Physics

1. Which one of the following represents the correct dimensions of the coefficient of viscosity ?
   (a) $[ML^{-1}T^{-2}]$ (b) $[MLT^{-1}]$
   (c) $[ML^{-1}T^{-1}]$ (d) $[ML^{-2}T^{-2}]$

2. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to :
   (a) x^2 (b) e^x
   (c) x (d) $\log_e x$

3. A ball is released from the top of a tower of height h m. It takes T s to reach the ground. What is the position of the ball in $T/3$ s ?
   (a) $h/9$ m from the ground
   (b) $7h/9$ m from the ground
   (c) $8h/9$ m from the ground
   (d) $17h/18$ m from the ground

4. If $\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \vec{\mathbf{B}} \times \vec{\mathbf{A}}$, then the angle between A and B is :
   (a) π (b) $\pi/3$
   (c) $\pi/2$ (d) $\pi/4$

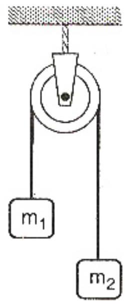
5. A projectile can have the same range R for two angles of projection. If T_1 and T_2 be the times of flights in the two cases, then the product of the two times of flights is directly proportional to :
   (a) $\frac{1}{R^2}$ (b) $\frac{1}{R}$
   (c) R (d) R^2

6. Which of the following statements is false for a particle moving in a circle with a constant angular speed ?
   (a) The velocity vector is tangent to the circle
   (b) The acceleration vector is tangent to the circle
   (c) The acceleration vector points to the centre of the circle
   (d) The velocity and acceleration vectors are perpendicular to each other

7. An automobile travelling with a speed of 60 km/h, can brake to stop within a distance of 20 m. If the car is going twice as fast, *i. e.*, 120 km/h, the stopping distance will be :
   (a) 20 m (b) 40 m
   (c) 60 m (d) 80 m

8. A machine gun fires a bullet of mass 40 g with a velocity 1200 ms^{-1}. The man holding it, can exert a maximum force of 144 N on the gun. How many bullets can he fire per second at the most ?
   (a) One (b) Four
   (c) Two (d) Three

9. Two masses $m_1 = 5$ kg and $m_2 = 4.8$ kg tied to a string are hanging over a light frictionless pulley. What is the acceleration of the masses when lift is free to move ? ($g = 9.8$ m/s^2)



   (a) 0.2 m/s^2
   (b) 9.8 m/s^2
   (c) 5 m/s^2
   (d) 4.8 m/s^2

10. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table ?
    (a) 7.2 J (b) 3.6 J
    (c) 120 J (d) 1200 J

11. A block rests on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of static friction between the block and the plane is 0.8. If the frictional force on the block is 10 N, the mass of the block (in kg) is : (take $g = 10$ m/s^2)
    (a) 2.0 (b) 4.0
    (c) 1.6 (d) 2.5

12. A force $\vec{F} = (5\hat{i} + 3\hat{j} + 2\hat{k})$N is applied over a particle which displaces it from its origin to the point $\vec{r} = (2\hat{i} - \hat{j})$ m. The work done on the particle in joules is :
(a) -7 (b) $+7$
(c) $+10$ (d) $+13$

13. A body of mass m accelerates uniformly from rest to v_1 in time t_1. The instantaneous power delivered to the body as a function of time t is :
(a) $\frac{mv_1t}{t_1}$ (b) $\frac{mv_1^2t}{t_1^2}$
(c) $\frac{mv_1t^2}{t_1}$ (d) $\frac{mv_1^2t}{t_1}$

14. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane, it follows that :
(a) its velocity is constant
(b) its acceleration is constant
(c) its kinetic energy is constant
(d) it moves in a straight line

15. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same which one of the following will not be affected ?
(a) Moment of inertia
(b) Angular momentum
(c) Angular velocity
(d) Rotational kinetic energy

16. A ball is thrown from a point with a speed v_0 at an angle of projection θ. From the same point and at the same instant, a person starts running with a constant speed $\frac{v_0}{2}$ to catch the ball. Will the person be able to catch the ball ? If yes, what should be the angle of projection ?
(a) Yes, 60° (b) Yes, 30°
(c) No (d) Yes, 45°

17. One solid sphere A and another hollow sphere B are of same mass and same outer radii. Their moment of inertia about their diameters are respectively I_A and I_B such that :
(a) $I_A = I_B$ (b) $I_A > I_B$
(c) $I_A < I_B$ (d) $\frac{I_A}{I_B} = \frac{d_A}{d_B}$
where d_A and d_B are their densities.

18. A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is :
(a) gx (b) $\frac{gR}{R-x}$
(c) $\frac{gR^2}{R+x}$ (d) $\left(\frac{gR^2}{R+x}\right)^{1/2}$

19. The time period of an earth satellite in circular orbit is independent of :
(a) the mass of the satellite
(b) radius of its orbit
(c) both the mass and radius of the orbit
(d) neither the mass of the satellite nor the radius of its orbit

20. If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the earth, is :
(a) $2mgR$ (b) $\frac{1}{2}mgR$
(c) $\frac{1}{4}mgR$ (d) mgR

21. Suppose the gravitational force varies inversely as the nth power of distance. Then the time period of a planet in circular orbit of radius R around the sun will be proportional to :
(a) $R^{\left(\frac{n+1}{2}\right)}$ (b) $R^{\left(\frac{n-1}{2}\right)}$
(c) R^n (d) $R^{\left(\frac{n-2}{2}\right)}$

22. A wire fixed at the upper end stretches by length l by applying a force F. The work done in stretching is :
(a) $\frac{F}{2l}$ (b) Fl
(c) $2Fl$ (d) $\frac{Fl}{2}$

23. Spherical balls of radius R are falling in a viscous fluid of viscosity η with a velocity v. The retarding viscous force acting on the spherical ball is :
(a) directly proportional to R but inversely proportional to v
(b) directly proportional to both radius R and velocity v
(c) inversely proportional to both radius R and velocity v
(d) inversely proportional to R but directly proportional to velocity v

24. If two soap bubbles of different radii are connected by a tube :

(a) air flows from the bigger bubble to the smaller bubble till the sizes become equal
(b) air flows from bigger bubble to the smaller bubble till the sizes are interchanged
(c) air flows from the smaller bubble to the bigger
(d) there is no flow of air

25. The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is $(4/3) \times 1000$ kg/m^3. What relationship between t and t_0 is true ?

(a) $t = t_0$ (b) $t = t_0/2$
(c) $t = 2t_0$ (d) $t = 4t_0$

26. A particle at the end of a spring executes simple harmonic motion with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T, then :

(a) $T = t_1 + t_2$ (b) $T^2 = t_1^2 + t_2^2$
(c) $T^{-1} = t_1^{-1} + t_2^{-1}$ (d) $T^{-2} = t_1^{-2} + t_2^{-2}$

27. The total energy of a particle, executing simple harmonic motion is :

(a) $\propto x$ (b) $\propto x^2$
(c) independent of x (d) $\propto x^{1/2}$

where x is the displacement from the mean position.

28. The displacement y of a particle in a medium can be expressed as :

$y = 10^{-6} \sin\left(100t + 20x + \frac{\pi}{4}\right)$m, where t is in second and x in metre. The speed of the wave is :

(a) 2000 m/s (b) 5 m/s
(c) 20 m/s (d) 5π m/s

29. A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force $F(t)$ proportional to $\cos \omega t$ $(\omega \neq \omega_0)$ is applied to the oscillator. The time displacement of the oscillator will be proportional to :

(a) $\frac{m}{\omega_0^2 - \omega^2}$ (b) $\frac{1}{m(\omega_0^2 - \omega^2)}$
(c) $\frac{1}{m(\omega_0^2 + \omega^2)}$ (d) $\frac{m}{\omega_0^2 + \omega^2}$

30. In forced oscillation of a particle, the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force, then :

(a) $\omega_1 = \omega_2$
(b) $\omega_1 > \omega_2$
(c) $\omega_1 < \omega_2$ when damping is small and $\omega_1 > \omega_2$ when damping is large
(d) $\omega_1 < \omega_2$

31. One mole of ideal monoatomic gas ($\gamma = 5/3$) is mixed with one mole of diatomic gas ($\gamma = 7/5$). What is γ for the mixture ? γ denotes the ratio of specific heat at constant pressure, to that at constant volume :

(a) 3/2 (b) 23/15
(c) 35/23 (d) 4/3

32. If the temperature of the sun were to increase from T to $2T$ and its radius from R to $2R$, then the ratio of the radiant energy received on earth to what it was previously, will be :

(a) 4 (b) 16
(c) 32 (d) 64

33. Which of the following statements is correct for any thermodynamic system ?

(a) The internal energy changes in all processes
(b) Internal energy and entropy are state functions
(c) The change in entropy can never be zero
(d) The work done in an adiabatic process is always zero

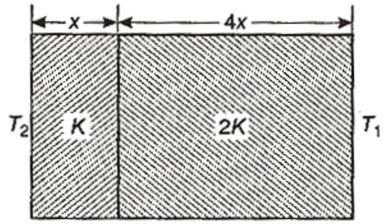
34. Two thermally insulated vessels 1 and 2 are filled with air at temperatures (T_1, T_2), volume (V_1, V_2) and pressure (P_1, P_2) respectively. If the valve joining the two vessels is opened, the temperature inside the vessel at equilibrium will be :

(a) $T_1 + T_2$
(b) $\frac{(T_1 + T_2)}{2}$
(c) $\frac{T_1T_2(P_1V_1 + P_2V_2)}{P_1V_1T_2 + P_2V_2T_1}$
(d) $\frac{T_1T_2(P_1V_1 + P_2V_2)}{P_1V_1T_1 + P_2V_2T_2}$

35. A radiation of energy E falls normally on a perfectly reflecting surface. The momentum transferred to the surface is :

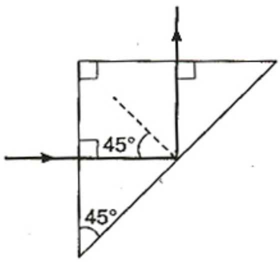
(a) E/c (b) $2E/c$
(c) Ec (d) E/c^2

36. The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and $2K$ and thickness x and $4x$, respectively are T_2 and T_1 $(T_2 > T_1)$. The rate of heat transfer through the slab, in a steady state is $\left(\frac{A(T_2 - T_1)K}{x}\right) f$, with f equals to :



(a) 1 (b) 1/2
(c) 2/3 (d) 1/3

37. A light ray is incident perpendicular to one face of a 90° prism and is totally internally reflected at the glass-air interface. If the angle of reflection is 45°, we conclude that the refractive index n :



(a) $n < \frac{1}{\sqrt{2}}$ (b) $n > \sqrt{2}$
(c) $n > \frac{1}{\sqrt{2}}$ (d) $n < \sqrt{2}$

38. A plano-convex lens of refractive index 1.5 and radius of curvature 30 cm is silvered at the curved surface. Now, this lens has been used to form the image of an object. At what distance from this lens, an object be placed in order to have a real image of the size of the object ?

(a) 20 cm (b) 30 cm
(c) 60 cm (d) 80 cm

39. The angle of incidence at which reflected light is totally polarized for reflection from air to glass (refractive index n), is :

(a) $\sin^{-1}(n)$ (b) $\sin^{-1}(1/n)$
(c) $\tan^{-1}(1/n)$ (d) $\tan^{-1}(n)$

40. The maximum number of possible interference maxima for slit-separation equal to twice the wavelength in Young's double-slit experiment, is :

(a) infinite (b) five
(c) three (d) zero

41. An electromagnetic wave of frequency $\nu = 3.0$ MHz passes from vacuum into a dielectric medium with permittivity $\varepsilon = 4.0$. Then :

(a) wavelength is doubled and the frequency remains unchanged
(b) wavelength is doubled and frequency becomes half
(c) wavelength is halved and frequency remains unchanged
(d) wavelength and frequency both remain unchanged

42. Two spherical conductors B and C having equal radii and carrying equal charges in them repel each other with a force F when kept apart at some distance. A third spherical conductor having same radius as that of B but uncharged, is brought in contact with B, then brought in contact with C and finally removed away from both. The new force of repulsion between B and C is :

(a) $\frac{F}{4}$ (b) $\frac{3F}{4}$
(c) $\frac{F}{8}$ (d) $\frac{3F}{8}$

43. A charged particle q is shot towards another charged particle Q which is fixed, with a speed v. It approaches Q upto a closest distance r and then returns. If q was given a speed $2v$, the closest distance of approach would be :

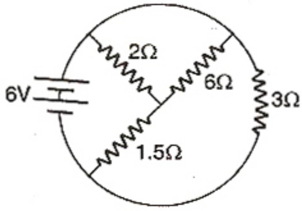
(a) r (b) $2r$
(c) $r/2$ (d) $r/4$

44. Four charges equal to $-Q$ are placed at the four corners of a square and a charge q is at its centre. If the system is in equilibrium, the value of q is :

(a) $-\frac{Q}{4}(1 + 2\sqrt{2})$ (b) $\frac{Q}{4}(1 + 2\sqrt{2})$
(c) $-\frac{Q}{2}(1 + 2\sqrt{2})$ (d) $\frac{Q}{2}(1 + 2\sqrt{2})$

45. Alternating current can not be measured by DC ammeter because :
(a) AC cannot pass through DC ammeter
(b) AC changes direction
(c) average value of current for complete cycle is zero
(d) DC ammeter will get damaged

46. The total current supplied to the circuit by the battery is :



(a) 1 A (b) 2 A
(c) 4 A (d) 6 A

47. The resistance of the series combination of two resistances is S. When they are joined in parallel, the total resistance is P. If $S = nP$, then the minimum possible value of n is :
(a) 4 (b) 3
(c) 2 (d) 1

48. An electric current is passed through a circuit containing two wires of the same material, connected in parallel. If the lengths and radii of the wires are in the ratio of 4/3 and 2/3, then the ratio of the currents passing through the wire will be :
(a) 3 (b) 1/3
(c) 8/9 (d) 2

49. In a metre bridge experiment, null point is obtained at 20 cm from one end of the wire when resistance X is balanced against another resistance Y. If $X < Y$, then where will be the new position of the null point from the same end, if one decides to balance a resistance of $4X$ against Y ?
(a) 50 cm (b) 80 cm
(c) 40 cm (d) 70 cm

50. The thermistors are usually made of :
(a) metals with low temperature coefficient of resistivity
(b) metals with high temperature coefficient of resistivity
(c) metal oxides with high temperature coefficient of resistivity
(d) semiconducting materials having low temperature coefficient of resistivity

51. Time taken by a 836 W heater to heat one litre of water from 10°C to 40°C is :
(a) 50 s (b) 100 s
(c) 150 s (d) 200 s

52. The thermo-emf of a thermocouple varies with the temperature θ of the hot junction as $E = a\,\theta + b\theta^2$ in volts where the ratio a/b is 700°C. If the cold junction is kept at 0°C, then the neutral temperature is :
(a) 700°C
(b) 350°C
(c) 1400°C
(d) no neutral temperature is possible for this thermocouple

53. The electrochemical equivalent of metal is 3.3×10^{-7} kg per coulomb. The mass of the metal liberated at the cathode when a 3 A current is passed for 2 s, will be :
(a) 19.8×10^{-7} kg
(b) 9.9×10^{-7} kg
(c) 6.6×10^{-7} kg
(d) 1.1×10^{-7} kg

54. A current i A flows along an infinitely long straight thin walled tube, then the magnetic induction at any point inside the tube is :
(a) infinite (b) zero
(c) $\frac{\mu_0}{4\pi} \cdot \frac{2i}{r}$ T (d) $\frac{2i}{r}$ T

55. A long wire carries a steady current. It is bent into a circle of one turn and the magnetic field at the centre of the coil is B. It is then bent into a circular loop of n turns. The magnetic field at the centre of the coil will be :
(a) nB (b) n^2B
(c) $2nB$ (d) $2n^2B$

56. The magnetic field due to a current carrying circular loop of radius 3 cm at a point on the axis at a distance of 4 cm from the centre is 54 μT. What will be its value at the centre of the loop ?
(a) 250 μT (b) 150 μT
(c) 125 μT (d) 75 μT

57. Two long conductors, separated by a distance d carry currents I_1 and I_2 in the same direction. They exert a force F on each other. Now the current in one of them is increased to two times and its direction is reversed. The distance is also increased to $3d$. The new value of the force between them is :
(a) $-2F$ (b) $F/3$
(c) $-2F/3$ (d) $-F/3$

58. The length of a magnet is large compared to its width and breadth. The time period of its oscillation in a vibration magnetometer is 2 s. The magnet is cut along its length into three equal parts and three parts are then placed on each other with their like poles together. The time period of this combination will be :

(a) 2 s (b) 2/3 s
(c) $2\sqrt{3}$ s (d) $2/\sqrt{3}$ s

59. The materials suitable for making electromagnets should have :

(a) high retentivity and high coercivity
(b) low retentivity and low coercivity
(c) high retentivity and low coercivity
(d) low retentivity and high coercivity

60. In an *LCR* series ac circuit, the voltage across each of the components. *L*, *C* and *R* is 50 V. The voltage across the *LC* combination will be :

(a) 50 V (b) $50\sqrt{2}$ V
(c) 100 V (d) 0 (zero)

61. A coil having *n* turns and resistance *R* Ω is connected with a galvanometer of resistance 4*R* Ω. This combination is moved in time *t* seconds from a magnetic field W_1 weber to W_2 weber. The induced current in the circuit is :

(a) $\frac{W_2 - W_1}{5Rnt}$ (b) $-\frac{n(W_2 - W_1)}{5Rt}$
(c) $-\frac{(W_2 - W_1)}{Rnt}$ (d) $-\frac{n(W_2 - W_1)}{Rt}$

62. In a uniform magnetic field of induction *B*, a wire in the form of semicircle of radius *r* rotates about the diameter of the circle with angular frequency ω. If the total resistance of the circuit is *R*, the mean power generated per period of rotation is :

(a) $\frac{B\pi r^2\omega}{2R}$ (b) $\frac{(B\pi r^2\omega)^2}{8R}$
(c) $\frac{(B\pi r\omega)^2}{2R}$ (d) $\frac{(B\pi r\omega^2)^2}{8R}$

63. In an *LCR* circuit, capacitance is changed from *C* to 2*C*. For the resonant frequency to remain unchanged, the inductance should be changed from *L* to :

(a) 4*L* (b) 2*L*
(c) *L*/2 (d) *L*/4

64. A metal conductor of length 1m rotates vertically about one of its ends at angular velocity 5 radians per second. If the horizontal component of earth's magnetic field is 0.2×10^{-4} T, then the emf developed between the two ends of the conductor is :

(a) 5 μV (b) 50 μV
(c) 5 mV (d) 50 mV

65. According to Einstein's photoelectric equation, the plot of the kinetic energy of the emitted photoelectrons from a metal *Vs* the frequency, of the incident radiation gives a straight line whose slope :

(a) depends on the nature of the metal used
(b) depends on the intensity of the radiation
(c) depends both on the intensity of the radiation and the metal used
(d) is the same for all metals and independent of the intensity of the radiation

66. The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately :

(a) 540 nm (b) 400 nm
(c) 310 nm (d) 220 nm

67. A charged oil drop is suspended in uniform field of 3×10^4 V/m so that it neither falls nor rises. The charge on the drop will be :
(take the mass of the charge $= 9.9 \times 10^{-15}$ kg and $g = 10$ m/s^2)

(a) 3.3×10^{-18} C (b) 3.2×10^{-18} C
(c) 1.6×10^{-18} C (d) 4.8×10^{-18} C

68. A nucleus disintegrates into two nuclear parts which have their velocities in the ratio 2 : 1. The ratio of their nuclear sizes will be :

(a) $2^{1/3} : 1$ (b) $1 : 3^{1/2}$
(c) $3^{1/2} : 1$ (d) $1 : 2^{1/3}$

69. The binding energy per nucleon of deuteron (^2_1H) and helium nucleus (^4_2He) is 1.1 MeV and 7 MeV respectively. If two deuteron nuclei react to form a single helium nucleus, then the energy released is :

(a) 13.9 MeV (b) 26.9 MeV
(c) 23.6 MeV (d) 19.2 MeV

70. An α-particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of the closest approach is of the order of :

(a) 1 Å (b) 10^{-10} cm
(c) 10^{-12} cm (d) 10^{-15} cm

71. When *npn* transistor is used as an amplifier :
(a) electrons move from base to collector
(b) holes move from emitter to base
(c) electrons move from collector to base
(d) holes move from base to emitter

72. For a transistor amplifier in common emitter configuration for load impedance of 1 kΩ ($h_{fe} = 50$ and $h_{oe} = 25 \mu$ A/V), the current gain is :
(a) – 5.2 (b) – 15.7
(c) – 24.8 (d) – 48.78

73. A piece of copper and another of germanium are cooled from room temperature to 77 K, the resistance of :
(a) each of them increases
(b) each of them decreases
(c) copper decreases and germanium increases
(d) copper increases and germanium decreases

74. The manifestation of band structure in solids is due to :
(a) Heisenberg's uncertainty principle
(b) Pauli's exclusion principle
(c) Bohr's correspondence principle
(d) Boltzmann's law

75. When *p-n* junction diode is forward biased, then :
(a) the depletion region is reduced and barrier height is increased
(b) the depletion region is widened and barrier height is reduced
(c) both the depletion region and barrier height are reduced
(d) both the depletion region and barrier height are increased

Chemistry

76. Which of the following sets of quantum numbers is correct for an electron in $4f$ orbital ?
(a) $n = 4, l = 3, m = +4, s = +1/2$
(b) $n = 4, l = 4, m = -4, s = -1/2$
(c) $n = 4, l = 3, m = +1, s = +1/2$
(d) $n = 3, l = 2, m = -2, s = +1/2$

77. Consider the ground state of Cr atom ($Z = 24$). The numbers of electrons with the azimuthal quantum numbers, $l = 1$ and 2 are, respectively :
(a) 12 and 4 (b) 12 and 5
(c) 16 and 4 (d) 16 and 5

78. Which one of the following ions has the highest value of ionic radius ?
(a) Li^+ (b) B^{3+}
(c) O^{2-} (d) F^-

79. The wavelength of the radiation emitted, when in a hydrogen atom electron falls from infinity to stationary state 1, would be (Rydberg constant $= 1.097 \times 10^7$ m^{-1}) :
(a) 91 nm (b) 192 nm
(c) 406 nm (d) 9.1×10^{-8} nm

80. The correct order of bond angles (smallest first) in H_2S, NH_3, BF_3 and SiH_4 is :
(a) $H_2S < SiH_4 < NH_3 < BF_3$
(b) $NH_3 < H_2S < SiH_4 < BF_3$
(c) $H_2S < NH_3 < SiH_4 < BF_3$
(d) $H_2S < NH_3 < BF_3 < SiH_4$

81. Which one of the following sets of ions represents the collection of isoelectronic species ?
(a) K^+, Ca^{2+}, Sc^{3+}, Cl^-
(b) Na^+, Ca^{2+}, Sc^{3+}, F^-
(c) K^+, Cl^-, Mg^{2+}, Sc^{3+}
(d) Na^+, Mg^{2+}, Al^{3+}, Cl^-
(Atomic numbers F = 9, Cl = 17, Na = 11, Mg = 12, Al = 13, K = 19, Ca = 20, Sc = 21)

82. Among Al_2O_3, SiO_2, P_2O_3 and SO_2 the correct order of acid strength is :
(a) $SO_2 < P_2O_3 < SiO_2 < Al_2O_3$
(b) $SiO_2 < SO_2 < Al_2O_3 < P_2O_3$
(c) $Al_2O_3 < SiO_2 < SO_2 < P_2O_3$
(d) $Al_2O_3 < SiO_2 < P_2O_3 < SO_2$

83. The bond order in NO is 2.5 while that in NO^+ is 3. Which of the following statements is true for these two species ?
(a) Bond length in NO^+ is greater than in NO
(b) Bond length in NO is greater than in NO^+
(c) Bond length in NO^+ is equal to that in NO
(d) Bond length is unpredictable

84. The formation of the oxide ion $O^{2-}(g)$ requires first an exothermic and then an endothermic step as shown below
$O(g) + e^- = O^-(g)$; $\Delta H° = -142$ kJ mol^{-1}
$O(g)^- + e^- = O^{2-}(g)$; $\Delta H° = 844$ kJ mol^{-1}

This is because :
(a) oxygen is more electronegative
(b) oxygen has high electron affinity
(c) O^- ion will tend to resist the addition of another electron
(d) O^- ion has comparatively larger size than oxygen atom

85. The states of hybridization of boron and oxygen atoms in boric acid (H_3BO_3) are respectively :
(a) sp^2 and sp^2 (b) sp^2 and sp^3
(c) sp^3 and sp^2 (d) sp^3 and sp^3

86. Which one of the following has the regular tetrahedral structure ?
(a) XeF_4 (b) SF_4
(c) BF_4^- (d) $[Ni(CN)_4]^{2-}$
(Atomic numbers B = 5, S = 16, Ni = 28, Xe = 54)

87. Of the following outer electronic configurations of atoms, the highest oxidation state is achieved by which one of them ?
(a) $(n-1)\,d^8ns^2$ (b) $(n-1)\,d^5ns^1$
(c) $(n-1)\,d^3ns^2$ (d) $(n-1)\,d^5ns^2$

88. As the temperature is raised from 20°C to 40°C, the average kinetic energy of neon atoms changes by a factor of which of the following ?
(a) 1/2 (b) $\sqrt{313/293}$
(c) 313/293 (d) 2

89. The maximum number of 90° angles between bond pair-bond pair of electrons is observed in :
(a) dsp^3 hybridization
(b) sp^3d hybridization
(c) dsp^2 hybridization
(d) sp^3d^2 hybridization

90. Which one of the following aqueous solutions will exhibit highest boiling point ?
(a) 0.01 M Na_2SO_4
(b) 0.01 M KNO_3
(c) 0.015 M urea
(d) 0.015 M glucose

91. Which among the following factors is the most important in making fluorine the strongest oxidizing agent ?
(a) Electron affinity
(b) Ionization enthalpy
(c) Hydration enthalpy
(d) Bond dissociation energy

92. In van der Waals' equation of state of the gas law, the constant 'b' is a measure of :
(a) intermolecular repulsions
(b) intermolecular attraction
(c) volume occupied by the molecules
(d) intermolecular collisions per unit volume

93. The conjugate base of $H_2PO_4^-$ is :
(a) PO_4^{3-} (b) P_2O_5
(c) H_3PO_4 (d) HPO_4^{2-}

94. 6.02×10^{20} molecules of urea are present in 100 mL of its solution. The concentration of urea solution is :
(a) 0.001 M (b) 0.01 M
(c) 0.02 M (d) 0.1 M
(Avogadro constant, $N_A = 6.02 \times 10^{23}\ mol^{-1}$)

95. To neutralise completely 20 mL of 0.1M aqueous solution of phosphorous acid (H_3PO_3), the volume of 0.1M aqueous KOH solution required is :
(a) 10 m L (b) 20 m L
(c) 40 m L (d) 60 m L

96. For which of the following parameters the structural isomers C_2H_5OH and CH_3OCH_3 would be expected to have the same values ? (Assume ideal behaviour) :
(a) Heat of vaporisation
(b) Vapour pressure at the same temperature
(c) Boiling points
(d) Gaseous densities at the same temperature and pressure

97. Which of the following liquid pairs shows a positive deviation from Raoult's law ?
(a) Water — hydrochloric acid
(b) Benzene — methanol
(c) Water — nitric acid
(d) Acetone — chloroform

98. Which one of the following statements is false ?
(a) Raoult's law states that the vapour pressure of a component over a solution is proportional to its mole fraction
(b) The osmotic pressure (π) of a solution is given by the equation $\pi = MRT$, where M is the molarity of the solution
(c) The correct order of osmotic pressure for 0.01 M aqueous solution of each compound is $BaCl_2 > KCl > CH_3COOH >$ sucrose
(d) Two sucrose solutions of same molality prepared in different solvents will have the same freezing point depression

99. What type of crystal defect is indicated in the diagram below ?

Na^+, Cl^-, Na^+, Cl^-, Na^+, Cl^-

Cl^- □ Cl^- Na^+ □ Na^+

Na^+ Cl^- □ Cl^-, Na^+ Cl^-

Cl^- Na^+ Cl^- Na^+ □ Na^+

(a) Frenkel defect
(b) Schottky defect
(c) Interstitial defect
(d) Frenkel and Schottky defects

100. An ideal gas expands in volume from 1×10^{-3} m^3 to 1×10^{-2} m^3 at 300K against a constant pressure of 1×10^5 Nm^{-2}. The work done is :

(a) – 900 J (b) –900 k J
(c) 270 k J (d) 900 k J

101. In a hydrogen-oxygen fuel cell, combustion of hydrogen occurs to :

(a) generate heat
(b) create potential difference between the two electrodes
(c) produce high purity water
(d) remove adsorbed oxygen from electrode surfaces

102. In a first order reaction, the concentration of the reactant, decreases from 0.8 M to 0.4 M in 15 min. The time taken for the concentration to change from 0.1 M to 0.025 M is :

(a) 30 min (b) 15 min
(c) 7.5 min (d) 60 min

103. What is the equilibrium expression for the reaction

$$P_4(s) + 5O_2(g) \rightleftharpoons P_4O_{10}(s) ?$$

(a) $K_c = \frac{[P_4O_{10}]}{[P_4][O_2]^5}$ (b) $K_c = \frac{[P_4O_{10}]}{5[P_4][O_2]}$

(c) $K_c = [O_2]^5$ (d) $K_c = \frac{1}{[O_2]^5}$

104. For the reaction,
$CO(g) + Cl_2(g) \rightleftharpoons COCl_2(g)$, the K_p / K_c is equal to :

(a) $1/RT$ (b) RT
(c) $\sqrt{RT}$ (d) 1.0

105. The equilibrium constant for the reaction

$$N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$$

at temperature T is 4×10^{-4}. The value of K_c for the reaction :
$NO(g) \rightleftharpoons \frac{1}{2}N_2(g) + \frac{1}{2}O_2(g)$ at the same temperature is :

(a) 2.5×10^2 (b) 50
(c) 4×10^{-4} (d) 0.02

106. The rate equation for the reaction $2A + B \rightarrow C$ is found to be :

$$\text{rate} = k[A][B]$$

The correct statement in relation to this reaction is that the :

(a) unit of k must be s^{-1}
(b) $t_{1/2}$ is a constant
(c) rate of formation of C is twice the rate of disappearance of A
(d) value of k is independent of the initial concentrations of A and B

107. Consider the following $E°$ values :

$$E°_{Fe^{3+}/Fe^{2+}} = +0.77 \text{ V}$$

$$E°_{Sn^{2+}/Sn} = -0.14 \text{ V}$$

Under standard conditions the potential for the reaction
$Sn(s) + 2Fe^{3+}(aq) \rightarrow 2Fe^{2+}(aq) + Sn^{2+}(aq)$ is :

(a) 1.68 V (b) 1.40 V
(c) 0.91 V (d) 0.63 V

108. The molar solubility (in mol L^{-1}) of a sparingly soluble salt MX_4 is 's'. The corresponding solubility product is K_{sp}. s is given in terms of K_{sp} by the relation :

(a) $s = (K_{sp}/128)^{1/4}$ (b) $s = (128\,K_{sp})^{1/4}$

(c) $s = (256\,K_{sp})^{1/5}$ (d) $s = (K_{sp}/256)^{1/5}$

109. The standard emf of a cell, involving one electron change is found to be 0.591 V at 25°C. The equilibrium constant of the reaction is (F = 96,500 C mol^{-1}, :

(a) 1.0×10^1 (b) 1.0×10^5
(c) 1.0×10^{10} (d) 1.0×10^{30}

110. The enthalpies of combustion of carbon and carbon monoxide are –393.5 and –283k J mol^{-1} respectively. The enthalpy of formation of carbon monoxide per mole is :

(a) 110.5 k J (b) 676.5 k J
(c) – 676.5 k J (d) – 110.5 k J

111. The limiting molar conductivities $\wedge°$ for NaCl, KBr and KCl are 126, 152 and 150 S cm^2 mol^{-1} respectively. The $\wedge°$ for NaBr is :

(a) 128 S cm^2 mol^{-1} (b) 176 S cm^2 mol^{-1}
(c) 278 S cm^2 mol^{-1} (d) 302 S cm^2 mol^{-1}

112. In a cell that utilises the reaction

$Zn(s) + 2H^+(aq) \longrightarrow Zn^{2+}(aq) + H_2(g)$

addition of H_2SO_4 to cathode compartment, will :

(a) lower the E and shift equilibrium to the left
(b) lower the E and shift the equilibrium to the right
(c) increase the E and shift the equilibrium to the right
(d) increase the E and shift the equilibrium to the left

113. Which one of the following statements regarding helium is incorrect ?

(a) It is used to fill gas balloons instead of hydrogen because it is lighter and non-inflammable
(b) It is used as a cryogenic agent for carrying out experiments at low temperatures
(c) It is used to produce and sustain powerful superconducting magnets
(d) It is used in gas-cooled nuclear reactors

114. Identify the correct statement regarding enzymes :

(a) Enzymes are specific biological catalysts that can normally function at very high temperatures ($T \sim 1000$ K).
(b) Enzymes are normally heterogeneous catalysts that are very specific in their action
(c) Enzymes are specific biological catalysts that cannot be poisoned
(d) Enzymes are specific biological catalysts that possess well defined active sites.

115. One mole of magnesium nitride on the reaction with an excess of water gives :

(a) one mole of ammonia
(b) one mole of nitric acid
(c) two moles of ammonia
(d) two moles of nitric acid

116. Which one of the following ores is best concentrated by froth-floatation method ?

(a) Magnetite (b) Cassiterite
(c) Galena (d) Malachite

117. Beryllium and aluminium exhibit many properties which are similar. But, the two elements differ in :

(a) exhibiting maximum covalency in compounds
(b) forming polymeric hydrides
(c) forming covalent halides
(d) exhibiting amphoteric nature in their oxides

118. Aluminium chloride exists as dimer, Al_2Cl_6 in solid state as well as in solution of non-polar solvents such as benzene. When dissolved in water, it gives :

(a) $Al^{3+} + 3Cl^-$
(b) $[Al(H_2O)_6]^{3+} + 3Cl^-$
(c) $[Al(OH)_6]^{3-} + 3HCl$
(d) $Al_2O_3 + 6HCl$

119. The soldiers of Napolean army while at Alps during freezing winter suffered a serious problem as regards to the tin buttons of their uniforms. White metallic tin buttons got converted to grey powder. This transformation is related to :

(a) a change in the crystalline structure of tin
(b) an interaction with nitrogen of the air at very low temperatures
(c) a change in the partial pressure of oxygen in the air
(d) an interaction with water vapour contained in the humid air

120. The $E^\circ_{M^{3+}/M^{2+}}$ values for Cr, Mn, Fe and Co are -0.41, $+1.57$, $+0.77$ and $+1.97$ V respectively. For which one of these metals the change in oxidation state from $+2$ to $+3$ is easiest ?

(a) Cr (b) Mn
(c) Fe (d) Co

121. Excess of KI reacts with $CuSO_4$ solution and then $Na_2S_2O_3$ solution is added to it. Which of the statements is incorrect for this reaction ?

(a) Cu_2I_2 is formed
(b) CuI_2 is formed
(c) $Na_2S_2O_3$ is oxidised
(d) Evolved I_2 is reduced

122. Among the properties (A) reducing (B) oxidising (C) complexing, the set of properties shown by CN^- ion towards metal species is :

(a) A, B (b) B, C
(c) C, A (d) A, B, C

123. The co-ordination number of a central metal atom in a complex is determined by :

(a) the number of ligands around a metal ion bonded by sigma bonds
(b) the number of ligands around a metal ion bonded by pi-bonds
(c) the number of ligands around a metal ion bonded by sigma and pi-bonds both
(d) the number of only anionic ligands bonded to the metal ion

124. Which one of the following complexes is an outer orbital complex ?
(a) $[Fe(CN)_6]^{4-}$ (b) $[Mn(CN)_6]^{4-}$
(c) $[Co(NH_3)_6]^{3+}$ (d) $[Ni(NH_3)_6]^{2+}$
(Atomic numbers Mn = 25, Fe = 26, Co = 27, Ni = 28)

125. Co-ordination compounds have great importance in biological systems. In this context which of the following statements is incorrect ?
(a) Chlorophylls are green pigments in plants and contain calcium
(b) Haemoglobin is the red pigment of blood and contains iron
(c) Cyanocobalamin is vitamin B_{12} and contains cobalt
(d) Carboxypeptidase-A is an enzyme and contains zinc

126. Cerium ($Z = 58$) is an important member of the lanthanides. Which of the following statements about cerium is incorrect ?
(a) The common oxidation states of cerium are + 3 and + 4
(b) The +3 oxidation state of cerium is more stable than the + 4 oxidation state
(c) The + 4 oxidation state of cerium is not known in solutions
(d) Cerium (IV) acts as an oxidising agent

127. Which one of the following has largest number of isomers ?
(a) $[Ru(NH_3)_4Cl_2]^+$
(b) $[Co(NH_3)_5Cl]^{2+}$
(c) $[Ir(PR_3)_2H(CO)]^{2+}$
(d) $[Co(en)_2Cl_2]^+$
(R = alkyl group, en = ethylenediamine)

128. The correct order of magnetic moments (spin only values in BM) among the following is :
(a) $[MnCl_4]^{2-} > [CoCl_4]^{2-} > [Fe(CN)_6]^{4-}$
(b) $[MnCl_4]^{2-} > [Fe(CN)_6]^{4-} > [CoCl_4]^{2-}$
(c) $[Fe(CN)_6]^{4-} > [MnCl_4]^{2-} > [CoCl_4]^{2-}$
(d) $[Fe(CN)_6]^{4-} > [CoCl_4]^{2-} > [MnCl_4]^{2-}$
(Atomic numbers Mn = 25, Fe = 26, Co = 27)

129. Consider the following nuclear reactions :
$^{238}_{92}M \rightarrow ^{x}_{y}N + 2\,^{4}_{2}He\ ;\ ^{x}_{y}N \rightarrow ^{A}_{B}L + 2\beta^+$
The number of neutrons in the element L is :
(a) 142 (b) 144
(c) 140 (d) 146

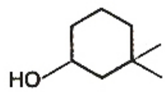
130. The half-life of a radioisotope is four hours. If the initial mass of the isotope was 200g, the mass remaining after 24 hours undecayed is :
(a) 1.042 g (b) 2.084 g
(c) 3.125 g (d) 4.167 g

131. The compound formed in the positive test for nitrogen with the Lassaigne solution of an organic compound is :
(a) $Fe_4[Fe(CN)_6]_3$
(b) $Na_3[Fe(CN)_6]$
(c) $Fe(CN)_3$
(d) $Na_4[Fe(CN)_5NOS]$

132. The ammonia evolved from the treatment of 0.30g of an organic compound for the estimation of nitrogen was passed in 100 mL of 0.1 M sulphuric acid. The excess of acid required 20 mL of 0.5 M sodium hydroxide solution for complete neutralization. The organic compound is :
(a) acetamide (b) benzamide
(c) urea (d) thiourea

133. Which one of the following has the minimum boiling point ?
(a) *n*-butane (b) 1-butyne
(c) 1-butene (d) Isobutene

134. The IUPAC name of the compound

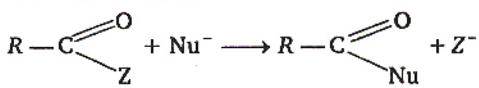


is :
(a) 3,3-dimethyl-1-hydroxy cyclohexane
(b) 1,1-dimethyl-3-hydroxy cyclohexane
(c) 3,3-dimethyl-1-cyclohexanol
(d) 1,1-dimethyl-3-cyclohexanol

135. Which one of the following does not have sp^2 hybridised carbon ?
(a) Acetone (b) Acetic acid
(c) Acetonitrile (d) Acetamide

136. Which of the following will have a meso-isomer also ?
(a) 2-chlorobutane
(b) 2,3-dichlorobutane
(c) 2,3-dichloropentane
(d) 2-hydroxypropanoic acid

137. Rate of the reaction

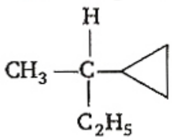


is fastest when Z is :
(a) Cl (b) NH_2
(c) OC_2H_5 (d) $OCOCH_3$

138. Amongst the following compounds, the optically active alkane having lowest molecular mass is :

(a) $CH_3-CH_2-CH_2-CH_3$

(b) $CH_3-CH_2-CH(CH_3)-CH_3$

(c) 

(d) $CH_3-CH_2-C\equiv CH$

139. Consider the acidity of the carboxylic acids :

(i) PhCOOH

(ii) o-$NO_2C_6H_4COOH$

(iii) p-$NO_2C_6H_4COOH$

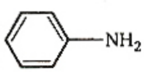
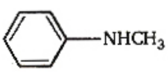
(iv) m-$NO_2C_6H_4COOH$

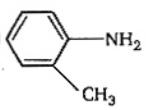
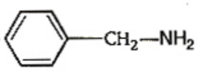
Which of the following order is correct ?

(a) i > ii > iii > iv (b) ii > iv > iii > i

(c) ii > iv > i > iii (d) ii > iii > iv > i

140. Which of the following is the strongest base ?

(a) 
(b) 

(c) 
(d) 

141. Which base is present in RNA but not in DNA ?

(a) Uracil (b) Cytosine

(c) Guanine (d) Thymine

142. The compound formed on heating chlorobenzene with chloral in the presence of concentrated sulphuric acid is :

(a) gammexane (b) DDT

(c) freon (d) hexachloroethane

143. On mixing ethyl acetate with aqueous sodium chloride, the composition of the resultant solution is :

(a) $CH_3COOC_2H_5 + NaCl$

(b) $CH_3COONa + C_2H_5OH$

(c) $CH_3COCl + C_2H_5OH + NaOH$

(d) $CH_3Cl + C_2H_5COONa$

144. Acetyl bromide reacts with excess of CH_3MgI followed by treatment with a saturated solution of NH_4Cl gives :

(a) acetone

(b) acetamide

(c) 2-methyl-2-propanol

(d) acetyl iodide

145. Which one of the following is reduced with zinc and hydrochloric acid to give the corresponding hydrocarbon ?

(a) Ethyl acetate (b) Acetic acid

(c) Acetamide (d) Butan-2-one

146. Which one of the following undergoes reaction with 50% sodium hydroxide solution to give the corresponding alcohol and acid ?

(a) Phenol (b) Benzaldehyde

(c) Butanal (d) Benzoic acid

147. Among the following compounds which can be dehydrated very easily ?

(a) $CH_3CH_2CH_2CH_2CH_2OH$

(b) $CH_3CH_2CH_2CH(OH)CH_3$

(c) $CH_3CH_2C(CH_3)(OH)CH_2CH_3$

(d) $CH_3CH_2CH(CH_3)CH_2CH_2OH$

148. Which of the following compounds is not chiral ?

(a) 1-chloropentane

(b) 2-chloropentane

(c) 1-chloro-2-methyl pentane

(d) 3-chloro-2-methyl pentane

149. Insulin production and its action in human body are responsible for the level of diabetes. This compound belongs to which of the following categories ?

(a) A co-enzyme (b) A hormone

(c) An enzyme (d) An antibiotic

150. The smog is essentially caused by the presence of :

(a) O_2 and O_3

(b) O_2 and N_2

(c) oxides of sulphur and nitrogen

(d) O_3 and N_2

Mathematics

1. Let $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ be a relation on the set $A = \{1, 2, 3, 4\}$. The relation R is :
(a) a function (b) transitive
(c) not symmetric (d) reflexive

2. The range of the function $f(x) = {}^{7-x}P_{x-3}$ is :
(a) $\{1, 2, 3\}$ (b) $\{1, 2, 3, 4, 5, 6\}$
(c) $\{1, 2, 3, 4\}$ (d) $\{1, 2, 3, 4, 5\}$

3. Let z, w be complex numbers such that $\bar{z} + i\,\bar{w} = 0$ and $\arg zw = \pi$. Then $\arg z$ equals :
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) $\frac{3\pi}{4}$ (d) $\frac{5\pi}{4}$

4. If $z = x - iy$ and $z^{1/3} = p + iq$, then $\left(\frac{x}{p} + \frac{y}{q}\right) \Big/ (p^2 + q^2)$ is equal to :
(a) 1 (b) -1
(c) 2 (d) -2

5. If $|z^2 - 1| = |z|^2 + 1$, then z lies on :
(a) the real axis (b) the imaginary axis
(c) a circle (d) an ellipse

6. Let $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$. The only correct statement about the matrix A is :
(a) A is a zero matrix
(b) $A = (-1)I$, where I is a unit matrix
(c) A^{-1} does not exist
(d) $A^2 = I$

7. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and (10) $B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$.
If B is the inverse of matrix A, then α is :
(a) -2 (b) 1
(c) 2 (d) 5

8. If $a_1, a_2, a_3, \ldots, a_n, \ldots$ are in GP, then the value of the determinant
$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$, is :
(a) 0 (b) 1
(c) 2 (d) -2

9. Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation :
(a) $x^2 + 18x + 16 = 0$
(b) $x^2 - 18x + 16 = 0$
(c) $x^2 + 18x - 16 = 0$
(d) $x^2 - 18x - 16 = 0$

10. If $(1 - p)$ is a root of quadratic equation $x^2 + px + (1 - p) = 0$, then its roots are :
(a) 0, 1 (b) -1, 1
(c) 0, -1 (d) -1, 2

11. Let $S(K) = 1 + 3 + 5 + \ldots + (2K - 1) = 3 + K^2$. Then which of the following is true ?
(a) $S(1)$ is correct
(b) $S(K) \Rightarrow S(K + 1)$
(c) $S(K) \not\Rightarrow S(K + 1)$
(d) Principle of mathematical induction can be used to prove the formula.

12. How many ways are there to arrange the letters in the word GARDEN with the vowels in alphabetical order ?
(a) 120 (b) 240
(c) 360 (d) 480

13. The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty, is :
(a) 5 (b) 21
(c) 3^8 (d) ${}^8C_3$

14. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of 'q' is :
(a) $\frac{49}{4}$ (b) 12
(c) 3 (d) 4

15. The coefficient of the middle term in the binomial expansion in powers of x of $(1 + \alpha x)^4$ and of $(1 - \alpha x)^6$ is the same, if α equals :
(a) $-\frac{5}{3}$ (b) $\frac{10}{3}$
(c) $-\frac{3}{10}$ (d) $\frac{3}{5}$

16. The coefficient of x^n in expansion of $(1 + x)(1 - x)^n$ is :
(a) $(n - 1)$ (b) $(-1)^n (1 - n)$
(c) $(-1)^{n-1} (n - 1)^2$ (d) $(-1)^{n-1} n$

17. If $s_n = \sum_{r=0}^{n} \frac{1}{^nC_r}$ and $t_n = \sum_{r=0}^{n} \frac{r}{^nC_r}$, then $\frac{t_n}{s_n}$ is equal to :

(a) $\frac{n}{2}$ (b) $\frac{n}{2} - 1$

(c) $n - 1$ (d) $\frac{2n-1}{2}$

18. Let T_r be the rth term of an A P whose first term is a and common difference is d. If for some positive integers $m, n, m \neq n, T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then $a - d$ equals :

(a) 0 (b) 1

(c) $\frac{1}{mn}$ (d) $\frac{1}{m} + \frac{1}{n}$

19. The sum of the first n terms of the series $1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + 5^2 + 2 \cdot 6^2 + \ldots$ is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum is :

(a) $\frac{3n(n+1)}{2}$ (b) $\frac{n^2(n+1)}{2}$

(c) $\frac{n(n+1)^2}{4}$ (d) $\left[\frac{n(n+1)}{2}\right]^2$

20. The sum of series $\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots$ is :

(a) $\frac{(e^2-1)}{2}$ (b) $\frac{(e-1)^2}{2e}$

(c) $\frac{(e^2-1)}{2e}$ (d) $\frac{(e^2-2)}{e}$

21. Let α, β be such that $\pi < \alpha - \beta < 3\pi$. If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\frac{\alpha-\beta}{2}$ is :

(a) $-\frac{3}{\sqrt{130}}$ (b) $\frac{3}{\sqrt{130}}$

(c) $\frac{6}{65}$ (d) $-\frac{6}{65}$

22. If $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$, then the difference between the maximum and minimum values of u^2 is given by :

(a) $2(a^2+b^2)$ (b) $2\sqrt{a^2+b^2}$

(c) $(a+b)^2$ (d) $(a-b)^2$

23. The sides of a triangle are $\sin\alpha$, $\cos\alpha$ and $\sqrt{1+\sin\alpha\cos\alpha}$ for some $0 < \alpha < \frac{\pi}{2}$. Then the greatest angle of the triangle is :

(a) 60° (b) 90°

(c) 120° (d) 150°

24. A person standing on the bank of a river, observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60° and when he retires 40 m away from the tree the angle of elevation becomes 30°. The breadth of the river is :

(a) 20 m (b) 30 m

(c) 40 m (d) 60 m

25. If $f : R \to S$, defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$, is onto , then the interval of S is :

(a) [0, 3] (b) [– 1, 1]

(c) [0, 1] (d) [– 1, 3]

26. The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then :

(a) $f(x+2) = f(x-2)$

(b) $f(2+x) = f(2-x)$

(c) $f(x) = f(-x)$

(d) $f(x) = -f(-x)$

27. The domain of the function

$$f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}} \text{ is :}$$

(a) [2, 3] (b) [2, 3)

(c) [1, 2] (d) [1, 2)

28. If $\lim_{x\to\infty}\left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x} = e^2$, then the values of a and b are :

(a) $a \in R, b \in R$ (b) $a = 1, b \in R$

(c) $a \in R, b = 2$ (d) $a = 1, b = 2$

29. Let $f(x) = \frac{1-\tan x}{4x-\pi}$, $x \neq \frac{\pi}{4}$, $x \in \left[0, \frac{\pi}{2}\right]$. If $f(x)$ is continuous in $\left[0, \frac{\pi}{2}\right]$, then $f\left(\frac{\pi}{4}\right)$ is :

(a) 1 (b) 1/2

(c) –1/2 (d) – 1

30. If $x = e^{y + e^{y + \ldots \text{to} \infty}}$, $x > 0$, then $\frac{dy}{dx}$ is :

(a) $\frac{x}{1+x}$ (b) $\frac{1}{x}$

(c) $\frac{1-x}{x}$ (d) $\frac{1+x}{x}$

31. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa, is :

(a) (2, 4) (b) (2, – 4)

(c) $\left(-\frac{9}{8}, \frac{9}{2}\right)$ (d) $\left(\frac{9}{8}, \frac{9}{2}\right)$

32. A function $y = f(x)$ has a second order derivative $f'' = 6(x-1)$. If its graph passes through the point (2, 1) and at that point the tangent to the graph is $y = 3x - 5$, then the function is :

(a) $(x-1)^2$ (b) $(x-1)^3$

(c) $(x+1)^3$ (d) $(x+1)^2$

33. The normal to the curve $x = a(1+\cos\theta)$, $y = a\sin\theta$ at 'θ' always passes through the fixed point :

(a) $(a, 0)$ (b) $(0, a)$

(c) (0, 0) (d) (a, a)

34. If $2a + 3b + 6c = 0$, then at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval :

(a) (0, 1) (b) (1, 2)

(c) (2, 3) (d) (1, 3)

35. $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n} e^{r/n}$ is :

(a) e (b) $e - 1$

(c) $1 - e$ (d) $e + 1$

36. If $\int \frac{\sin x}{\sin(x-\alpha)}\, dx = Ax + B \log \sin(x-\alpha) + c$, then value of (A, B) is :

(a) $(\sin\alpha, \cos\alpha)$ (b) $(\cos\alpha, \sin\alpha)$

(c) $(-\sin\alpha, \cos\alpha)$ (d) $(-\cos\alpha, \sin\alpha)$

37. $\int \frac{dx}{\cos x - \sin x}$ is equal to :

(a) $\frac{1}{\sqrt{2}} \log \left|\tan\left(\frac{x}{2} - \frac{\pi}{8}\right)\right| + c$

(b) $\frac{1}{\sqrt{2}} \log \left|\cot\left(\frac{x}{2}\right)\right| + c$

(c) $\frac{1}{\sqrt{2}} \log \left|\tan\left(\frac{x}{2} - \frac{3\pi}{8}\right)\right| + c$

(d) $\frac{1}{\sqrt{2}} \log \left|\tan\left(\frac{x}{2} + \frac{3\pi}{8}\right)\right| + c$

38. The value of $\int_{-2}^{3} |1 - x^2|\, dx$ is :

(a) $\frac{28}{3}$ (b) $\frac{14}{3}$

(c) $\frac{7}{3}$ (d) $\frac{1}{3}$

39. The value of $\int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{1 + \sin 2x}}\, dx$ is :

(a) 0 (b) 1

(c) 2 (d) 3

40. If $\int_0^{\pi} x f(\sin x)\, dx = A \int_0^{\pi/2} f(\sin x)\, dx$, then A is equal to :

(a) 0 (b) π

(c) $\frac{\pi}{4}$ (d) 2π

41. If $f(x) = \frac{e^x}{1+e^x}$, $I_1 = \int_{f(-a)}^{f(a)} x\, g\{x(1-x)\}\, dx$ and $I_2 = \int_{f(-a)}^{f(a)} g\{x(1-x)\}\, dx$, then the value of $\frac{I_2}{I_1}$ is :

(a) 2 (b) – 3

(c) – 1 (d) 1

42. The area of the region bounded by the curves $y = |x - 2|$, $x = 1$, $x = 3$ and the x-axis is :

(a) 1 (b) 2

(c) 3 (d) 4

43. The differential equation for the family of curves $x^2 + y^2 - 2ay = 0$, where a is an arbitrary constant, is :

(a) $2(x^2 - y^2)y' = xy$

(b) $2(x^2 + y^2)y' = xy$

(c) $(x^2 - y^2)y' = 2xy$

(d) $(x^2 + y^2)y' = 2xy$

44. The solution of the differential equation $y\, dx + (x + x^2 y)\, dy = 0$ is :

(a) $-\frac{1}{xy} = c$ (b) $-\frac{1}{xy} + \log y = c$

(c) $\frac{1}{xy} + \log y = c$ (d) $\log y = cx$

45. Let $A(2, -3)$ and $B(-2, 1)$ be vertices of a triangle ABC. If the centroid of this triangle moves on the line $2x + 3y = 1$, then the locus of the vertex C is the line :

(a) $2x + 3y = 9$ (b) $2x - 3y = 7$

(c) $3x + 2y = 5$ (d) $3x - 2y = 3$

46. The equation of the straight line passing through the point (4, 3) and making intercepts on the co-ordinate axes whose sum is – 1, is :

(a) $\frac{x}{2} + \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$

(b) $\frac{x}{2} - \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$

(c) $\frac{x}{2}+\frac{y}{3}=1$ and $\frac{x}{-2}+\frac{y}{1}=1$

(d) $\frac{x}{2}-\frac{y}{3}=1$ and $\frac{x}{-2}+\frac{y}{1}=1$

47. If the sum of the slopes of the lines given by $x^2-2cxy-7y^2=0$ is four times their product, then c has the value :

(a) 1 (b) -1
(c) 2 (d) -2

48. If one of the lines given by $6x^2-xy+4cy^2=0$ is $3x+4y=0$, then c equals :

(a) 1 (b) -1
(c) 3 (d) -3

49. If a circle passes through the point (a, b) and cuts the circle $x^2+y^2=4$ orthogonally, then the locus of its centre is :

(a) $2ax+2by+(a^2+b^2+4)=0$
(b) $2ax+2by-(a^2+b^2+4)=0$
(c) $2ax-2by+(a^2+b^2+4)=0$
(d) $2ax-2by-(a^2+b^2+4)=0$

50. A variable circle passes through the fixed point $A(p, q)$ and touches x-axis. The locus of the other end of the diameter through A is :

(a) $(x-p)^2=4qy$ (b) $(x-q)^2=4py$
(c) $(y-p)^2=4qx$ (d) $(y-q)^2=4px$

51. If the lines $2x+3y+1=0$ and $3x-y-4=0$ lie along diameters of a circle of circumference 10π, then the equation of the circle is :

(a) $x^2+y^2-2x+2y-23=0$
(b) $x^2+y^2-2x-2y-23=0$
(c) $x^2+y^2+2x+2y-23=0$
(d) $x^2+y^2+2x-2y-23=0$

52. The intercept on the line $y=x$ by the circle $x^2+y^2-2x=0$ is AB. Equation of the circle on AB as a diameter is :

(a) $x^2+y^2-x-y=0$
(b) $x^2+y^2-x+y=0$
(c) $x^2+y^2+x+y=0$
(d) $x^2+y^2+x-y=0$

53. If $a\neq 0$ and the line $2bx+3cy+4d=0$ passes through the points of intersection of the parabolas $y^2=4ax$ and $x^2=4ay$, then :

(a) $d^2+(2b+3c)^2=0$
(b) $d^2+(3b+2c)^2=0$
(c) $d^2+(2b-3c)^2=0$
(d) $d^2+(3b-2c)^2=0$

54. The eccentricity of an ellipse with its centre at the origin, is $\frac{1}{2}$. If one of the directrices is $x=4$, then the equation of the ellipse is :

(a) $3x^2+4y^2=1$ (b) $3x^2+4y^2=12$
(c) $4x^2+3y^2=12$ (d) $4x^2+3y^2=1$

55. A line makes the same angle θ with each of the x and z axis. If the angle β, which it makes with y-axis, is such that $\sin^2\beta=3\sin^2\theta$, then $\cos^2\theta$ equals :

(a) $\frac{2}{3}$ (b) $\frac{1}{5}$
(c) $\frac{3}{5}$ (d) $\frac{2}{5}$

56. Distance between two parallel planes $2x+y+2z=8$ and $4x+2y+4z+5=0$ is :

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$
(c) $\frac{7}{2}$ (d) $\frac{9}{2}$

57. A line with direction cosines proportional to 2, 1, 2 meets each of the lines $x=y+a=z$ and $x+a=2y=2z$. The co-ordinates of each of the points of intersection are given by :

(a) $(3a, 3a, 3a)$, (a, a, a)
(b) $(3a, 2a, 3a)$, (a, a, a)
(c) $(3a, 2a, 3a)$, $(a, a, 2a)$
(d) $(2a, 3a, 3a)$, $(2a, a, a)$

58. If the straight lines $x=1+s$, $y=-3-\lambda s$, $z=1+\lambda s$ and $x=\frac{t}{2}$, $y=1+t$, $z=2-t$, with parameters s and t respectively, are co-planar, then λ equals :

(a) -2 (b) -1
(c) $-\frac{1}{2}$ (d) 0

59. The intersection of the spheres $x^2+y^2+z^2+7x-2y-z=13$ and $x^2+y^2+z^2-3x+3y+4z=8$ is the same as the intersection of one of the sphere and the plane :

(a) $x-y-z=1$ (b) $x-2y-z=1$
(c) $x-y-2z=1$ (d) $2x-y-z=1$

60. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of these are collinear. If the vector $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$ (λ being some non-zero scalar), then $\vec{a}+2\vec{b}+6\vec{c}$ equals :

(a) $\lambda\vec{a}$ (b) $\lambda\vec{b}$
(c) $\lambda\vec{c}$ (d) 0

61. A particle is acted upon by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ which displace it from a point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. The work done in standard units by the forces is given by :

(a) 40 (b) 30
(c) 25 (d) 15

62. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and λ is a real number, then the vectors $\vec{a}+2\vec{b}+3\vec{c}$, $\lambda\vec{b}+4\vec{c}$ and $(2\lambda-1)\vec{c}$ are non-coplanar for :

(a) all values of λ
(b) all except one value of λ
(c) all except two values of λ
(d) no value of λ

63. Let $\vec{u}, \vec{v}, \vec{w}$ be such that $|\vec{u}|=1, |\vec{v}|=2, |\vec{w}|=3$. If the projection $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and $\vec{v}, \vec{w}$ are perpendicular to each other, then $|\vec{u}-\vec{v}+\vec{w}|$ equals :

(a) 2 (b) $\sqrt{7}$
(c) $\sqrt{14}$ (d) 14

64. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be non-zero vectors such that $(\vec{a}\times\vec{b})\times\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the acute angle between the vectors $\vec{b}$ and $\vec{c}$, then $\sin\theta$ equals :

(a) $\frac{1}{3}$ (b) $\frac{\sqrt{2}}{3}$
(c) $\frac{2}{3}$ (d) $\frac{2\sqrt{2}}{3}$

65. Consider the following statements :
(1) Mode can be computed from histogram
(2) Median is not independent of change of scale
(3) Variance is independent of change of origin and scale
Which of these is/are correct ?
(a) Only (1) (b) Only (2)
(c) Only (1) and (2) (d) (1), (2) and (3)

66. In a series of $2n$ observations, half of them equal a and remaining half equal $-a$. If the standard deviation of the observations is 2, then $|a|$ equals :

(a) $\frac{1}{n}$ (b) $\sqrt{2}$
(c) 2 (d) $\frac{\sqrt{2}}{n}$

67. The probability that A speaks truth is 4/5 while this probability for B is 3/4. The probability that they contradict each other when asked to speak on a fact, is :

(a) $\frac{3}{20}$ (b) $\frac{1}{5}$
(c) $\frac{7}{20}$ (d) $\frac{4}{5}$

68. A random variable X has the probability distribution :

X:	1	2	3	4	5	6	7	8
$P(X)$:	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events $E=\{X \text{ is a prime number}\}$ and $F=\{X<4\}$, the probability $P(E\cup F)$ is :

(a) 0.87 (b) 0.77
(c) 0.35 (d) 0.50

69. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is :

(a) $\frac{37}{256}$ (b) $\frac{219}{256}$
(c) $\frac{128}{256}$ (d) $\frac{28}{256}$

70. With two forces acting at a point, the maximum effect is obtained when their resultant is 4N. If they act at right angles, then their resultant is 3N. Then the forces are :

(a) $(2+\sqrt{2})$ N and $(2-\sqrt{2})$ N
(b) $(2+\sqrt{3})$ N and $(2-\sqrt{3})$N
(c) $\left(2+\frac{1}{2}\sqrt{2}\right)$ N and $\left(2-\frac{1}{2}\sqrt{2}\right)$ N
(d) $\left(2+\frac{1}{2}\sqrt{3}\right)$ N and $\left(2-\frac{1}{2}\sqrt{3}\right)$ N

71. In a right angle ΔABC, $\angle A=90°$ and sides a, b, c are respectively, 5 cm, 4 cm and 3 cm. If a force $\vec{F}$ has moments 0, 9 and 16 in N cm unit respectively about vertices A, B and C, the magnitude of $\vec{F}$ is :

(a) 3 (b) 4
(c) 5 (d) 9

72. Three forces $\vec{P}, \vec{Q}$ and $\vec{R}$ acting along IA, IB and IC, where I is the incentre of a ΔABC, are in equilibrium. Then $\vec{P}:\vec{Q}:\vec{R}$ is :

(a) $\cos\frac{A}{2}:\cos\frac{B}{2}:\cos\frac{C}{2}$
(b) $\sin\frac{A}{2}:\sin\frac{B}{2}:\sin\frac{C}{2}$

(c) $\sec\frac{A}{2} : \sec\frac{B}{2} : \sec\frac{C}{2}$

(d) $\text{cosec}\frac{A}{2} : \text{cosec}\frac{B}{2} : \text{cosec}\frac{C}{2}$

73. A particle moves towards east from a point A to a point B at the rate of 4 km/h and then towards north from B to C at rate of 5 km/h. If $AB = 12$ km and $BC = 5$ km, then its average speed for its journey from A to C and resultant average velocity direct from A to C are respectively :

(a) $\frac{17}{4}$ km/h and $\frac{13}{4}$ km/h

(b) $\frac{13}{4}$ km/h and $\frac{17}{4}$ km/h

(c) $\frac{17}{9}$ km/h and $\frac{13}{9}$ km/h

(d) $\frac{13}{9}$ km/h and $\frac{17}{9}$ km/h

74. A velocity 1/4 m/s is resolved into two components along OA and OB making angles 30° and 45° respectively with the given velocity. Then the component along OB is :

(a) $\frac{1}{8}$ m/s (b) $\frac{1}{4}(\sqrt{3}-1)$ m/s

(c) $\frac{1}{4}$ m/s (d) $\frac{1}{8}(\sqrt{6}-\sqrt{2})$ m/s

75. If t_1 and t_2 are the times of flight of two particles having the same initial velocity u and range R on the horizontal, then $t_1^2 + t_2^2$ is equal to :

(a) u^2/g (b) $4u^2/g^2$

(c) $u^2/2g$ (d) 1

ANSWERS

PHYSICS AND CHEMISTRY

1.	(c)	2.	(a)	3.	(c)	4.	(a)	5.	(c)	6.	(b)	7.	(d)	8.	(d)
9.	(a)	10.	(b)	11.	(a)	12.	(b)	13.	(b)	14.	(c)	15.	(b)	16.	(a)
17.	(c)	18.	(d)	19.	(a)	20.	(b)	21.	(a)	22.	(d)	23.	(b)	24.	(c)
25.	(c)	26.	(b)	27.	(c)	28.	(b)	29.	(b)	30.	(a)	31.	(a)	32.	(d)
33.	(b)	34.	(c)	35.	(b)	36.	(d)	37.	(b)	38.	(a)	39.	(d)	40.	(b)
41.	(c)	42.	(d)	43.	(d)	44.	(b)	45.	(c)	46.	(c)	47.	(a)	48.	(b)
49.	(a)	50.	(c)	51.	(c)	52.	(d)	53.	(a)	54.	(b)	55.	(b)	56.	(a)
57.	(c)	58.	(b)	59.	(c)	60.	(d)	61.	(b)	62.	(b)	63.	(c)	64.	(b)
65.	(d)	66.	(c)	67.	(a)	68.	(d)	69.	(c)	70.	(c)	71.	(d)	72.	(d)
73.	(c)	74.	(b)	75.	(c)	76.	(c)	77.	(b)	78.	(c)	79.	(a)	80.	(c)
81.	(a)	82.	(d)	83.	(b)	84.	(c)	85.	(b)	86.	(c)	87.	(d)	88.	(c)
89.	(d)	90.	(a)	91.	(c)	92.	(c)	93.	(d)	94.	(b)	95.	(c)	96.	(d)
97.	(b)	98.	(d)	99.	(b)	100.	(a)	101.	(b)	102.	(a)	103.	(d)	104.	(a)
105.	(b)	106.	(c)	107.	(c)	108.	(d)	109.	(c)	110.	(d)	111.	(a)	112.	(c)
113.	(c)	114.	(d)	115.	(c)	116.	(c)	117.	(a)	118.	(b)	119.	(a)	120.	(a)
121.	(b)	122.	(c)	123.	(a)	124.	(d)	125.	(a)	126.	(c)	127.	(d)	128.	(a)
129.	(b)	130.	(c)	131.	(a)	132.	(c)	133.	(d)	134.	(c)	135.	(c)	136.	(b)
137.	(a)	138.	(c)	139.	(d)	140.	(d)	141.	(a)	142.	(b)	143.	(a)	144.	(c)
145.	(d)	146.	(b)	147.	(c)	148.	(a)	149.	(b)	150.	(c)				

MATHEMATICS

1.	(c)	2.	(a)	3.	(c)	4.	(d)	5.	(b)	6.	(d)	7.	(d)	8.	(a)
9.	(b)	10.	(c)	11.	(b)	12.	(c)	13.	(b)	14.	(a)	15.	(c)	16.	(b)
17.	(a)	18.	(a)	19.	(b)	20.	(b)	21.	(a)	22.	(d)	23.	(c)	24.	(a)
25.	(d)	26.	(b)	27.	(b)	28.	(b)	29.	(c)	30.	(c)	31.	(d)	32.	(b)
33.	(a)	34.	(a)	35.	(b)	36.	(b)	37.	(d)	38.	(a)	39.	(c)	40.	(b)
41.	(a)	42.	(a)	43.	(c)	44.	(b)	45.	(a)	46.	(d)	47.	(c)	48.	(d)
49.	(b)	50.	(a)	51.	(a)	52.	(a)	53.	(a)	54.	(b)	55.	(c)	56.	(c)
57.	(b)	58.	(a)	59.	(d)	60.	(d)	61.	(a)	62.	(c)	63.	(c)	64.	(d)
65.	(c)	66.	(c)	67.	(c)	68.	(b)	69.	(d)	70.	(c)	71.	(c)	72.	(a)
73.	(a)	74.	(d)	75.	(b)										

HINTS & SOLUTIONS

Physics

1. By Newton's formula

$$\eta = \frac{F}{A\,(\Delta v_x / \Delta z)}$$

$\therefore$ Dimensions of

$$\eta = \frac{\text{dimensions of force}}{\text{dimensions of area} \times \text{dimensions of velocity gradient}}$$

$$= \frac{[MLT^{-2}]}{[L^2]\,[T^{-1}]} = [ML^{-1}T^{-1}]$$

2. From given information $a = -kx$ where a is acceleration. x is displacement. and k is a proportionality constant.

$$\frac{v\,dv}{dx} = -k\,x$$

$$\Rightarrow \quad v\,dv = -k\,x\,dx$$

Let for any displacement from 0 to x, the velocity changes from v_0 to v.

$$\Rightarrow \quad \int_{v_0}^{v} v dv = -\int_0^x k\,x\,dx$$

$$\Rightarrow \quad \frac{v^2 - v_0^2}{2} = -\frac{k\,x^2}{2}$$

$$\Rightarrow \quad m\left(\frac{v^2 - v_0^2}{2}\right) = -\frac{mk\,x^2}{2}$$

$$\Rightarrow \quad \Delta K \propto x^2 \qquad [\Delta K \text{ is loss in KE}]$$

3. Second law of motion gives

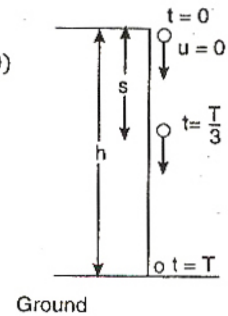
$$s = ut + \frac{1}{2}gT^2$$

or $h = 0 + \frac{1}{2}gT^2$ ($\because u = 0$)

$$\therefore\ T = \sqrt{\left(\frac{2h}{g}\right)}$$

At $t = \frac{T}{3}$ s,

$$s = 0 + \frac{1}{2}g\left(\frac{T}{3}\right)^2$$



$$\Rightarrow \quad s = \frac{1}{2}g \cdot \frac{T^2}{9}$$

$$\Rightarrow \quad s = \frac{g}{18} \times \frac{2h}{g} \qquad \left(\because T = \sqrt{\frac{2h}{g}}\right)$$

$$\therefore \quad s = \frac{h}{9}\ \text{m}$$

Hence, the position of ball from the ground

$$= h - \frac{h}{9} = \frac{8h}{9}\ \text{m}$$

4. $(\vec{A} \times \vec{B}) = (\vec{B} \times \vec{A})$ (given)

$$\Rightarrow \quad (\vec{A} \times \vec{B}) - (\vec{B} \times \vec{A}) = \vec{0}$$

$$(\vec{A} \times \vec{B}) + (\vec{A} \times \vec{B}) = \vec{0}$$

$$[\because (\vec{B} \times \vec{A}) = -(\vec{A} \times \vec{B})]$$

$$\Rightarrow \quad 2(\vec{A} \times \vec{B}) = \vec{0}$$

$$\Rightarrow \quad 2AB\sin\theta = 0$$

$$\Rightarrow \quad \sin\theta = 0$$

$$[\because |\vec{A}| = A \neq 0, |\vec{B}| = B \neq 0]$$

$$\therefore \quad \theta = 0 \text{ or } \pi$$

5. We know that range of projectile is same for complementary angles *i.e.*, for θ and $(90° - \theta)$.

$$\therefore \quad T_1 = \frac{2u\sin\theta}{g}$$

$$T_2 = \frac{2u\sin(90° - \theta)}{g} = \frac{2u\cos\theta}{g}$$

and $R = \dfrac{u^2\sin 2\theta}{g}$

Therefore, $T_1T_2 = \dfrac{2u\sin\theta}{g} \times \dfrac{2u\cos\theta}{g}$

$$= \frac{2u^2(2\sin\theta\cos\theta)}{g^2}$$

$$= \frac{2u^2(\sin 2\theta)}{g^2} = \frac{2R}{g}$$

$$\therefore \quad T_1T_2 \propto R$$

6. For a particle moving in a circle with constant angular speed, velocity vector is always tangent to the circle and the acceleration vector always points towards the centre of circle or is always along radius of the circle. Since, tangential vector is perpendicular to radial vector, therefore, velocity vector will be perpendicular to the acceleration vector. But in no case acceleration vector is tangent to the circle.

7. The braking retardation will remain same and assumed to be constant, let it be a.

From 3rd equation of motion, $v^2 = u^2 + 2as$

1st case : $0 = \left(60 \times \frac{5}{18}\right)^2 - 2a \times s_1$

$$\Rightarrow \quad s_1 = \frac{(60 \times 5/18)^2}{2a}$$

2nd case : $0 = \left(120 \times \frac{5}{18}\right)^2 - 2a \times s_2$

$$\Rightarrow \quad s_2 = \frac{(120 \times 5/18)^2}{2a}$$

$$\therefore \quad \frac{s_1}{s_2} = \frac{1}{4} \Rightarrow s_2 = 4s_1 = 4 \times 20 = 80 \text{ m}$$

8. The force exerted by machine gun on man's hand in firing a bullet

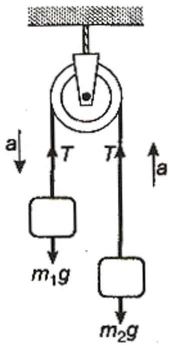
= change in momentum per second on a bullet or rate of change of momentum

$$= \left(\frac{40}{1000}\right) \times 1200 = 48 \text{ N}$$

The force exerted by man on machine gun= force exerted on man by machine gun = 144 N

Hence, number of bullets fired $= \frac{144}{48} = 3$

9. On releasing, the motion of the system will be according to figure.



$m_1 g - T = m_1 a$ ...(i)

and $T - m_2 g = m_2 a$ ...(ii)

On solving ;

$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$ ...(iii)

Here,

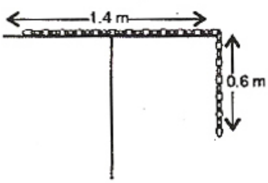
$m_1 = 5$ kg, $m_2 = 4.8$ kg,

$g = 9.8 \text{ m/s}^2$

$$\therefore a = \left(\frac{5 - 4.8}{5 + 4.8}\right) \times 9.8$$

$$= \frac{0.2}{9.8} \times 9.8 = 0.2 \text{ m/s}^2$$

10. Mass per unit length



$$= \frac{M}{L}$$

$$= \frac{4}{2} = 2 \text{ kg/m}$$

The mass of 0.6 m of chain

$$= 0.6 \times 2 = 1.2 \text{ kg}$$

The centre of mass of hanging part

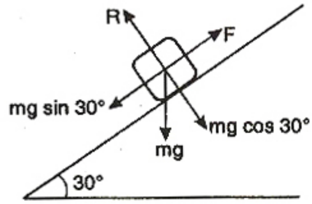
$$h = \frac{0.6 + 0}{2} = 0.3 \text{ m}$$

Hence, work done in pulling the chain on the table = work done against gravity force

$$W = mgh = 1.2 \times 10 \times 0.3 = 3.6 \text{ J}$$

11. Let the mass of block be m.

Frictional force in rest position



$$F = mg \sin 30°$$

[This is static frictional force and may be less than the limiting frictional force]

$$10 = m \times 10 \times \frac{1}{2}$$

$$\therefore \quad m = \frac{2 \times 10}{10} = 2 \text{ kg}$$

12. Work done is displacing the particle

$$W = \vec{\mathbf{F}} \cdot \vec{\mathbf{r}} = (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}).(2\hat{\mathbf{i}} - \hat{\mathbf{j}})$$

$$= 5 \times 2 + 3 \times (-1) + 2 \times 0 = 10 - 3$$

$$= 7 \text{ J}$$

13. Let the constant acceleration of body of mass m is a.

From equation of motion

$$v_1 = 0 + at_1$$

$$\Rightarrow \quad a = \frac{v_1}{t_1} \quad \text{...(i)}$$

At an instant t, the velocity v of the body

$$v = 0 + at$$

$$v = \frac{v_1}{t_1} t \quad \text{...(ii)}$$

Therefore, instantaneous power

$$P = Fv = mav \quad (\because F = ma)$$

$$= m\left(\frac{v_1}{t_1}\right) \times \left(\frac{v_1}{t_1} . t\right)$$

[from Eqs. (i) and (ii)]

$$= \frac{mv_1^2 t}{t_1^2}$$

14. When a force of constant magnitude acts on velocity of particle perpendicularly, then there is no change in the kinetic energy of particle. Hence, kinetic energy remains constant.

15. In free space, neither acceleration due to gravity nor external torque act on the rotating solid sphere. Therefore, taking the same mass of sphere if radius is increased then moment of inertia, rotational kinetic energy and angular velocity will change but according to law of conservation of momentum, angular momentum will not change.

16. Man will catch the ball if the horizontal component of velocity becomes equal to the constant speed of man *i.e.*,

$$v_0 \cos\theta = \frac{v_0}{2}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2}$$

$$\Rightarrow \quad \cos\theta = \cos 60°$$

$$\therefore \quad \theta = 60°$$

17. Let same mass and same outer radii of solid sphere and hollow sphere are M and R respectively.

The moment of inertia of solid sphere A about its diameter

$$I_A = \frac{2}{5}MR^2 \quad \text{...(i)}$$

Similarly, the moment of inertia of hollow sphere (spherical shell) B about its diameter

$$I_B = \frac{2}{3}MR^2 \quad \text{...(ii)}$$

It is clear from Eqs. (i) and (ii), $I_A < I_B$.

18. The gravitational force exerted on satellite at a height x is

$$F_G = \frac{GM_e m}{(R+x)^2}$$

where M_e = mass of earth

Since, gravitational force provides the necessary centripetal force, so,

$$\frac{GM_e m}{(R+x)^2} = \frac{mv_o^2}{(R+x)}$$

(where v_o is orbital speed of satellite)

$$\Rightarrow \quad \frac{GM_e m}{(R+x)} = mv_o^2$$

$$\Rightarrow \quad \frac{gR^2 m}{(R+x)} = mv_0^2 \quad \left(\because g = \frac{GM_e}{R^2}\right)$$

$$\Rightarrow \quad v_o = \sqrt{\left[\frac{gR^2}{(R+x)}\right]} = \left[\frac{gR^2}{(R+x)}\right]^{1/2}$$

19. Time period of satellite

$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM_e}}$$

where $R + h$ = orbital radius of satellite,

M_e = mass of earth

Thus, time period does not depend on mass of satellite.

20. Gravitational potential energy of body on earth's surface

$$U = -\frac{GM_e m}{R}$$

At a height h from earth's surface, its value is

$$U_h = -\frac{GM_e m}{(R+h)} = -\frac{GM_e m}{2R} \quad (\because h = R)$$

where M_e = mass of earth,

m = mass of body,

R = radius of earth

$\therefore$ Gain in potential energy

$$= U_h - U$$

$$= -\frac{GM_e m}{2R} - \left(-\frac{GM_e m}{R}\right)$$

$$= -\frac{GM_e m}{2R} + \frac{GM_e m}{R} = \frac{GM_e m}{2R} = \frac{gR^2 m}{2R}$$

$$\left(\because g = \frac{GM_e}{R^2}\right)$$

$$= \frac{1}{2} mgR$$

21. The necessary centripetal force required for a planet to move round the sun

= gravitational force exerted on it

i.e.,

$$\frac{mv^2}{R} = \frac{GM_e m}{R^n}$$

$$v = \left(\frac{GM_e}{R^{n-1}}\right)^{1/2}$$

Now,

$$T = \frac{2\pi R}{v} = 2\pi R \times \left(\frac{R^{n-1}}{GM_e}\right)^{1/2}$$

$$\Rightarrow \quad = 2\pi\left(\frac{R^2 \times R^{n-1}}{GM_e}\right)^{1/2}$$

$$= 2\pi\left(\frac{R^{(n+1)/2}}{(GM_e)^{1/2}}\right)$$

$$\therefore \quad T \propto R^{(n+1)/2}$$

22. Work done in stretching the wire

= potential energy stored

$$= \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2} \times \frac{F}{A} \times \frac{l}{L} \times AL$$

$$= \frac{1}{2} Fl$$

23. Retarding force acting on a ball falling into a viscous fluid

$$F = 6\pi\eta Rv$$

where R = radius of ball,
v = velocity of ball
and η = coefficient of viscosity

$\therefore$ $F \propto R$ and $F \propto v$

Or in words, retarding force is directly proportional to both R and v.

24. The excess pressure inside the soap bubble is inversely proportional to radius of soap bubble *i.e.*, $P \propto 1/r$, r being the radius of bubble. It follows that pressure inside a smaller bubble is greater than that inside a bigger bubble. Thus, if these two bubbles are connected by a tube, air will flow from smaller bubble to bigger bubble and the bigger bubble grows at the expense of the smaller one.

25. The time period of simple pendulum in air

$$T = t_0 = 2\pi\sqrt{\left(\frac{l}{g}\right)} \quad \text{...(i)}$$

l, being the length of simple pendulum.
In water, effective weight of bob
w' = weight of bob in air − upthrust

$$\Rightarrow \rho V g_{eff} = mg - m'g$$
$$= \rho Vg - \rho' Vg = (\rho - \rho')Vg$$

where ρ' = density of bob,
ρ = density of water

$$\therefore \quad g_{eff} = \left(\frac{\rho - \rho'}{\rho}\right) g = \left(1 - \frac{\rho'}{\rho}\right) g$$

$$\therefore \quad t = 2\pi\sqrt{\left[\frac{l}{\left(1 - \frac{\rho'}{\rho}\right) g}\right]} \quad \text{...(ii)}$$

$$\text{Thus, } \frac{t}{t_0} = \sqrt{\left[\frac{1}{\left(1 - \frac{\rho'}{\rho}\right)}\right]}$$

$$= \sqrt{\left(\frac{1}{1 - \frac{1000}{(4/3) \times 1000}}\right)} = \sqrt{\left(\frac{4}{4-3}\right)}$$

$$= 2 \Rightarrow t = 2t_0$$

26. Time period of spring

$$T = 2\pi\sqrt{\left(\frac{m}{k}\right)}$$

k, being the force constant of spring.
For first spring

$$t_1 = 2\pi\sqrt{\left(\frac{m}{k_1}\right)} \quad \text{...(i)}$$

For second spring

$$t_2 = 2\pi\sqrt{\left(\frac{m}{k_2}\right)} \quad \text{..(ii)}$$

The effective force constant in their series combination is

$$k = \frac{k_1 k_2}{k_1 + k_2}$$

$\therefore$ Time period of combination

$$T = 2\pi\sqrt{\left[\frac{m(k_1 + k_2)}{k_1 k_2}\right]}$$

$$\Rightarrow \quad T^2 = \frac{4\pi^2 m(k_1 + k_2)}{k_1 k_2} \quad \text{...(iii)}$$

From Eqs. (i) and (ii), we obtain

$$t_1^2 + t_2^2 = 4\pi^2\left(\frac{m}{k_1} + \frac{m}{k_2}\right)$$

$$\Rightarrow \quad t_1^2 + t_2^2 = 4\pi^2 m\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$\Rightarrow \quad t_1^2 + t_2^2 = \frac{4\pi^2 m(k_1 + k_2)}{k_1 k_2}$$

$$\therefore \quad t_1^2 + t_2^2 = T^2 \quad \text{[from Eq. (iii)]}$$

27. In simple harmonic motion when a particle is displaced to a position from its mean position, then its kinetic energy gets converted into potential energy. Hence, total energy of a particle remains constant or the total energy in simple harmonic motion does not depend on displacement x.

28. As given

$$y = 10^{-6} \sin\left(100t + 20x + \frac{\pi}{4}\right) \quad \text{...(i)}$$

Comparing it with

$$y = a \sin(\omega t + kx + \phi) \quad \text{...(ii)}$$

we find, $\omega = 100$ rad/s, $k = 20$/m

$$\therefore \quad v = \frac{\omega}{k} = \frac{100}{20} = 5 \text{ m/s}$$

29. Initial angular velocity of particle = ω_0
and at any instant t, angular velocity = ω
Therefore, for a displacement x, the resultant acceleration

$$f = (\omega_0^2 - \omega^2)\, x \quad \text{...(i)}$$

External force $F = m(\omega_0^2 - \omega^2)\, x$ ...(ii)

Since, $F \propto \cos \omega t$ (given)

$\therefore$ From Eq. (ii)

$$m(\omega_0^2 - \omega^2)\, x \propto \cos \omega t \quad \text{...(iii)}$$

Now, equation of simple harmonic motion

$$x = A\sin(\omega t + \phi) \quad \ldots(\text{iv})$$

at $t = 0$; $x = A$

$$\therefore \quad A = A\sin(0 + \phi)$$

$$\Rightarrow \quad \phi = \frac{\pi}{2}$$

$$\therefore \quad x = A\sin\left(\omega t + \frac{\pi}{2}\right) = A\cos\omega t \quad \ldots(\text{v})$$

Hence, from Eqs. (iii) and (v), we finally get

$$m(\omega_0^2 - \omega^2)A\cos\omega t \propto \cos\omega t$$

$$\Rightarrow \quad A \propto \frac{1}{m(\omega_0^2 - \omega^2)}$$

30. For amplitude of oscillation and energy to be maximum, frequency of force must be equal to the initial frequency and this is only possible in case of resonance.

In resonance state $\omega_1 = \omega_2$

31. Mayer's formula is

$$C_P - C_V = R$$

and $\gamma = \dfrac{C_P}{C_V}$

Therefore, using above two relations, we find

$$C_V = \frac{R}{\gamma - 1}$$

For a mole of monoatomic gas; $\gamma = \dfrac{5}{3}$

$$\therefore \quad C_V = \frac{R}{(5/3) - 1} = \frac{3}{2}R$$

For a mole of diatomic gas; $\gamma = \dfrac{7}{5}$

$$\therefore C_V = \frac{R}{(7/5) - 1} = \frac{5}{2}R$$

When these two moles are mixed, then heat required to raise the temperature to 1°C is

$$C_V = \frac{3}{2}R + \frac{5}{2}R = 4R$$

Hence, for one mole, heat required is

$$= \frac{4R}{2} = 2R$$

$$\therefore \quad C_V = 2R$$

$$\Rightarrow \quad \frac{R}{\gamma - 1} = 2R$$

$$\Rightarrow \quad \gamma = \frac{3}{2}$$

Alternative

$$\frac{n_1 + n_2}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$$

Here, $n_1 = 1, n_2 = 1, \gamma_1 = \dfrac{5}{3}, \gamma_2 = \dfrac{7}{5}$

$$\therefore \quad \frac{1+1}{\gamma - 1} = \frac{1}{\left(\frac{5}{3}\right) - 1} + \frac{1}{\left(\frac{7}{5}\right) - 1}$$

$$\Rightarrow \quad \frac{2}{\gamma - 1} = \frac{3}{2} + \frac{5}{2}$$

$$\Rightarrow \quad \frac{2}{\gamma - 1} = \frac{8}{2}$$

$$\Rightarrow \quad \frac{2}{\gamma - 1} = 4$$

$$\Rightarrow \quad \gamma = \frac{2}{4} + 1$$

Hence, $\gamma = \dfrac{3}{2}$

32. From Stefan law, the energy radiated by sun is given by. $P = \sigma e A T^4$, assuming $e = 1$ for sun.

In 1st case, $P_1 = \sigma e \times 4\pi R^2 \times T^4$

In 2nd case, $P_2 = \sigma e \times 4\pi(2R)^2 \times (2T)^4$

$$= \sigma e \times 4\pi R^2 \times T^4 \times 64 = 64P_1$$

The rate at which energy received at earth is,

$$E = \frac{P}{4\pi R_{SE}^2} \times A_E$$

where A_E = Area of earth

R_{SE} = Distance between sun and earth

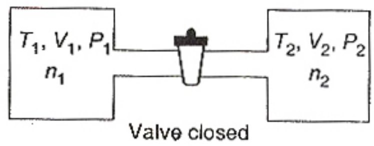
So, In 1st case, $E_1 = \dfrac{P_1}{4\pi R_{SE}^2} \times A_E$

$$E_2 = \frac{P_2}{4\pi R_{SE}^2} \times A_E = 64E_1$$

33. Internal energy does not change in isothermal process. ΔS can be zero for adiabatic process. Work done in adiabatic process may be non-zero.

34. $n_1 = \dfrac{P_1V_1}{RT_1}$

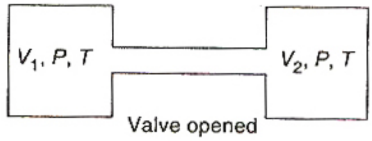
$n_2 = \dfrac{P_2V_2}{RT_2}$



[from ideal gas equation]

$n_1' = \dfrac{PV_1}{RT}$

$n_2' = \dfrac{PV_2}{RT}$



$n_1 + n_2 = n_1' + n_2'$ [moles of gas remain same]

$$\frac{1}{R}\left[\frac{P_1V_1}{T_1} + \frac{P_2V_2}{T_2}\right] = \frac{P}{RT}(V_1 + V_2)$$

$$\Rightarrow \quad \frac{P}{T} = \frac{P_1V_1T_2 + P_2V_2T_1}{T_1T_2(V_1 + V_2)} \quad \ldots(\text{i})$$

Internal energy of the system (air in two vessels) remains same before and after opening of valve, so

$$\frac{fn_1RT_1}{2} + \frac{fn_2RT_2}{2} = \frac{f(n_1 + n_2)RT}{2}$$

$$\left[U = \frac{fnRT}{2}\right]$$

$$\Rightarrow \quad n_1T_1 + n_2T_2 = (n_1 + n_2)T$$

$$\Rightarrow \quad \frac{P_1V_1 + P_2V_2}{R} = \left(\frac{P_1V_1}{T_1} + \frac{P_2V_2}{T_2}\right)\frac{T}{R}$$

$$\Rightarrow \quad T = \frac{(P_1V_1 + P_2V_2)T_1T_2}{P_1V_1T_2 + P_2V_2T_1}$$

From Eq. (i) we can find the equilibrium pressure also.

35. Initial momentum of surface

$$p_i = \frac{E}{c}$$

where c = velocity of light (constant).

Since, the surface is perfectly reflecting, so the same momentum will be reflected completely

Final momentum

$$p_f = \frac{E}{c} \quad \text{(negative value)}$$

∴ Change in momentum

$$\Delta p = p_f - p_i = -\frac{E}{c} - \frac{E}{c} = -\frac{2E}{c}$$

Thus, momentum transferred to the surface is

$$\Delta p' = |\Delta p| = \frac{2E}{c}$$

36. Let the temperature of common interface be T °C.

Rate of heat flow

$$H = \frac{Q}{t} = \frac{KA\,\Delta T}{l}$$

$$\therefore \quad H_1 = \left(\frac{Q}{t}\right)_1 = \frac{2KA\,(T - T_1)}{4x}$$

$$\text{and} \quad H_2 = \left(\frac{Q}{t}\right)_2 = \frac{KA\,(T_2 - T)}{x}$$

In steady state, the rate of heat flow should be same in whole system *i.e.*,

$$H_1 = H_2$$

$$\Rightarrow \quad \frac{2KA\,(T - T_1)}{4x} = \frac{KA\,(T_2 - T)}{x}$$

$$\Rightarrow \quad \frac{T - T_1}{2} = T_2 - T$$

$$\Rightarrow \quad T - T_1 = 2T_2 - 2T$$

$$\Rightarrow \quad T = \frac{2T_2 + T_1}{3} \quad \text{...(i)}$$

Hence, heat flow from composite slab is

$$H = \frac{KA\,(T_2 - T)}{x}$$

$$= \frac{KA}{x}\left(T_2 - \frac{2T_2 + T_1}{3}\right) = \frac{KA}{3x}(T_2 - T_1) \quad \text{...(ii)}$$

[from Eq. (i)]

Accordingly,

$$H = \left[\frac{A\,(T_2 - T_1)\,K}{x}\right] f \quad \text{...(iii)}$$

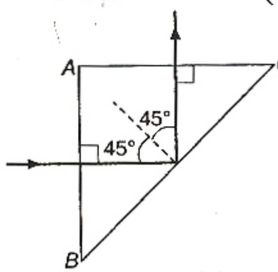
By comparing Eqs. (ii) and (iii), we get

$$\Rightarrow \quad f = \frac{1}{3}$$

37. For total internal reflection from glass-air interface, critical angle C must be less than angle of incidence.

i.e., $\quad C < i$

or $\quad C < 45° \quad (\because \angle i = 45°)$



$$\text{But } n = \frac{1}{\sin C} \Rightarrow C = \sin^{-1}\left(\frac{1}{n}\right)$$

$$\therefore \quad \sin^{-1}\left(\frac{1}{n}\right) < 45°$$

$$\Rightarrow \quad \frac{1}{n} < \sin 45°$$

$$\Rightarrow \quad n > \frac{1}{\sin 45°}$$

$$\Rightarrow \quad n > \frac{1}{(1/\sqrt{2})}$$

$$\Rightarrow \quad n > \sqrt{2}$$

38. A plano-convex lens behaves as a concave mirror if its one surface (curved) is silvered. The rays refracted from plane surface are reflected from curved surface and again refract from plane surface. Therefore, in this lens two refractions and one reflection occur.

Let the focal length of silvered lens is F.

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f} + \frac{1}{f_m}$$

$$= \frac{2}{f} + \frac{1}{f_m}$$

where, f = focal length of lens before silvering

f_m = focal length of spherical mirror

$$\frac{1}{F} = \frac{2}{f} + \frac{2}{R} \quad \text{...(i)}$$

$$(\because R = 2f_m)$$

Now, $$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{...(ii)}$$

Here, $R_1 = \infty$, $R_2 = 30$ cm

$$\therefore \quad \frac{1}{f} = (1.5 - 1)\left(\frac{1}{\infty} - \frac{1}{30}\right)$$

$$\Rightarrow \quad \frac{1}{f} = -\frac{0.5}{30} = -\frac{1}{60}$$

$$\Rightarrow \quad f = -60 \text{ cm}$$

Hence, from Eq. (i)

$$\frac{1}{F} = \frac{2}{60} + \frac{2}{30} = \frac{6}{60}$$

$$F = 10 \text{ cm}$$

Again given that,

size of object = size of image

$$\Rightarrow \quad O = I$$

$$\therefore \quad m = -\frac{v}{u} = \frac{I}{O} \Rightarrow \frac{v}{u} = -1$$

$$\Rightarrow \quad v = -u$$

Thus, from lens formula

$$\frac{1}{F} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{10} = \frac{1}{-u} - \frac{1}{u}$$

$$\frac{1}{10} = -\frac{2}{u}$$

$$\therefore \quad u = -20 \text{ cm}$$

Hence, to get a real image, object must be placed at a distance 20 cm on the left side of lens.

39. The particular angle of incidence for which reflected light is totally polarized for reflection from air to glass, is called the angle of polarisation (i_p) **(Brewster's law).**

Accordingly,

$$n = \tan i_p$$

$$\Rightarrow \quad i_p = \tan^{-1}(n)$$

where n = refractive index of glass.

40. For possible interference maxima on the screen, the condition is

$$d \sin\theta = n\lambda \quad \text{...(i)}$$

Given : d = slit - width = 2λ

$\therefore$ $2\lambda \sin\theta = n\lambda$

$\Rightarrow$ $2\sin\theta = n$

The maximum value of $\sin\theta$ is 1, hence,

$$n = 2 \times 1 = 2$$

Thus, Eq. (i) must be satisfied by 5 integer values *i.e.*, $-2, -1, 0, 1, 2$. Hence, the maximum number of possible interference maxima is 5.

41. In vacuum, $\varepsilon_0 = 1$

In medium, $\varepsilon = 4$

So, refractive index

$$\mu = \sqrt{\varepsilon/\varepsilon_0}$$

$$= \sqrt{4/1} = 2$$

Wavelength $$\lambda' = \frac{\lambda}{\mu} = \frac{\lambda}{2}$$

and wave velocity $$v = \frac{c}{\mu} = \frac{c}{2}$$

Hence, it is clear that wavelength and velocity will become half but frequency remains unchanged when the wave is passing through any medium.

42. Let the spherical conductors B and C have same charge as q. The electric force between them is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$$

r, being the distance between them.

When third uncharged conductor A is brought in contact with B, then charge on each conductor

$$q_A = q_B = \frac{q_A + q_B}{2}$$

$$= \frac{0 + q}{2} = \frac{q}{2}$$

When this conductor A is now brought in contact with C, then charge on each conductor

$$q_A = q_C = \frac{q_A + q_C}{2}$$

$$= \frac{(q/2) + q}{2}$$

$$= \frac{3q}{4}$$

Hence, electric force acting between B and C is

$$F' = \frac{1}{4\pi\varepsilon_0}\frac{q_B q_C}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{(q/2)(3q/4)}{r^2}$$

$$= \frac{3}{8}\left[\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}\right] = \frac{3F}{8}$$

43. Let a particle of charge q having velocity v approaches Q upto a closest distance r and if the velocity becomes $2v$, the closest distance will be r'.

The law of conservation of energy yields, kinetic energy of particle = electric potential energy between them at closest distance of approach.

or $$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}$$

or $$\frac{1}{2}mv^2 = k\frac{Qq}{r} \quad \text{....(i)}$$

$$\left(k = \text{constant} = \frac{1}{4\pi\varepsilon_0}\right)$$

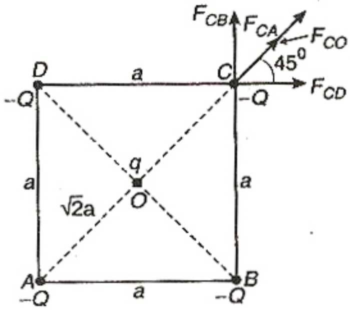
and $$\frac{1}{2}m(2v)^2 = k\frac{Qq}{r'} \quad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii),

$$\frac{\frac{1}{2}mv^2}{\frac{1}{2}m(2v)^2} = \frac{\frac{kQq}{r}}{\frac{kQq}{r'}}$$

$$\Rightarrow \quad \frac{1}{4} = \frac{r'}{r} \Rightarrow r' = \frac{r}{4}$$

44. The system is in equilibrium means the force experienced by each charge is 0. It is clear that charge placed at centre would be in equilibrium for any value of q, so we are



considering the equilibrium of charge placed at any corner.

$$F_{CD} + F_{CA}\cos 45° + F_{CO}\cos 45° = 0$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-Q)(-Q)}{a^2} + \frac{1}{4\pi\varepsilon_0}\frac{(-Q)(-Q)}{(\sqrt{2}a)^2}\times\frac{1}{\sqrt{2}}$$

$$+\frac{1}{4\pi\varepsilon_0}\frac{(-Q)q}{(\sqrt{2}a/2)^2}\times\frac{1}{\sqrt{2}} = 0$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{a^2} + \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{2a^2}\cdot\frac{1}{\sqrt{2}} - \frac{1}{4\pi\varepsilon_o}\frac{2Qq}{a^2}\times\frac{1}{\sqrt{2}} = 0$$

$$\Rightarrow \quad Q + \frac{Q}{2\sqrt{2}} - \sqrt{2}q = 0$$

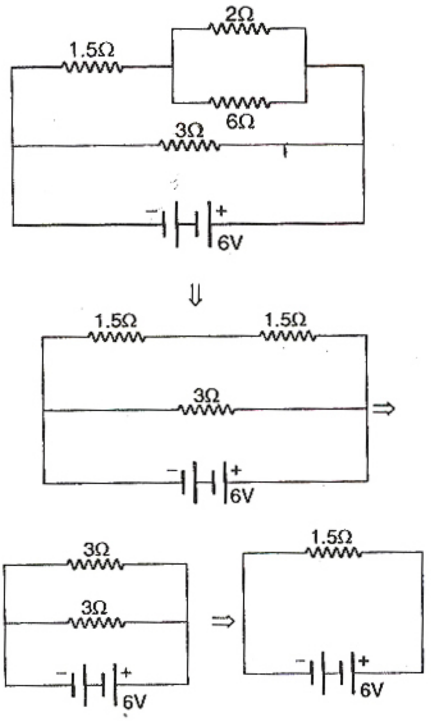
$$\Rightarrow \quad 2\sqrt{2}Q + Q - 4q = 0$$

$$\Rightarrow \quad 4q = (2\sqrt{2}+1)Q$$

$$\Rightarrow \quad q = (2\sqrt{2}+1)\frac{Q}{4}$$

45. The full cycle of alternating current consists of two half cycles. For one half, current is positive and for second half, current is negative. Therefore, for an AC cycle, the net value of current average out to zero. While the DC ammeter, read the average value. Hence, the alternating current cannot measured by DC ammeter.

46. The equivalent of the given circuit can be drawn as



Hence, current supplied by the battery is

$$i = \frac{V}{R} = \frac{6}{1.5} = 4\text{ A}$$

47. Let resistances are R_1 and R_2, then $S = R_1 + R_2$

and $$P = \frac{R_1R_2}{R_1+R_2}$$

$$\therefore \quad (R_1+R_2) = \frac{n \times R_1R_2}{R_1+R_2} \quad [\text{from } S = nP]$$

$$\Rightarrow \quad (R_1+R_2)^2 = nR_1R_2$$

$$\Rightarrow n = \left[\frac{R_1^2 + R_2^2 + 2R_1R_2}{R_1R_2}\right] = \left[\frac{R_1}{R_2} + \frac{R_2}{R_1} + 2\right]$$

We know,

Arithmatic Mean ≥ Geometric Mean

$$\frac{\frac{R_1}{R_2}+\frac{R_2}{R_1}}{2} \geq \sqrt{\frac{R_1}{R_2}\times\frac{R_2}{R_1}}$$

$$\Rightarrow \quad \frac{R_1}{R_2}+\frac{R_2}{R_1} \geq 2$$

So, n (min. value) $= 2 + 2 = 4$

48. Since, voltage remains same in parallel, so,

$$i \propto \frac{1}{R}$$

$$\Rightarrow \quad \frac{i_1}{i_2} = \frac{R_2}{R_1}$$

$$\frac{i_i}{i_2} = \frac{\rho l_2/A_2}{\rho l_1/A_1} \qquad \left(\because R = \frac{\rho l}{A}\right)$$

$$\Rightarrow \quad \frac{i_1}{i_2} = \frac{l_2}{l_1}\times\left(\frac{r_1}{r_2}\right)^2 \qquad (\because A = \pi r^2)$$

$$\Rightarrow \quad \frac{i_1}{i_2} = \frac{3}{4}\times\left(\frac{2}{3}\right)^2$$

Hence, $\frac{i_1}{i_2} = \frac{1}{3}$

49. Meter bridge is an arrangement which works on Wheatstone's principle, so the balancing condition is

$$\frac{R}{S} = \frac{l_1}{l_2}$$

where $l_2 = 100 - l_1$

Ist case : $R = X, S = Y, l_1 = 20$ cm,

$l_2 = 100 - 20 = 80$ cm

$$\therefore \quad \frac{X}{Y} = \frac{20}{80} \qquad \text{...(i)}$$

IInd case : Let the position of null point is obtained at a distance l from same end.

$\therefore R = 4X, S = Y, l_1 = l, l_2 = 100 - l$

So, from Eq. (i)

$$\frac{4X}{Y} = \frac{l}{100-l}$$

$$\Rightarrow \quad \frac{X}{Y} = \frac{l}{4(100-l)} \qquad \text{...(ii)}$$

Therefore, from Eqs. (i) and (ii)

$$\frac{l}{4(100-l)} = \frac{20}{80}$$

$$\Rightarrow \quad \frac{l}{4(100-l)} = \frac{1}{4}$$

$$\Rightarrow \quad l = 100 - l$$

$$\Rightarrow \quad 2l = 100$$

Hence, $l = 50$ cm

50. They are the resistors made up of semiconductors whose resistance decreases with the increase in temperature. This implies that they have negative and high temperature coefficient of resistivity.

They are usually made of metal oxides with high temperature coefficient of resistivity.

51. Let time taken in boiling the water by the heater is t s. Then

$$Q = ms\Delta T \Rightarrow \frac{Pt}{J} = ms\Delta T$$

$$\therefore \quad \frac{836}{4.2}t = 1\times 1000\,(40-10)$$

$$\frac{836}{4.2}t = 1000\times 30$$

$$t = \frac{1000\times 30\times 4.2}{836} = 150 \text{ s}$$

52. $$E = a\theta + b\theta^2 \qquad \text{(given)}$$

For neutral temperature (θ_n), $\frac{dE}{d\theta} = 0$

$$\Rightarrow \quad a + 2b\,\theta_n = 0$$

$$\Rightarrow \quad \theta_n = -\frac{a}{2b}$$

$$\therefore \quad \theta_n = -\frac{700}{2} \qquad \left(\because \frac{a}{b} = 700°\text{C}\right)$$

$$= -350°\text{C} < 0°\text{C}$$

But neutral temperature can never be negative (less than zero) *i.e.*, $\theta_n \nless 0°$C.

Hence, no neutral temperature is possible for this thermocouple.

53. Mass of substance liberated at cathode

$$m = zit$$

where, z = electro-chemical equivalent

$= 3.3\times 10^{-7}$ kg/C

i = current flowing = 3A,

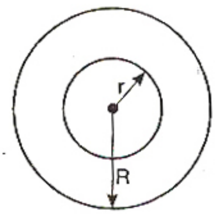
$t = 2$ s

$\therefore \; m = 3.3\times 10^{-7}\times 3\times 2 = 19.8\times 10^{-7}$ kg

54. Let R be the radius of a long thin cylindrical shell.

To calculate the magnetic induction at a distance r $(r < R)$ from the axis of cylinder, a circular shell of radius r is shown :

Since, no current is enclosed in the circle so, from Ampere's circuital law, magnetic induction is zero at every point of circle. Hence, the magnetic induction at any point inside the infinitely long straight thin walled tube (cylindrical) is zero.



55. The magnetic field at the centre of circular coil is

$$B = \frac{\mu_0 i}{2r}$$

where r = radius of circle $= \frac{l}{2\pi}$ $(\because l = 2\pi r)$

$$\therefore \quad B = \frac{\mu_0 i}{2} \times \frac{2\pi}{l}$$

$$= \frac{\mu_0 i \pi}{l} \quad \text{...(i)}$$

When wire of length l bents into a circular loops of n turns, then

$$l = n \times 2\pi r'$$

$$\Rightarrow \quad r' = \frac{l}{n \times 2\pi}$$

Thus, new magnetic field

$$B' = \frac{\mu_0 ni}{2r'} = \frac{\mu_0 ni}{2} \times \frac{n \times 2\pi}{l}$$

$$= \frac{\mu_0 i\pi}{l} \times n^2$$

$$= n^2 B \quad \text{[from Eq. (i)]}$$

56. The magnetic field at a point on the axis of a circular loop at a distance x from the centre is

$$B = \frac{\mu_0 iR^2}{2(R^2 + x^2)^{3/2}} \quad \text{...(i)}$$

Given : $B = 54\,\mu T$, $x = 4$ cm, $R = 3$ cm

Putting the given values in Eq. (i), we get

$$\therefore \quad 54 = \frac{\mu_0 i \times (3)^2}{2(3^2 + 4^2)^{3/2}}$$

$$\Rightarrow \quad 54 = \frac{9\mu_0 i}{2(25)^{3/2}} = \frac{9\mu_0 i}{2 \times (5)^3}$$

$$\therefore \quad \mu_0 i = \frac{54 \times 2 \times 125}{9}$$

$$\mu_0 i = 1500\,\mu\text{T-cm} \quad \text{...(ii)}$$

Now, putting $x = 0$ in Eq. (i), magnetic field at the centre of loop is

$$B = \frac{\mu_0 iR^2}{2R^3} = \frac{\mu_0 i}{2R} = \frac{1500}{2 \times 3}$$

$$= 250\,\mu\text{T}. \quad \text{[from Eq. (ii)]}$$

57. Force acting between two current carrying conductors

$$F = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d} l \quad \text{..(i)}$$

where d = distance between the conductors

l = length of each conductor

$$\text{Again} \quad F' = \frac{\mu_0}{2\pi} \frac{(-2I_1)(I_2)}{(3d)} \cdot l$$

$$= -\frac{\mu_0}{2\pi} \frac{2I_1 I_2}{3d} \cdot l \quad \text{...(ii)}$$

Thus, from Eqs. (i) and (ii)

$$\frac{F'}{F} = -\frac{2}{3}$$

$$\Rightarrow \quad F' = -\frac{2}{3} F$$

58. The time period of oscillations of magnet

$$T = 2\pi \sqrt{\left(\frac{I}{MH}\right)} \quad \text{...(i)}$$

where I = moment of inertia of magnet

$$= \frac{mL^2}{12}$$

(m, being the mass of magnet)

$$M = \text{pole strength} \times L$$

and H = horizontal component of earth's magnetic field.

When the three equal parts of magnet are placed on one another with their like poles together, then

$$I' = \frac{1}{12}\left(\frac{m}{3}\right) \times \left(\frac{L}{3}\right)^2 \times 3$$

$$= \frac{1}{12} \frac{mL^2}{9} = \frac{I}{9}$$

$$\text{and} \quad M' = \text{pole strength} \times \frac{L}{3} \times 3$$

$$= M$$

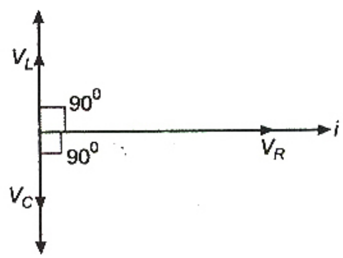
$$\text{Hence,} \quad T' = 2\pi \sqrt{\left(\frac{I/9}{MH}\right)}$$

$$\Rightarrow \quad T' = \frac{1}{3} \times T$$

$$T' = \frac{2}{3}\,\text{s}$$

59. Electromagnets are made of soft iron. The soft iron has high retentivity and low coercivity.

60. In an LCR series ac circuit, the voltage across inductor L leads the current by 90° and the voltage across capacitor C lags behind the current by 90°.



Hence, the voltage across LC combination will be zero.

61. The rate of change of flux or emf induced in the coil is $e = -n\frac{d\phi}{dt}$

$\therefore$ Induced current $i = \frac{e}{R'} = -\frac{n}{R'}\frac{d\phi}{dt}$

Given : $R' = R + 4R = 5R$, $d\phi = W_2 - W_1$, $dt = t$.
(Here, W_1 and W_2 are flux associated with one turn.)

Putting the given values in Eq. (i), we get

$$\therefore \quad i = -\frac{n}{5R}\frac{(W_2 - W_1)}{t}$$

62. The flux associated with coil of area A and magnetic induction B is

$$\phi = BA\cos\theta$$
$$= \frac{1}{2}B\pi r^2 \cos\omega t \quad \left[\because A = \frac{1}{2}\pi r^2\right]$$

$$\therefore \quad e_{induced} = -\frac{d\phi}{dt}$$
$$= -\frac{d}{dt}\left(\frac{1}{2}B\pi r^2 \cos\omega t\right)$$
$$= \frac{1}{2}B\pi r^2 \omega \sin\omega t$$

$$\therefore \quad \text{Power } P = \frac{e_{induced}^2}{R}$$
$$= \frac{B^2\pi^2 r^4 \omega^2 \sin^2\omega t}{4R}$$

Hence,

$$P_{mean} = \langle P \rangle$$
$$= \frac{B^2\pi^2 r^4\omega^2}{4R}\cdot\frac{1}{2} \quad \left[\because \langle \sin\omega t \rangle = \frac{1}{2}\right]$$
$$= \frac{(B\pi r^2\omega)^2}{8R}$$

63. In the condition of resonance

$$X_L = X_C$$

or $$\omega L = \frac{1}{\omega C} \quad \text{...(i)}$$

Since, resonant frequency remains unchanged,

so, $\sqrt{LC}$ = constant

or LC = constant

$\therefore$ $L_1C_1 = L_2C_2$

$\Rightarrow$ $L \times C = L_2 \times 2C$

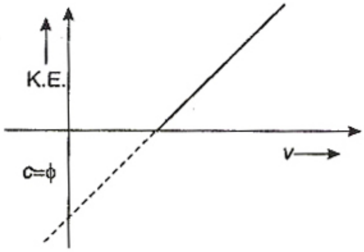
$\Rightarrow$ $L_2 = \frac{L}{2}$

64. The emf induced between ends of conductor

$$e = \frac{1}{2}B\omega L^2$$
$$= \frac{1}{2}\times 0.2\times 10^{-4}\times 5\times(1)^2$$
$$= 0.5\times 10^{-4}\text{ V}$$
$$= 5\times 10^{-5}\text{ V} = 50\,\mu\text{V}$$

65. Einstein's photoelectric equation is

$$KE_{max} = h\nu - \phi \quad \text{...(i)}$$



The equation of line is

$$y = mx + c \quad \text{...(ii)}$$

Comparing above two equations

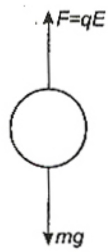
$$m = h,\ c = -\phi$$

Hence, slope of graph is equal to Planck's constant (non-variable) and does not depend on intensity of radiation.

66. $\frac{hc}{\lambda_0} = \phi$

$$\Rightarrow \lambda_{max} = \frac{hc}{\phi} = \frac{6.6\times 10^{-34}\times 3\times 10^8}{4\times 1.6\times 10^{-19}}$$
$$= 310\text{ nm}$$

67. In steady state,
electric force on drop
= weight of drop



$\therefore$ $qE = mg$

$\Rightarrow$ $q = \frac{mg}{E}$

$$= \frac{9.9\times 10^{-15}\times 10}{3\times 10^4}$$
$$= 3.3\times 10^{-18}\text{ C}$$

68. Law of conservation of momentum gives

$$m_1v_1 = m_2v_2$$

$\Rightarrow$ $\frac{m_1}{m_2} = \frac{v_2}{v_1}$

But $m = \frac{4}{3}\pi r^3\rho$

or $m \propto r^3$

$\therefore$ $\frac{m_1}{m_2} = \frac{r_1^3}{r_2^3} = \frac{v_2}{v_1}$

$\Rightarrow$ $\frac{r_1}{r_2} = \left(\frac{1}{2}\right)^{1/3}$

$\therefore$ $r_1 : r_2 = 1 : 2^{1/3}$

69. As given

$$_1H^2 + _1H^2 \longrightarrow _2He^4 + \text{energy}$$

The binding energy per nucleon of a deuteron ($_1H^2$)

$$= 1.1 \text{ MeV}$$

∴Total binding energy of one deuteron nuclus

$$= 2 \times 1.1 = 2.2 \text{ MeV}$$

The binding energy per nucleon of helium ($_2He^4$)

$$= 7 \text{ MeV}$$

∴Total binding energy

$$= 4 \times 7 = 28 \text{ MeV}$$

Hence, energy released in the above process

$$= 28 - 2 \times 2.2$$
$$= 28 - 4.4 = 23.6 \text{ MeV}$$

70. According to law of conservation of energy, kinetic energy of α-particle

= potential energy of α-particle at distance of closest approach

i.e., $$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$$

$$\therefore \; 5 \text{ MeV} = \frac{9 \times 10^9 \times (2e) \times (92e)}{r}$$

$$\left(\because \frac{1}{2}mv^2 = 5 \text{ MeV}\right)$$

$$\Rightarrow \quad r = \frac{9 \times 10^9 \times 2 \times 92 \times (1.6 \times 10^{-19})^2}{5 \times 10^6 \times 1.6 \times 10^{-19}}$$

$$\therefore \quad r = 5.3 \times 10^{-14} \text{ m} \approx 10^{-12} \text{cm}$$

71. When forward bias is applied on *npn*-transistor, then it works as an amplifier. In forward biased *npn*-transistor, electrons move from emitter to base and holes move from base to emitter.

72. For a transistor amplifier in common emitter configuration, current gain

$$A_i = -\frac{h_{fe}}{1 + h_{oe} R_L}$$

where h_{fe} and h_{oe} are hybrid parameters of a transistor.

$$\therefore \quad A_i = -\frac{50}{1 + 25 \times 10^{-6} \times 1 \times 10^3}$$
$$= -48.78$$

73. We know that resistance of conductor is directly proportional to temperature (*i.e.*, $R \propto \Delta t$), while resistance of semiconductor is inversely proportional to temperature $\left(i.e., R \propto \frac{1}{\Delta t}\right)$.

Therefore, it is clear that resistance of conductor decreases with decrease in temperature or *vice-versa*, while in case of semiconductor, resistance increases with decrease in temperature or *vice-versa*.

Since, copper is pure conductor and germanium is a semiconductor hence, due to decrease in temperature, resistance of conductor decreases while that of semiconductor increases.

74. According to Pauli's exclusion principle, the electronic configuration of number of subshells existing in a shell and number of electrons entering each subshell is found. Hence, on the basis of Pauli's exclusion principle, the manifestation of band structure in solids can be explained.

75. When *p*-end of *p-n* junction is connected to positive terminal of battery and *n*-end to negative terminal of battery, then *p-n* junction is said to be in forward bias. In forward bias, the more numbers of electrons go from *n*-region to *p*-region and more numbers of holes go from *p*-region to *n*-region. Therefore, major current due to both types of carriers takes place through the junction causing, more recombination of electron hole pairs thus causing reduction in height of depletion region and barrier potential.

Chemistry

76. Any sub-orbit is represented as *nl* such that *n* is the principal quantum number (in the form of values) and *l* is the azimuthal quantum number (its name).

Value of $l < n$, | l | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| | s | p | d | f | g |

Value of m : $-l, -l+1 \ldots\ldots 0, \ldots\ldots +l$

Value of s : $+\frac{1}{2}$ or $-\frac{1}{2}$

Thus for $4f$: $n = 4$, $l = 3$, m = any value between -3 to $+3$.

77. EC of Cr (Z = 24) is

	n	l
$1s^2$	1	0
$2s^2$	2	0
$2p^6$	2	1
$3s^2$	3	0
$3p^6$	3	1
$3d^5$	3	2
$4s^1$	4	0

Thus electrons with $l = 1$, are 12
with $l = 2$, are 5

78. All the ions belong to same period thus for them cations will be smaller than anions. Now, O^{2-} and F^- are isoelectronic and $r_n \propto \frac{1}{Z}$

Thus ionic radius of $O^{2-}(Z = 8) > F^-(Z = 9)$

79. $\frac{1}{\lambda} = \bar{v}_H = \bar{R}_H \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

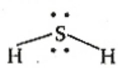
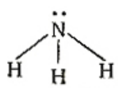
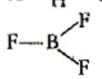
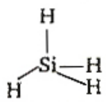
$$= 1.097 \times 10^7 \left[\frac{1}{1^2} - \frac{1}{\infty^2}\right]$$

$$\therefore \quad \lambda = \frac{1}{1.097 \times 10^7} \text{ m} = 9.11 \times 10^{-8} \text{ m}$$

$$= 91.1 \times 10^{-9} \text{ m}$$

$$= 91.1 \text{ nm} \qquad (1\text{nm} = 10^{-9} \text{ m})$$

80.

Species	Structure	lp	bp	VSEPR	Bond angle
H_2S		2	2	lp-lp lp-bp bp-bp	90°
NH_3		1	3	lp-bp bp-bp	107°
BF_3		0	3	bp-bp	120°
SiH_4		0	4	bp-bp	109° 28°

Thus bond angle $H_2S < NH_3 < SiH_4 < BF_3$

81.

Species	Z	Electron gained or lost in the formation	Electrons
K^+	19	− 1	18
Ca^{2+}	20	− 2	18
Sc^{3+}	21	− 3	18
Cl^-	17	+ 1	18

In this set all have 18 electrons, thus isoelectronic.

82. While moving along a group from top to bottom, acidic nature of oxides decreases and along a period left to right, acidic nature increases.

	Al	Si	P	S
Z	13	14	15	16
	Al_2O_3	SiO_2	P_2O_3	SO_2
	amphoteric	acidic		max. acidic

Thus $Al_2O_3 < SiO_2 < P_2O_3 < SO_2$

83. Bond length is inversely proportional to bond-order. Bond-order in $NO^+ = 3$

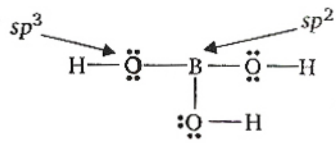
$NO = 2.5$

Thus bond length in $NO > NO^+$

84. $O^-(g) + e^- \longrightarrow O^{2-}(g)$, $\Delta H° = 844 \text{ kJ mol}^{-1}$

This process is unfavourable in the gas phase because the resulting increase in electron-electron repulsion overweighs the stability gained by achieving the noble gas configuration.

85. H_3BO_3 has structure

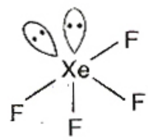


Boron has three bonds thus sp^2 hybridised. Each oxygen has two bonds and two lone pair hence sp^3 hybridised.

86. Tetrahedral structure is associated with sp^3 hybridised central atom without any lone pair.

The structure of all the compounds given are as follows :

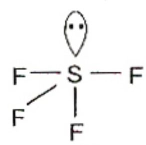
XeF_4



distorted octahedral

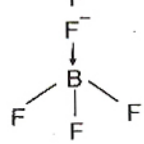
$[Ni(CN)_4]^{2-}$ —— square planar

SF_4



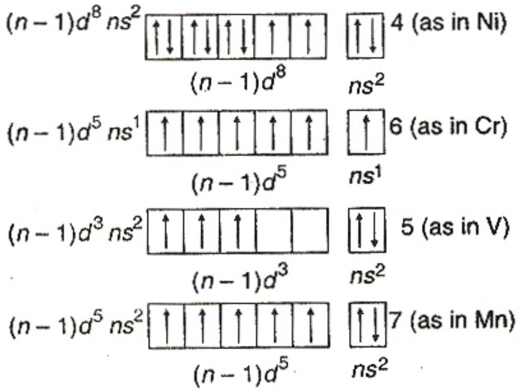
distorted trigonal bipyramidal

BF_4^-



tetrahedral

87.

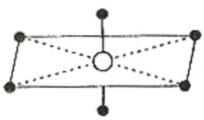


88. Average KE $= \frac{3}{2} RT / N_0$

$(\text{KE}) \propto T$

$\therefore (\text{KE})_{313} / (\text{KE})_{293} = \frac{313}{293}$

89. sp^3d^2 hybridisation has octahedral structure such that four hybrid orbitals are at 90° w.r.t. each other and others two at 90° with first four.



90. Boiling point

$= T_o$ (Solvent) + ΔT_b (Elevation in bp)

$\Delta T_b = miK_b$

where, m is the molality (≈ Molarity M)

i, the van't Hoff factor $= [1 + (y - 1)x]$

K_b, molal elevation constant.

Thus $\Delta T_b \propto i$

Assume 100% ionisation

(a) $mi\ (Na_2SO_4) = 0.01 \times 3 = 0.03$

(b) $mi\ (KNO_3) = 0.01 \times 2 = 0.02$

(c) mi (urea) $= 0.015$

(d) mi (glucose) $= 0.015$

91. F_2 has the most negative $\Delta G°$ value which is dependent on hydration enthalpy.

92. van der Waals' equation for one mol of a gas is

$$\left[P + \frac{a}{V^2}\right][V - b] = RT$$

where b is volume correction. It arises due to finite size of molecules.

93. H_3PO_4 is a tribasic acid, thus ionising in three steps :

I. $H_3PO_4 \rightleftharpoons H^+ + H_2PO_4^-$

II. $H_2PO_4^- \rightleftharpoons H^+ + HPO_4^{2-}$

III. $HPO_4^{2-} \rightleftharpoons H^+ + PO_4^{3-}$

Conjugate base is formed when an acid loses its proton. Thus HPO_4^{2-} is the conjugate base of $H_2PO_4^-$ (which is an acid in step II, but is the conjugate base of H_3PO_4 in step I).

94. Avogadro's number

$N_A = 6.02 \times 10^{23} = 1$ mol

$\therefore 6.02 \times 10^{20}$ molecules $= 0.001$ mol in 100 mL (0.1 L) solution

$\therefore$ Molar concentration

$= \frac{\text{mol}}{\text{volume in L}} = \frac{0.001}{0.1} = 0.01$ M

95. H_3PO_3 is a dibasic acid (containing two ionisable protons attached to O directly).

$H_3PO_3 \rightleftharpoons 2H^+ + HPO_4^{2-}$

$\therefore$ 0.1 M H_3PO_3 = 0.2 N H_3PO_3

and 0.1 M KOH = 0.1 N KOH

$N_1V_1 = N_2V_2$

(KOH) (H_3PO_3)

$0.1\, V_1 = 0.2 \times 20$

$V_1 = 40$ mL

96. In CH_3CH_2OH, there is intermolecular H-bonding while it is absent in isomeric ether CH_3OCH_3

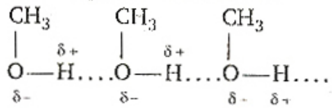
- Larger heat is required to vaporise CH_3CH_2OH as compared to CH_3OCH_3, thus (a) is incorrect.
- CH_3CH_2OH is less volatile than CH_3OCH_3, thus vapour pressures are different, thus (b) is incorrect.
- bp of $CH_3CH_2OH > CH_3OCH_3$, thus (c) is incorrect.

Density $= \frac{\text{mass}}{\text{volume}}$, due to ideal behaviour at a given temperature and pressure volume and molar mass are same. Hence, they have same vapour density.

97. Water and hydrochloric acid ; and water and nitric acid form miscible solutions. They show negative deviation.

In case of CH_3COCH_3 and $CHCl_3$, there is interaction between them, thus force of attraction between CH_3COCH_3 ... $CHCl_3$ is larger than between $CHCl_3 \ldots CHCl_3$ or $CH_3COCH_3 \ldots CH_3COCH_3$ and thus vapour pressure is less than expected.—a negative deviation.

In case of CH_3OH, there is association by intermolecular H—bonding. When benzene is added to CH_3OH, H—bonding breaks and thus force of attraction between CH_3OH and benzene molecules is smaller than between CH_3OH or benzene molecules (in pure state). Vapour pressure of mixture is greater than expected—a positive deviation.



98. (a) $p_A = X_A\, p^\circ_A$ true

(b) $\pi = iMRT = MRT$

true (if van't Hoff factor $i = 1$)

(c) $i = [1 + (y - 1)\,x]$

y = number of ions

x = degree of ionisation

$i = 3$ for $BaCl_2$ $x = 1$ (strong electrolyte)

$i = 2$ for KCl $x = 1$ (strong electrolyte)

$i = (1 + x)$ for CH_3COOH $x << 1$ (weak)

$i = 1$ for sucrose (non-electrolyte)

i (for $BaCl_2$) > KCl > CH_3COOH > sucrose

Thus (c) is also true.

(d) $\Delta T_f = K_f m$.

K_f is dependent on solvent.

Thus freezing points [$= T$ (solvent) $- \Delta T_f$) are different.

Thus (d) is false.

99. When equal number of cations and anions (such that charges are equal) are missing ($1Na^+$, 1 Cl^-/1 Fe^{2+}, $2Cl^-$)

It is a case of Schottky defect.

100. Work done due to change in volume against constant pressure is

$$W = -P\,(V_2 - V_1)$$

$$= -1 \times 10^5\ Nm^{-2}\ (1 \times 10^{-2} - 1 \times 10^{-3})\ m^3$$

$$= -900\ Nm = -900\ J \qquad (1\ Nm = 1\ J)$$

101. Any cell (like fuel cell), works when potential difference is developed.

102. Order = 1

Concentration changes from 0.8 M to 0.4 M in (50%) 15 minutes thus half-life is $= 15$ minutes $= T_{50}$

A change from 0.1 M to 0.025 M is 75% and for first order reaction

$$T_{75} = 2 \times T_{50} = 2 \times 15 = 30\ \text{min}$$

OR

$T_{50} = 15$ min

$$k = \frac{2.303 \log 2}{T_{50}} = \frac{2.303 \log 2}{15}$$

$$a = 0.1\ M$$

$$(a - x) = 0.025\ M$$

For first order :

$$k = \frac{2.303}{t} \log\left(\frac{a}{a - x}\right)$$

$$\frac{2.303 \log 2}{15} = \frac{2.303}{t} \log \frac{0.1}{0.025}$$

$$= \frac{2.303}{t} \log 4$$

$$\therefore \frac{2.303 \log 2}{15} = \frac{2 \times 2.303 \log 2}{t}$$

$$\therefore \quad t = 30\ \text{min}$$

103. In the expression for equilibrium constant (K_p or K_c) species in solid state are not written (*i. e.*, their molar concentrations are taken as 1)

$$P_4(s) + 5O_2(g) \rightleftharpoons P_4O_{10}(s)$$

Thus, $K_c = \dfrac{1}{[O_2]^5}$

104. $K_p = K_c\,(RT)^{\Delta n}$

Δn = Sum of coefficients of gaseous products – that of gaseous reactants.

$$CO\ (g) + Cl_2\ (g) \longrightarrow COCl_2\ (g)$$

$$\therefore \quad \Delta n = 1 - 2 = -1$$

$$\therefore \quad K_p = K_c\ (RT)^{-1}$$

$$\therefore \quad \frac{K_p}{K_c} = (RT)^{-1} = \frac{1}{(RT)}$$

105. $N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$

$$K_c = \frac{[NO]^2}{[N_2]\,[O_2]} = 4 \times 10^{-4}$$

$$NO \rightleftharpoons \frac{1}{2} N_2\ (g) + \frac{1}{2} O_2\ (g)$$

$$K_c' = \frac{[N_2]^{1/2}\,[O_2]^{1/2}}{[NO]}$$

$$= \sqrt{\frac{1}{K_c}} = \sqrt{\frac{1}{4 \times 10^{-4}}} = 50$$

106. $2A + B \longrightarrow C$

Rate $= k\,[A]\,[B]$

It represents second-order reaction.

Thus unit of k is $M^{-1}\ s^{-1}$

$\therefore$ (a) is false

T_{50} is dependent of concentration but not constant

$\therefore$ (b) is false

$-\dfrac{1}{2}\dfrac{d[A]}{dt} = \dfrac{d[C]}{dt}$, thus (c) is correct

107. $Sn(s) + 2Fe^{3+}(aq) \longrightarrow 2Fe^{2+}(aq) + Sn^{2+}(aq)$

$$E^\circ_{cell} = E^\circ_{ox} + E^\circ_{red}$$

$$= E^\circ_{Sn/Sn^{2+}} + E^\circ_{Fe^{3+}/Fe^{2+}}$$

Given $\quad E^\circ_{Sn^{2+}/Sn} = -0.14\ V$

$\therefore \quad E^\circ_{Sn/Sn^{2+}} = +0.14$ V

$E^\circ_{Fe^{3+}/Fe^{2+}} = 0.77$ V

$\therefore \quad E^\circ_{cell} = 0.14 + 0.77 = 0.91$ V

108. For the solute $A_xB_y \rightleftharpoons xA + yB$

$$K_{sp} = x^x y^y (s)^{x+y}$$

$$MX_4 \rightleftharpoons M^{4+} + 4X^-$$

$x = 1, y = 4$

$\therefore \quad K_{sp} = (4)^4 (1)^1 (s)^5 = 256\, s^5$

$\therefore \quad s = \left(\frac{K_{sp}}{256}\right)^{1/5}$

109. Relation between K_{eq} and E_{cell} is

$$E^\circ_{cell} = \frac{2.303\,RT}{nF} \log K_{eq}$$

$$E^\circ_{cell} = \frac{0.0591}{n} \log K_{eq} \quad \text{(at 298 K)}$$

$$0.591 = \frac{0.0591}{1} \log K_{eq}$$

$\therefore \quad \log K_{eq} = 10$

$\therefore \quad K_{eq} = 1 \times 10^{10}$

110. I : $C(s) + O_2(g) \longrightarrow CO_2(g)$ $\Delta H = -393.5$ kJ

II : $CO(g) + \frac{1}{2} O_2(g) \longrightarrow CO_2(g)$, $\Delta H = -283.0$ kJ

I–II gives

III : $C(s) + \frac{1}{2} O_2(g) \longrightarrow CO(g)$, $\Delta H = -110.5$ kJ

This equation III also represents formation of one mol of CO and thus enthalpy change is the heat of formation of CO (g).

111. By Kohlrausch's law

$$\Lambda^\circ_{NaBr} = \Lambda^\circ_{NaCl} + \Lambda^\circ_{KBr} - \Lambda^\circ_{KCl}$$
$$= 126 + 152 - 150$$
$$= 128 \text{ S cm}^2 \text{ mol}^{-1}$$

112. $Zn(s) + 2H^+ \longrightarrow Zn^{2+}(aq) + H_2(g)$

Reaction quotient $Q = \frac{[Zn^{2+}]}{[H^+]^2}$

Corresponding cell is

$$\underset{\text{anode}}{\overset{-}{Zn}} | Zn^{2+} (C_1) || H^+ (aq) | \underset{\text{cathode}}{\overset{+}{Pt}} (H_2)$$

and $\quad E_{cell} = E^\circ_{cell} - \frac{0.0591}{2} \log K$

$$= E^\circ_{cell} - \frac{0.0591}{2} \log \frac{[Zn^{2+}]}{[H^+]^2}$$

$$= E^\circ_{cell} + \frac{0.0591}{2} \log \frac{[H^+]^2}{[Zn^{2+}]}$$

If H_2SO_4 is added to cathodic compartment, (towards reactant side), then Q decreases (due to increase in H^+).

Hence, equilibrium is displaced towards right and E_{cell} increases.

113. Helium is not used to produce and sustain powerful superconducting magnets. All others are the uses of helium.

114. Normal optimum temperature of enzymes is between 25° C to 40°C hence (a) is false. Similarly (b) and (c) are also false. Enzymes have well defined active sites and their actions are specific in nature.

115. $\underset{1\text{ mol}}{Mg_3N_2} (s) + 6H_2O (l) \longrightarrow 3\, Mg(OH)_2 + \underset{2\text{ mol}}{2NH_3(g)}$

116. Froth-floatation is used to concentrate sulphide ores [Galena (PbS)].

117. Be $(Z = 4)$ has maximum covalency of 4 while Al $(Z = 13)$ has maximum covalency of 6.

118. $AlCl_3$ is covalent but in water, it become ionic due to large hydration energy of Al^{3+}.

$$AlCl_3 + 6H_2O \rightleftharpoons [Al(H_2O)_6]^{3+} + 3Cl^-$$

119. As temperature decreases, white tin (β-form) changes to grey tin (α-form).

$$\alpha\text{-Sn} \overset{13.2^\circ C}{\rightleftharpoons} \beta\text{-Sn}$$

α-Sn has a much lower density.

120. $E^\circ_{Cr^{3+}/Cr^{2+}} = -0.41$ V

$E^\circ_{Mn^{3+}/Mn^{2+}} = +1.57$ V

$E^\circ_{Fe^{3+}/Fe^{2+}} = +0.77$ V

$E^\circ_{Co^{3+}/Co^{2+}} = +1.97$ V

More negative value of E°_{red} indicates better reducing agent thus easily oxidised. Thus oxidation of Cr^{2+} to Cr^{3+} is the easiest.

121. $CuSO_4 + 2KI \longrightarrow \underset{\text{unstable}}{CuI_2} + K_2SO_4$

$$2CuI_2 \longrightarrow Cu_2I_2 + I_2$$

Thus CuI_2 is not formed.

122. CN^- is a better complexing agent (C) as well as a reducing agent (A)

Thus properties (A) and (C) are shown.

Property (C) : $Ni^{2+} + 4CN^- \longrightarrow [Ni(CN)_4]^{2-}$

Property (*A*) : $\overset{II}{CuCl_2} + 5KCN \longrightarrow$

$K_3[\overset{I}{Cu}(CN)_4] + \frac{1}{2}(CN)_2 + 2KCl$

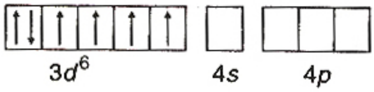
(CN^- reduces Cu^{2+} to Cu^+)

123. Co-ordination number is the maximum covalency shown by a metal or metal ion. It is the maximum number of ligands attached to metal by sigma bonds or co-ordinate bonds.

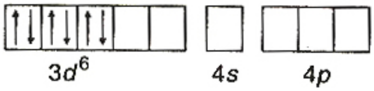
124.

(a)

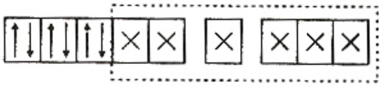
Fe^{2+} [Ar]



Fe^{2+} in strong ligand CN^- [Ar]



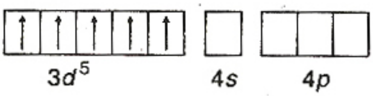
$[Fe(CN)_6]^{4-}$ [Ar]



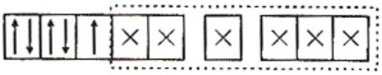
d^2sp^3 inner *d*-orbital complex

(b)

Mn^{2+} [Ar]



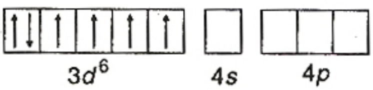
$[Mn(CN)_6^{4-}]$ [Ar]



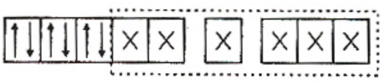
d^2sp^3 inner *d*-orbital complex

(c)

Co^{3+} [Ar]



$[Co(NH_3)_6]^{3+}$ [Ar]

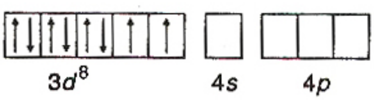


NH_3 is a strong ligand

d^2sp^3 inner *d*-orbital complex

(d)

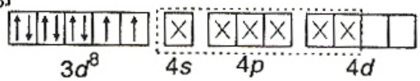
Ni^{2+} [Ar]



In this case also NH_3 is a strong ligand but electrons remain unpaired since only one orbital is left vacant in 3*d*. Thus

$[Ni(NH_3)_6]^{2+}$

[Ar]



sp^3d^2 outer *d*-orbital complex

125. Chlorophyll contains Mg, hence (a) is incorrect statement.

126. + 3 and + 4 states are shown by Ce in aqueous solution. Thus statement (c) is incorrect.

127. $[Co(en)_2Cl_2]$ forms optical and geometrical isomers.

128. As in Q. 124, number of unpaired electrons in $[Fe(CN)_6]^{4-}$ is zero.

Thus magnetic moment

$$= \sqrt{n(n+2)} = 0 \text{ BM}$$

(n = unpaired electrons)

n in $[MnCl_4]^{2-} = 5, \sqrt{35}$ BM

n in $[CoCl_4]^{2-} = 3, \sqrt{15}$ BM

129. ${}^{238}_{92}M \longrightarrow {}^{X}_{Y}N + 2\,{}^{4}_{2}He$

$$X = 230$$

$$Y = 88$$

${}^{230}_{88}N \longrightarrow {}^{A}_{B}L + 2\,{}^{0}_{1}e\ (\beta^+)$

$\therefore \quad A = 230$

$= n + p$

$\therefore \quad B = 86 = p$

$\therefore \quad n = 144$

130. If y = number of half-lives,

$$\therefore \quad y = \frac{\text{total time}}{\text{half-life}} = \frac{24}{4} = 6$$

C = amount left after y half-life

C_0 = initial amount

$$\therefore \quad C = C_0\left(\frac{1}{2}\right)^y = 200\left(\frac{1}{2}\right)^6 = 3.125 \text{ g}$$

131. If nitrogen is present in organic compound then sodium extract contains $Na_4[Fe(CN)_6]$.

$$Na + C + N \xrightarrow{\text{fuse}} NaCN$$

$$FeSO_4 + 6NaCN \longrightarrow \underset{(A)}{Na_4[Fe(CN)_6]} + Na_2SO_4$$

A changes to Prussian blue $Fe_4[Fe(CN)_6]_3$ on reaction with $FeCl_3$.

$4FeCl_3 + 3Na_4[Fe(CN)_6] \longrightarrow$

$Fe_4[Fe(CN)_6]_3 + 12NaCl$

132. Let unreacted 0.1 M (= 0.2 N) $H_2SO_4 = V'$ mL

$\therefore$ 20 mL of 0.5 M NaOH

$= V'$ mL of 0.2 N H_2SO_4

$\therefore \quad 20 \times 0.5 = V' \times 0.2$

$\therefore V' = 50$ mL

Used $H_2SO_4 = 100 - 50 = 50$ mL

$$\% \text{ Nitrogen} = \frac{1.4\,NV}{w}$$

where N = normality of H_2SO_4

V = volume of H_2SO_4 used

$$\therefore \quad \% \text{ Nitrogen} = \frac{1.4 \times 0.2 \times 50}{0.30} = 46.67\%$$

% of nitrogen in

(a) $CH_3CONH_2 = \frac{14 \times 100}{59} = 23.73\%$

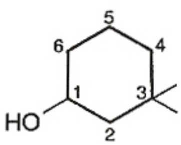
(b) $C_6H_5CONH_2 = \frac{14 \times 100}{122} = 11.48\%$

(c) $NH_2CONH_2 = \frac{28 \times 100}{60} = 46.67\%$

(d) $NH_2\ CSNH_2 = \frac{28 \times 100}{76} = 36.84\%$

133. Isobutene $(CH_3)_2C{=}CH_2$ has minimum force of attraction (due to steric hindrance). Thus minimum boiling point.

134.



Carbon with —OH group is given C_1 thus it is 3,3-dimethyl-1-cyclohexanol.

135. (a) $\underset{sp^3}{CH_3}-\underset{sp^2}{\overset{O}{\overset{\|}{C}}}-\underset{sp^3}{CH_3}$ (b) $\underset{sp^3}{CH_3}-\underset{sp^2}{\overset{O}{\overset{\|}{C}}}-OH$

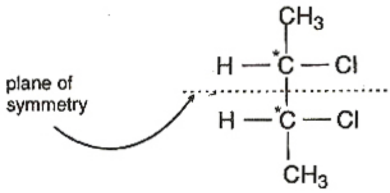
(c) $\underset{sp^3}{CH_3}-\underset{sp}{C}\equiv N$ (d) $\underset{sp^3}{CH_3}-\underset{sp^2}{\overset{O}{\overset{\|}{C}}}-NH_2$

Acetonitrile does not contain sp^2 hybridised carbon.

136. (a) $CH_3-\overset{*}{C}H(Cl)-CH_2CH_3$

One asymmetric carbon atom, forms *d*-and *l*-optical isomers.

(b) Two asymmetric carbon atoms, forms *d*-, *l*- and *meso* forms.



meso due to internal compensation

(c) $CH_3-\overset{*}{C}H(Cl)-\overset{*}{C}H(Cl)-CH_2CH_3$

Two asymmetric carbon atoms but does not have symmetry. Hence, *meso* form is not formed.

(d) $CH_3-\overset{*}{C}H(OH)-COOH$

One asymmetric carbon atom, *meso* form is not formed.

137. Cl^- is the best leaving group being the weakest nucleophile out of NH_2^-, Cl^-, $\bar{O}-C_2H_5$ and $O^--\overset{O}{\overset{\|}{C}}-CH_3$

Note : If acid HX is weak, its conjugate base X^- is strong and *vice-versa*.

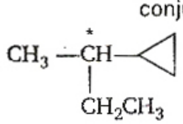
$NH_3 < C_2H_5OH < CH_3COOH < HCl$

Acidic strength increasing $\longrightarrow$

$NH_2^- > C_2H_5O^- > CH_3COO^- > Cl^-$

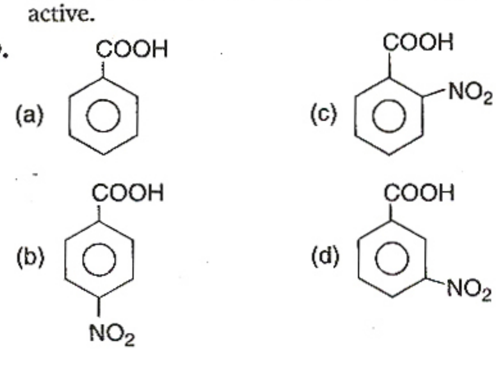
$\longleftarrow$ Basic strength increasing conjugate base

138.



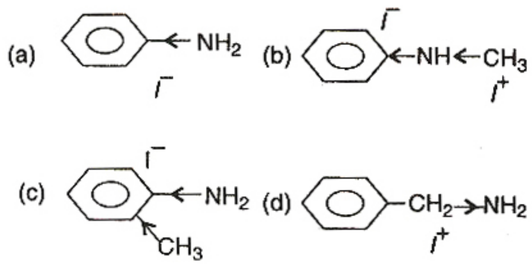
contains asymmetric carbon thus optically active.

139.



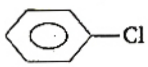
—NO_2 group at any position shows electron withdrawing effect thus acid strength is increased. But *o*-nitro benzoate ion is stabilised by intramolecular H-bonding like forces hence its acid strength is maximum, Thus
acid strength (II) > (III) > (IV) > (I)
The effect is more at *para* position than *meta*.

140. CH_3— (an electron releasing (+*I*) group) increases electron density at N-atom hence basic nature is increased.



C_6H_5 decreases electron density at N-atom thus basic nature is decreased. (Lone-pair on N in aniline compounds is delocalised along with π-electrons in benzene).
Thus (d) is the strongest base.

141. Uracil is present in RNA but not in DNA.

142. $CCl_3CHO + 2$  $\xrightarrow{\text{conc. } H_2SO_4}$ DDT

Chloral — Chlorobenzene

143. Aqueous NaCl is neutral hence there is no reaction between ethyl acetate and aqueous NaCl.

144. $CH_3-\overset{O}{\overset{\|}{C}}-Br + CH_3MgI \longrightarrow CH_3-\overset{O}{\overset{\|}{C}}-CH_3$

$$CH_3-\overset{O}{\overset{\|}{C}}-CH_3 + CH_3MgI$$

$$\rightarrow CH_3-\underset{CH_3}{\overset{CH_3}{C}}-OMgI \xrightarrow{aq.\ NH_4Cl} CH_3-\underset{CH_3}{\overset{CH_3}{C}}-OH$$

2-methyl-2-propanol

145. Carbonyl compounds are reduced to corresponding alkanes with (Zn + conc. HCl). It is called Clemmensen reduction.

$$CH_3CH_2\cdot\overset{O}{\overset{\|}{C}}-CH_3 \xrightarrow{Zn(Hg) + HCl} CH_3CH_2CH_2CH_3$$

146. *A* + NaOH→ alcohol + acid
Thus it is Cannizaro reaction.
A is thus aldehyde without H at α – carbon.
(as C_6H_5CHO, HCHO)

$$2C_6H_5CHO + NaOH \longrightarrow C_6H_5CH_2OH + C_6H_5COONa$$

147. Dehydration of alcohol is in order

$$1° < 2° < 3°$$

Thus (c), a 3° alcohol is dehydrated very easily.

148. Chiral carbon has all the four different groups attached to it.
(a) $CH_3CH_2CH_2CH_2Cl$ no chiral carbon atom
(b) $CH_3\overset{*}{C}HCH_2CH_2CH_3$ (with Cl on the starred carbon) one chiral carbon atom
(c) $CH_3CH_2CH_2\overset{*}{C}HCH_2Cl$ (with CH_3 on the starred carbon) one chiral carbon atom
(d) $CH_3\overset{*}{C}H-\overset{*}{C}HCH_2CH_3$ (with CH_3 and Cl on the starred carbons) two chiral carbon atom

149. Insulin is a hormone built up of two polypeptide chains.

150. NO, NO_2, SO_2 and SO_3 are responsible for smog (environmental pollution).

Mathematics

1. **Key Idea :** *A correspondence which is one-one or many-one is called a function, a one to many or many to many correspondence cannot be a function.*

Let $R = \{(1, 3), (4, 2), (2, 4), (2,3), (3, 1)\}$ is a relation on the set $A = \{1, 2, 3, 4\}$, then

(a) Since $(2, 4) \in R$ and $(2, 3) \in R$, so R is not a function.

(b) Since $(1, 3) \in R$ and $(3, 1) \in R$ but $(1, 1) \notin R$. So R is not transitive.

(c) Since $(2, 3) \in R$ but $(3, 2) \notin R$, so R is not symmetric.

(d) Since $(4, 4) \notin R$, so R is not reflexive,

Hence the option (c) is correct.

Note : A relation which is not symmetric, is known as antisymmetric relation.

2. The given function $f(x) = {}^{7-x}P_{x-3}$ would be defined, if

(i) $7 - x > 0 \Rightarrow x < 7$ (ii) $x - 3 \geq 0 \Rightarrow x \geq 3$

(iii) $(x - 3) \leq (7 - x)$

$\Rightarrow \quad 2x \leq 10 \Rightarrow x \leq 5$

$\Rightarrow \quad x = 3, 4, 5$

Hence Range of $f(x) = \{{}^4P_0, {}^3P_1, {}^2P_2\}$.

Range of $f(x) = \{1, 3, 2\}$.

Note : If $f : A \to B$, then the set of elements of B which have pre-image is known as range of $f(x)$ while set B is known as co-domain of $f(x)$.

3. Since $\bar{z} + i\,\bar{w} = 0 \Rightarrow \bar{z} = -i\,\bar{w}$

$\Rightarrow \quad z = iw \Rightarrow w = -iz$

Also $\arg(zw) = \pi$

$\Rightarrow \quad \arg(-iz^2) = \pi$

$\Rightarrow \quad \arg(-i) + 2\arg(z) = \pi$

$\Rightarrow \quad -\frac{\pi}{2} + 2\arg(z) = \pi$

$(\because \arg(-i) = -\frac{\pi}{2})$

$\Rightarrow \quad 2\arg(z) = \frac{3\pi}{2}$

$\Rightarrow \quad \arg(z) = \frac{3\pi}{4}$

4. $z^{1/3} = p + iq$

$\Rightarrow \quad (x - iy)^{1/3} = (p + iq) \quad (\because z = x - iy)$

$\Rightarrow \quad (x - iy) = (p + iq)^3$

$\Rightarrow (x - iy) = p^3 + (iq)^3 + 3p^2qi + 3pq^2i^2$

$\Rightarrow \quad (x - iy) = p^3 - iq^3 + 3p^2qi - 3pq^2$

$\Rightarrow (x - iy) = (p^3 - 3pq^2) + i\,(3p^2q - q^3)$

On comparing both sides, we get

$x = (p^3 - 3pq^2)$ and $-y = 3p^2q - q^3$

$\Rightarrow x = p\,(p^2 - 3q^2)$ and $y = q\,(q^2 - 3p^2)$

$\Rightarrow \quad \frac{x}{p} = (p^2 - 3q^2)$ and $\frac{y}{q} = (q^2 - 3p^2)$

Now, $\quad \frac{x}{p} + \frac{y}{q} = p^2 - 3q^2 + q^2 - 3p^2$

$\Rightarrow \quad \frac{x}{p} + \frac{y}{q} = -2p^2 - 2q^2$

$\Rightarrow \quad \frac{x}{p} + \frac{y}{q} = -2(p^2 + q^2)$

$\Rightarrow \quad \frac{x/p + y/q}{(p^2 + q^2)} = -2$

5. Given that

$|z^2 - 1| = |z|^2 + 1$

$\Rightarrow \quad |z^2 + (-1)| = |z^2| + |-1|$

It shows that the origin, -1 and z^2 lies on a line and z^2 and -1 lies on one side of the origin, therefore z^2 is a negative number. Hence z will be purely imaginary. So we can say that z lies on y-axis.

Alternate Solution

We know that, if

$|z_1 + z_2| = |z_1| + |z_2|$

$\Rightarrow \quad \arg(z_1) = \arg(z_2)$

$\because \quad |z^2 + (-1)| = |z^2| + |-1|$

$\Rightarrow \quad \arg(z^2) = \arg(-1)$

$\Rightarrow \quad 2\arg(z) = \pi \qquad (\because \arg(-1) = \pi)$

$\Rightarrow \quad \arg(z) = \frac{\pi}{2}$

$\Rightarrow z$ lies on y-axis (imaginary axis).

6. The given matrix $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

(a) It is clear that A is not a zero matrix.

(b) $(-1)I = -1\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \neq A$

i.e., $(-1)I \neq A$

(c) $|A| = 0\begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} - 0\begin{vmatrix} 0 & 0 \\ -1 & 0 \end{vmatrix} - 1\begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix}$

$= 0 - 0 - 1(-1) = 1$

Since, $|A| \neq 0$. So A^{-1} exists.

(d) $A^2 = A \cdot A$

$$= \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \quad A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad A^2 = I$$

7. Since B is inverse of A, *i.e.* $B = A^{-1}$.

So, $$10\,A^{-1} = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$$

$$\Rightarrow \quad 10\,A^{-1}A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} A$$

$$\Rightarrow \quad 10\,I = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \quad (\because A^{-1}A = I)$$

$$\Rightarrow \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ -5+\alpha & 5+\alpha & -5+\alpha \\ 0 & 0 & 10 \end{bmatrix}$$

$$\Rightarrow \quad -5 + \alpha = 0$$

$$\Rightarrow \quad \alpha = 5$$

8. Since $a_1, a_2, \ldots\ldots, a_n$ are in GP

Then, $a_n = a_1 r^{n-1}$

$\Rightarrow \quad \log a_n = \log a_1 + (n-1)\log r$

$a_{n+1} = a_1 r^n$

$\Rightarrow \quad \log a_{n+1} = \log a_1 + n\log r$

$a_{n+2} = a_1 r^{n+1}$

$\Rightarrow \quad \log a_{n+2} = \log a_1 + (n+1)\log r$

. . . .

. . . .

. . . .

$a_{n+8} = a_1 r^{n+7}$

$\Rightarrow \quad \log a_{n+8} = \log a_1 + (n+7)\log r$

Now, $$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

$$= \begin{vmatrix} \log a_1 + (n-1)\log r & \log a_1 + n\log r & \log a_1 + (n+1)\log r \\ \log a_1 + (n+2)\log r & \log a_1 + (n+3)\log r & \log a_1 + (n+4)\log r \\ \log a_1 + (n+5)\log r & \log a_1 + (n+6)\log r & \log a_1 + (n+7)\log r \end{vmatrix}$$

Now, $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Rightarrow \begin{vmatrix} \log a_1 + (n-1)\log r & \log a_1 + n\log r & \log a_1 + (n+1)\log r \\ 3\log r & 3\log r & 3\log r \\ 3\log r & 3\log r & 3\log r \end{vmatrix} = 0$$

(since two rows are identical)

9. **Key Idea :** *If α and β are the roots of an equation, then the equation will be*

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0.$$

Let α and β be two numbers whose arithmetic mean is 9 and geometric mean is 4.

$\therefore \quad \alpha + \beta = 18 \quad \ldots$(i)

and $\quad \alpha\beta = 16 \quad \ldots$(ii)

$\therefore$ Required equation is

$$x^2 - (\alpha + \beta)x + (\alpha\beta) = 0$$

$$\Rightarrow \quad x^2 - 18x + 16 = 0$$

[using Eqs. (i) and (ii)]

Note : If a and b are two numbers, then their arithmetic mean $= \dfrac{a+b}{2}$

and geometric mean $= \sqrt{ab}$.

10. Since $(1-p)$ is the root of quadratic equation

$$x^2 + px + (1-p) = 0 \quad \ldots\text{(i)}$$

So, $(1-p)$ satisfied the above equation.

$\therefore \quad (1-p)^2 + p(1-p) + (1-p) = 0$

$(1-p)[1 - p + p + 1] = 0$

$\Rightarrow \quad (1-p)(2) = 0$

$\Rightarrow \quad p = 1$

On putting this value of p in Eq. (i)

$x^2 + x = 0$

$\Rightarrow \quad x(x+1) = 0$

$\Rightarrow \quad x = 0, -1$

11. $S(K) = 1 + 3 + 5 + \ldots + (2K - 1) = 3 + K^2$

Put $K = 1$ in both sides

$\therefore$ LHS = 1 and RHS = 3 + 1 = 4

$\Rightarrow$ LHS $\neq$ RHS

Put $(K + 1)$ in both sides on the place of K

LHS $= 1 + 3 + 5 + \ldots + (2K - 1) + (2K + 1)$

RHS $= 3 + (K + 1)^2 = 3 + K^2 + 2K + 1$

Let LHS = RHS

$$1 + 3 + 5 + \ldots + (2K - 1) + (2K + 1) = 3 + K^2 + 2K + 1$$

$$\Rightarrow 1 + 3 + 5 + \ldots + (2K - 1) = 3 + K^2$$

If $S(K)$ is true, then $S(K + 1)$ is also true.

Hence, $S(K) \Rightarrow S(K + 1)$

12. **Key Idea :** *The number of permutations of n things of which p are alike of one kind, q are alike of second kind and remaining all are distinct, is* $\frac{n!}{p!\,q!}$.

Total number of ways in which all letters can be arranged in alphabetical order = 6!.

There are two vowels in the word GARDEN.

Total number of ways in which these two vowels can be arranged = 2!

$\therefore$ Total number of required ways

$$= \frac{6!}{2!} = 360$$

13. **Key Idea :** *The total number of ways of dividing n identical items among r persons, each one of whom receives at least one item, is* ${}^{n-1}C_{r-1}$.

The required number of ways $= {}^{8-1}C_{3-1}$

$$= {}^7C_2 = \frac{7!}{2!\,5!}$$

$$= \frac{7 \cdot 6}{2 \cdot 1} = 21$$

14. Since 4 is one of the roots of equation $x^2 + px + 12 = 0$. So it must satisfy the equation.

$\therefore$ $16 + 4p + 12 = 0$

$\Rightarrow$ $4p = -28$

$\Rightarrow$ $p = -7$

The other equation is $x^2 - 7x + q = 0$ whose roots are equal. Let roots are α and α of above equation .

$\therefore$ Sum of roots $= \alpha + \alpha = \frac{7}{1}$

$\Rightarrow$ $2\alpha = 7$

$\Rightarrow$ $\alpha = \frac{7}{2}$

and product of roots $= \alpha \cdot \alpha = q$

$\Rightarrow$ $\alpha^2 = q$

$\Rightarrow$ $\left(\frac{7}{2}\right)^2 = q$

$\Rightarrow$ $q = \frac{49}{4}$

15. The coefficient of x in the middle term of expansion of $(1 + \alpha x)^4 = {}^4C_2 \cdot \alpha^2$

The coefficient of x in the middle term of the expansion of $(1 - \alpha x)^6 = {}^6C_3\,(-\alpha)^3$

According to question, ${}^4C_2\,\alpha^2 = {}^6C_3\,(-\alpha)^3$

$\Rightarrow$ $\frac{4!}{2!\,2!}\alpha^2 = -\frac{6!}{3!\,3!}\alpha^3$

$\Rightarrow$ $6\alpha^2 = -20\,\alpha^3$

$\Rightarrow$ $\alpha = -\frac{6}{20}$

$\Rightarrow$ $\alpha = -\frac{3}{10}$

16. The coefficient of x^n in the expansion of $(1 + x)(1 - x)^n$

= Coefficient of x^n in the expansion of $(1 - x)^n$ + coefficient of x^{n-1} in the expansion of $(1 - x)^{n-1}$

$$= (-1)^n \frac{n!}{n!\,0!} + (-1)^{n-1} \frac{n!}{1!\,(n-1)!}$$

$$= (-1)^n \left(\frac{n!}{n!\cdot 0!} - \frac{n!}{1!\,(n-1)!}\right)$$

$$= (-1)^n (1 - n)$$

17. Given that, $s_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$

$$s_n = \sum_{r=0}^{n} \frac{1}{{}^nC_{n-r}} \quad (\because {}^nC_r = {}^nC_{n-r})$$

$\Rightarrow$ $n\,s_n = \sum_{r=0}^{n} \frac{n}{{}^nC_{n-r}}$

$\Rightarrow$ $n\,s_n = \sum_{r=0}^{n} \left[\frac{n-r}{{}^nC_{n-r}} + \frac{r}{{}^nC_{n-r}}\right]$

$\Rightarrow$ $n\,s_n = \sum_{r=0}^{n} \frac{n-r}{{}^nC_{n-r}} + \sum_{r=0}^{n} \frac{r}{{}^nC_r}$

$\Rightarrow$ $n\,s_n = \left(\frac{n}{{}^nC_n} + \frac{n-1}{{}^nC_{n-1}} + \ldots + \frac{1}{{}^nC_n}\right) + \sum_{r=0}^{n} \frac{r}{{}^nC_r}$

$\Rightarrow$ $ns_n = t_n + t_n$

$\Rightarrow$ $n\,s_n = 2t_n$

$\Rightarrow$ $\frac{t_n}{s_n} = \frac{n}{2}$

18. Given that, $T_m = \frac{1}{n}$

$\Rightarrow \quad a + (m-1)d = \frac{1}{n}$ ...(i)

and $\quad T_n = \frac{1}{m}$

$\Rightarrow \quad a + (n-1)d = \frac{1}{m}$ ...(ii)

On solving Eqs. (i) and (ii), we get

$$a = \frac{1}{mn} \text{ and } d = \frac{1}{mn}$$

So, $a - d = \frac{1}{mn} - \frac{1}{mn} = 0$

19. The sum of n terms of given series $= \frac{n(n+1)^2}{2}$, if n is even.

Let n is odd *i.e.*, $n = 2m + 1$

Then, $S_{2m+1} = S_{2m} + (2m+1)$th term

$= \frac{2m(2m+1)^2}{2} + (2m+1)^2$

$= \frac{(n-1)n^2}{2}$ nth term

$= \frac{(n-1)n^2}{2} + n^2 \quad (\because n \text{ is odd} = 2m+1)$

$= n^2\left[\frac{n-1+2}{2}\right] = \frac{(n+1)n^2}{2}$

20. We know that

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ldots\ldots \infty \quad \ldots(i)$$

$$e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \ldots \infty \quad \ldots(ii)$$

On adding Eqs. (i) and (ii)

$$e + e^{-1} = 2 + \frac{2}{2!} + \frac{2}{4!} + \ldots \infty$$

$\Rightarrow \quad \frac{e^2+1}{e} - 2 = \frac{2}{2!} + \frac{2}{4!} + \ldots \infty$

$\Rightarrow \quad \frac{e^2+1-2e}{e} = 2\left[\frac{1}{2!} + \frac{1}{4!} + \ldots \infty\right]$

$\Rightarrow \quad \frac{(e-1)^2}{2e} = \frac{1}{2!} + \frac{1}{4!} + \ldots \infty$

21. Given that,

$$\sin\alpha + \sin\beta = -\frac{21}{65} \quad \ldots(i)$$

and $\quad \cos\alpha + \cos\beta = -\frac{27}{65}$ ...(ii)

Squaring Eqs. (i) and (ii) and then adding, we get

$(\sin\alpha + \sin\beta)^2 + (\cos\alpha + \cos\beta)^2$

$= \left(-\frac{21}{65}\right)^2 + \left(-\frac{27}{65}\right)^2$

$\sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta + \cos^2\alpha + \cos^2\beta$

$+ 2\cos\alpha\cos\beta = \frac{1170}{4225}$

$\Rightarrow 2 + 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta) = \frac{1170}{4225}$

$\Rightarrow \quad 2 + 2\cos(\alpha - \beta) = \frac{1170}{4225}$

$\Rightarrow \quad 2[1 + \cos(\alpha-\beta)] = \frac{1170}{4225}$

$\Rightarrow \quad 2\left[2\cos^2\left(\frac{\alpha-\beta}{2}\right)\right] = \frac{1170}{4225}$

$\Rightarrow \quad \cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{1170}{4 \times 4225}$

$\Rightarrow \quad \cos^2\left(\frac{\alpha-\beta}{2}\right) = \frac{9}{130}$

$\Rightarrow \quad \cos\left(\frac{\alpha-\beta}{2}\right) = -\frac{3}{\sqrt{130}}$

$(\because \pi < \alpha - \beta < 3\pi)$

22. $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$

$u^2 = a^2\cos^2\theta + b^2\sin^2\theta + a^2\sin^2\theta + b^2\cos^2\theta$
$+ 2\sqrt{(a^2\cos^2\theta + b^2\sin^2\theta)}\sqrt{(a^2\sin^2\theta + b^2\cos^2\theta)}$

$u^2 = a^2 + b^2 + 2\sqrt{x(a^2+b^2-x)}$

[where $x = a^2\cos^2\theta + b^2\sin^2\theta$]

$u^2 = (a^2+b^2) + 2\sqrt{(a^2+b^2)x - x^2}$

$\frac{du^2}{d\theta} = \frac{1}{\sqrt{(a^2+b^2)x - x^2}}(a^2+b^2-2x) \times \frac{dx}{d\theta}$

and $\frac{dx}{d\theta} = (b^2 - a^2)\sin 2\theta$

$\frac{du^2}{d\theta} = \frac{(a^2+b^2-2x)}{\sqrt{(a^2+b^2)x - x^2}} \times (b^2-a^2)\sin 2\theta$

For maxima and minima put $\frac{du^2}{d\theta} = 0$

$a^2 + b^2 = 2[a^2\cos^2\theta + b^2\sin^2\theta]$ and $\sin 2\theta = 0$

$\Rightarrow \quad \cos 2\theta(b^2 - a^2) = 0$

$\Rightarrow \quad \theta = 0, \cos 2\theta = 0$

$\Rightarrow \quad 2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4}$

u^2 will be minimum at $\theta = 0$ and will be maximum at $\theta = \frac{\pi}{4}$

$\therefore \; u^2_{min} = (a+b)^2$ and $u^2_{max} = 2(a^2+b^2)$

Hence, $u^2_{max} - u^2_{min} = 2(a^2+b^2) - (a+b)^2$
$= (a-b)^2$

23. **Key Idea :** *If a, b and c are the sides of a triangle and C is a largest angle, then*

$$\cos C = \frac{a^2+b^2-c^2}{2ab}.$$

Let $a = \sin\alpha$, $b = \cos\alpha$, $c = \sqrt{1 + \sin\alpha\cos\alpha}$ then

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\Rightarrow \cos C = \frac{\sin^2\alpha + \cos^2\alpha - 1 - \sin\alpha\cos\alpha}{2\sin\alpha\cos\alpha}$$

$$\Rightarrow \cos C = -\frac{\sin\alpha\cos\alpha}{2\sin\alpha\cos\alpha}$$

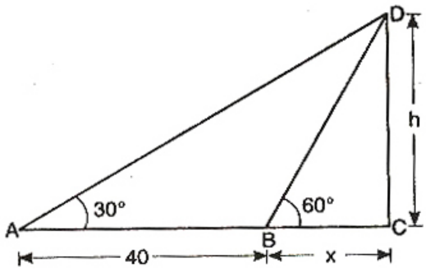
$$\Rightarrow \cos C = -\frac{1}{2} = \cos 120°$$

$$\Rightarrow \angle C = 120°$$

24. Let $CD\ (= h)$ be the height of the tree and $BC\ (= x)$ be the width of the river.

Now, in $\Delta\, BCD$

$$\tan 60° = \frac{CD}{BC}$$



$$\Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3} \quad \text{...(i)}$$

Now, in $\Delta\, ACD$,

$$\tan 30° = \frac{CD}{AC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{40 + x} \Rightarrow h\sqrt{3} = 40 + x$$

$$\Rightarrow 3x = 40 + x \quad \text{[using Eq. (i)]}$$

$$\Rightarrow 2x = 40$$

$$\Rightarrow x = 20 \text{ m}$$

25. Key Idea : *We know that*

$$-\sqrt{a^2 + b^2} \le a\cos\theta + b\sin\theta \le \sqrt{a^2 + b^2}.$$

Since $-2 \le \sin x - \sqrt{3}\cos x \le 2$

$$\Rightarrow -1 \le \sin x - \sqrt{3}\cos x + 1 \le 3$$

$\therefore$ Range of $f(x) = [-1, 3]$.

26. Since graph is symmetrical about the line $x = 2$.

$$\Rightarrow f(2 + x) = f(2 - x)$$

27. The function $f(x) = \dfrac{\sin^{-1}(x-3)}{\sqrt{9 - x^2}}$ will be defined, if

(I) $-1 \le (x - 3) \le 1 \Rightarrow 2 \le x \le 4$ ...(i)

(II) $9 - x^2 > 0 \Rightarrow -3 < x < 3$ ...(ii)

From relations (i) and (ii), we get

$$2 \le x < 3$$

$\therefore$ Domain of the given function $= [2, 3)$.

28. $\lim\limits_{x\to\infty}\left(1 + \dfrac{a}{x} + \dfrac{b}{x^2}\right)^{2x}$

$$= \lim_{x\to\infty}\left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x\left(\frac{a/x + b/x^2}{a/x + b/x^2}\right)}$$

$$= \lim_{x\to\infty} e^{2x(a/x + b/x^2)}$$

$$\left(\because \lim_{x\to\infty}(1 + x)^{1/x} = e\right)$$

$$= \lim_{x\to\infty} (e)^{2(a + b/x)} = e^{2a}$$

But $\lim\limits_{x\to\infty}\left(1 + \dfrac{a}{x} + \dfrac{b}{x^2}\right)^{2x} = e^2$

$$\Rightarrow e^{2a} = e^2$$

$$\Rightarrow a = 1 \text{ and } b \in R$$

29. Key Idea : *A function $f(x)$ is said to be continuous at $x = a$, if $f(a) = \lim\limits_{x\to a} f(a)$*

Since, $f(x) = \dfrac{1 - \tan x}{4x - \pi}$

$$\lim_{x\to\pi/4} f(x) = \lim_{x\to\pi/4}\left(\frac{1 - \tan x}{4x - \pi}\right)$$

By L' Hospital's rule

$$= \lim_{x\to\pi/4}\left(\frac{-\sec^2 x}{4}\right) = \frac{-\sec^2(\pi/4)}{4} = -\frac{2}{4}$$

$$\Rightarrow \lim_{x\to\pi/4} f(x) = -\frac{1}{2}$$

Also, $f(x)$ is continuous in $[0, \pi/2]$, so $f(x)$ will be continuous at $\pi/4$.

$\therefore$ Value of function = Value of limit

$$\Rightarrow f\left(\frac{\pi}{4}\right) = -\frac{1}{2}$$

30. $x = e^{y + e^{y + \dots \infty}}$

$\therefore x = e^{y + x}$

Taking log on both sides, we get

$$\log x = (y + x)$$

Differentiate w.r. to x, we get

$$\frac{1}{x} = \frac{dy}{dx} + 1 \Rightarrow \frac{dy}{dx} = \frac{1 - x}{x}$$

31. Equation of parabola is $y^2 = 18x$.

Differentiate w.r. to t, we get

$$2y\frac{dy}{dt} = 18\frac{dx}{dt}$$

$$2 \cdot 2y = 18 \qquad \left(\because \frac{dy}{dt} = 2\frac{dx}{dt}\right)$$

$$y = \frac{9}{2}$$

$\therefore$ From equation of parabola

$$\left(\frac{9}{2}\right)^2 = 18x$$

$$\Rightarrow \quad \frac{81}{4} = 18x \Rightarrow x = \frac{81}{4 \times 18}$$

$$\Rightarrow \quad x = \frac{9}{8}$$

$\therefore$ Point is $\left(\frac{9}{8}, \frac{9}{2}\right)$.

32. $f''(x) = 6(x-1)$

$\Rightarrow \quad f'(x) = 3(x-1)^2 + c \quad$ ...(i)

At the point (2, 1) the tangent to graph is

$$y = 3x - 5.$$

Slope of tangent $= 3 \Rightarrow f'(2) = 3$

$\therefore \quad f'(2) = 3(2-1)^2 + c = 3$

$\Rightarrow \quad 3 + c = 3 \Rightarrow c = 0$

$\therefore$ From Eq. (i)

$$f'(x) = 3(x-1)^2$$

$\Rightarrow \quad f(x) = (x-1)^3 + k \quad$ ...(ii)

Since graph passes through (2, 1)

$\therefore \quad 1 = (2-1)^2 + k$

$$k = 0$$

$\therefore$ Equation of function is

$$f(x) = (x-1)^3.$$

33. Key Idea : *Equation of normal at* (x_1, y_1) *is*

$$(y - y_1) = -\frac{1}{\left(\frac{dy}{dx}\right)_{(x_1, y_1)}}(x - x_1).$$

We have, $x = a(1 + \cos\theta), y = a\sin\theta$

$$\frac{dx}{d\theta} = a(-\sin\theta), \frac{dy}{d\theta} = a\cos\theta$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{\cos\theta}{\sin\theta}$$

$\therefore$ Equation of normal at $[a(1+\cos\theta), a\sin\theta]$

$$(y - a\sin\theta) = \frac{\sin\theta}{\cos\theta}[x - a(1+\cos\theta)]$$

It is clear that in the given options normal passes through the point $(a, 0)$.

34. Let $\quad f'(x) = ax^2 + bx + c$

$$\Rightarrow \quad f(x) = \frac{ax^3}{3} + \frac{bx^2}{2} + cx + d$$

$$\Rightarrow \quad f(x) = \frac{2ax^3 + 3bx^2 + 6cx + 6d}{6}$$

$$\Rightarrow \quad f(1) = \frac{2a + 3b + 6c + 6d}{6} = \frac{6d}{6} = d$$

$$(\because 2a + 3b + 6c = 0)$$

$$f(0) = \frac{6d}{6} = d$$

$\therefore \quad f(0) = f(1)$

$\Rightarrow f'(x)$ will vanish at least once between 0 and 1.

$\therefore$ One of the roots of $ax^2 + bx + c = 0$ lies between 0 and 1.

35. Key Idea : $\lim_{n \to \infty} \sum_{r=1}^{n} \frac{1}{n} e^{r/n} = \int_0^1 e^x \, dx$.

Now, $\quad \lim_{n \to \infty} \sum_{r=1}^{n} \frac{1}{n} e^{r/n}$

$$= \int_0^1 e^x \, dx = [e^x]_0^1$$

$$= e - 1$$

36. Let $I = \int \frac{\sin x}{\sin(x - \alpha)} dx$

Put $x - \alpha = t \Rightarrow dx = dt$

$$I = \int \frac{\sin(t + \alpha)}{\sin t} dt$$

$$I = \int \frac{\sin t \cos\alpha + \cos t \sin\alpha}{\sin t} dt$$

$$I = \int \cos\alpha \, dt + \int \sin\alpha \frac{\cos t}{\sin t} dt$$

$$I = \cos\alpha \int 1 \, dt + \sin\alpha \int \frac{\cos t}{\sin t} dt$$

$I = \cos\alpha\,(t) + \sin\alpha \log \sin t + c_1$

$I = \cos\alpha\,(x - \alpha) + \sin\alpha \log\sin(x - \alpha) + c_1$

$I = x\cos\alpha + \sin\alpha \log\sin(x - \alpha) - \alpha\cos\alpha + c_1$

$I = x\cos\alpha + \sin\alpha \log\sin(x - \alpha) + c$

But

$$\int \frac{\sin x}{\sin(x - \alpha)} dx = Ax + B\log\sin(x - \alpha) + c$$

$\therefore x\cos\alpha + \sin\alpha \log\sin(x - \alpha) + c$

$$= Ax + B\log\sin(x - \alpha) + c$$

On comparing, we get

$$A = \cos\alpha, B = \sin\alpha$$

Alternate Solution

$$\because \int \frac{\sin x}{\sin(x - \alpha)} dx = Ax + B\log\sin(x - \alpha) + c$$

On differentiating both sides with respect to x, we get

$$\frac{\sin x}{\sin(x - \alpha)} = A + B\frac{\cos(x - \alpha)}{\sin(x - \alpha)}$$

$\Rightarrow \sin x = A\sin(x - \alpha) + B\cos(x - \alpha)$

$\Rightarrow \sin x = A(\sin x \cos\alpha - \cos x \sin\alpha) + B(\cos x \cos\alpha + \sin x \sin\alpha)$

$\Rightarrow \sin x = \sin x\,(A\cos\alpha + B\sin\alpha) + \cos x\,(B\cos\alpha - A\sin\alpha)$

On comparing, we get

$A\cos\alpha + B\sin\alpha = 1$ ...(i)

and $B\cos\alpha - A\sin\alpha = 0$ ...(ii)

On solving Eqs. (i) and (ii), we get

$A = \cos\alpha,\ B = \sin\alpha$

37. $I = \int \frac{dx}{\cos x - \sin x}$

$= \frac{1}{\sqrt{2}} \int \frac{dx}{\left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right)}$

$= \frac{1}{\sqrt{2}} \int \frac{dx}{\cos\left(x + \frac{\pi}{4}\right)} = \frac{1}{\sqrt{2}} \int \sec\left(x + \frac{\pi}{4}\right) dx$

$= \frac{1}{\sqrt{2}} \log\left|\tan\left(\frac{\pi}{4} + \frac{x}{2} + \frac{\pi}{8}\right)\right| + c$

$= \frac{1}{\sqrt{2}} \log\left|\tan\left(\frac{x}{2} + \frac{3\pi}{8}\right)\right| + c$

38. $\int_{-2}^{3} |1 - x^2|\, dx = \int_{-2}^{-1} (x^2 - 1)\, dx + \int_{-1}^{1} (1 - x^2)\, dx + \int_{1}^{3} (x^2 - 1)\, dx$

$= \left[\frac{x^3}{3} - x\right]_{-2}^{-1} + \left[x - \frac{x^3}{3}\right]_{-1}^{1} + \left[\frac{x^3}{3} - x\right]_{1}^{3}$

$= \left[-\frac{1}{3} + 1 + \frac{8}{3} - 2\right] + \left[1 - \frac{1}{3} + 1 - \frac{1}{3}\right] + \left[9 - 3 - \frac{1}{3} + 1\right]$

$= \frac{4}{3} + \frac{4}{3} + \frac{20}{3}$

$= \frac{28}{3}$

39. Let

$I = \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{\sin^2 x + \cos^2 x + 2\sin x \cos x}}\, dx$

$I = \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{(\sin x + \cos x)^2}}\, dx$

$I = \int_0^{\pi/2} (\sin x + \cos x)\, dx$

$I = [-\cos x + \sin x]_0^{\pi/2}$

$I = -\cos\frac{\pi}{2} + \sin\frac{\pi}{2} + \cos 0 - \sin 0$

$I = -0 + 1 + 1 - 0 = 2$

40. **Key Idea :** $\int_0^a f(x)\, dx = \int_0^a f(a - x)\, dx.$

Let $I = \int_0^{\pi} x\, f(\sin x)\, dx$ ...(i)

$I = \int_0^{\pi} (\pi - x)\, f[\sin(\pi - x)]\, dx$

$I = \int_0^{\pi} (\pi - x)\, f(\sin x)\, dx$ ...(ii)

On adding Eq. (i) and Eq. (ii)

$2I = \int_0^{\pi} (x + \pi - x)\, f(\sin x)\, dx$

$2I = \pi \int_0^{\pi} f(\sin x)\, dx$

$2I = 2\pi \int_0^{\pi/2} f(\sin x)\, dx$

$I = \pi \int_0^{\pi/2} f(\sin x)\, dx$

$\Rightarrow A \int_0^{\pi/2} f(\sin x)\, dx = \pi \int_0^{\pi/2} f(\sin x)\, dx$

$\Rightarrow A = \pi$

41. Given that $f(x) = \frac{e^x}{1 + e^x}$

$\therefore\ f(a) = \frac{e^a}{1 + e^a}$

and $f(-a) = \frac{1}{1 + e^a}$

$\therefore\ f(a) + f(-a) = 1$

$\Rightarrow\ f(a) = 1 - f(-a)$

Let $f(-a) = t$

$\Rightarrow\ f(a) = 1 - t$

Now, $I_1 = \int_t^{1-t} x\, g[x(1 - x)]\, dx$ ....(i)

$I_1 = \int_t^{1-t} (1 - x)\, g([x(1 - x)]\, dx$ ...(ii)

On adding Eqs. (i) and (ii)

$2I_1 = \int_t^{1-t} g[x(1 - x)]\,(1 - x + x)\, dx$

$2I_1 = \int_t^{1-t} g[x(1 - x)]\, dx = I_2$

$\Rightarrow\ \frac{I_2}{I_1} = \frac{2}{1} = 2$

42. Required area $= \int_1^3 y\, dx$

$= \int_1^3 |x - 2|\, dx$

$= \int_1^2 -(x - 2)\, dx + \int_2^3 (x - 2)\, dx$

$= \int_1^2 (2 - x)\, dx + \int_2^3 (x - 2)\, dx$

$$= \left[2x - \frac{x^2}{2}\right]_1^2 + \left[\frac{x^2}{2} - 2x\right]_2^3$$

$$= (4-2) - \left(2 - \frac{1}{2}\right) + \left(\frac{9}{2} - 6\right) - (2-4)$$

$$= 2 - \frac{3}{2} - \frac{3}{2} + 2$$

$$= 4 - 3 = 1$$

43. The equation of the family of curves is

$$x^2 + y^2 - 2ay = 0 \quad \text{...(i)}$$

On differentiating w.r.t. to x

$$2x + 2yy' - 2ay' = 0$$

$$\Rightarrow \quad 2x + 2yy' = 2ay'$$

$$\Rightarrow \quad \frac{2x + 2yy'}{y'} = 2a \quad \text{...(ii)}$$

From Eq. (i)

$$2a = \frac{x^2 + y^2}{y}$$

On putting this value in Eq. (ii)

$$\frac{2x + 2yy'}{y'} = \frac{x^2 + y^2}{y}$$

$$\Rightarrow \quad 2xy + 2y^2y' = x^2y' + y^2y'$$

$$\Rightarrow \quad (x^2 - y^2)y' = 2xy$$

44. $y\,dx + (x + x^2y)\,dy = 0$

$$\Rightarrow \quad y\,dx + x\,dy = -x^2y\,dy$$

$$\Rightarrow \quad \frac{y\,dx + x\,dy}{x^2y^2} = -\frac{1}{y}\,dy$$

$$\Rightarrow \quad d\left(-\frac{1}{xy}\right) = -\frac{1}{y}\,dy$$

On integrating, we get

$$-\frac{1}{xy} = -\log y + c$$

$$\Rightarrow \quad -\frac{1}{xy} + \log y = c$$

45. **Key Idea :** *If $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ be the vertices of a triangle, then the co-ordinates of the centroid of a triangle are $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.*

Let (x, y) be the co-ordinates of vertex C and (x_1, y_1) be the co-ordinates of centroid of the triangle.

$$\therefore \quad x_1 = \frac{x + 2 - 2}{3} \text{ and } y_1 = \frac{y - 3 + 1}{3}$$

$$\Rightarrow \quad x_1 = \frac{x}{3} \text{ and } y_1 = \frac{y - 2}{3}$$

Since the centroid lies on the line $2x + 3y = 1$. So, x_1 and y_1 satisfied the equation of line.

$$\therefore \quad 2x_1 + 3y_1 = 1$$

$$\Rightarrow \quad \frac{2x}{3} + \frac{3(y-2)}{3} = 1$$

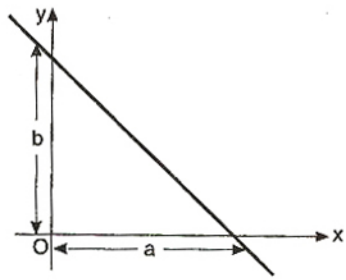
$$\Rightarrow \quad 2x + 3y - 6 = 3$$

$$\Rightarrow \quad 2x + 3y = 9$$

This equation is locus of the vertex C.

46. **Key Idea :** *If a and b are intercepts on the x-axis and y-axis respectively, then the equation of the line is $\frac{x}{a} + \frac{y}{b} = 1$.*

Let a and b be intercepts on the co-ordinate axes.



$$\therefore \quad a + b = -1$$

$$\Rightarrow \quad b = -a - 1 = -(a+1)$$

Equation of line is $\frac{x}{a} + \frac{y}{b} = 1$

$$\Rightarrow \quad \frac{x}{a} - \frac{y}{a+1} = 1 \quad \text{...(i)}$$

Since, this line passes through (4, 3).

$$\therefore \quad \frac{4}{a} - \frac{3}{a+1} = 1 \Rightarrow \frac{4a + 4 - 3a}{a(a+1)} = 1$$

$$\Rightarrow \quad a + 4 = a^2 + a$$

$$\Rightarrow \quad a^2 = 4 \Rightarrow a = \pm 2$$

$\therefore$ Equation of line is

$$\frac{x}{2} - \frac{y}{3} = 1 \text{ or } \frac{x}{-2} + \frac{y}{1} = 1 \quad \text{[from Eq. (i)]}$$

47. **Key Idea :** *Let m_1 and m_2 be the slopes of lines represented by $ax^2 + 2hxy + by^2 = 0$, then*

$$m_1 + m_2 = -\frac{2h}{b} \text{ and } m_1m_2 = \frac{a}{b}.$$

The given pair of lines is $x^2 - 2cxy - 7y^2 = 0$

On comparing with $ax^2 + 2hxy + by^2 = 0$,

we get

$a = 1, 2h = -2c, b = -7$

$$m_1 + m_2 = -\frac{2h}{b} = -\frac{2c}{7} \text{ and } m_1m_2 = \frac{a}{b} = -\frac{1}{7}$$

Given that, $m_1 + m_2 = 4m_1m_2$

$$\Rightarrow \quad -\frac{2c}{7} = -\frac{4}{7} \Rightarrow c = \frac{4}{2} = 2$$

48. The pair of lines is $6x^2 - xy + 4cy^2 = 0$.

On comparing with $ax^2 + 2hxy + by^2 = 0$, we get

$$a = 6,\ 2h = -1,\ b = 4c$$

$$\therefore m_1 + m_2 = -\frac{2h}{b} = \frac{1}{4c} \text{ and } m_1 m_2 = \frac{a}{b} = \frac{6}{4c}.$$

One line of given pair of lines is

$$3x + 4y = 0$$

Slope of line $= -\frac{3}{4} = m_1$ (say)

$$\frac{3}{4} + m_2 = \frac{1}{4c}$$

$$= \frac{1}{4c} + \frac{3}{4}$$

$$\therefore \left(-\frac{3}{4}\right)\left(\frac{1}{4c} + \frac{3}{4}\right) = \frac{6}{4c}$$

$$\Rightarrow -\frac{3}{4}\left(\frac{1 + 3c}{4c}\right) = \frac{6}{4c}$$

$$\Rightarrow 1 + 3c = \frac{-6 \times 4}{3}$$

$$\Rightarrow 1 + 3c = -8$$

$$\Rightarrow 3c = -9 \Rightarrow c = -3$$

Alternate Solution

$\because$ $3x + 4y = 0$ is one of the two lines. Hence $y = -\frac{3x}{4}$ will satisfy the equation $6x^2 - xy + 4cy^2 = 0$

$$\therefore 6x^2 - x\left(\frac{-3x}{4}\right) + 4c\left(\frac{-3x}{4}\right)^2 = 0$$

$$\Rightarrow 6x^2 + \frac{3x^2}{4} + 4c\frac{9x^2}{16} = 0$$

$$\Rightarrow x^2(24 + 3 + 9c) = 0 \Rightarrow 9c = -27$$

$$\Rightarrow c = -3$$

49. **Key Idea :** *If two circles* $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ *and* $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ *cut orthogonally, then*

$$2g_1g_2 + 2f_1f_2 = c_1 + c_2.$$

Let the equation of circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

It cuts the circle $x^2 + y^2 = 4$ orthogonally, if

$$2g \cdot 0 + 2f \cdot 0 = c - 4$$

$$\Rightarrow c = 4$$

$\therefore$ Equation of circle is

$$x^2 + y^2 + 2gx + 2fy + 4 = 0$$

$\because$ It passes through the point (a, b)

$$\therefore a^2 + b^2 + 2ag + 2fb + 4 = 0$$

Locus of centre $(-g, -f)$ will be

$$a^2 + b^2 - 2xa - 2yb + 4 = 0$$

$$\Rightarrow 2ax + 2by - (a^2 + b^2 + 4) = 0$$

Alternate Solution

Let the centre of required circle is $(-g, -f)$. This circle cuts the circle $x^2 + y^2 = 4$ orthogonally. The centre and radius of circle $x^2 + y^2 = 4$ are $(0, 0)$ and 2 respectively.

$$\therefore g^2 + f^2 = 4 + (a + g)^2 + (b + f)^2$$

$$\Rightarrow g^2 + f^2 = 4 + a^2 + g^2 + 2ag + 2bf + b^2 + f^2$$

$$\Rightarrow 4 + a^2 + b^2 + 2ag + 2bf = 0$$

So the locus of centre is

$$2ax + 2by - (a^2 + b^2 + 4) = 0.$$

50. **Key Idea :** *If* $A(x_1, y_1)$ *and* $B(x_2, y_2)$ *be the co-ordinates of end points diameter of a circle, then the equation of circle is*

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0.$$

In a circle AB is as a diameter where the co-ordinates of A are (p, q) and let the co-ordinate of B are (x_1, y_1).

Equation of circle in diameter form is

$$(x - p)(x - x_1) + (y - q)(y - y_1) = 0$$

$$\Rightarrow x^2 - (p + x_1)x + px_1 + y^2 - (y_1 + q)y + qy_1 = 0$$

$$\Rightarrow x^2 - (p + x_1)x + y^2 - (y_1 + q)y + px_1 + qy_1 = 0$$

Since, this circle touches x-axis

$$\therefore y = 0$$

$$\Rightarrow x^2 - (p + x_1)x + px_1 + qy_1 = 0$$

Also the discriminant of above equation will be equal to zero because circle touches x-axis.

$$\therefore (p + x_1)^2 = 4(px_1 + qy_1)$$

$$\Rightarrow p^2 + x_1^2 + 2px_1 = 4px_1 + 4qy_1$$

$$\Rightarrow x_1^2 - 2px_1 + p^2 = 4qy_1$$

Therefore the locus of point B is

$$(x - p)^2 = 4qy.$$

51. The lines $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$ are diameters of circle.

On solving these equations, we get

$$x = 1 \text{ and } y = -1$$

Therefore the centre of circle is $(1, -1)$ and circumference of circle $= 10\pi$

$$\Rightarrow 2\pi r = 10\pi$$

$$\Rightarrow r = 5$$

$\therefore$ Equation of circle is

$$(x - x_1)^2 + (y - y_1)^2 = r^2$$

$$\Rightarrow (x - 1)^2 + (y + 1)^2 = 5^2$$

$\Rightarrow \quad x^2 + 1 - 2x + y^2 + 2y + 1 = 25$

$\Rightarrow \quad x^2 + y^2 - 2x + 2y - 23 = 0$

52. The equation of line is $y = x$ and equation of circle is $x^2 + y^2 - 2x = 0$

On solving Eqs. (i) and (ii), we get

$\Rightarrow x^2 + x^2 - 2x = 0 \Rightarrow 2x^2 - 2x = 0$

$\Rightarrow \quad 2x(x-1) = 0$

$\Rightarrow x = 0, x = 1$ when $x = 0, y = 0$

when $x = 1, y = 1$

Let co-ordinates of A are (0, 0) and co-ordinates of B are (1, 1).

$\therefore$ Equation of circle (AB as a diameter) is

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

$\Rightarrow (x - 0)(x - 1) + (y - 0)(y - 1) = 0$

$\Rightarrow x(x-1) + y(y-1) = 0$

$\Rightarrow x^2 - x + y^2 - y = 0$

$\Rightarrow x^2 + y^2 - x - y = 0$

Alternative Solution :

Equation of a circle passing through the points of intersection of $x^2 + y^2 - 2x = 0$ and $y = x$ is, $(x^2 + y^2 - 2x) + \lambda(x - y) = 0$

i.e., $x^2 + y^2 + (\lambda - 2)x - \lambda y = 0$

Its centre is $\left(\frac{2-\lambda}{2}, \frac{\lambda}{2}\right)$

$\because AB$ is diameter of required circle.

$\therefore \left(\frac{2-\lambda}{2}, \frac{\lambda}{2}\right)$ must lies on $y = x$.

$\Rightarrow \quad \frac{\lambda}{2} = \frac{2-\lambda}{2} \Rightarrow \lambda = 1$

$\therefore$ The equation of required circle is

$$x^2 + y^2 - x - y = 0.$$

53. The equation of parabolas are

$$y^2 = 4ax \text{ and } x^2 = 4ay.$$

On solving these, we get

$$x = 0 \quad \text{and} \quad x = 4a$$

Also $y = 0$ and $y = 4a$.

$\therefore$ The point of intersection of parabolas are $A(0, 0)$ and $B(4a, 4a)$.

Also line $2bx + 3cy + 4d = 0$ passes through A and B.

$\therefore \quad d = 0 \quad$ ...(i)

and $\quad 2b \cdot 4a + 3c \cdot 4a + 4d = 0$

$\Rightarrow \quad 2ab + 3ac + d = 0$

$\Rightarrow \quad a(2b + 3c) = 0 \quad (\because d = 0)$

$\Rightarrow \quad 2b + 3c = 0 \quad$ ...(ii)

On squaring Eqs. (i) and (ii) and then adding, we get

$$d^2 + (2b + 3c)^2 = 0$$

54. Key Idea : *The equation of directrix of an ellipse* $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ *is* $x = \pm \frac{a}{e}$.

Since directrix is $x = 4$, then major axis of an ellipse is along x-axis.

$\therefore \quad \frac{a}{e} = 4 \Rightarrow a = 4e \Rightarrow 4 \times \frac{1}{2}$

$\Rightarrow \quad a = 2 \quad \left(\because e = \frac{1}{2}\right)$

Also we know that $b^2 = a^2(1 - e^2)$

$$b^2 = 4\left(1 - \frac{1}{4}\right) = 4 \times \frac{3}{4}$$

$$b^2 = 3$$

$\therefore$ Equation of ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$

$\Rightarrow \quad 3x^2 + 4y^2 = 12$

55. Key Idea : *A line make angles* α, β *and* γ *with x-axis, y-axis and z-axis respectively, then*

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.$$

A line makes angle θ with x-axis and z-axis and β with y-axis.

$$\therefore l = \cos\theta, m = \cos\beta, n = \cos\theta$$

We know that, $l^2 + m^2 + n^2 = 1$

$\Rightarrow \quad \cos^2\theta + \cos^2\beta + \cos^2\theta = 1$

$\Rightarrow \quad 2\cos^2\theta = 1 - \cos^2\beta$

$\Rightarrow \quad 2\cos^2\theta = \sin^2\beta \quad$ ...(i)

But $\quad \sin^2\beta = 3\sin^2\theta \quad$ ...(ii)

$\therefore$ From Eqs. (i) and (ii)

$$3\sin^2\theta = 2\cos^2\theta$$

$\Rightarrow \quad 3(1 - \cos^2\theta) = 2\cos^2\theta$

$\Rightarrow \quad 3 - 3\cos^2\theta = 2\cos^2\theta$

$\Rightarrow \quad 3 = 5\cos^2\theta$

$\Rightarrow \quad \cos^2\theta = \frac{3}{5}$

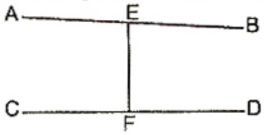
56. Key Idea : *Distance between two parallel planes* $ax + by + cz + d_1 = 0$ *and* $ax + by + cz + d_2 = 0$ *is* $\left|\frac{d_2 - d_1}{\sqrt{a^2 + b^2 + c^2}}\right|$.

The distance between $4x + 2y + 4z - 16 = 0$

and $4x + 2y + 4z + 5 = 0$ is $\left|\frac{5+16}{\sqrt{16+4+16}}\right|$

$$= \left|\frac{21}{\sqrt{36}}\right| = \frac{21}{6} = \frac{7}{2}$$

57. Let the equation of line AB is



$$\frac{x-0}{1} = \frac{y+a}{1} = \frac{z-0}{1} = k \text{ (say)}$$

$\therefore$ Co-ordinates of E are $(k, k-a, k)$.

Also the equation of other line CD is

$$\frac{x+a}{2} = \frac{y-0}{1} = \frac{z-0}{1} = \lambda \text{ (say)}$$

$\therefore$ Co-ordinates of F are $(2\lambda - a, \lambda, \lambda)$

Direction Ratio of EF are $(k - 2\lambda + a)$, $(k - \lambda - a)$, $(k - \lambda)$.

$$\therefore \quad \frac{k-2\lambda+a}{2} = \frac{k-\lambda-a}{1} = \frac{k-\lambda}{2}$$

On solving first and second fraction.

$$\frac{k-2\lambda+a}{2} = \frac{k-\lambda-a}{1}$$

$$\Rightarrow \quad k - 2\lambda + a = 2k - 2\lambda - 2a$$

$$\Rightarrow \quad k = 3a$$

On solving second and third fraction

$$\frac{k-\lambda-a}{1} = \frac{k-\lambda}{2}$$

$$\Rightarrow \quad 2k - 2\lambda - 2a = k - \lambda$$

$$\Rightarrow \quad k - \lambda = 2a$$

$$\Rightarrow \quad \lambda = k - 2a = 3a - 2a$$

$$\Rightarrow \quad \lambda = a$$

$\therefore$ Co-ordinates of $E = (3a, 2a, 3a)$

and co-ordinates of $F = (a, a, a)$.

58. The given straight line is

$$x = 1 + s,\ y = -3 - \lambda s,\ z = 1 + \lambda s$$

or $$\frac{x-1}{1} = \frac{y+3}{-\lambda} = \frac{z-1}{\lambda} = s$$

Also given equation of another straight line is

$$x = \frac{t}{2},\ y = 1 + t,\ z = 2 - t$$

or $$\frac{x-0}{1} = \frac{y-1}{2} = \frac{z-2}{-2} = t$$

These two lines are coplanar, if

$$\begin{vmatrix} 1-0 & -3-1 & 1-2 \\ 1 & -\lambda & \lambda \\ 1 & 2 & -2 \end{vmatrix} = 0$$

$$\Rightarrow \quad \begin{vmatrix} 1 & -4 & -1 \\ 1 & -\lambda & \lambda \\ 1 & 2 & -2 \end{vmatrix} = 0$$

$$\Rightarrow 1\begin{vmatrix} -\lambda & \lambda \\ 2 & -2 \end{vmatrix} + 4\begin{vmatrix} 1 & \lambda \\ 1 & -2 \end{vmatrix} - 1\begin{vmatrix} 1 & -\lambda \\ 1 & 2 \end{vmatrix} = 0$$

$$\Rightarrow (2\lambda - 2\lambda) + 4(-2 - \lambda) - 1(2 + \lambda) = 0$$

$$\Rightarrow \quad -8 - 4\lambda - 2 - \lambda = 0$$

$$\Rightarrow \quad -10 = 5\lambda \Rightarrow \lambda = -2$$

59. **Key Idea :** *Equation of plane of intersection of two spheres S and S' is $S - S' = 0$.*

Equation of two spheres are

$x^2 + y^2 + z^2 + 7x - 2y - z - 13 = 0$ and

$x^2 + y^2 + z^2 - 3x + 3y + 4z - 8 = 0$. If these sphere intersect, then $S - S' = 0$ represents the equation of common plane of intersection.

$$\therefore \quad (x^2 + y^2 + z^2 + 7x - 2y - z - 13)$$
$$- (x^2 + y^2 + z^2 - 3x + 3y + 4z - 8) = 0$$

$$\Rightarrow x^2 + y^2 + z^2 + 7x - 2y - z - 13 - x^2$$
$$- y^2 - z^2 + 3x - 3y - 4z + 8 = 0$$

$$\Rightarrow \quad 10x - 5y - 5z - 5 = 0$$

$$\Rightarrow \quad 2x - y - z = 1$$

60. If $\vec{a} + 2\vec{b}$ is collinear with $\vec{c}$, then

$$\vec{a} + 2\vec{b} = t\,\vec{c} \quad \text{...(i)}$$

Also, if $\vec{b} + 3\vec{c}$ is collinear with $\vec{a}$, then

$$\vec{b} + 3\vec{c} = \lambda\,\vec{a} \quad \text{...(ii)}$$

$$\Rightarrow \quad \vec{b} = \lambda\,\vec{a} - 3\vec{c}$$

On putting this value in Eq. (i)

$$\vec{a} + 2(\lambda\,\vec{a} - 3\vec{c}) = t\,\vec{c}$$

$$\Rightarrow \quad \vec{a} + 2\lambda\,\vec{a} - 6\,\vec{c} = t\,\vec{c}$$

$$\Rightarrow \quad (\vec{a} - 6\,\vec{c}) = t\,\vec{c} - 2\lambda\,\vec{a}$$

On comparing, we get

$$1 = -2\lambda \Rightarrow \lambda = -\frac{1}{2}$$

and $$-6 = t \Rightarrow t = -6$$

From Eq. (i)

$$\vec{a} + 2\vec{b} = -6\,\vec{c}$$

$$\Rightarrow \quad \vec{a} + 2\vec{b} + 6\,\vec{c} = 0$$

61. Total force, $\vec{F} = (4\hat{i} + \hat{j} - 3\hat{k}) + (3\hat{i} + \hat{j} - \hat{k})$

$$\vec{F} = 7\hat{i} + 2\hat{j} - 4\hat{k}$$

The particle is displaced from

$A(\hat{i} + 2\hat{j} + 3\hat{k})$ to $B(5\hat{i} + 4\hat{j} + \hat{k})$.

∴ Displacement

$$\vec{AB} = (5\hat{i} + 4\hat{j} + \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$$
$$= 4\hat{i} + 2\hat{j} - 2\hat{k}$$

Work done $= \vec{F} \cdot \vec{AB}$

$$= (7\hat{i} + 2\hat{j} - 4\hat{k}) \cdot (4\hat{i} + 2\hat{j} - 2\hat{k})$$
$$= 28 + 4 + 8 = 40 \text{ unit.}$$

62. Key Idea : *If* $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ *and* $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ *are coplanar, then*

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0.$$

The three vectors $(\vec{a} + 2\vec{b} + 3\vec{c}), (\lambda\vec{b} + 4\vec{c})$ and $(2\lambda - 1)\vec{c}$ are coplanar, if

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & \lambda & 4 \\ 0 & 0 & 2\lambda - 1 \end{vmatrix} = 0$$

$\Rightarrow \quad (2\lambda - 1)(\lambda) = 0$

$\Rightarrow \quad \lambda = 0, \frac{1}{2}$

∴ These three vectors are non-coplanar for all except two values of $\lambda \left(i.e., \; 0, \frac{1}{2}\right)$.

63. $|\vec{u}| = 1, |\vec{v}| = 2, |\vec{w}| = 3$

The projection of $\vec{v}$ along $\vec{u} = \dfrac{\vec{v} \cdot \vec{u}}{|\vec{u}|}$

and the projection of $\vec{w}$ along $\vec{u} = \dfrac{\vec{w} \cdot \vec{u}}{|\vec{u}|}$

So, $\dfrac{\vec{v} \cdot \vec{u}}{|\vec{u}|} = \dfrac{\vec{w} \cdot \vec{u}}{|\vec{u}|}$

$\Rightarrow \quad \vec{v} \cdot \vec{u} = \vec{w} \cdot \vec{u}$

and $\vec{v}, \vec{w}$ are perpendicular to each other

∴ $\quad \vec{v} \cdot \vec{w} = 0$

Now, $|\vec{u} - \vec{v} + \vec{w}|^2 = |\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 - 2\vec{u} \cdot \vec{v} + 2\vec{u} \cdot \vec{w} - 2\vec{v} \cdot \vec{w}$

$\Rightarrow \quad |\vec{u} - \vec{v} + \vec{w}|^2 = 1 + 4 + 9 - 2\vec{u} \cdot \vec{v} + 2\vec{v} \cdot \vec{u}$

$\Rightarrow \quad |\vec{u} - \vec{v} + \vec{w}|^2 = 1 + 4 + 9$

$\Rightarrow \quad |\vec{u} - \vec{v} + \vec{w}| = \sqrt{14}$

64. Since, $\frac{1}{3}|\vec{b}||\vec{c}| \; \vec{a} = (\vec{a} \times \vec{b}) \times \vec{c}$

We know that

$$(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a}$$

∴ $\frac{1}{3}|\vec{b}||\vec{c}| \; \vec{a} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a}$

On comparing, we get

$$\frac{1}{3}|\vec{b}||\vec{c}| = -\vec{b} \cdot \vec{c} \text{ and } \vec{a} \cdot \vec{c} = 0$$

$\Rightarrow \frac{1}{3} bc = -bc \cos\theta \Rightarrow \quad \cos\theta = -\frac{1}{3}$

$\Rightarrow \cos^2\theta = \frac{1}{9} \quad \Rightarrow 1 - \sin^2\theta = \frac{1}{9}$

$\Rightarrow \sin^2\theta = 1 - \frac{1}{9} = \frac{8}{9}$

$\Rightarrow \quad \sin\theta = \frac{2\sqrt{2}}{3} \qquad (\because \; 0 \le \theta \le \pi)$

65. In the given statements only first and second statements are correct.

66. In the $2n$ observations, half of them equal to a and remaining half equal to $-a$. Then the mean of total $2n$ observations is equal to zero.

∴ $\quad SD = \sqrt{\dfrac{\Sigma(x - \bar{x})^2}{N}}, \quad 2 = \sqrt{\dfrac{\Sigma x^2}{2n}}$

$\Rightarrow \quad 4 = \dfrac{\Sigma x^2}{2n} \Rightarrow 4 = \dfrac{2na^2}{2n}$

$\Rightarrow \quad a^2 = 4 \quad \Rightarrow |a| = 2$

67. The probability of speaking truth by A, $P(A) = \frac{4}{5}$. The probability of not speaking truth by A, $P(\bar{A}) = 1 - \frac{4}{5} = \frac{1}{5}$. The probability of speaking truth by B, $P(B) = \frac{3}{4}$.

The probability of not speaking truth of B, $P(\bar{B}) = \frac{1}{4}$.

The probability that they contradict each other

$$= P(A) \times P(\bar{B}) + P(\bar{A}) \times P(B)$$
$$= \frac{4}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{3}{4}$$
$$= \frac{1}{5} + \frac{3}{20} = \frac{7}{20}$$

68. $E = \{X \text{ is a prime number}\} = \{2, 3, 5, 7\}$

$$P(E) = P(X = 2) + P(X = 3) + P(X = 5) + P(X = 7)$$

$$P(E) = 0.23 + 0.12 + 0.20 + 0.07 = 0.62$$

and $F = \{X < 4\} = \{1, 2, 3\}$

$$P(F) = P(X = 1) + P(X = 2) + P(X = 3)$$

$$P(F) = 0.15 + 0.23 + 0.12 = 0.5$$

$E \cap F = \{X$ is prime number as well as $< 4\}$ $= \{2, 3\}$

$$P(E \cap F) = P(X = 2) + P(X = 3) = 0.23 + 0.12 = 0.35$$

$\therefore$ Required probability

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$P(E \cup F) = 0.62 + 0.5 - 0.35$$

$$P(E \cup F) = 0.77$$

69. Given that mean $= 4, \Rightarrow np = 4$

and variance $= 2$

$$npq = 2 \Rightarrow 4q = 2$$

$$\Rightarrow \quad q = \frac{1}{2}$$

$$\therefore p = 1 - q = 1 - \frac{1}{2} = \frac{1}{2}$$

Also $n = 8$

Probability of 2 successes $= P(X = 2) = {}^8C_2\, p^2 q^6$

$$= \frac{8!}{2! \times 6!} \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^6 = 28 \times \frac{1}{2^8}$$

$$= \frac{28}{256}$$

70. Let P and Q are forces. We know that

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

When $\theta = 0°$, $R = 4$ N

$$R = 4\text{ N} = \sqrt{P^2 + Q^2 + 2PQ}$$

$$P + Q = 4 \qquad \text{...(i)}$$

When $\theta = 90°$, $R = 3$ N

$$P^2 + Q^2 = 9 \qquad \text{...(ii)}$$

From Eq. (i)

$$(P + Q)^2 = 16$$

$$\Rightarrow \quad P^2 + Q^2 + 2PQ = 16$$

$$\Rightarrow \quad 9 + 2PQ = 16 \qquad \text{[using (ii)]}$$

$$\Rightarrow \quad 2PQ = 7$$

Now, $(P - Q)^2 = P^2 + Q^2 - 2PQ$

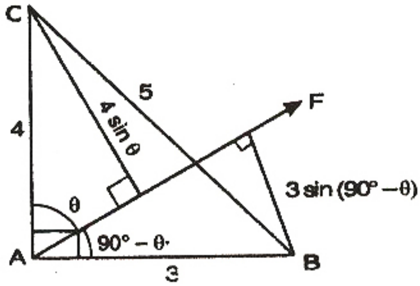
$$\Rightarrow \quad (P - Q)^2 = 9 - 7$$

$$P - Q = \sqrt{2} \qquad \text{...(iii)}$$

On solving Eqs. (i) and (iii)

$$P = \left(2 + \frac{1}{2}\sqrt{2}\right)\text{ N and } Q = \left(2 - \frac{1}{2}\sqrt{2}\right)\text{ N}$$

71. Moment about A of force $\vec{F} = 0$



$\Rightarrow$ Force $\vec{F}$ passes through vertex A.

Let $\vec{F}$ makes angle θ with AC and $90° - \theta$ with AB.

Moment about B of force $\vec{F} = 9$

$$\Rightarrow \quad F \cdot 3\cos\theta = 9 \Rightarrow F\cos\theta = 3 \qquad \text{...(i)}$$

Moment about C of force $\vec{F} = 16$

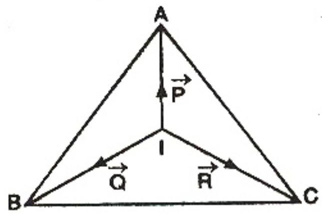
$$F \cdot 4\sin\theta = 16 \Rightarrow F \cdot \sin\theta = 4 \qquad \text{...(ii)}$$

On squaring Eqs. (i) and (ii) and then adding

$$F^2 = 3^2 + 4^2$$

$$F = 5$$

72. Three forces $\vec{P}$, $\vec{Q}$ and $\vec{R}$ acting along IA, IB and IC are in equilibrium :



$$\angle AIB = \pi - \frac{\angle A + \angle B}{2} = \pi - \left(\frac{\pi}{2} - \frac{C}{2}\right) = \frac{\pi}{2} + \frac{C}{2}$$

Similarly, $\angle BIC = \frac{\pi}{2} + \frac{A}{2}$

and $\angle AIC = \frac{\pi}{2} + \frac{B}{2}$

By Lami's theorem

$$\frac{P}{\sin\angle BIC} = \frac{Q}{\sin\angle AIC} = \frac{R}{\sin\angle AIB}$$

$$\Rightarrow \frac{P}{\sin(\pi/2 + A/2)} = \frac{Q}{\sin(\pi/2 + B/2)} = \frac{R}{\sin(\pi/2 + C/2)}$$

$$\Rightarrow \frac{P}{\cos A/2} = \frac{Q}{\cos B/2} = \frac{R}{\cos C/2} = \lambda \text{ (say)}$$

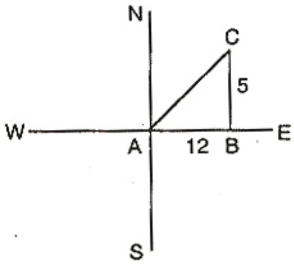
$$\Rightarrow P = \lambda \cos A/2, Q = \lambda \cos B/2, R = \lambda \cos C/2$$

$$\therefore \quad P : Q : R = \cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}$$

73. Given $AB = 12$ km (s_1)

and $BC = 5$ km (s_2)

Speed from A to $B = 4$ km/h



Time taken, $t_1 = \frac{12}{4} = 3$ h.

Speed from B to $C = 5$ km/h

Time taken to complete distance from B to C

$$t_2 = \frac{5}{5} = 1 \text{ h}$$

Average speed

$$= \frac{\text{total distance}}{\text{total time}} = \frac{s_1 + s_2}{t_1 + t_2} = \frac{12 + 5}{3 + 1}$$

$$= \frac{17}{4} \text{ km/h}$$

$$AC = \sqrt{(AB)^2 + (BC)^2}$$

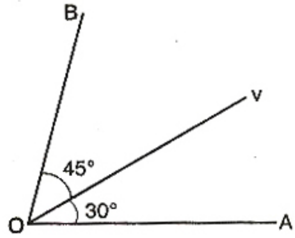
$$= \sqrt{144 + 25} = \sqrt{169}, \; AC = 13 \text{ km}$$

$$\text{Average velocity} = \frac{\text{distance } AC}{\text{total time}}$$

$$= \frac{13}{4} \text{ km/h}$$

74. $v = \frac{1}{4}$ m/s^2

Component of v along OB



$$= \frac{\frac{1}{4}\sin 30^\circ}{\sin(30^\circ + 45^\circ)} = \frac{\frac{1}{4}\sin 30^\circ}{\sin 75^\circ}$$

$$= \frac{\frac{1}{4}\cdot\frac{1}{2}}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = \frac{\sqrt{2}}{4(\sqrt{3}+1)} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{\sqrt{2}(\sqrt{3}-1)}{4(3-1)} = \frac{1}{8}(\sqrt{6} - \sqrt{2}) \text{ m/s}$$

75. If two particles having same initial velocity u and range R, then their direction must be opposite *i. e.*, the direction of projection of them are α and $90^\circ - \alpha$.

$$\therefore \; t_1 = \frac{2u \sin\alpha}{g} \text{ and } t_2 = \frac{2u \sin(90^\circ - \alpha)}{g},$$

$$\Rightarrow \qquad t_2 = \frac{2u\cos\alpha}{g}$$

$$\text{Now, } t_1^2 + t_2^2 = \frac{(2u\sin\alpha)^2}{g^2} + \frac{(2u\cos\alpha)^2}{g^2}$$

$$= \frac{4u^2}{g^2}(\sin^2\alpha + \cos^2\alpha) = \frac{4u^2}{g^2}$$