Note: filtuce solutions to IIT-JEE, 2005 Screening Test is based on Screening Test paper created using memory retention of select filitec students appeared in this test and hence may not exactly be the same as the original paper. However, every effort has been made to reproduce the original paper in the interest of the aspiring students.

## fIITJEG solutions to III-IEE, 2005 Screening

1. The locus of $z$ which lies in shaded region is best represented by
(A) $\mathrm{z}:|\mathrm{z}+1|>2,|\arg (\mathrm{z}+1)|<\pi / 4$
(B) $\mathrm{z}:|\mathrm{z}-1|>2,|\arg (\mathrm{z}-1)|<\pi / 4$
(C) $\mathrm{z}:|\mathrm{z}+1|<2,|\arg (\mathrm{z}+1)|<\pi / 2$
(D) $\mathrm{z}:|\mathrm{z}-1|<2,|\arg (\mathrm{z}-1)|<\pi / 2$


Ans. A
Sol. The points $(1,0),(\sqrt{2}-1,-\sqrt{2})$ and $(\sqrt{2}-1, \sqrt{2})$ are equidistant from the point $(-1,0)$.
The shaded area belongs to the region outside the sector of circle $|z+1|=2$, lying between the line rays $\arg (z+1)=\frac{\pi}{4}$ and $\arg (z+1)=\frac{-\pi}{4}$.
2. In an equilateral triangle, 3 coins of radii 1 unit each are kept so that they touch each other and also the sides of the triangle. Area of the triangle is
(A) $4+2 \sqrt{3}$
(B) $6+4 \sqrt{3}$
(C) $12+\frac{7 \sqrt{3}}{4}$
(D) $3+\frac{7 \sqrt{3}}{4}$


Ans. B
Sol. The line joining the vertex of the triangle and the centre of the coin makes angle $\frac{\pi}{6}$ with the sides of the triangle. The length of each of the sides of the equilateral triangle is $2+2 \cot \frac{\pi}{6}=2(1+\sqrt{3})$.
Hence its area is $\frac{\sqrt{3}}{4} 4(1+\sqrt{3})^{2}=6+4 \sqrt{3}$.
3. If $a, b, c$ are integers not all equal and $w$ is a cube root of unity $(w \neq 1)$, then the minimum value of $\left|\mathrm{a}+\mathrm{bw}+\mathrm{cw}^{2}\right|$ is
(A) 0
(B) 1
(C) $\frac{\sqrt{3}}{2}$
(D) $\frac{1}{2}$

Ans. B
Sol. $\left|a+b w+c w^{2}\right|=\sqrt{\left(a-\frac{b}{2}-\frac{c}{2}\right)^{2}+\frac{3}{4}(c-b)^{2}}=\sqrt{\frac{1}{2}\left((a-b)^{2}+(b-c)^{2}+(c-a)^{2}\right)}$.
This is minimum when $\mathrm{a}=\mathrm{b}$ and $(\mathrm{b}-\mathrm{c})^{2}=(\mathrm{c}-\mathrm{a})^{2}=1 \Rightarrow$ The minimum value is 1 .
4. A rectangle with sides $2 \mathrm{~m}-1$ and $2 \mathrm{n}-1$ is divided into squares of unit length by drawing parallel lines as shown in the diagram, then the number of rectangles possible with odd side lengths is
(A) $(\mathrm{m}+\mathrm{n}+1)^{2}$
(B) $4^{\mathrm{m}+\mathrm{n}-1}$
(C) $m^{2} n^{2}$
(D) $m n(m+1)(n+1)$


Ans. C
Sol. There are 2 m vertical (numbered $1,2, \ldots, 2 \mathrm{~m}$ ) and 2 n horizontal lines (numbered $1,2, \ldots .2 \mathrm{n}$ ).
To form the required rectangle we must select two horizontal lines, one even numbered and one odd numbered and similarly two vertical lines. The number of rectangles is then ${ }^{m} C_{1} \cdot{ }^{m} C_{1} \cdot{ }^{n} C_{1} \cdot{ }^{n} C_{1}=m^{2} n^{2}$.

## Alternate solution:

Number of rectangles possible is $(1+3+5+\ldots .+(2 m-1))(1+3+5+\ldots .+(2 n-1))=m^{2} n^{2}$.
5. A circle is given by $x^{2}+(y-1)^{2}=1$, another circle $C$ touches it externally and also the $x$-axis, then the locus of its centre is
(A) $\left\{(\mathrm{x}, \mathrm{y}): \mathrm{x}^{2}=4 \mathrm{y}\right\} \cup\{(\mathrm{x}, \mathrm{y}): \mathrm{y} \leq 0\}$
(B) $\left.\left\{(\mathrm{x}, \mathrm{y}): \mathrm{x}^{2}+(\mathrm{y}-1)^{2}=4\right\} \cup\{\mathrm{x}, \mathrm{y}): \mathrm{y} \leq 0\right\}$
(C) $\left\{(\mathrm{x}, \mathrm{y}): \mathrm{x}^{2}=\mathrm{y}\right\} \cup\{(0, \mathrm{y}): \mathrm{y} \leq 0\}$
(D) $\left\{(\mathrm{x}, \mathrm{y}): \mathrm{x}^{2}=4 \mathrm{y}\right\} \cup\{(0, \mathrm{y}): \mathrm{y} \leq 0\}$

Ans. D
Sol. Let the circle touching the $x$-axis be $x^{2}+y^{2}-2 a x-2 b y+a^{2}=0$ with centre at $(a, b)$ and radius $b$.
Since it touches the circle $x^{2}+(y-1)^{2}=1,|b+1|=\sqrt{a^{2}+(b-1)^{2}}$.
$\Rightarrow \mathrm{b}^{2}+2 \mathrm{~b}+1=\mathrm{a}^{2}+\mathrm{b}^{2}-2 \mathrm{~b}+1$
$\Rightarrow 4 b=a^{2}$ so that locus of $(a, b)$ is $x^{2}=4 y$. If the centre of the circle lies on the $y$-axis, then $y \leq 0$.
6. $\cos (\alpha-\beta)=1$ and $\cos (\alpha+\beta)=1 / \mathrm{e}$, where $\alpha, \beta \in[-\pi, \pi]$. Pairs of $\alpha, \beta$ which satisfy both the equations is/ are
(A) 0
(B) 1
(C) 2
(D) 4

Ans. D
Sol. For $\cos (\alpha-\beta)=1, \alpha=\beta$ so that $\cos (\alpha+\beta)=1 / \mathrm{e} \Rightarrow \alpha+\beta= \pm \cos ^{-1} 1 / \mathrm{e}$
$\Rightarrow 2 \alpha= \pm \cos ^{-1}\left(\frac{1}{\mathrm{e}}\right) \in[-2 \pi, 2 \pi] . \Rightarrow \alpha, \beta$ can be satisfied by 4 sets of values.
7. In $\triangle \mathrm{ABC}, \mathrm{a}, \mathrm{b}, \mathrm{c}$ are the lengths of its sides and $\mathrm{A}, \mathrm{B}, \mathrm{C}$ are the angles of triangle ABC . The correct relation is given by
(A) $(\mathrm{b}-\mathrm{c}) \sin \left(\frac{\mathrm{B}-\mathrm{C}}{2}\right)=\mathrm{a} \cos \frac{\mathrm{A}}{2}$
(B) $(\mathrm{b}-\mathrm{c}) \cos \frac{\mathrm{A}}{2}=\mathrm{a} \sin \left(\frac{\mathrm{B}-\mathrm{C}}{2}\right)$
(C) $(\mathrm{b}+\mathrm{c}) \sin \left(\frac{\mathrm{B}+\mathrm{C}}{2}\right)=\mathrm{a} \cos \frac{\mathrm{A}}{2}$
(D) $(\mathrm{b}-\mathrm{c}) \cos \left(\frac{\mathrm{A}}{2}\right)=2 \mathrm{a} \sin \left(\frac{\mathrm{B}+\mathrm{C}}{2}\right)$

Ans. B
Sol. Here $\frac{b-c}{a}=\frac{\sin B-\sin C}{\sin A}=\frac{2 \sin \frac{B-C}{2} \cos \frac{B+C}{2}}{2 \sin \frac{A}{2} \cos \frac{A}{2}}=\frac{\sin \left(\frac{B-C}{2}\right)}{\cos \frac{A}{2}}$.
8. The value of $\binom{30}{0}\binom{30}{10}-\binom{30}{1}\binom{30}{11}+\binom{30}{2}\binom{30}{12} \ldots \ldots+\binom{30}{20}\binom{30}{30}$ is, where $\binom{\mathrm{n}}{\mathrm{r}}={ }^{\mathrm{n}} \mathrm{C}_{\mathrm{r}}$.
(A) $\binom{30}{10}$
(B) $\binom{30}{15}$
(C) $\binom{60}{30}$
(D) $\binom{31}{10}$

Ans. A
Sol. The given expression is the coefficient of $x^{20}$ in the product $(1+x)^{30}(1-x)^{30}=\left(1-x^{2}\right)^{30}$ $\Rightarrow$ the given expression $={ }^{30} \mathrm{C}_{10}$.
9. A variable plane at a distance of 1 unit from the origin cuts the co-ordinate axes at $A, B$ and $C$. If the centroid $D$ $(x, y, z)$ of triangle $A B C$ satisfies the relation $\frac{1}{x^{2}}+\frac{1}{y^{2}}+\frac{1}{z^{2}}=k$, then the value of $k$ is
(A) 3
(B) 1
(C) $1 / 3$
(D) 9

## Ans. D

Sol. Let $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ be the variable plane so that $\left|\frac{1}{\sqrt{\frac{1}{a^{2}}+\frac{1}{b^{2}}+\frac{1}{c^{2}}}}\right|=1$.
The plane meets the coordinate axes at $\mathrm{A}(\mathrm{a}, 0,0), \mathrm{B}(0, b, 0), \mathrm{C}(0,0, c)$. The centroid D of the triangle ABC is $\left(\frac{\mathrm{a}}{3}, \frac{\mathrm{~b}}{3}, \frac{\mathrm{c}}{3}\right)$
$\Rightarrow \mathrm{x}=\frac{\mathrm{a}}{3}, \mathrm{y}=\frac{\mathrm{b}}{3}, \mathrm{z}=\frac{\mathrm{c}}{3}$ and $\frac{1}{\mathrm{a}^{2}}+\frac{1}{\mathrm{~b}^{2}}+\frac{1}{\mathrm{c}^{2}}=1 \Rightarrow \frac{1}{\mathrm{x}^{2}}+\frac{1}{\mathrm{y}^{2}}+\frac{1}{\mathrm{z}^{2}}=9$.
10. If $\int_{\sin x}^{1} t^{2}(f(t)) d t=(1-\sin x)$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is
(A) $1 / 3$
(B) $1 / \sqrt{3}$
(C) 3
(D) $\sqrt{3}$

Ans. C
Sol. Differentiating both sides with respect to x , we get
$-\sin ^{2} x f(\sin x) \cdot \cos x=-\cos x \Rightarrow f(\sin x)=\frac{1}{\sin ^{2} x}$
$\Rightarrow \mathrm{f}(\mathrm{x})=\frac{1}{\mathrm{x}^{2}} \Rightarrow \mathrm{f}\left(\frac{1}{\sqrt{3}}\right)=3$.
11. In the quadratic equation $\mathrm{ax}^{2}+\mathrm{bx}+\mathrm{c}=0$, if $\Delta=\mathrm{b}^{2}-4 \mathrm{ac}$ and $\alpha+\beta, \alpha^{2}+\beta^{2}, \alpha^{3}+\beta^{3}$ are in G.P. where $\alpha, \beta$ are the roots of $\mathrm{ax}^{2}+b x+c=0$, then
(A) $\Delta \neq 0$
(B) $b \Delta=0$
(C) $\mathrm{c} \Delta=0$
(D) $\Delta=0$

## Ans. C

Sol. We have $\left(\alpha^{2}+\beta^{2}\right)^{2}=(\alpha+\beta)\left(\alpha^{3}+\beta^{3}\right) \Rightarrow \alpha \beta(\alpha-\beta)^{2}=0$
$\Rightarrow \mathrm{c} \Delta=0$.
12. A six faced fair dice is thrown until 1 comes, then the probability that 1 comes in even no. of trials is
(A) $5 / 11$
(B) $5 / 6$
(C) $6 / 11$
(D) $1 / 6$

## Ans. A

Sol. The required probability $=\frac{1}{6} \cdot \frac{5}{6}+\frac{1}{6} \cdot\left(\frac{5}{6}\right)^{3}+\ldots \ldots$
$=\frac{1}{6} \cdot \frac{5}{6}\left[1+\left(\frac{5}{6}\right)^{2}+\ldots\right]=\frac{5}{11}$.
13. If $f(x)$ is a twice differentiable function and given that $f(1)=1, f(2)=4, f(3)=9$, then
(A) $\mathrm{f}^{\prime \prime}(\mathrm{x})=2$, for $\forall \mathrm{x} \in(1,3)$
(B) $\mathrm{f}^{\prime \prime}(\mathrm{x})=\mathrm{f}^{\prime}(\mathrm{x})=5$ for some $\mathrm{x} \in(2,3)$
(C) $\mathrm{f}^{\prime \prime}(\mathrm{x})=3, \forall \mathrm{x} \in(2,3)$
(D) $f^{\prime \prime}(x)=2$, for some $x \in(1,3)$

Ans. D
Sol. Let $g(x)=f(x)-x^{2}$.
We have $g(1)=0, g(2)=0, g(3)=0$.
Hence by Rolle's theorem $\mathrm{g}^{\prime}(\mathrm{x})=0$ for some $\mathrm{c} \in(1,2)$

$$
\text { and } \quad g^{\prime}(x)=0 \quad \text { for some } d \in(2,3)
$$

Again, by Rolle's theorem $g^{\prime \prime}(x)=0$ at some $x \in(c, d)$
$\Rightarrow \mathrm{f}^{\prime \prime}(\mathrm{x})=2$ for some values $\mathrm{x} \in(1,3)$.
14. $\int_{-2}^{0}\left(x^{3}+3 x^{2}+3 x+3+(x+1) \cos (x+1)\right) d x$ is equal to
(A) -4
(B) 0
(C) 4
(D) 6

## Ans. C

Sol. Here $\mathrm{I}=\int_{-2}^{0}\left[\mathrm{x}^{3}+3 \mathrm{x}^{2}+3 \mathrm{x}+3+(\mathrm{x}+1) \cos (\mathrm{x}+1)\right] \mathrm{dx}$
Put $\mathrm{x}+1=\mathrm{t}$
$=\int_{-1}^{1}\left[\left(t^{3}+t \cos t\right)+2\right] d t$
$=\int_{-1}^{1} 2 \mathrm{dt}=4$.
15. If $\mathrm{P}(\mathrm{x})$ is a polynomial of degree less than or equal to 2 and S is the set of all such polynomials so that $\mathrm{P}(1)=1$, $\mathrm{P}(0)=0$ and $\mathrm{P}^{\prime}(\mathrm{x})>0 \quad \forall \mathrm{x} \in[0,1]$, then
(A) $S=\phi$
(B) $S=\left\{(1-a) x^{2}+a x \quad 0<a<2\right.$
(C) $(1-a) x^{2}+a x a \in(0, \infty)$
(D) $S=\left\{(1-a) x^{2}+a x \quad 0<a<1\right.$

Ans. B
Sol. Let the polynomial be $\mathrm{P}(\mathrm{x})=\mathrm{ax}^{2}+\mathrm{bx}+\mathrm{c}$
$P(0)=0 \Rightarrow c=0$ and $P(1)=1 \Rightarrow a+b=1$ so that
$\mathrm{P}^{\prime}(\mathrm{x})=2(1-\mathrm{b}) \mathrm{x}+\mathrm{b}>0 \forall \mathrm{x}$
$\Rightarrow \mathrm{b} \in(0,2)$.
$\Rightarrow S=\left\{(1-a) x^{2}+a x, a \in(0,2)\right\}$
16. The minimum area of triangle formed by the tangent to the ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$ and coordinate axes is
(A) ab sq. units
(B) $\frac{a^{2}+b^{2}}{2}$ sq. units
(C) $\frac{(a+b)^{2}}{2}$ sq. units
(D) $\frac{a^{2}+a b+b^{2}}{3}$ sq. units

## Ans. A

Sol. A tangent of the given ellipse is $y=m x+\sqrt{a^{2} m^{2}+b^{2}}$.
It meets the axes at $\left(\frac{-\sqrt{a^{2} \mathrm{~m}^{2}+\mathrm{b}^{2}}}{\mathrm{~m}}, 0\right)$ and $\left(0, \sqrt{\mathrm{a}^{2} \mathrm{~m}^{2}+\mathrm{b}^{2}}\right)$.
Hence the area of the triangle is $\frac{1}{2}\left|\frac{a^{2} m^{2}+b^{2}}{m}\right|=\frac{1}{2}\left|a^{2} m+\frac{b^{2}}{m}\right| \geq a b$.

## Alternate:

The equation of tangent at $(a \cos \theta, b \sin \theta)$ is $\frac{x \cos \theta}{a}+\frac{y \sin \theta}{b}=1$.
It meets the coordinate axes at $\mathrm{A} \equiv(0, \mathrm{~b} \operatorname{cosec} \theta), \mathrm{B} \equiv(\mathrm{a} \sec \theta, 0)$.
Area of triangle $=\frac{a b}{2 \sin \theta \cos \theta}=\frac{a b}{\sin 2 \theta} \geq a b$.
17. If the functions $f(x)$ and $g(x)$ are defined on $R \rightarrow R$ such that
$f(x)=\left\{\begin{array}{ll}0, & x \in \text { rational } \\ x, & x \in \text { irrational }\end{array}, g(x)=\left\{\begin{array}{ll}0, & x \in \text { irrational } \\ x, & x \in \text { rational }\end{array}\right.\right.$, then $(f-g)(x)$ is
(A) one-one and onto
(B) neither one-one nor onto
(C) one-one but not onto
(D) onto but not one-one

## Ans. A

Sol. Let $h(x)=f(x)-g(x)= \begin{cases}x ; & x \in \text { irrational } \\ -x ; & x \in \text { rational }\end{cases}$
$\Rightarrow$ the function $\mathrm{h}(\mathrm{x})$ is one-one and onto.
18. The area bounded by the parabolas $y=(x+1)^{2}$ and $y=(x-1)^{2}$ and the line $y=1 / 4$ is
(A) 4 sq. units
(B) $1 / 6$ sq. units
(C) $4 / 3$ sq. units
(D) $1 / 3$ sq. units

Ans. D
Sol. The parabolas meet at $(0,1)$ and intersect the line $y=1 / 4$ at $x=-\frac{3}{2},-\frac{1}{2}, \frac{1}{2}$ and $\frac{3}{2}$.
Hence the required area $=2\left[\int_{0}^{1 / 2}(x-1)^{2} d x\right]-\frac{1}{4}=\left.\frac{2}{3}(x-1)^{3}\right|_{0} ^{1 / 2}-\frac{1}{4}=\frac{1}{3}$
19. The function given by $y=\| x|-1|$ is differentiable for all real numbers except the points
(A) $\{0,1,-1\}$
(B) $\pm 1$
(C) 1
(D) -1

Ans. A
Sol. From the graph, the function is not differentiable at $\mathrm{x}=-1,0,1$.

20. If $y=y(x)$ and it follows the relation $x \cos y+y \cos x=\pi$, then $y^{\prime \prime}(0)$
(A) 1
(B) -1
(C) $\pi$
(D) $-\pi$

Ans. C
Sol. $\quad \mathrm{x} \cos \mathrm{y}+\mathrm{y} \cos \mathrm{x}=\pi, \mathrm{y}(0)=\pi$.
$\Rightarrow-x \sin y \frac{d y}{d x}+\cos y-y \sin x+\cos x \frac{d y}{d x}=0 \Rightarrow y^{\prime}(0)=1$.
Again differentiating and using $\mathrm{y}^{\prime}(0)=1$ and $\mathrm{y}(0)=\pi$, we get $\mathrm{y}^{\prime \prime}(0)=\pi$.
21. The solution of primitive integral equation $\left(x^{2}+y^{2}\right) d y=x y d x$, is $y=y(x)$. If $y(1)=1$ and $y\left(x_{0}\right)=e$, then $x_{0}$ is
(A) $\sqrt{2\left(\mathrm{e}^{2}-1\right)}$
(B) $\sqrt{2\left(\mathrm{e}^{2}+1\right)}$
(C) $\sqrt{3} \mathrm{e}$
(D) $\sqrt{\frac{\mathrm{e}^{2}+1}{2}}$

Ans. C
Sol. We have $\frac{d y}{d x}=\frac{x y}{x^{2}+y^{2}}$.
Solving the homogenous differential equation by writing $y=v x$, we get
$-\frac{x^{2}}{2 y^{2}}+\ln y=-\frac{1}{2}$.
For $\mathrm{y}=\mathrm{e}, \frac{-\mathrm{x}_{0}^{2}}{2 \mathrm{e}^{2}}+\ln \mathrm{e}=-\frac{1}{2} \Rightarrow \mathrm{x}_{0}{ }^{2}=3 \mathrm{e}^{2} \Rightarrow \mathrm{x}_{0}=\sqrt{3} \mathrm{e}$.
22. $A=\left[\begin{array}{ccc}1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4\end{array}\right], I=\left[\begin{array}{lll}1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{array}\right]$ and $A^{-1}=\left[\frac{1}{6}\left(A^{2}+c A+d I\right)\right]$, then the value of $c$ and $d$ are
(A) $-6,-11$
(B) 6,11
(C) $-6,11$
(D) $6,-11$

## Ans. C

Sol. We evaluate $A^{2}$ and $A^{3}$ and write the given equation as $A A^{-1}=I=\frac{1}{6}\left[A^{3}+c A^{2}+d A\right]$.
Comparing the corresponding elements on both the sides we get
$\mathrm{c}=-6, \mathrm{~d}=11$.
Alternatively, we may use Cayley Hamilton Theorem.
23. If $\mathrm{P}=\left[\begin{array}{cc}\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{array}\right], \mathrm{A}=\left[\begin{array}{ll}1 & 1 \\ 0 & 1\end{array}\right]$ and $\mathrm{Q}=\mathrm{PAP}^{\mathrm{T}}$ and $\mathrm{x}=\mathrm{P}^{\mathrm{T}} \mathrm{Q}^{2005} \mathrm{P}$, then x is equal to
(A) $\left[\begin{array}{cc}1 & 2005 \\ 0 & 1\end{array}\right]$
(B) $\left[\begin{array}{c}4+2005 \sqrt{3} \\ 2005\end{array}\right.$
$\left.\begin{array}{l}6015 \\ 2005 \sqrt{3}\end{array}\right]$
(C) $\frac{1}{4}\left[\begin{array}{cc}2+\sqrt{3} & 1 \\ -1 & 2-\sqrt{3}\end{array}\right]$
(D) $\frac{1}{4}\left[\begin{array}{cc}2005 & 2-\sqrt{3} \\ 2+\sqrt{3} & 2005\end{array}\right]$

Ans. A
Sol. $\quad P^{T} P=I$
$\mathrm{Q}=\mathrm{PAP}^{\mathrm{T}}$ so that
$\mathrm{x}=\mathrm{P}^{\mathrm{T}} \mathrm{Q}^{2005} \mathrm{P}=\mathrm{P}^{\mathrm{T}}\left(\mathrm{PAP}^{\mathrm{T}}\right)^{2005} \mathrm{P}$
$=\mathrm{P}^{\mathrm{T}} \mathrm{PAP}^{\mathrm{T}}\left(\mathrm{PAP}^{\mathrm{T}}\right)^{2004} \mathrm{P}$
$=\mathrm{A}^{2005}=\left[\begin{array}{cc}1 & 2005 \\ 0 & 1\end{array}\right]$
24. Tangent to the curve $y=x^{2}+6$ at a point $P(1,7)$ touches the circle $x^{2}+y^{2}+16 x+12 y+c=0$ at a point $Q$.

Then the coordinates of Q are
(A) $(-6,-11)$
(B) $(-9,-13)$
(C) $(-10,-15)$
(D) $(-6,-7)$

Ans. D
Sol. Equation of tangent to the parabola at $(1,7)$ is
$x-\frac{(y+7)}{2}+6=0 \Rightarrow 2 x-y+5=0$.
$\Rightarrow$ Centre $\equiv(-8,-6)$
Equation of $\mathrm{CQ}=\mathrm{x}+2 \mathrm{y}+\mathrm{k}=0$
$-8-12+\mathrm{k}=0 \Rightarrow \mathrm{k}=20$
$P Q \equiv 4 x-2 y+10=0$
$C Q \equiv x+2 y+20=0$
$=5 \mathrm{x}+30=0 \Rightarrow \mathrm{x}=-6$
$\Rightarrow-6+2 y+20=0 \Rightarrow y=-7$
Hence the point of contact is $(-6,-7)$.
25. If $\mathrm{f}(\mathrm{x})$ is a continuous and differentiable function and $\mathrm{f}\left(\frac{1}{\mathrm{n}}\right)=0 \forall \mathrm{n} \geq 1$ and $\mathrm{n} \in \mathrm{I}$, then
(A) $f(x)=0, x \in(0,1]$
(B) $\mathrm{f}(0)=0, \mathrm{f}^{\prime}(0)=0$
(C) $\mathrm{f}^{\prime}(0)=0=\mathrm{f}^{\prime \prime}(0), \mathrm{x} \in(0,1]$
(D) $f(0)=0$ and $f^{\prime}(0)$ need not to be zero

## Ans. B

Sol. Given $\mathrm{f}\left(\frac{1}{\mathrm{n}}\right)=0 \quad \forall \mathrm{n} \geq 1$ and $\mathrm{n} \in \mathrm{I}$.
This indicates that $f(x)$ has a wavy behaviour.
Amplitude of the wave either (a) is constant (b) increases or (c) decreases.
In case of (a) and (b), function will not be differentiable at 0 .
$\Rightarrow$ Amplitude has to decrease such that $\mathrm{f}^{\prime}(0)=0$.
26. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero, non-coplanar vectors and $\overrightarrow{b_{1}}=\vec{b}-\frac{\vec{b} \cdot \vec{a}}{|\vec{a}|^{2}} \vec{a}, \vec{b}_{2}=\vec{b}+\frac{\vec{b} \cdot \vec{a}}{|\vec{a}|^{2}} \vec{a}$, $\overrightarrow{\mathrm{c}}_{1}=\overrightarrow{\mathrm{c}}-\frac{\overrightarrow{\mathrm{c}} \cdot \overrightarrow{\mathrm{a}}}{|\overrightarrow{\mathrm{a}}|^{2}} \overrightarrow{\mathrm{a}}+\frac{\overrightarrow{\mathrm{b}} \cdot \overrightarrow{\mathrm{c}}}{|\overrightarrow{\mathrm{c}}|^{2}} \overrightarrow{\mathrm{~b}}_{1}, \quad \overrightarrow{\mathrm{c}}_{2}=\overrightarrow{\mathrm{c}}-\frac{\overrightarrow{\mathrm{c}} \cdot \overrightarrow{\mathrm{a}}}{|\overrightarrow{\mathrm{a}}|^{2}} \overrightarrow{\mathrm{a}}-\frac{\overrightarrow{\mathrm{b}}}{1} \cdot \overrightarrow{\mathrm{c}} \overrightarrow{\mathrm{b}}_{1}, \overrightarrow{\mathrm{c}_{3}}=\overrightarrow{\mathrm{c}}-\frac{\overrightarrow{\mathrm{c}} \cdot \overrightarrow{\mathrm{a}}}{|\overrightarrow{\mathrm{c}}|^{2}} \overrightarrow{\mathrm{a}}+\frac{\overrightarrow{\mathrm{b}} \cdot \overrightarrow{\mathrm{c}}}{|\overrightarrow{\mathrm{c}}|^{2}} \overrightarrow{\mathrm{~b}}_{1}, \quad \overrightarrow{\mathrm{c}}_{4}=\overrightarrow{\mathrm{c}}-\frac{\overrightarrow{\mathrm{c}} \cdot \overrightarrow{\mathrm{a}}}{|\overrightarrow{\mathrm{c}}|^{2}} \overrightarrow{\mathrm{a}}-\frac{\overrightarrow{\mathrm{b}} \cdot \overrightarrow{\mathrm{c}}}{|\overrightarrow{\mathrm{b}}|^{2}} \overrightarrow{\mathrm{~b}}_{1}$, then the set of orthogonal vectors is
(A) $\left(\vec{a}, \vec{b}_{1}, \overrightarrow{\mathrm{c}}_{3}\right)$
(B) $\left(\overrightarrow{\mathrm{a}}, \overrightarrow{\mathrm{b}}_{1}, \overrightarrow{\mathrm{c}}_{2}\right)$
(C) $\left(\vec{a}, \vec{b}_{1}, \overrightarrow{\mathrm{c}}_{1}\right)$
(D) $\left(\vec{a}, \vec{b}_{2}, \vec{c}_{2}\right)$

## Ans. B

Sol. Obviously $\vec{a} \cdot \vec{b}_{1}=\left(\vec{b} \cdot \vec{a}-\frac{\vec{b} \cdot \vec{a}}{|\vec{a}|^{2}} \vec{a} \cdot \vec{a}\right)=0$
$\& \overrightarrow{\mathrm{a}} \cdot \overrightarrow{\mathrm{c}}_{2}=0$ and $\overrightarrow{\mathrm{b}}_{1} \cdot \overrightarrow{\mathrm{c}}_{2}=0$.
$\Rightarrow\left(\overrightarrow{\mathrm{a}}, \overrightarrow{\mathrm{b}}_{1}, \overrightarrow{\mathrm{c}}_{2}\right)$ are orthogonal vectors.
27. For the primitive integral equation $y d x+y^{2} d y=x d y ; x \in R, y>0, y=y(x), y(1)=1$, then $y(-3)$ is
(A) 3
(B) 2
(C) 1
(D) 5

Ans. A
Sol. $y \frac{d x-x d y}{y^{2}}=-d y$
$\frac{x}{y}=-y+c$
$y(1)=1 \Rightarrow c=2$
$y^{2}-2 y+x=0$
$y(-3)$ :
$y^{2}-2 y-3=0 \Rightarrow(y-3)(y+1)=0$
$y=3,-1$
28. $X$ and $Y$ are two sets and $f: X \rightarrow Y$. If $\{f(c)=y ; c \subset X, y \subset Y\}$ and $\left\{f^{1}(d)=x ; d \subset Y, x \subset X\right\}$, then the true statement is
(A) $f\left(f^{1}(b)\right)=b$
(B) $f^{1}(f(a))=a$
(C) $f\left(f^{1}(b)\right)=b, b \subset y$
(D) $f^{1}(f(a))=a, a \subset x$

Ans. D
Sol. The given data is shown in the figure
Since $\mathrm{f}^{-1}(\mathrm{~d})=\mathrm{x}$
$\Rightarrow \mathrm{f}(\mathrm{x})=\mathrm{d}$
Now, if $\mathrm{a} \subset \mathrm{x}, \mathrm{f}(\mathrm{a}) \subset \mathrm{d}$
$\Rightarrow \mathrm{f}^{-1}(\mathrm{f}(\mathrm{a}))=\mathrm{a}$.


Analyse your performance in Screening Test for evaluation of your preparation for Mains. A comprehensive analysis of your preparation on different topics would be couriered to you. Fill this sheet as per answers you have made in the IIT-JEE Screening Examination as per the sequencing provided in the solution booklet and send to nearest fIITJEE's office immediately.

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## fIITJE€ solutions to III-IEE, 2005 Screening

## PHYSICS

57. Find current in $2 \Omega$ resistor
(A) 0
(B) 2 A
(C) 4 A
(D) 1 A


Ans. A
Sol. According to Kirchoff's junction rule no current passes through $2 \Omega$ resistor. $\therefore \mathrm{i}=0$
58. In Young's double slit experiment the angular position of a point on the central maxima whose intensity is one fourth of maximum intensity
(A) $\sin ^{-1}(\lambda / d)$
(B) $\sin ^{-1}(\lambda / 2 d)$
(C) $\sin ^{-1}(\lambda / 3 d)$
(D) $\sin ^{-1}(\lambda / 4 d)$

Ans. C
Sol. $\quad I=I_{\text {max }} \cos ^{2} \phi / 2$
$\Rightarrow \phi=2 \pi / 3$ and $\frac{2 \pi}{\lambda} \mathrm{~d} \sin \theta=\frac{2 \pi}{3}$
$\therefore \theta=\sin ^{-1}\left(\frac{\lambda}{3 \mathrm{~d}}\right)$

59. Ratio of area of hole to beaker is 0.1 . Height of liquid in beaker is 3 m , and hole is at the height of 52.5 cm from the bottom of beaker, find the square of the velocity of liquid coming out from the hole
(A) $50(\mathrm{~m} / \mathrm{s})^{2}$
(B) $50.5(\mathrm{~m} / \mathrm{s})^{2}$
(C) $51(\mathrm{~m} / \mathrm{s})^{2}$
(D) $42(\mathrm{~m} / \mathrm{s})^{2}$

Ans. A
Sol. $u^{2}=\frac{2 g h}{\left[1-\left(\frac{A_{0}}{A}\right)^{2}\right]}=50(\mathrm{~m} / \mathrm{s})^{2}$

60. In the figure shown, a cubical block is held stationary against a rough wall by applying force ' $F$ ' then incorrect statement among the following is
(A) frictional force, $\mathrm{f}=\mathrm{Mg}$
(B) $\mathrm{F}=\mathrm{N}, \mathrm{N}$ is normal reaction
(C) F does not apply any torque
(D) N does not apply any torque


Ans. D
Sol. For equilibrium, $\mathrm{f}=\mathrm{Mg}$
$\mathrm{F}=\mathrm{N}$
For rotational equilibrium normal will shift downward.
Hence torque due to friction about centre of mass $=$ Torque due to Normal
 reaction about centre of mass.
61. Three infinitely charged sheets are kept parallel to $x-y$ plane having charge densities as shown. Then the value of electric field at ' $P$ ' is
(A) $\frac{-4 \sigma}{\epsilon_{0}} \hat{k}$
(B) $\frac{4 \sigma}{\epsilon_{0}} \hat{\mathrm{k}}$
(C) $\frac{-2 \sigma}{\epsilon_{0}} \hat{k}$
(D) $\frac{2 \sigma}{\epsilon_{0}} \hat{\mathrm{k}}$


Ans. C
Sol. $\quad \overrightarrow{\mathrm{E}}_{\mathrm{P}}=\frac{\sigma}{2 \epsilon_{0}}(-\hat{\mathrm{k}})+\frac{(-2 \sigma)}{2 \epsilon_{0}}(\hat{\mathrm{k}})+\frac{(-\sigma)}{2 \epsilon_{0}}(\hat{\mathrm{k}})$

$$
=\frac{-2 \sigma}{\epsilon_{0}} \hat{k}
$$

62. A cylindrical conducting rod is kept with its axis along positive $z$-axis, where a uniform magnetic field exists parallel to z -axis. The current induced in the cylinder is
(A) zero
(B) clockwise as seen from $+z$ axis
(C) anti-clockwise as seen from $+z$ axis
(D) opposite to the direction of magnetic field.

Ans. A
Sol. Since B is constant
$\therefore \frac{\mathrm{d} \phi}{\mathrm{dt}}=0$
$\therefore \mathrm{i}=0$
63. A circular disc of radius $\mathrm{R} / 3$ is cut from a circular disc of radius R and mass 9 M as shown. Then moment of inertia of remaining disc about ' $O$ ' perpendicular to the plane of the disc is
(A) $4 \mathrm{MR}^{2}$
(B) $9 \mathrm{MR}^{2}$
(C) $\frac{37}{9} \mathrm{MR}^{2}$
(D) $\frac{40}{9} \mathrm{MR}^{2}$

Ans. A
Sol. $\quad I_{0}=\frac{9 \mathrm{MR}^{2}}{2}-\left[\frac{\mathrm{M}(\mathrm{R} / 3)^{2}}{2}+M\left(\frac{2 \mathrm{R}}{3}\right)^{2}\right]=4 \mathrm{MR}^{2}$

64. Depict the shown $\mathrm{v}-\mathrm{x}$ graph in $\mathrm{a}-\mathrm{x}$ graph.

(A)

(B)

(C)

(D)


Ans. A
Sol. Equation of curve is
$\frac{\mathrm{V}}{\mathrm{v}_{0}}+\frac{\mathrm{x}}{\mathrm{x}_{0}}=1$
$\therefore \mathrm{v}=\left(1-\frac{\mathrm{x}}{\mathrm{x}_{0}}\right) \mathrm{v}_{0}$

$\therefore a=\frac{d v}{d t}=-\frac{v_{0}}{x_{0}}(v)=-\frac{v_{0}^{2}}{x_{0}}\left(1-\frac{x^{\prime}}{x_{0}}\right)$
Alternative: $a=-v\left(\frac{d v}{d x}\right)$; but $d v / d x$ is negative and $v$ is decreasing with the increase in $x$.
Hence ' $a$ ' should increase with increase of ' $x$ '.
65. A particle is confined to rotate in a circular path with decreasing linear speed, then which of the following is correct?
(A) $\overrightarrow{\mathrm{L}}$ (angular momentum) is conserved about the centre.
(B) only direction of angular momentum $\overrightarrow{\mathrm{L}}$ is conserved.
(C) It spirals towards the centre.
(D) its acceleration is towards the centre.

Ans. B
66. The atomic number $(Z)$ of an element whose $k_{\alpha}$ wavelength is $\lambda$ is 11 . The atomic number of an element whose $\mathrm{k}_{\alpha}$ wavelength is $4 \lambda$ is equal to
(A) 6
(B) 11
(C) 44
(D) 4

Ans. A
Sol. $\quad(Z-1)^{2} \lambda=\mathrm{constant}$

$$
\therefore\left(10^{2}\right) \lambda=4 \lambda(Z-1)^{2} \Rightarrow Z=6
$$

67. The graph shown in the figure represents energy density $E_{\lambda}$ versus $\lambda$ for three sources sun, welding arc, tungsten filament. For $\lambda_{\max }$, correct combination will be
(A) 1 - Tungsten, $2-$ Welding arc, $3-$ Sun
(B) 1 - Sun, 2 - Tungsten, 3 - Welding arc.
(C) 1 - Sun, 2 - Welding arc, 3 - Tungsten
(D) 1 - Welding arc, 2 - Sun, 3 - Tungsten


Ans. A
Sol. Temperature of sun would be maximum out of the given three

$$
\begin{aligned}
& \text { as } \lambda_{\mathrm{m}} \mathrm{~T}=\text { constant } \\
& \lambda_{\mathrm{m}} \text { for Sun is minimum }
\end{aligned}
$$

68. $\mathrm{T}_{1}$ is the time period of simple pendulum. The point of suspension moves vertically upwards according to $\mathrm{y}=$ $\mathrm{kt}^{2}$, where $\mathrm{k}=1 \mathrm{~m} / \mathrm{s}^{2}$. New time period is $\mathrm{T}_{2}$, then $\frac{\mathrm{T}_{1}^{2}}{\mathrm{~T}_{2}^{2}}=?\left(\mathrm{~g}=10 \mathrm{~m} / \mathrm{s}^{2}\right)$
(A) $4 / 5$
(B) $6 / 5$
(C) $5 / 6$
(D) 1

Ans. B
Sol. Acceleration of the point of suspension

$$
\begin{aligned}
& \mathrm{a}=\frac{\mathrm{d}^{2} \mathrm{y}}{\mathrm{dt}^{2}}=2 \mathrm{k}=2 \mathrm{~m} / \mathrm{s}^{2} \\
& \mathrm{~T}=2 \pi \sqrt{\frac{\mathrm{~L}}{\mathrm{~g}_{\text {eff }}}} \Rightarrow \mathrm{T}_{1}=2 \pi \sqrt{\frac{\mathrm{~L}}{10}} \text { and } \mathrm{T}_{2}=2 \pi \sqrt{\frac{\mathrm{~L}}{12}} \\
& \therefore \frac{\mathrm{~T}_{1}^{2}}{\mathrm{~T}_{2}^{2}}=\frac{6}{5}
\end{aligned}
$$

69. Which of the following does not have the same dimension?
(A) Electric flux, Electric field, Electric dipole moment
(B) Pressure, stress, Young's modulus
(C) Electromotive force, Potential difference, Electric voltage.
(D) Heat, Potential energy, Work done

## Ans. A

70. A capacitor $(\mathrm{C}=4.0 \mu \mathrm{~F})$ is connected through a resistor $(\mathrm{R}=2.5 \mathrm{M} \Omega)$ across a battery of negligible internal resistance of voltage 12 volts. The time after which the potential difference across the capacitor becomes three times to that of resistor is $(\ln 2=0.693)$
(A) 13.86 sec .
(B) 6.48 sec .
(C) 3.24 sec .
(D) 20.52 sec .

Ans. A
Sol. $\quad \mathrm{q}=\mathrm{C} \varepsilon\left(1-\mathrm{e}^{-\frac{\mathrm{t}}{\mathrm{RC}}}\right) \Rightarrow \mathrm{i}=\frac{\varepsilon}{\mathrm{R}} \mathrm{e}^{-\frac{\mathrm{t}}{\mathrm{RC}}}$
$3 \mathrm{~V}_{\mathrm{R}}=\mathrm{V}_{\mathrm{C}}$
$\Rightarrow \varepsilon\left(1-\mathrm{e}^{-\frac{\mathrm{t}}{\mathrm{RC}}}\right)=3 \varepsilon \mathrm{e}^{-\frac{\mathrm{t}}{\mathrm{RC}}} \Rightarrow \mathrm{e}^{-\mathrm{tRC}}=1 / 4$
$\mathrm{t} / \mathrm{RC}=2 \ln 2 \therefore \mathrm{t}=20 \times(0.693)=13.86 \mathrm{sec}$
71. A photon of energy 10.2 eV collides inelastically with a Hydrogen atom in ground state. After a certain time interval of few micro seconds another photon of energy 15.0 eV collides inelastically with the same hydrogen atom, then the observation made by a suitable detector is
(A) 1 photon with energy 10.2 eV and an electron with energy 1.4 eV
(B) 2 photon with energy 10.2 eV
(C) 2 photon with energy 1.4 eV
(D) one photon with energy 3.4 eV and 1 electron with energy 1.4 eV

Ans. A
Sol. $\quad 10.2 \mathrm{eV}$ photon on collision will excite H -atom to first excited state but Hydrogen atom will return to ground state before next collision. Second photon will provide ionization energy to Hydrogen atom, i.e., electron will be ejected with energy $=1.4 \mathrm{eV}$
72. In a resonance tube with tuning fork of frequency 512 Hz , first resonance occurs at water level equal to 30.3 cm and second resonance occurs at 63.7 cm . The maximum possible error in the speed of sound is
(A) $51.2 \mathrm{~cm} / \mathrm{s}$
(B) $102.4 \mathrm{~cm} / \mathrm{s}$
(C) $204.8 \mathrm{~cm} / \mathrm{s}$
(D) $153.6 \mathrm{~cm} / \mathrm{s}$

## Ans. C

Sol. $\quad \ell_{1}+e=\frac{\mathrm{v}}{4 \mathrm{f}}$ and $\ell_{2}+\mathrm{e}=\frac{3 \mathrm{v}}{4 \mathrm{f}}$
$\ell_{2}-\ell_{1}=\frac{2 v}{4 \mathrm{f}}$
$\frac{\Delta\left(\ell_{2}-\ell_{1}\right)}{\left(\ell_{2}-\ell_{1}\right)}=\frac{\Delta v}{v}$
$\Delta \mathrm{v}=2 \mathrm{f} \Delta\left(\ell_{2}-\ell_{1}\right)=2 \mathrm{f}\left(\Delta \ell_{1}+\Delta \ell_{2}\right) \quad$ (For maximum error)
$=2 \times 512 \times 0.2=204.8 \mathrm{~cm} / \mathrm{s}$.
73. A thin concave and a thin convex lens are in contact. The ratio of the magnitude of power of two lenses is $2 / 3$ and focal length of combination is 30 cm , then the focal length of individual lenses are
(A) $-15 \mathrm{~cm}, 10 \mathrm{~cm}$
(B) $-75 \mathrm{~cm}, 50 \mathrm{~cm}$
(C) $75 \mathrm{~cm},-50 \mathrm{~cm}$
(D) $75 \mathrm{~cm}, 50 \mathrm{~cm}$

Ans. A
Sol. $\quad\left|\frac{\mathrm{P}_{\text {concave }}}{\mathrm{P}_{\text {convex }}}\right|=\frac{2}{3}$
$\frac{1}{\mathrm{~F}}=\frac{1}{\mathrm{f}_{\text {concave }}}+\frac{1}{\mathrm{f}_{\text {convex }}}$ $\frac{1}{30}=\frac{-2}{3 \mathrm{f}}+\frac{1}{\mathrm{f}}=\frac{1}{3 \mathrm{f}} \Rightarrow \mathrm{f}=10 \mathrm{~cm}$, where f is focal length of convex lens
74. Which of the following process does not occur through convection
(A) Boiling of water
(B) Land breeze and Sea breeze
(C) Circulation of air around furnace
(D) Heating of glass bulb through filament

Ans. D
Sol. Heating of glass bulb is by radiation.
75. A tank of height 33.25 cm is completely filled with liquid ( $\mu=1.33$ ). An object is placed at the bottom of tank on the axis of concave mirror as shown in the figure. Image of the object is formed 25 cm below the surface of the liquid, then focal length of the mirror is
(A) 10 cm
(B) 15 cm
(C) 20 cm
(D) 25 cm


Ans. C
Sol. After first refraction, position of the image $=\frac{33.25}{1.33}=25 \mathrm{~cm}$
From reflection, $\frac{1}{\mathrm{f}}=\frac{1}{\mathrm{v}}-\frac{1}{25+15}=\frac{1}{\mathrm{v}}-\frac{1}{40}$
From second refraction position of the object $=\frac{25}{1.33}$
$\frac{1}{\mathrm{f}}=-\frac{1}{15+\frac{25}{1.33}}-\frac{1}{40} \Rightarrow \mathrm{f}=-18.31 \mathrm{~cm}$
Hence magnitude of focal length of convex lens is 18.31 cm . The nearest possible matching answer is 20 cm .
76. In YDSE, an electron beam is used to obtain interference pattern. If speed of electron is increased then
(A) no interference pattern will be observed.
(B) distance between two consecutive fringes will increase.
(C) distance between two consecutive fringes will decrease.
(D) distance between two consecutive fringes remains same.

## Ans. C

Sol. $\lambda=\frac{h}{\mathrm{mv}}$
$v$ is increased, $\lambda$ is decreased.
$\beta=\lambda \mathrm{D} / \mathrm{d} \Rightarrow \beta$ decreases.
77. A spherical body of area A and emissivity $\mathrm{e}=0.6$ is kept inside a perfectly black body. Total heat radiated by the body at temperature $T$
(A) $0.4 \mathrm{e} \mathrm{AT}^{4}$
(B) $0.8 \mathrm{eAT}^{4}$
(C) $0.6 \mathrm{eAT}^{4}$
(D) $1.0 \mathrm{eAT}^{4}$

Ans. D
Sol. When a non black body is placed inside a hollow enclosure the total radiation from the body is the sum of what it would emit in the open (with $\mathrm{e}<1$ ) and the part $(1-\mathrm{a})$ of the incident radiation from the walls reflected by it. The two add up to a black body radiation. Hence the total radiation emitted by the body is $1.0 \sigma \mathrm{AT}^{4}$.
Probably in the examination paper ' $\sigma$ ' is misprinted as ' $e$ '
78. An open organ pipe resonated with frequency ' $f_{1}$ ' and $2^{\text {nd }}$ harmonic. Now one end is closed and the frequency is slowly increased then it resonates with frequency $f_{2}$ and nth harmonic then
(A) $n=3, f_{2}=\frac{3}{4} f_{1}$
(B) $n=5, f_{2}=\frac{3}{4} f_{1}$
(C) $\mathrm{n}=3, \mathrm{f}_{2}=\frac{5}{4} \mathrm{f}_{1}$
(D) $\mathrm{n}=5, \mathrm{f}_{2}=\frac{5}{4} \mathrm{f}_{1}$

Ans. D
Sol. $\mathrm{f}_{1}=\frac{1}{\ell} \sqrt{\frac{B}{\rho}}$

$$
\begin{aligned}
& \mathrm{f}_{2}=\frac{\mathrm{n}}{4 \ell} \sqrt{\frac{\mathrm{~B}}{\rho}} \\
& \frac{\mathrm{f}_{1}}{\mathrm{f}_{2}}=\frac{4}{\mathrm{n}} \Rightarrow \mathrm{f}_{2}=\frac{\mathrm{n}}{4} \mathrm{f}_{1}
\end{aligned}
$$

For the first resonance $\mathrm{n}=5, \mathrm{f}_{2}=\frac{5}{4} \mathrm{f}_{1}$ (as frequency increases)
79. Temperature of a gas is $20^{0} \mathrm{C}$ and pressure is changed from $1.01 \times 10^{5} \mathrm{~Pa}$ to $1.165 \times 10^{5} \mathrm{~Pa}$. If volume is decreased isothermally by $10 \%$. Bulk modulus of gas is
(A) $1.55 \times 10^{5}$
(B) $0.155 \times 10^{5}$
(C) $1.4 \times 10^{5}$
(D) $1.01 \times 10^{5}$

## Ans. A

Sol. $\quad B=-\Delta \mathrm{P} /(\Delta \mathrm{V} / \mathrm{V})=-\frac{(1.165-1.01) \times 10^{5}}{0.1}=1.55 \times 10^{5}$
80. A galvanometer with resistance $100 \Omega$ is converted to ammeter with a resistance of $0.1 \Omega$. The galvanometer shows full scale deflection with a current of $100 \mu \mathrm{~A}$. Then the minimum current in the circuit for full scale deflection of galvanometer will be
(A) 100.1 mA
(B) 10.01 mA
(C) 1.001 mA
(D) 0.1001 mA

## Ans. A

Sol. $\quad 0.1=\frac{100 \mathrm{R}^{\prime}}{100+\mathrm{R}^{\prime}} \Rightarrow \mathrm{R}^{\prime}=\frac{100}{1001}$


$$
\begin{aligned}
& (100)\left(100 \times 10^{-6}\right)=\mathrm{R}^{\prime}\left(\mathrm{I}-100 \times 10^{-6}\right) \\
& \therefore \mathrm{I}=100.1 \mathrm{~mA}
\end{aligned}
$$

81. One calorie is defined as the heat required to raise the temperature of 1 gm of water by $1^{0} \mathrm{C}$ in a certain interval of temperature and at certain pressure. The temperature interval and pressure is
(A) $13.5^{\circ} \mathrm{C}$ to $14.5^{\circ} \mathrm{C} \& 76 \mathrm{~mm}$ of Hg
(B) $6.5^{\circ} \mathrm{C}$ to $7.5^{\circ} \mathrm{C} \& 76 \mathrm{~mm}$ of Hg
(C) $14.5^{\circ} \mathrm{C}$ to $15.5^{\circ} \mathrm{C} \& 760 \mathrm{~mm}$ of Hg
(D) $98.5^{\circ} \mathrm{C}$ to $99.5^{\circ} \mathrm{C} \& 760 \mathrm{~mm}$ of Hg

## Ans. C

Sol. By definition.
82. If a star converts all of its Helium into oxygen nucleus, find the amount of energy released per nucleus of oxygen. $\mathrm{He}=4.0026 \mathrm{amu}, \mathrm{O}=15.9994 \mathrm{amu}$
(A) 7.26 MeV
(B) 7 MeV
(C) 10.24 MeV
(D) 5.12 MeV

Ans. C
Sol. $\quad \mathrm{E}=\Delta \mathrm{mc}^{2}=[4 \times 4.0026-15.9994] \times 931.5=10.24 \mathrm{MeV}$
83. Two litre of water at initial temperature of $27^{\circ} \mathrm{C}$ is heated by a heater of power 1 kW . If the lid of kettle is opened, then heat is lost at the constant rate of $160 \mathrm{~J} / \mathrm{s}$. Find the time required to raise the temperature of water to $77^{\circ} \mathrm{C}$ with the lid open (Specific heat of water $4.2 \mathrm{~kJ} / \mathrm{kg}$ )
(A) 5 min 40 sec
(B) 14 min 20 sec
(C) 8 min 20 sec
(D) 16 min 10 sec

## Ans. C

Sol. Rate of heat gain $=1000-160=840 \mathrm{~J} / \mathrm{s}$
$\therefore \quad$ Required time $=\frac{2 \times 4.2 \times 10^{3} \times(77-27)}{840}=500 \mathrm{sec}=8 \mathrm{~min} 20 \mathrm{sec}$
84. Ideal gas is contained in a thermally insulated and rigid container and it is heated through a resistance $100 \Omega$ by passing a current of 1 A for five minutes, then change in internal energy of the gas is
(A) 0 kJ
(B) 30 kJ
(C) 10 kJ
(D) 20 kJ

Ans. B
Sol. $\quad \Delta \mathrm{W}=0 \therefore \Delta \mathrm{Q}=\Delta \mathrm{U}$
$\Delta \mathrm{Q}=\Delta \mathrm{U}=\mathrm{I}^{2} \mathrm{R} \Delta \mathrm{t}=(1)^{2}(100)(5 \times 60)=30 \mathrm{~kJ}$

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## fIITJEG sofutions to IIT-IEE, 2005 Screening

## CHEMISTRY

29. When Phenyl Magnesium Bromide reacts with tert. butanol, which of the following is formed?
(A) Tert. butyl methyl ether
(B) Benzene
(C) Tert. butyl benzene
(D) Phenol

Ans. B
Sol. $\mathrm{PhMgBr}+\mathrm{Me}_{3} \mathrm{COH} \longrightarrow \mathrm{Ph}-\mathrm{H}+\mathrm{Me}_{3} \mathrm{COMgBr}$
30. 1-bromo-3-chlorocyclobutane when treated with two equivalents of Na , in the presence of ether which of the
following will be formed?
(A)

(B)

(C)

(D)


Ans. D
Sol. It is an intramolecular Wurtz reaction.

31.


What is X ?
(A) $\mathrm{CH}_{3} \mathrm{COOH}$
(B) $\mathrm{BrCH}_{2} \cdot \mathrm{COOH}$
(C) $\left(\mathrm{CH}_{3} \mathrm{CO}\right)_{2} \mathrm{O}$
(D) $\mathrm{CHO}-\mathrm{COOH}$

Ans. C
Sol.

$$
\mathrm{p}-\mathrm{MeO}-\mathrm{Ph}-\mathrm{CHO}+(\mathrm{AcO})_{2} \mathrm{O} \xrightarrow{\mathrm{CH}_{3} \mathrm{COONa}} \mathrm{p}-\mathrm{MeO}
$$

32. Cyclohexene is best prepared from cyclohexanol by which of the following:
(A) conc. $\mathrm{H}_{3} \mathrm{PO}_{4}$
(B) conc. $\mathrm{HCl} / \mathrm{ZnCl}_{2}$
(C) conc. HCl
(D) conc. HBr

Ans.
A
Sol.

$\mathrm{H}_{3} \mathrm{PO}_{4}$ acts as dehydrating agent.
33.
 on hydrolysis in aqueous acetone gives,




It mainly gives:
(A) K and L
(B) Only K
(C) L and M
(D) Only M

Ans. A
Sol. $\quad \mathrm{S}_{\mathrm{N}} 1$ and $\mathrm{S}_{\mathrm{N}} 2$, both reactions are possible due to aqueous acetone solution.
34. For 1-methoxy-1,3-butadiene, which of the following resonating structure is the least stable?
(A)

(B)


Ans. C
Sol. Point of difference is nature of carbanion. $2^{\circ}$ carbanions are less stable than $1^{\circ}-$ carbanions generally.
35. But-2-one can be converted to propanoic acid by which of the following:
(A) $\mathrm{NaOH}, \mathrm{NaI} / \mathrm{H}^{+}$
(B) Fehling Solution
(C) $\mathrm{NaOH}, \mathrm{I}_{2} / \mathrm{H}^{+}$
(D) Tollen's reagent

## Ans. C

Sol.


Iodoform test.
36. Two forms of D - glucopyranose, are called.
(A) Enantiomers
(B) Anomers
(C) Epimers
(D) Diastereomers

Ans. B
Sol. $\quad$ D - glucopyranose is cyclic form of glucose. Around C - 1 (Newly formed chiral centre, due to cycle formation) two isomers are observed. They are called as $\alpha$ and $\beta$ - Anomers.
37. Which of the following pair is expected to exhibit same colour in solution?
(A) $\mathrm{VOCl}_{2} ; \mathrm{FeCl}_{2}$
(B) $\mathrm{CuCl}_{2} ; \mathrm{VOCl}_{2}$
(C) $\mathrm{MnCl}_{2} ; \mathrm{FeCl}_{2}$
(D) $\mathrm{FeCl}_{2} ; \mathrm{CuCl}_{2}$

Ans. B
Sol. $\quad \mathrm{V}^{4+}$ and $\mathrm{Cu}^{2+}$ both have one unpaired electron available.
38. Which of the following isomers of phosphorus is thermodynamically most stable?
(A) Red
(B) White
(C) Black
(D) Yellow

## Ans. C

Sol. Due to layered structure in Black phosphorous, it is most stable.
39. A metal nitrate gives black ppt. with KI and on adding excess of KI it gives orange colour. It is:
(A) $\mathrm{Hg}^{+2}$
(B) $\mathrm{Bi}^{+3}$
(C) $\mathrm{Sn}^{+2}$
(D) $\mathrm{Pb}^{+2}$

## Ans. B

Sol. $\mathrm{Bi}^{+3}+\mathrm{KI} \longrightarrow \underset{\text { Black }}{\mathrm{BiI}_{3} \downarrow}$

$$
\mathrm{BiI}_{3}+\mathrm{KI} \longrightarrow \longrightarrow \text { } \mathrm{K}\left[\mathrm{BiI}_{4}\right]
$$

40. Which of the following will not be oxidised by $\mathrm{O}_{3}$ ?
(A) KI
(B) $\mathrm{FeSO}_{4}$
(C) $\mathrm{KMnO}_{4}$
(D) $\mathrm{K}_{2} \mathrm{MnO}_{4}$

Ans. C
Sol. $\mathrm{KMnO}_{4}$ can't be oxidised by any oxidising agents. Mn is in maximum possible oxidation state of VI.
41. Which type of isomerism is shown by $\mathrm{Co}\left(\mathrm{NH}_{3}\right)_{4} \mathrm{Br}_{2} \mathrm{Cl}$ ?
(A) Geometrical and Ionisation
(B) Optical and Ionisation
(C) Geometrical and Optical
(D) Geometrical only

## Ans. A

Sol. $\left[\mathrm{Co}\left(\mathrm{NH}_{3}\right)_{4}(\mathrm{Br})_{2}\right] \mathrm{Cl}$ can show both Geometrical and Ionisation isomerism.
42. Which of the following FCC structure contains cations in alternate tetrahedral voids?
(A) NaCl
(B) ZnS
(C) $\mathrm{Na}_{2} \mathrm{O}$
(D) $\mathrm{CaF}_{2}$

Ans. B
Sol. In ZnS , Anions $\left(\mathrm{S}^{-2}\right)$ are placed in fcc manner and cations $\left(\mathrm{Zn}^{+2}\right)$ are placed in alternate tetrahedral voids.
43. The elevation in boiling point, when 13.44 g of freshly prepared $\mathrm{CuCl}_{2}$ are added to one kilogram of water, is. [Some useful data, $\mathrm{Kb}=0.52 \mathrm{~kg} \mathrm{~K} \mathrm{~mol}^{-1}$, molecular weight of $\mathrm{CuCl}_{2}=134.4 \mathrm{gm}$ ].
(A) 0.05
(B) 0.1
(C) 0.16
(D) 0.21

## Ans. C

Sol. $\quad \Delta \mathrm{T}_{\mathrm{b}}=\mathrm{i} \times \mathrm{K}_{\mathrm{b}} \times \mathrm{m}=3 \times 0.52 \times\left(\frac{13.44}{134.4} \times \frac{1000}{1000}\right)=0.16$
44. The half cell reactions for rusting of iron are:

$$
2 \mathrm{H}^{+}+\frac{1}{2} \mathrm{O}_{2}+2 \mathrm{e}^{-} \longrightarrow \mathrm{H}_{2} \mathrm{O} ; \mathrm{E}^{0}=+1.23 \mathrm{~V}, \quad \mathrm{Fe}^{2+}+2 \mathrm{e}^{-} \longrightarrow \mathrm{Fe}_{(\mathrm{S})} ; \mathrm{E}^{0}=-0.44 \mathrm{~V}
$$

$\Delta \mathrm{G}^{0}$ (in kJ ) for the reaction is:
(A) -76
(B) -322
(C) -122
(D) -176

## Ans. B

Sol. $2 \mathrm{H}^{+}+\frac{1}{2} \mathrm{O}_{2}+2 \mathrm{e}^{-} \longrightarrow \mathrm{H}_{2} \mathrm{O} ; \quad \mathrm{E}^{\mathrm{o}}=+1.23 \mathrm{~V}$
$\mathrm{Fe}^{+2}+2 \mathrm{e}^{-} \longrightarrow \mathrm{Fe}_{(\mathrm{s})} ; \mathrm{E}^{\mathrm{o}}=-0.44 \mathrm{~V}$
$\mathrm{Fe}_{(\mathrm{s})}+2 \mathrm{H}^{+}+\frac{1}{2} \mathrm{O}_{2} \longrightarrow \mathrm{Fe}^{+2}+\mathrm{H}_{2} \mathrm{O} ; \mathrm{E}_{\text {cell }}^{\mathrm{o}}=1.67 \mathrm{~V}$
$\Delta \mathrm{G}^{\mathrm{o}}=-\mathrm{nFE} \mathrm{E}_{\text {cell }}^{\mathrm{o}}=-2 \times 96.500 \times 1.67=-322 \mathrm{~kJ}$
45. The number of radial nodes in 3 s and 2 p respectively are:
(A) 2 and 0
(B) 1 and 2
(C) 0 and 2
(D) 2 and 1

## Ans. A

Sol. Number of radial nodes $=\mathrm{n}-\ell-1$
so, for 3s: $3-0-1=2$
For 2 p : $2-1-1=0$

## IIT-JEE-2005-S-4

46. Which of the following ore contains both Copper and Iron?
(A) Cuprite
(B) Chalcocite
(C) Chalcopyrite
(D) Malachite

Ans. C
Sol. Chalcopyrite $\left(\mathrm{CuFeS}_{2}\right)$
47. A pale blue liquid which obtained by equi molar mixture of two gases at $-30^{\circ} \mathrm{C}$ is:
(A) $\mathrm{N}_{2} \mathrm{O}$
(B) $\mathrm{N}_{2} \mathrm{O}_{3}$
(C) $\mathrm{N}_{2} \mathrm{O}_{4}$
(D) $\mathrm{N}_{2} \mathrm{O}_{5}$

Ans. B
Sol. $\quad \mathrm{NO}+\mathrm{NO}_{2} \xrightarrow{-30^{\circ} \mathrm{C}} \mathrm{N}_{2} \mathrm{O}_{3}$
Pale blue colour
48. Which of the following is obtained when 4 - Methylbenzenesulphonicacid is hydrolysed with excess of sodium acetate?
(A)

(B)

(C)

(D)


Ans. C
Sol.


Above reaction is acid base reaction.
49. $\mathrm{CH}_{3} \cdot \mathrm{NH}_{2}\left(0.1\right.$ mole, $\left.\mathrm{K}_{\mathrm{b}}=5 \times 10^{-4}\right)$ is added to 0.08 moles of HCl and the solution is diluted to one litre,
resulting hydrogen ion concentration is:
(A) $1.6 \times 10^{-11}$
(B) $8 \times 10^{-11}$
(C) $5 \times 10^{-5}$
(D) $8 \times 10^{-2}$

Ans. B
Sol.


$$
\begin{aligned}
& {\left[\mathrm{OH}^{-}\right]=\mathrm{K}_{\mathrm{b}} \frac{\left[\mathrm{CH}_{3} \mathrm{NH}_{2}\right]}{\left[\mathrm{CH}_{3} \mathrm{NH}_{3}^{+}\right]}} \\
& {\left[\mathrm{OH}^{-}\right]=\frac{5 \times 10^{-4} \times 0.02}{0.08}=\frac{5}{4} \times 10^{-4}} \\
& {\left[\mathrm{H}^{+}\right]=\frac{\mathrm{K}_{\mathrm{w}}}{\left[\mathrm{OH}^{-}\right]}=\frac{10^{-14} \times 4}{5 \times 10^{-4}}=8 \times 10^{-11}}
\end{aligned}
$$

50. Which silicates is formed from $\left[\mathrm{SiO}_{4}\right]^{4-}$, tetrahedral units by sharing 3 oxygen atoms?
(A) Sheet silicates
(B) Pyro silicates
(C) Linear chain silicates
(D) 3 dimensional silicates

## Ans. A

Sol.

51. Which gas is evolved when $\mathrm{PbO}_{2}$ is treated with conc. $\mathrm{HNO}_{3}$ ?
(A) $\mathrm{NO}_{2}$
(B) $\mathrm{O}_{2}$
(C) $\mathrm{N}_{2}$
(D) $\mathrm{N}_{2} \mathrm{O}$

## Ans. B

Sol. $\quad \mathrm{PbO}_{2}+2 \mathrm{HNO}_{3} \longrightarrow \mathrm{~Pb}\left(\mathrm{NO}_{3}\right)_{3}+\mathrm{H}_{2} \mathrm{O}+\frac{1}{2} \mathrm{O}_{2}$
52. If helium and methane are allowed to diffuse out of the container under the similar conditions of temperature and pressure, then the ratio of rate of diffusion of helium to methane is:
(A) 2.0
(B) 1.0
(C) 0.5
(D) 4.0

Ans. A
Sol. $\quad \frac{\mathrm{r}_{\mathrm{He}}}{\mathrm{r}_{\mathrm{CH}_{4}}}=\sqrt{\frac{16}{4}}=2: 1$
53. Which of the following contains maximum number of lone pairs on the central atom?
(A) $\mathrm{ClO}_{3}^{-}$
(B) $\mathrm{XeF}_{4}$
(C) $\mathrm{SF}_{4}$
(D) $\mathrm{I}_{3}^{-}$

Ans. D
Sol.




54. Which of the following is correct for lyophilic sols?
(A) They are irreversible
(B) They are formed by inorganic substances
(C) They are readily coagulated by addition of electrolytes
(D) They are self stabilized

Ans. D
Sol. Lyophilic sols are solvent loving in nature. Due to this property, such kind of sols are self stabilised.
55. Which of the following statement is incorrect about order of reaction?
(A) Order of reaction is determined experimentally
(B) It is the sum of power of concentration terms in the rate law expression
(C) It does not necessarily depend on stoichiometric coefficients
(D) Order of the reaction can not have fractional value.

Ans. D
Sol. Order of reaction is determined experimentally. It may be fractional.
56. One mole of monoatomic ideal gas expands adiabatically at initial temperature T against a constant external pressure of 1 atm . from one litre to two litre. Find out the final temperature ( $\mathrm{R}=0.0821 \mathrm{lt}$. $\mathrm{atm} \mathrm{K}^{-1} \mathrm{~mole}^{-1}$ )
(A) T
(B) $\frac{\mathrm{T}}{\frac{5}{\frac{5}{3}-1}}$
(2)
(C) $\mathrm{T}-\frac{2}{3 \times 0.0821}$
(D) $\mathrm{T}+\frac{2}{3 \times 0.0821}$

Ans. C
Sol. Work done against constant external pressure $=P_{\text {ext }}\left(\mathrm{V}_{2}-\mathrm{V}_{1}\right)$
In adiabatic condition $\Delta q=0$ therefore $w=\Delta u$
$\therefore-\mathrm{P}_{\text {ext }}\left(\mathrm{V}_{2}-\mathrm{V}_{1}\right)=\frac{3}{2} \mathrm{R}\left(\mathrm{T}_{2}-\mathrm{T}_{1}\right)$ [Expansion work is negative]
On solving, $\mathrm{T}_{2}=\mathrm{T}_{1}-\frac{2}{3 \times 0.0821}$.

