2 (2) ± 3

(3) ± 4 (4) ± 1

Sol. **(4)**

$\frac{dy}{dx}=|x|=2 \Rightarrow x=\pm 2 \Rightarrow y=\int_0^2 |t|\,dt=2$ for x = 2

and $y = \int_0^{-2} |t|\,dt=-2$ for x = – 2

∴ tangents are y – 2 = 2 (x – 2) ⇒ y = 2x – 2

and y + 2 = 2 (x + 2) ⇒ y = 2x + 2

Putting y = 0, we get x = 1 and – 1.

41. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, ….. , is

(1) $\frac{7}{9}\left(99-10^{-20}\right)$ (2) $\frac{7}{81}\left(179+10^{-20}\right)$

(3) $\frac{7}{9}\left(99+10^{-20}\right)$ (4) $\frac{7}{81}\left(179-10^{-20}\right)$

Sol. **(2)**

t_r = 0.777 ….. r times

$= 7\,(10^{-1} + 10^{-2} + 10^{-3} + \ldots.. + 10^{-r})$

$= \frac{7}{9}\left(1-10^{-r}\right)$

$$S_{20} = \sum_{r=1}^{20} t_r = \frac{7}{9}\left(20-\sum_{r=1}^{20}10^{-r}\right) = \frac{7}{9}\left(20-\frac{1}{9}\left(1-10^{-20}\right)\right) = \frac{7}{81}\left(179+10^{-20}\right)$$

42. Consider :

Statement – I : $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a fallacy.

Statement – II : $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is a tautology.

(1) Statement - I is True; Statement -II is true; Statement-II is **not** a correct explanation for Statement-I

(2) Statement -I is True; Statement -II is False.

(3) Statement -I is False; Statement -II is True

(4) Statement -I is True; Statement -II is True; Statement-II is a **correct** explanation for Statement-I

Sol. **(1)**

S1:

p	q	~p	~q	p^~q	~p^q	(p^~q)^(~p^q)
T	T	F	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	F	F

Fallacy

S2:

p	q	~ p	~ q	$p \Rightarrow q$	$\sim q \Rightarrow \sim p$	$(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Tautology

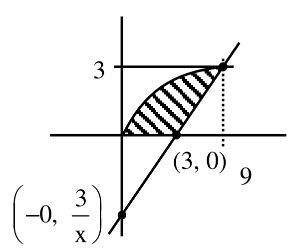
S2 is not an explanation of S1

43. The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x-axis, and lying in the first quadrant is

(1) 36 (2) 18

(3) $\frac{27}{4}$ (4) 9

Sol. **(4)**



$2\sqrt{x} = x - 3$

$4x = x^2 - 6x + 9$

$x^2 - 10x + 9$

$x = 9, x = 1$

$$\int_0^3 \left[(2y+3)-y^2\right]dy$$

$$\left[y^2+3y-\frac{y^3}{3}\right]_0^3 = 9 + 9 - 9 = 9$$

44. The expression $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$ can be written as

(1) secA cosecA + 1 (2) tanA + cotA

(3) secA + cosecA (4) sinA cosA + 1

Sol. **(1)**

$$\frac{1}{\cot A(1-\cot A)}-\frac{\cot^2 A}{(1-\cot A)}=\frac{1-\cot^3 A}{\cot A(1-\cot A)}=\frac{\operatorname{cosec}^2 A+\cot A}{\cot A} = 1 + \sec A \operatorname{cosec} A$$

45. The real number k for which the equation, $2x^3 + 3x + k = 0$ has two distinct real roots in [0, 1]

(1) lies between 2 and 3
(2) lies between −1 and 0
(3) does not exist
(4) lies between 1 and 2

Sol. **(3)**

If $2x^3 + 3x + k = 0$ has 2 distinct real roots in [0, 1], then $f'(x)$ will change sign

but $f'(x) = 6x^2 + 3 > 0$

So no value of k exists.

46. $\lim\limits_{x\to 0}\dfrac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to

(1) $\frac{1}{2}$
(2) 1
(3) 2
(4) $-\frac{1}{4}$

Sol. **(3)**

$$\lim_{x\to 0}\frac{(1-\cos 2x)}{x(\tan 4x)}(3+\cos x)$$

$$\lim_{x\to 0} 2\left(\frac{\sin x}{x}\right)^2\cdot\frac{1}{4}\left(\frac{4x}{\tan 4x}\right)(3+\cos x) = 2\times 1\times\frac{1}{4}\times 1\times(3+1)=2.$$

47. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is

(1) 5
(2) 10
(3) 8
(4) 7

Sol. **(1)**

$^{n+1}C_3 - {}^nC_3 = 10 \Rightarrow {}^nC_2 = 10 \Rightarrow n = 5.$

48. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\dfrac{dP}{dx}=100-12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is

(1) 3000
(2) 3500
(3) 4500
(4) 2500

Sol. **(2)**

$$\int_{2000}^{P} dP = \int_0^{25}\left(100-12\sqrt{x}\right)dx$$

$$(P - 2000) = 25 \times 100 - \frac{12\times 2}{3}(25)^{3/2}$$

P = 3500.

49. **Statement – I :** The value of the integral $\displaystyle\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}}$ is equal to $\dfrac{\pi}{6}$.

Statement – II : $\displaystyle\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$.

(1) Statement - I is True; Statement -II is true; Statement-II is **not** a correct explanation for Statement-I
(2) Statement -I is True; Statement -II is False.

(3) Statement -I is False; Statement -II is True
(4) Statement -I is True; Statement -II is True; Statement-II is a **correct** explanation for Statement-I

Sol. **(3)**

$$I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}\, dx$$

$$2I = \frac{\pi}{6}$$

$$I = \frac{\pi}{12}.$$

50. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A| = 4$, then α is equal to

(1) 11 (2) 5
(3) 0 (4) 4

Sol. **(1)**

$$P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$$

$|\text{Adj } A| = |A|^2$
$|\text{Adj } A| = 16$
$1\,(12 - 12) - \alpha\,(4 - 6) + 3\,(4 - 6) = 16.$
$2\alpha - 6 = 16.$
$2\alpha = 22.$
$\alpha = 11.$

51. The number of values of k, for which the system of equations

$(k + 1)x + 8y = 4k$
$kx + (k + 3)y = 3k - 1$

has no solution, is

(1) 1 (2) 2
(3) 3 (4) infinite

Sol. **(1)**

For no solution

$$\frac{k+1}{k} = \frac{8}{k+3} \neq \frac{4k}{3k-1} \qquad \ldots (1)$$

$\Rightarrow (k + 1)\,(k + 3) - 8k = 0$
or $k^2 - 4k + 3 = 0 \Rightarrow k = 1, 3$
But for k = 1, equation (1) is not satisfied
Hence k = 3.

52. If $y = \sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at x = 1 is equal to

(1) $\frac{1}{2}$ (2) 1
(3) $\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$

Sol. **(4)**

$y = \sec(\tan^{-1}x)$

$\frac{dy}{dx} = \sec\left(\tan^{-1}x\right)\tan\left(\tan^{-1}x\right)\cdot\frac{1}{1+x^2}$

$\left.\frac{dy}{dx}\right|_{x=1} = \sqrt{2}\times1\times\frac{1}{2} = \frac{1}{\sqrt{2}}$.

53. If the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar, then k can have

(1) exactly one value
(2) exactly two values
(3) exactly three values
(4) any value

Sol. **(2)**

$\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$

$1(1+2k) + 1(1+k^2) - 1(2-k) = 0$

$k^2 + 1 + 2k + 1 - 2 + k = 0$

$k^2 + 3k = 0$

$(k)(k+3) = 0$

2 values of k.

54. Let A and B be two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is

(1) 220
(2) 219
(3) 211
(4) 256

Sol. **(2)**

$A \times B$ will have 8 elements.

$2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2 = 256 - 1 - 8 - 28 = 219$.

55. If the vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is

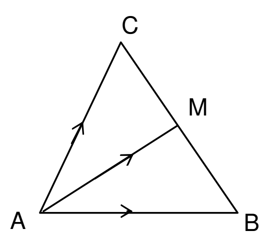
(1) $\sqrt{72}$
(2) $\sqrt{33}$
(3) $\sqrt{45}$
(4) $\sqrt{18}$

Sol. **(2)**

$\overrightarrow{AM} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2}$

$\overrightarrow{AM} = 4\hat{i} - \hat{j} + 4\hat{k}$

$\left|\overrightarrow{AM}\right| = \sqrt{16+16+1} = \sqrt{33}$



56. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is

(1) $\frac{13}{3^5}$
(2) $\frac{11}{3^5}$
(3) $\frac{10}{3^5}$
(4) $\frac{17}{3^5}$

Sol. **(2)**

P (correct answer) = 1/3

$${}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^1 + {}^5C_5\left(\frac{1}{3}\right)^5$$

$$\frac{5\times 2}{(3)^5}+\frac{1}{(3)^5}=\frac{11}{3^5}.$$

57. If z is a complex number of unit modulus and argument θ, then $\arg\left(\frac{1+z}{1+\bar{z}}\right)$ equals

(1) $\frac{\pi}{2}-\theta$ (2) θ
(3) $\pi-\theta$ (4) $-\theta$

Sol. **(2)**
$|z| = 1 \Rightarrow z\bar{z}=1$
$$\frac{1+z}{1+\bar{z}}=\frac{1+z}{1+\frac{1}{z}}=z.$$

58. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, a, b, c ∈ R, have a common root, then a : b : c is
(1) 3 : 2 : 1 (2) 1 : 3 : 2
(3) 3 : 1 : 2 (4) 1 : 2 : 3

Sol. **(4)**
For equation $x^2 + 2x + 3 = 0$
both roots are imaginary.
Since a, b, c ∈ R.
If one root is common then both roots are common
Hence, $\frac{a}{1}=\frac{b}{2}=\frac{c}{3}$
a : b : c = 1 : 2 : 3.

59. Distance between two parallel planes 2x + y + 2z = 8 and 4x + 2y + 4z + 5 = 0 is
(1) $\frac{5}{2}$ (2) $\frac{7}{2}$
(3) $\frac{9}{2}$ (4) $\frac{3}{2}$

Sol. **(2)**
4x + 2y + 4z = 16
4x + 2y + 4z = – 5
$$d_{min}=\frac{21}{\sqrt{36}}=\frac{21}{6}=\frac{7}{2}.$$

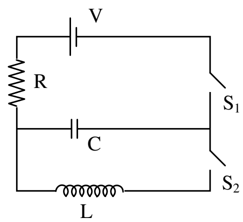
60. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$ is
(1) 120 (2) 210
(3) 310 (4) 4

Sol. **(2)**
$$\left(\frac{(x^{1/3}+1)(x^{2/3}-x^{1/3}+1)}{x^{2/3}-x^{1/3}+1}-\frac{1}{\sqrt{x}}\cdot\frac{(\sqrt{x}+1)(\sqrt{x}-1)}{(\sqrt{x}-1)}\right)^{10}=(x^{1/3}-x^{-1/2})^{10}$$

$T_{r+1} = (-1)^r\, {}^{10}C_r\, x^{\frac{20-5r}{6}} \Rightarrow r = 4$
${}^{10}C_4 = 210.$

PART C – PHYSICS

61. In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open. (q is charge on the capacitor and τ = RC is capacitive time constant). Which of the following statement is correct?



(1) At $t = \tau$, $q = CV/2$
(2) At $t = 2\tau$, $q = CV(1 - e^{-2})$
(3) At $t = \frac{\tau}{2}$, $q = CV(1-e^{-1})$
(4) Work done by the battery is half of the energy dissipated in the resistor.

Sol. **(2)**
Charge on the capacitor at any time 't' is
$q = CV\left(1-e^{-t/\tau}\right)$
at $t = 2\tau$
$q = CV\left(1-e^{-2}\right)$

62. A diode detector is used to detect an amplitude modulated wave of 60% modulation by using a condenser of capacity 250 pico farad in parallel with a load resistance 100 kilo ohm. Find the maximum modulated frequency which could be detected by it.

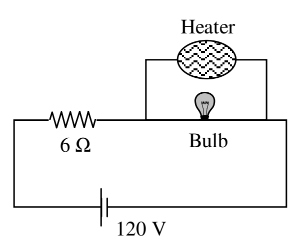
(1) 10.62 kHz
(2) 5.31 MHz
(3) 5.31 kHz
(4) 10.62 MHz

Sol. **(3)**
$$f_C = \frac{1}{2\pi RC} = \frac{1}{2\times3.14\times100\times10^3\times250\times10^{-12}} = 6.37 \text{ kHz}$$
f_C = cut off frequency
As we know that $f_m \ll f_C$
$\therefore$ (3) is correct
Note: The maximum frequency of modulation must be less than f_m, where
$$f_m = f_C\frac{\sqrt{1-m^2}}{m}$$
m $\Rightarrow$ modulation index

63. The supply voltage to a room is 120 V. The resistance of the lead wires is 6 Ω. A 60 W bulb is already switched on. What is the decrease of voltage across the bulb, when a 240 W heater is switched on in parallel to the bulb?

(1) 2.9 Volt
(2) 13.3 Volt
(3) 10.04 Volt
(4) zero volt

Sol. **(3)**



Resistance of bulb = $\frac{120\times120}{60} = 240\Omega$

Resistance of Heater = $\frac{120\times120}{240} = 60\Omega$

Voltage across bulb before heater is switched on, $V_1 = \frac{120}{246} \times 240$

Voltage across bulb after heater is switched on, $V_2 = \frac{120}{54} \times 48$

Decrease in the voltage is $V_1 - V_2 = 10.04$ (approximately)
Note: Here supply voltage is taken as rated voltage.

64. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is:

(1) $\frac{Mg}{k}\left(1 - \frac{LA\sigma}{M}\right)$ (2) $\frac{Mg}{k}\left(1 - \frac{LA\sigma}{2M}\right)$

(3) $\frac{Mg}{k}\left(1 + \frac{LA\sigma}{M}\right)$ (4) $\frac{Mg}{k}$

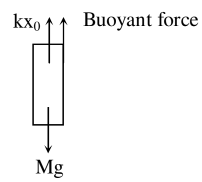
(Here k is spring constant)

Sol. **(2)**

At equilibrium $\Sigma F = 0$

$$kx_0 + \left(\frac{AL}{2}\sigma g\right) - Mg = 0$$

$$x_0 = Mg\left[1 - \frac{LA\sigma}{2M}\right]$$



65. Two charges, each equal to q, are kept at x = −a and x = a on the x-axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement ($y \ll a$) along the y-axis, the net force acting on the particle is proportional to:

(1) $-y$ (2) $\frac{1}{y}$

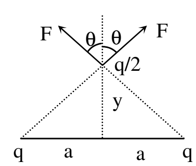
(3) $-\frac{1}{y}$ (4) y

Sol. **(4)**

$$F_{net} = 2F\cos\theta$$

$$= 2\frac{k \cdot q \cdot q/2}{\left(\sqrt{a^2 + y^2}\right)^2} \cdot \frac{y}{\sqrt{a^2 + y^2}}$$

$$= \frac{kq^2 y}{a^3} \; (y \ll a)$$



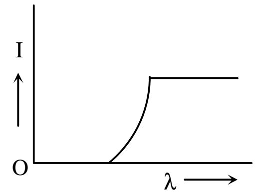
66. A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is:

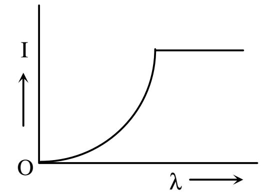
(1) $I_0/2$ (2) $I_0/4$

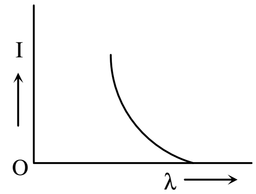
(3) $I_0/8$ (4) I_0

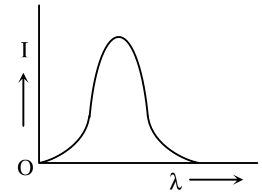
Sol. **(2)**

67. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows:

(1) 

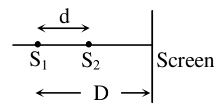
(2) 

(3) 

(4) 

Sol. **(3)**

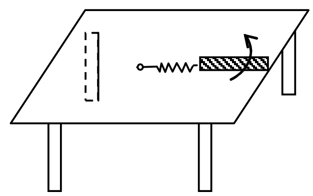
68. Two coherent point sources S_1 and S_2 are separated by a small distance 'd' as shown. The fringes obtained on the screen will be:



(1) straight lines
(2) semi-circles
(3) concentric circles
(4) points

Sol. **(3)**

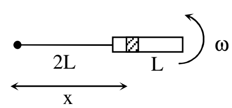
69. A metallic rod of length 'ℓ' is tied to a string of length 2ℓ and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field 'B' in the region, the e.m.f. induced across the ends of the rod is:



(1) $\frac{3B\omega\ell^2}{2}$
(2) $\frac{4B\omega\ell^2}{2}$
(3) $\frac{5B\omega\ell^2}{2}$
(4) $\frac{2B\omega\ell^2}{2}$

Sol. **(3)**

$$de = B(\omega x)\cdot dx$$



$$e = B\omega\int_{2L}^{3L} x\,dx$$

$$= \frac{5B\omega L^2}{2}$$

70. In a hydrogen like atom electron makes transition from an energy level with quantum number n to another with quantum number (n – 1). If n >> 1, the frequency of radiation emitted is proportional to

(1) $\frac{1}{n^2}$
(2) $\frac{1}{n^{3/2}}$
(3) $\frac{1}{n^3}$
(4) $\frac{1}{n}$

Sol. **(3)**

$$\nu \propto \left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

$$\propto \frac{(2n-1)}{n^2(n-1)^2}$$

$$\propto \frac{1}{n^3} \quad (\text{since } n \gg 1)$$

71. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T, density of liquid is ρ and L is its latent heat of vaporization.

(1) $\sqrt{T/\rho L}$ (2) $T/\rho L$

(3) $2T/\rho L$ (4) $\rho L/T$

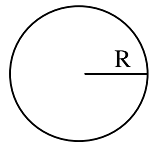
Sol. **(3)**

$\rho 4\pi R^2 \Delta R L = T\, 4\pi[R^2 - (R - \Delta R)^2]$

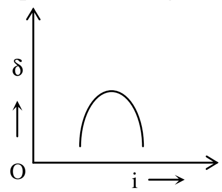
$\rho R^2 \Delta R L = T[R^2 - R^2 + 2R\Delta R - \Delta R^2]$

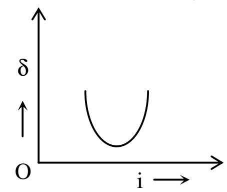
$\rho R^2 \Delta R L = T 2R\Delta R$ (ΔR is very small)

$R = \frac{2T}{\rho L}$.



72. The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by:

(1)


(2)


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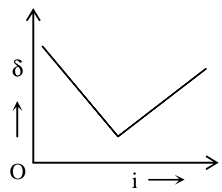

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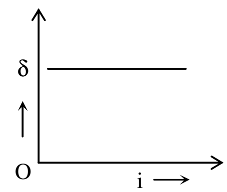


Sol. **(2)**

73. Let $[\varepsilon_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then:

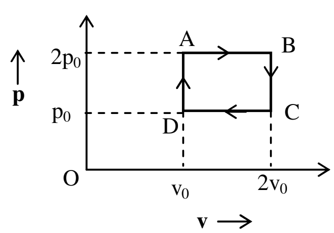
(1) $[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2]$ (2) $[\varepsilon_0] = [M^{-1}L^2T^{-1}A^{-2}]$

(3) $[\varepsilon_0] = [M^{-1}L^2T^{-1}A]$ (4) $[\varepsilon_0] = [M^{-1}L^{-3}T^2A]$

Sol. **(1)**

73. $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = F$

$\varepsilon_0 = \frac{[A^2T^2]}{[MLT^{-2}L^2]} = [M^{-1}L^{-3}A^2T^4]$

74.


The above p-v diagram represents the thermodynamic cycle of an engine, operating with an ideal monoatomic gas. The amount of heat extracted from the source in a single cycle is

(1) $\left(\frac{13}{2}\right)p_0v_0$ (2) $\left(\frac{11}{2}\right)p_0v_0$

(3) $4p_0v_0$ (4) p_0v_0

Sol. **(2)**

Heat is extracted from the source in path DA and AB is

$$\Delta Q = \frac{3}{2}R\left(\frac{P_0V_0}{R}\right) + \frac{5}{2}R\left(\frac{2P_0V_0}{R}\right) = \frac{13}{2}P_0V_0$$

75. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1 %. What is the fundamental frequency of steel if density and elasticity of steel are 7.7×10^3 kg/m^3 and 2.2×10^{11}N/m^2 respectively?

(1) 178.2 Hz (2) 200.5 Hz
(3) 770 Hz (4) 188.5 Hz

Sol. **(1)**

Fundamental frequency $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$

$= \frac{1}{2\ell}\sqrt{\frac{T}{A\rho}}$

$= \frac{1}{2\ell}\sqrt{\frac{\text{stress}}{\rho}} = \frac{1}{2\times1.5}\sqrt{\frac{2.2\times10^{11}\times10^{-2}}{7.7\times10^{3}}}$.

76. This question has statement I and statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement- I: Higher the range, greater is the resistance of ammeter.
Statement- II: To increase the range of ammeter, additional shunt needs to be used across it.
(1) Statement – I is true, Statement – II is true, Statement – II is not the correct explanation of Statement–I.
(2) Statement – I is true, statement – II is false.
(3) Statement – I is false, Statement – II is true
(4) Statement – I is true, Statement – II is true, Statement – II is the correct explanation of statement- I .

Sol. **(3)**

For Ammeter, $S = \frac{I_gG}{I - I_g}$

So for I to increase, S should decrease, so additional S can be connected across it.

77. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of 2R?

(1) $\frac{2GmM}{3R}$ (2) $\frac{GmM}{2R}$
(3) $\frac{GmM}{3R}$ (4) $\frac{5GmM}{6R}$

Sol. **(4)**

$T.\,E_f = -\frac{GMm}{6R}$

$T.\,E_i = -\frac{GMm}{R}$

$\Delta W = T.E_f - T.E_i = \frac{5GMm}{6R}$

78. A projectile is given an initial velocity of $\left(\hat{i} + 2\hat{j}\right)$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If g = 10 m/s^2, the equation of its trajectory is:

(1) $y = 2x - 5x^2$ (2) $4y = 2x - 5x^2$
(3) $4y = 2x - 25x^2$ (4) $y = x - 5x^2$

Sol. **(1)**

$x = t$

$y = 2t - 5t^2$

Equation of trajectory is $y = 2x - 5x^2$

79. Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found that by connecting them together the potential on each one can be made zero. Then :

(1) $3C_1 = 5C_2$ (2) $3C_1 + 5C_2 = 0$

(3) $9C_1 = 4C_2$ (4) $5C_1 = 3C_2$

Sol. **(1)**

$120C_1 = 200C_2$

$6C_1 = 10C_2$

$3C_1 = 5C_2$

80. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip?

(1) $\frac{r\omega_0}{3}$ (2) $\frac{r\omega_0}{2}$

(3) $r\omega_0$ (4) $\frac{r\omega_0}{4}$

Sol. **(2)**

From conservation of angular momentum about any fix point on the surface

$mr^2\omega_0 = 2mr^2\omega$

$\therefore \quad \omega = \frac{\omega_0}{2}$

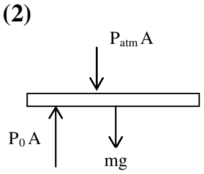
$\therefore \quad V_{CM} = \frac{\omega_0 r}{2}$

81. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency:

(1) $\frac{1}{2\pi}\frac{V_0 M P_0}{A^2\gamma}$ (2) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$

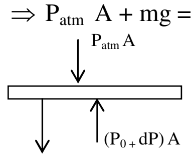
(3) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$ (4) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$

Sol. **(2)**



FBD of piston at equilibrium

$\Rightarrow P_{atm}\ A + mg = P_0\ A$ ...(1)



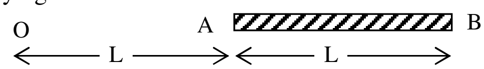
FBD of piston when piston is pushed down a distance x

$$P_{atm} + mg - (P_0 + dP)A = m\frac{d^2x}{dt^2} \qquad ...(2)$$

Process is adiabatic $\Rightarrow PV^\gamma = C \Rightarrow -dP = \frac{\gamma P dV}{V}$

Using 1, 2, 3 me get $f = \frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$

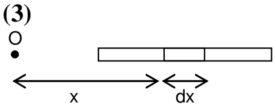
82. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at a distance L from the end A is:



(1) $\frac{3Q}{4\pi\varepsilon_0 L}$ (2) $\frac{Q}{4\pi\varepsilon_0 L\ln 2}$

(3) $\frac{Q\ln 2}{4\pi\varepsilon_0 L}$ (4) $\frac{Q}{8\pi\varepsilon_0 L}$

Sol. **(3)**



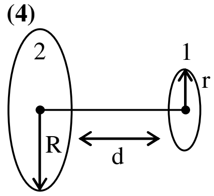
$$V = \int_{x=L}^{x=2L} \frac{k}{x}\left(\frac{Q}{L}\right)dx = \frac{Q\ln 2}{4\pi\varepsilon_0 L}$$

83. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is

(1) 6×10^{-11} weber (2) 3.3×10^{-11} weber

(3) 6.6×10^{-9} weber (4) 9.1×10^{-11} weber

Sol. **(4)**



Let M_{12} be the coefficient of mutual induction between loops

$\phi_1 = M_{12}i_2$

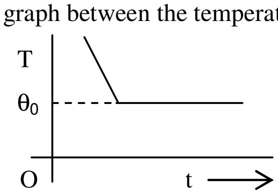
$$\Rightarrow \frac{\mu_0 i_2 R^2}{2(d^2+R^2)^{3/2}}\pi r^2 = M_{12}i_2$$

$$\Rightarrow M_{12} = \frac{\mu_0 R^2 \pi r^2}{2(d^2+R^2)^{3/2}}$$

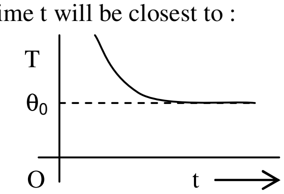
$\phi_2 = M_{12}i_1 \Rightarrow \phi_2 = 9.1\times10^{-11}$ weber

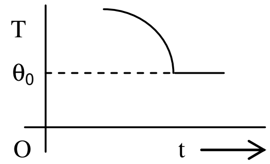
84. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0 the graph between the temperature T of the metal and time t will be closest to :

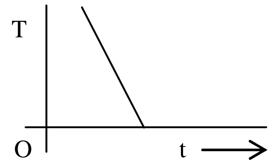
(1)



(2)

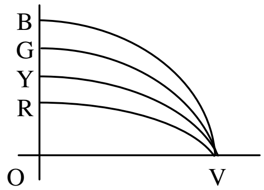


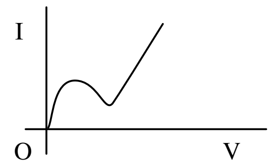
(3)


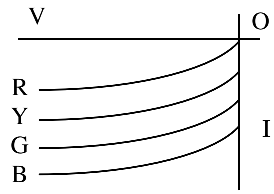
(4)


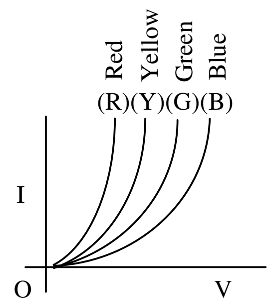
Sol. **(2)**
The temperature goes on decreasing with time (non-linearly) The rate of decrease will be more initially which is depicted in the second graph.

85. The I – V characteristic of an LED is

(1)


(2)


(3)


(4)


Sol. **(4)**
For LED, in forward bias, intensity increases with voltage.

86. This question has Statement I and Statement II. Of the four choices given after the Statements, choose the one that best describes the two Statements.
Statement – I : A point particle of mass m moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$ then $f = \left(\frac{m}{M+m}\right)$.
Statement – II : Maximum energy loss occurs when the particles get stuck together as a result of the collision.
(1) Statement – I is true, Statement – II is true, Statement – II is not a correct explanation of Statement – I.
(2) Statement – I is true, Statement – II is false.
(3) Statement – I is false, Statement – II is true
(4) Statement – I is true, Statement – II is true, Statement – II is a correct explanation of Statement – I.

Sol. **(3)**
Loss of energy is maximum when collision is inelastic as in an inelastic collision there will be maximum deformation.

KE in COM frame is $\frac{1}{2}\left(\frac{Mm}{M+m}\right)V_{rel}^2$

$KE_i = \frac{1}{2}\left(\frac{Mm}{M+m}\right)V^2 \qquad KE_f = 0 \ (\because V_{rel} = 0)$

Hence loss in energy is $\frac{1}{2}\left(\frac{Mm}{M+m}\right)V^2$

$\Rightarrow f = \frac{M}{M+m}$

87. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude is 5s. In another 10s it will decrease to α times its original magnitude, where α equals.

(1) 0.81 (2) 0.729
(3) 0.6 (4) 0.7

Sol. **(2)**

$A = A_0 e^{-kt}$

$\Rightarrow\ 0.9A_0 = A_0 e^{-5k}$

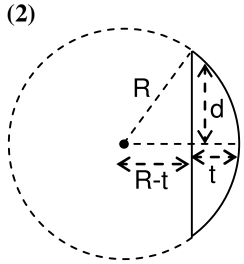
and $\alpha A_0 = A_0 e^{-15k}$

solving $\Rightarrow \alpha = 0.729$

88. Diameter of plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m/s, the focal length of the lens is :

(1) 20 cm (2) 30 cm
(3) 10 cm (4) 15 cm

Sol. **(2)**



$$R^2 = d^2 + (R-t)^2$$

$$R^2 - d^2 = R^2\left\{1-\frac{t}{R}\right\}^2$$

$$1-\frac{d^2}{R^2} = 1-\frac{2t}{R}$$

$$R = \frac{(3)^2}{2\times(0\cdot 3)} = \frac{90}{6} = 15\,\text{cm}$$

$$\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f} = \left(\frac{3}{2}-1\right)\left(\frac{1}{15}\right)$$

$$f = 30\,\text{cm}$$

89. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is :

(1) 6 V/m (2) 9 V/m
(3) 12 V/m (4) 3 V/m

Sol. **(1)**

$E_0 = CB_0$

$= 3 \times 10^8 \times 20 \times 10^{-9}$

$= 6$ V/m

90. Two short bar magnets of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2 respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid - point O of the line joining their centres is close to (Horizontal component of earth's magnetic induction is 3.6×10^{-5} Wb/m^2)

(1) 2.56×10^{-4} Wb/m^2
(2) 3.50×10^{-4} Wb/m^2
(3) 5.80×10^{-4} Wb/m^2
(4) 3.6×10^{-5} Wb/m^2

Sol. **(1)**

$$B_{net} = B_{M_1} + B_{M_2} + B_H$$

$$= \frac{\mu_0 M_1}{4\pi x^3} + \frac{\mu_0 M_2}{4\pi x^3} + B_H$$

$$= \frac{\mu_0}{4\pi x^3}\left(M_1 + M_2\right) + B_H$$

$$= \frac{10^{-7}}{10^{-3}} \times 2.2 + 3.6 \times 10^{-5}$$

$$= 2.56 \times 10^{-4} \quad \text{Wb/m}^2$$