

Percentage

A fraction with its denominator as '100' is called a **percentage**. Percentage means per hundred.

So it is a fraction of the form

$\frac{6}{100}$, $\frac{37}{100}$ and $\frac{151}{100}$ and these fractions can be expressed as 6%, 37% and 151% respectively.

In such a fraction, the numerator is called rate percent.

To express % as a fraction or a decimal, divide x by 100.

If the price of an item increases by r%, then the reduction in consumption, so that the expenditure remains the same is

$$\left(\frac{r}{r+100}\right) \times 100\%$$

If the price of the commodity decreases by r%, the increase in consumption, so that the expenditure remains the same is

$$\left(\frac{r}{100-r}\right) \times 100\%$$

If the value is first increased by % and then by y%, the final increase is

$$\left(x + y + \frac{xy}{100}\right)\%$$

If there is a decrease instead of increase, a negative sign is attached to the corresponding rate percent.

If the value of a number is first increased by % and later it is decreased by % then net change is always a decrease which is equal to

$$\left(\frac{x^2}{100}\right)\%$$

If pass marks in an examination is % and

if a student secures y marks and fails by z marks, then the maximum mark

$$= \frac{100(y+z)}{x}$$

A candidate scores x% in an examination fails by 'a' marks while another candidate who scores y% gets 'b' marks more than the minimum required for a pass, then the

$$\text{maximum mark} = \frac{100(a+b)}{y-x}$$

If the length of a rectangle is increased by x% and the breadth is decreased by y%, then the area is increased or decreased by

$\left(x - y - \frac{xy}{100}\right)\%$ according to the (+) ve or (-) ve sign obtained.

If the present population is P which increases R% annually, then

(i) the population after n years

$$= P \left(\frac{100+R}{100}\right)^n$$

(ii) the population n years ago

$$= P \left(\frac{100}{100+R}\right)^n$$

If the present value of a machine is P which depreciates at R% per annum, then

(i) the value of the machine after n years

$$= P \left(\frac{100-R}{100}\right)^n$$

(ii) the value of the machine n years ago

$$= P \left(\frac{100}{100-R}\right)^n$$

If $x\%$ students failed in a particular subject, $y\%$ students failed in another subject, and $z\%$ students failed in both subjects, then the pass present = $100+z-(x+y)$

Fractional Equivalentents of important percents

| | | | |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $1\% = \frac{1}{100}$ | $2\% = \frac{1}{50}$ | $4\% = \frac{1}{25}$ | $5\% = \frac{1}{20}$ |
| $40\% = \frac{2}{5}$ | $60\% = \frac{3}{5}$ | $80\% = \frac{4}{5}$ | $100\% = 1$ |
| $6\frac{1}{4}\% = \frac{1}{16}$ | $12\frac{1}{2}\% = \frac{1}{8}$ | $25\% = \frac{1}{4}$ | $37\frac{1}{2}\% = \frac{3}{8}$ |
| $8\frac{1}{3}\% = \frac{1}{12}$ | $16\frac{2}{3}\% = \frac{1}{6}$ | $33\frac{1}{3}\% = \frac{1}{3}$ | $66\frac{2}{3}\% = \frac{2}{3}$ |

| | | |
|---------------------------------|---------------------------------|----------------------------------|
| $8\% = \frac{2}{25}$ | $10\% = \frac{1}{10}$ | $20\% = \frac{1}{5}$ |
| $50\% = \frac{1}{2}$ | $62\frac{1}{2}\% = \frac{5}{8}$ | $75\% = \frac{3}{4}$ |
| $83\frac{1}{3}\% = \frac{5}{6}$ | $87\frac{1}{2}\% = \frac{7}{8}$ | $133\frac{1}{3}\% = \frac{4}{3}$ |

SOLVED EXAMPLES:

1. Find $33\frac{1}{3}\%$ of 600

Ans: $33\frac{1}{3}\%$ of 600 = $\frac{1}{3} \times 600 = 200$

2. What percent of 144 is 36?

Ans: Let $x\%$ of 144 = 36

$$(ie) \frac{x}{100} \times 144 = 36$$

$$(ie) x = \frac{36 \times 100}{144} = 25$$

3. 2.5 is 5% of what?

Ans : Let the number be x

$$\therefore 5\% \text{ of } x = 2.5$$

$$\frac{5}{100} \times x = 2.5 \Rightarrow x = 50$$

4 In an examination 36% are pass marks. If an examinee gets 17 marks and fails by 10 marks, what are the maximum marks?

Ans : Pass mark = $(17+10) = 27$

Let maximum marks be x

$$\text{Then } 36\% \text{ of } x = 27 \text{ or } \frac{36}{100} \times x = 27$$

$$x = \frac{27 \times 100}{36} = 75$$

Hence, maximum marks = 75

The answer can be arrived quickly by

$$\text{Maximum marks} = \frac{100(17 + 10)}{36}$$

$$= \frac{100 \times 27}{36} = 75$$

5. Subtracting 40% of a number from the number, we get the result as 30. Find the number.

Ans: Let the number be x

$$\therefore x - \frac{40}{100}x = 30 \text{ (ie) } x \left(1 - \frac{2}{5}\right) = 30$$

$$x = 30 \times \frac{5}{3} = 50$$

6. If the price of sugar be increased by 25%, find by how much percent must its consumption be decreased to keep the expenditure fixed on sugar?

Ans:

Decrease in consumption

$$= \left(\frac{25}{100 + 25}\right) 100\% = \frac{25 \times 100}{125}\% = 20\%$$

7. The salary of a worker was first increased by 10% and thereafter decreased by 5%. What was the effect in his salary?

$$\text{Ans: } \% \text{ effect} = \left(10 - 5 - \frac{10 \times 5}{100}\right)\%$$

\therefore His salary is increased by 4.5% (because the sign is +ve.)

8. The value of a machine depreciates at the rate of 10% per annum. If its present value is Rs. 81,000 what will be its worth after 2 years?

Ans: The value of the machine after

$$2 \text{ years} = \text{Rs. } 81,000 \times \left(1 - \frac{10}{100}\right)^2$$

$$= \text{Rs. } 81000 \times \frac{9}{10} \times \frac{9}{10} = \text{Rs. } 65,610$$

9. Due to fall of 10% in the rate of sugar, 500 gm more sugar can be purchased for Rs. 140. Find the original rate and reduced rate.

Ans : Money spent originally = Rs. 140

$$\begin{aligned} \text{Less Money to be spent now} \\ &= 10\% \text{ of } 140 \\ &= \text{Rs. } 14 \end{aligned}$$

\therefore Rs. 14 now yield 500gm sugar

\therefore Present rate of sugar = Rs. 28 per kg.

If the present value is Rs. 90, the original value = Rs. 100

If the present value is Rs. 28 the original value

$$\begin{aligned} &= \text{Rs. } \frac{100}{90} \times 28 \\ &= \text{Rs. } 31.11 \end{aligned}$$

10. In an examination, 42% students failed in History and 52% failed in Geography. If 17% students failed in both subjects, find the percentage of those students who passed in both the subjects.

Ans:- Pass percent = $100 + 17 - (42 + 52)$

$$\begin{aligned} &= 117 - 94 \\ &= 23 \end{aligned}$$

PRACTICE TEST

- 65% of 7 + 35% of 3 = ?% of 56
a) 1 b) 10 c) 50 d) 100
- What is 20% of a number whose 200% is 360?
a) 72 b) 36 c) 52 d) 144

3. What percent of $\frac{4}{7}$ is $\frac{2}{35}$?

- a) 2.5% b) 1000%
c) 25% d) 10%

4. The total income of A and B is Rs. 6000. A spends 60% of his income and B spends 80% of his income. If their savings are equal, then the income of A is,

- a) Rs. 3500 b) Rs. 2000
c) Rs. 4000 d) Rs. 3000

5. With an increase of Rs. 2,000, Vishnu's monthly salary became Rs. 12,000. What is the percent increase in his salary?

- a) 20 b) 25 c) 40 d) 80

6. If 75% of the students in a school are boys and the number of girls is 420, the number of boys is

- a) 1176 b) 1350
c) 1260 d) 1125

7. The salary of a worker is first increased by 10% and thereafter it was reduced by 10%. What was the change in his salary?

- a) 1% increase b) 5% increase
c) no change d) 1% decrease

8. A water tank contains 5% salt by weight. x litres of fresh water is added to 40 litres of tank water, so that the solution contains 2% salt. The value of x is

- a) 40 b) 50 c) 55 d) 60

9. The population of a town increases 5% annually. If it is 15,435 now, what was it 2 years ago?

- a) 14,000 b) 13,473
c) 12,345 d) 10,145

10. Navin spends 15% of his salary on cloths, 30% on food and 10% on transport. After this if he is left with Rs. 900/- what is his salary?

- a) Rs. 1,500 b) Rs. 2000
c) Rs. 1,635 d) Rs. 2500

11. When the price of an article was reduced by 15% the sale of the article is increased by

- 20%. What was the effect on the sales?
 a) 2% increase b) 1% increase
 c) 2% decrease d) 1% decrease
12. In an election between two candidates, the one gets 35% of the votes polled is defeated by 15000 votes. The number of votes casted by the winning candidate is
 a) 15,000 b) 1,75,000
 c) 32,500 d) 52,500
13. In an examination, 70% students passed in English and 75% in Hindi while 20% failed in both the subjects. If 260 students passed in both the subjects, the total number of students is
 a) 400 b) 500
 c) 340 d) 460
14. If the radius of a circle is diminished by 10%, the area is diminished by
 a) 36% b) 20%
 c) 19% d) 10%
15. The price of an article is cut by 10%. In order to restore it to its former value, the new price must be increased by
 a) $10\frac{1}{3}\%$ b) 11%
 c) $11\frac{1}{9}\%$ d) $12\frac{1}{9}\%$
16. The breadth of a rectangular field is 60% of its length. If the perimeter of the field is 800m, What is the area of the field?
 a) 37,500 sq.m. b) 4,800 sq.m
 c) 18,750 sq.m d) 40,000 sq.m
17. In a factory, 60% of the employees are males. Among them 20% are matriculates and the remaining are graduates. Among the females 40% are matriculates and the remaining are graduates. If the total number of female employees in the factory is 640, how many graduates are there in the factory?
 a) 1024 b) 896
 c) 1,152 d) 1000
18. In an employment exchange, 40% of the job seekers are graduates, 20% are post-graduates and remaining 6000 are non-graduates. How many post-graduate job seekers are there?
 a) 3,000 b) 6,000
 c) 9,000 d) 12,000
19. A company hired a salesman on a monthly salary of Rs. 3,000. In addition to it, the salesman was entitled for 20% commission on the monthly sale. How much sale the salesman should do if he wants his monthly income as Rs. 10,000?
 a) Rs. 50,000 b) Rs. 15,000
 c) Rs. 35,000 d) Rs. 21,000
20. In a public sector company, 30% employees opted for pension and 50% employees opted for provident fund. The remaining employees were uncertain. If the difference between those who opted for provident fund and those who were uncertain was 1440, how many employees were there in the company?
 a) 7,200 b) 2,400
 c) 2,880 d) 4,800
21. Prasanna spends 25% of her monthly income on petrol for her car, $\frac{2}{3}$ rd of the remaining income on house hold items, rent, etc. If she is left with Rs. 1,800 with her at the end of the month how much does she spend on petrol?
 a) Rs. 1,800 b) Rs. 720
 c) Rs. 2,500 d) Rs. 1,440
22. Rajesh earns Rs. 2,300 per month. He spends Rs. 1,200 on food, Rs. 630 on conveyance, 10% of his monthly income on other incidentals and saves the remaining amount. How much money will he save in one year?
 a) Rs. 2300 b) Rs. 2880
 c) Rs. 2600 d) Rs. 2400
23. In an examination, Hari got 8 marks less than 80% of the full marks and Ravi got 5 marks more than 70% of the full marks. Hari beats Ravi by 2 marks. The marks scored by Ravi is
 a) 90 b) 110 c) 130 d) 140

ANSWERS TO PRACTICE

1. (b) 2. (b) 3. (d) 4. (b) 5. (a) 6. (c) 7. (d) 8.(d)
 9. (a) 10. (b) 11. (a) 12. (c) 13. (a) 14. (c) 15. (c) 16.(a)
 17. (c) 18. (a) 19. (c) 20. (d) 21.(a) 22. (b) 23. (b)

Profit & Loss

Cost Price: The price for which an article is purchased is called the Cost Price (C.P.)

Selling price : The price at which an article is sold is called the Selling Price (S.P.)

Profit (Gain) : The difference between the selling price and the cost price (when S.P. is more than C.P) is called the Profit.

Loss:The difference between the cost price and selling price (when C.P. is more than S.P.) is called the Loss.

Points to remember:

1. $\text{Gain} = (\text{S.P}) - (\text{C.P}); \text{Loss} = (\text{C.P}) - (\text{S.P})$

2. $\text{Gain}\% = \frac{\text{Gain} \times 100}{\text{C.P.}} = \left(\frac{\text{SP} - \text{CP}}{\text{CP}} \right) \times 100$

$\text{Loss}\% = \frac{\text{Loss} \times 100}{\text{C.P.}} = \left(\frac{\text{CP} - \text{SP}}{\text{CP}} \right) \times 100$

3. When the cost price and gain percent are given,

$$\text{S.P.} = \text{C.P.} \times \left(\frac{100 + \text{Gain}\%}{100} \right)$$

4. When the cost price and loss percent are given

$$\text{S.P.} = \text{C.P.} \times \left(\frac{100 - \text{Loss}\%}{100} \right)$$

5. When the selling price and gain percent are given

$$\text{C.P.} = \text{S.P.} \times \left(\frac{100}{100 + \text{Gain}\%} \right)$$

6. When the selling price and loss percent are given

$$\text{C.P.} = \text{S.P.} \times \frac{100}{(100 - \text{Loss}\%)}$$

7. The discount percent is calculated on the marked price.

Discount percent

$$= \frac{\text{Discount}}{\text{Marked price}} \times 100$$

8. If there are two successive profits of $x\%$ and $y\%$ in a transaction then the resultant profit percent is

$$x + y + \frac{xy}{100}$$

9. If there is a profit of $x\%$ and loss of $y\%$ in a transaction, then the resultant profit and loss percent is

$$x - y - \frac{xy}{100} \text{ according to the (+)ve and the (-)ve signs respectively.}$$

10. If cost price of x articles is equal to the selling price of y articles, then profit percent.

$$= \left(\frac{x - y}{y} \right) \times 100$$

SOLVED EXAMPLES

1. A man buys a toy for Rs. 25 and sells it for Rs. 30. Find his gain percent.

Ans: $\text{Gain} = 30 - 25 = \text{Rs. } 5$

$$\text{Gain \%} = \frac{5}{25} \times 100 = \text{Rs. } 20\%$$

2. By selling a watch for Rs. 144 a man loses 10%. At what price should he sell it in order to gain 10%?

Ans: $\text{S.P.} = \text{Rs. } 144; \text{Loss} = 10\%$

$$\therefore \text{C.P.} = \text{Rs} \left(\frac{100}{100 - 10} \right) \times 144$$

$$= \text{Rs. } \frac{100}{90} \times 144 = \text{Rs. } 160$$

Gain required = 15%

$$\therefore \text{S.P.} = \text{Rs. } \frac{115}{100} \times 160 = \text{Rs. } 184$$

Short cut:

$$\text{Required SP} = \frac{144}{90} \times 115 = \text{Rs. } 184$$

3. I sold a book at a profit of 16%. Had I sold it for Rs. 18 more, 20% would have been gained. Find the cost price.

Ans: Here 120% of C.P. – 116% of C.P.
= Rs. 18

$$\therefore 4\% \text{ of cost} = \text{Rs. } 18$$

$$\therefore \text{C.P.} = \frac{18 \times 100}{4} = \text{Rs. } 450$$

Formula :

$$\text{C.P.} = \frac{\text{More gain} \times 100}{\text{Diff. in percentage profit}}$$

$$= \frac{18 \times 100}{20 - 16} = \text{Rs. } 450$$

4. A shopkeeper mixes two varieties of tea, one costing Rs. 35 per kg. and another at Rs. 45 per kg. in the ratio 3:2. If he sells the mixed variety at Rs. 41.60 per kg. what is his gain or loss percent?

Ans: - C.P. of 5 kg. mix

$$= \text{Rs. } (35 \times 3 + 45 \times 2) = \text{Rs. } 195$$

$$\text{S.P. of 5 kg. mix} = \text{Rs. } (41.60 \times 5) \\ = \text{Rs. } 208$$

$$\text{Gain} = \text{Rs. } (208 - 195) = \text{Rs. } 13$$

$$\text{Gain\%} = \frac{13}{195} \times 100 = 6 \frac{2}{3}\%$$

5. By selling a table for Rs. 40 instead of Rs. 50, 5% more is lost. Find the cost of the table.

Ans : Let the C.P. be Rs. x

Then if S.P.=Rs. 40, loss=Rs. ($x-40$)

If S.P = Rs. 50, loss = Rs. ($x-50$)

$$\therefore (X - 40) - (X - 50) = \frac{5}{100} \times X$$

$$\Rightarrow 10 = \frac{5}{100} \times X \Rightarrow X = \text{Rs. } 200$$

6. A grocer sells rice at a profit of 10% and uses weights which are 20% less than the marked weight. The total gain earned by him will be

Ans: Let us consider a packet of rice marked 1 Kg.

Then its actual weight=80% of 1Kg.=0.8 Kg.

Let C.P. of 1 Kg be Rs. x

Then C.P. of 0.8Kg = Rs. $0.8x$

Now, S.P. = 110% of C.P. of 1 Kg

$$= \frac{110}{100} \times X = \text{Rs. } 1.1X$$

$$\text{Gain \%} = \frac{0.3X}{0.8X} \times 100\% = 37.5\%$$

7. The cost price of 10 articles is equal to the selling price of 9 articles. Find the gain percent.

Ans: Let the cp of 1 article = Rs. 1

Then CP of 9 articles = Rs. 9

CP of 10 articles = Rs. 10

\therefore SP of 9 articles = Rs. 10

$$\text{Gain} = \left(\frac{10-9}{9} \right) 100 = \frac{100}{9} = 11 \frac{1}{9}\%$$

8. A boy buys oranges at Rs. 2 for 3 oranges and sells them at one rupee each. To make a profit of Rs. 10 he must sell:

Ans: Suppose he sells x oranges.

$$\text{CP of } x \text{ oranges} = \text{Rs. } \frac{2}{3}x$$

$$\text{SP of } x \text{ oranges} = \text{Rs. } x$$

$$\text{Profit on } x \text{ oranges} = x - \frac{2x}{3} = \frac{x}{3}$$

$$\therefore \frac{x}{3} = 10 \Rightarrow x = 30$$

9. A man sells two horses for Rs. 4000 each neither losing nor gaining in the deal. If he sold one horse at a gain of 25%, the other horse would be sold at a loss of:

Ans: Let the other horse be sold at % loss

$$\begin{aligned} \text{Then } 25 - x - \frac{25x}{100} &= 0 \\ \Rightarrow x &= 20\% \end{aligned}$$

10. A discount series of 10%, 20% and 40% is equal to a single discount of

Ans: Equivalent single discount

$$= 100 - \frac{60}{100} \times \frac{80}{100} \times 90 = 56.8\%$$

PRACTICE TEST

1. By selling an article for Rs. 100, one gains Rs. 10. Then the gain percent is

- a) 10% b) 9%
c) $11\frac{1}{9}\%$ d) $10\frac{1}{2}\%$

2. A loss of 5% was suffered by selling a plot for Rs. 4,085. The cost price of the plot was:

- a) Rs. 4350 b) Rs. 4259.25
c) Rs. 4200 d) Rs. 4300

3. A dealer sold a mixer for Rs. 420 at a loss of 12.5%. At what price should he have sold it to gain 12.5%?

- a) Rs. 620 b) Rs. 540
c) Rs. 650 d) Rs. 750

4. On selling 33m. of cloth, a person gained an amount equivalent to the S.P. of 11m. of cloth. The profit in the deal is

- a) 50% b) 20%
c) 70% d) 30%

5. There are two consecutive discounts of 35% and 10% on a saree. If a person paid Rs. 1170 for that, then what was the original price of the saree before the discounts?

- a) Rs. 2,000 b) Rs. 1,800
c) Rs. 1,900 d) Rs. 1,700

6. A dishonest dealer claims to sell his goods at cost price, but he uses a weight of 960 gm for the kg weight. His gain percent is

- a) 4% b) $4\frac{1}{6}\%$
c) $2\frac{1}{2}\%$ d) $3\frac{3}{4}\%$

7. By selling a vehicle for Rs. 36,300, a person gains 21% profit. What was his gain in Rupees?

- a) Rs. 3,000 b) Rs. 7,623
c) Rs. 3,600 d) Rs. 6,300

8. Hameed bought a calculator for Rs. 520 and sold it with 15% profit on the price he bought. At what price did he sell the calculator?

- a) Rs. 598/- b) Rs. 542/-
c) Rs. 528/- d) Rs. 780/-

9. Ramesh purchased four old cycles at the rate of Rs. 625 for each. He spent Rs. 175 on each cycle for repairing and painting. At what price should he sell each cycle in order to make 35% profit on the money he spent?

- a) Rs. 1,019 b) Rs. 1,000
c) Rs. 1,080 d) Rs. 844

10. Raghu bought 10kg. of sugar at the rate of Rs. 14 per kg and 15kg. of sugar at the rate of Rs. 16 per kg. He mixed the two varieties and sold the mixture at the rate of Rs. 18 per kg. What was his total gain by doing so?

- a) Rs. 50 b) Rs. 70
c) Rs. 40 d) Rs. 80

11. Ravi buys a radio three-fourth of its value and sells it for 20% more than its value. What will be the profit percent?

- a) 50% b) 40%
c) 60% d) 70%

12. A man buys pencils at 10 for Rs. 3 and sells at 8 for Rs. 3. His gain percent is

- a) 20 b) 25 c) 30 d) 27

13. A fruit seller buys lemons at 2 for a rupee and sells them at 5 for three rupees. His gain percent is
 a) 10% b) 15%
 c) 20% d) 12%
14. Toffee are bought at a rate of 8 for one rupee. To gain 60% they must be sold at
 a) 6 for Re. 1 b) 5 for Re. 1
 c) 9 for Re. 2 d) 24 for Re. 5
15. By selling sugar at Rs. 5.58 per kg. a man loses 7%. To gain 7% it must be sold at the rate of Rs.
 a) 5.62 per kg b) 6.42 per kg
 c) 7.32 per kg d) 6.62 per kg
16. A tradesman's prices are 20% above C.P. He allows his customers some discount on his bill and makes a profit of 8%. The rate of discount is :
 a) 10% b) 12%
 c) 14% d) 16%
17. An article was sold at a loss of 5%. If it were sold for Rs. 30 more, the gain would have been 1.25%. The cost price of the article is
 a) Rs. 488 b) Rs. 480
 c) Rs. 460 d) Rs. 420
18. Anitha sold a painting at a profit of 11%. Had she sold it for Rs.175 more, she would have gained 18%. The C.P of the painting is
 a) Rs. 2250 b) Rs. 2350
 c) Rs. 2500 d) Rs. 2550
19. Pradeep bought a toy with 20% discount on its labelled price. He sold it for Rs. 468 at 4% profit on the labelled price. At what price did he buy the toy ?
 a) Rs. 450 b) Rs. 360
 c) Rs. 390 d) Rs. 380
20. Arun purchased a bag with 25% discount on the labelled price. At what percentage profit on the price he bought should he sell it to make 20% profit on the labelled price?
 a) 60% b) 40%
 c) 45% d) 50%
21. Nimesh bought a cycle for Rs. 1,850. He spent Rs.380 for buying different accessories. Approximately, at what price should he sell the cycle to make 20% profit in the transaction?
 a) Rs. 2900 b) Rs. 2676
 c) Rs. 3000 d) Rs. 3125
22. A man buys an article and sells it at a profit of 20%. If he bought at 20% less and sold it for Rs. 75 less, he would have gained 25%. What is the cost price ?
 a) Rs. 375 b) Rs. 425
 c) Rs. 350 d) Data inadequate
23. A man sells a car to his friend at 10% loss. If the friend sells it for Rs. 54,000 and gains 20%, the C.P. of the car was:
 a) Rs. 25,000 b) Rs. 37,500
 c) Rs. 50,000 d) Rs. 60,000
24. Listed price of an article is Rs. 65. A customer pays Rs. 56.16 for it. He was given two successive discounts. If the first discount is 10% find the second .
 a) 8% b) 6% c) 5% d) 4%
25. A single discount equivalent to a discount series of 20%, 10% and 25% is
 a) 55% b) 54%
 c) 46% d) 42%
26. A trader allows two successive discounts of 20% and 10%. if he sells an article for Rs. 108, then the market price of the article is
 a) Rs. 140 b) Rs. 142
 c) Rs. 148 d) Rs. 150

ANSWERS TO PRACTICE

1. (c) 2. (d) 3. (b) 4. (a) 5. (a) 6. (b) 7. (d) 8.(a)
 9. (c) 10. (b) 11. (c) 12. (b) 13. (c) 14. (b) 15. (b) 16.(a)
 17. (b) 18. (c) 19. (b) 20. (a) 21.(b) 22. (a) 23. (c) 24. (d)
 25. (c) 26. (d)

Average

An average, or an arithmetic mean, is the sum of 'n' different data divided by 'n'

$$\text{Average} = \frac{\text{sum of data}}{\text{No. of data}}$$

$$\text{No. of data} = \frac{\text{sum of data}}{\text{Average}}$$

$$\text{Sum} = \text{Average} \times \text{No. of data}$$

Points to remember:

1. Age of new entrant = New average + No. of old members x change in average
2. Age of one who left = New average - No. of old members x change in average
3. Age of new person = Age of the removed person + No. of members x change in average

In all the above three cases, if there is a decrease in the average, the sign of change in average will be negative.

4. If a certain distance is covered at x km/hr and the same distance is covered by y km/hr, then the average speed during the whole journey is

$$\frac{2xy}{x+y} \text{ km/hr}$$

Examples

1. The average age of 30 boys of a class is equal to 14 years. When the age of the class teacher is included the average becomes 15 years. Find the age of the class teacher.

$$\text{Total age of 30 boys} = 14 \times 30 = 420 \text{ years}$$

$$\text{Total age when the teacher is included}$$

$$= 15 \times 31 = 465 \text{ years}$$

$$\therefore \text{Age of the class teacher}$$

$$= 465 - 420 = 45 \text{ years}$$

Direct Formula

$$\begin{aligned} \text{Age of new entrant} &= \text{New average} + \text{No.} \\ &\text{of old members} \times \text{change in average} \\ &= 15 + 30(15-14) = 45 \text{ years.} \end{aligned}$$

2. The average weight of 8 men is increased by 1.5 g. when one of the men who weighs 65 kg is replaced by a new man. The weight of the new man is:

$$\begin{aligned} \text{Weight of the new man} &= \text{Weight of the man} \\ &\text{replaced} + (\text{Number} \times \text{change in average}) \\ &= 65 + (8 \times 1.5) = 65 + 12 = 77 \text{ kg.} \end{aligned}$$

3. The average of 11 results is 50. If the average of first six results is 49 and that of last six is 52, find the sixth result.

$$\text{The sum of 11 results} = 11 \times 50 = 550$$

$$\text{The sum of first 6 results} = 6 \times 49 = 294$$

$$\text{The sum of last 6 results} = 6 \times 52 = 312$$

$$\text{Sixth results} = 294 + 312 - 550 = 56$$

4. There were 35 students in a hostel. If the number of students increased by 7, the expenses of the mess were increased by Rs. 42 per day, while the average expenditure per head diminished by Re. 1. The original expenditure of the mess was:

Ans: Let the original expenditure per head be Rs. x

$$\text{Then } 35x + 42 = (x+1) 42$$

$$35x + 42 = 42x - 42 \text{ or } x = 12$$

5. The average expenditure of a man for the first five months was Rs. 120 and for the next seven months is Rs. 130. What was his monthly average income if he saved Rs. 290 in that years.

$$\text{Total income for 12 months.}$$

$$= \text{Rs. } (120 \times 5 + 130 \times 7 + 290)$$

$$= \text{Rs. } 1800$$

∴ Average monthly income

$$= \frac{1800}{12} = \text{Rs. } 150$$

6. There are 50 boys in a class. Their average weight is 45 kg. When one boy leaves the class, the average reduces by 100 gms. Find the weight of the boy who left the class.

Weight of the boy left = New average -
No. of old members x change in average

$$= 44.9 - 50 \times (-0.1) = 44.9 + 5 = 49.9 \text{ kg.}$$

7. The average attendance in a school for the first 4 days of the week is 30 and for the first 5 days of the week is 32. The attendance on the fifth day is:

Total attendance for the first 4 days

$$= 4 \times 30 = 120$$

Total attendance for the first 5 days

$$= 5 \times 32 = 160$$

Attendance on the fifth day

$$= 160 - 120 = 40$$

PRACTICE TEST

1. The marks obtained by a student in five subjects are 68, 73, 62, 85 and 79. Find the average score.

- a) 73 b) 73.4
c) 75 d) 74.5

2. The average income of a group of 9 workers is Rs. 137.30 and that of another group of 7 workers is Rs. 95.06. The average income of all the persons is:

- a) Rs. 118.82 b) Rs. 116.18
c) Rs. 125.18 d) Rs. 128.15

3. There are 40 boys in a class. One boy weighing 40 kg goes away, and at the same time another boy joins the class. If the average weight of the class is thus increased by 100 gm, the weight of the new boy is.

- a) 39.9 kg b) 44.1 kg
c) 40.1 kg d) 44 kg

4. My average expenses for 4 days is Rs. 6.00. I spend Rs. 7.70 on first day and Rs.

6.30 on second day. If I spent Rs. 10 on third day, what did I spend on the 4th day?

- a) Rs. 2/- b) Rs. 3/-
c) Rs. 4/- d) Nothing

5. The average temperature on Tuesday, Wednesday and Thursday was 37° centigrade. The average temperature on Wednesday, Thursday and Friday was 38° centigrade. If the temperature on Friday was 39° centigrade, the temperature on Tuesday was:

- a) 35° C b) 36° C
c) 37° C d) 38° C

6. The average age of students in two classes of 40 students each is 10 years and 8 years respectively. The average age of students in both the classes taken together is:

- a) 8 years b) 9 years
c) 10 years d) 11 years

7. The average age of 50 soldiers in troop is 25 years. If the captain's age is included, the average age of all of them still remains the same. What is the captain's age in years?

- a) 25 b) More than 25
c) Less than 25 d) Cannot be
determined

8. Two towns A and B are some distance apart. A girl cycles from A to B at a speed of 10 km/hr and then back from B to A at the rate of 15 km/hr. The average speed during the journey is:

- a) 12.5 km/hr b) 15 km/hr
c) 12 km/hr d) 13.5 km/hr

9. An employee's average contribution to his provident fund for the first 9 months was Rs. 3,500 each and for each of the remaining 3 months, the contribution was Rs. 5,500. By what amount was his total contribution short of Rs. 58,000?

- a) Rs. 4,000 b) Rs. 16,500
c) Rs. 8,000 d) Rs. 10,000

10. What fraction must be subtracted from the sum of $\frac{1}{4}$ and $\frac{1}{6}$ to have an average of

$\frac{1}{12}$ of these the two fractions?

- a) $\frac{1}{3}$ b) $\frac{1}{2}$ c) $\frac{1}{4}$ d) $\frac{1}{8}$

11. The average marks of 12 students was calculated as 40. But it was later found that marks of one student had been entered wrongly as 42 instead of 54 and of another as 74 instead of 50. The correct average is:

- a) 39 b) 40 c) 41 d) 43

12. The average salary of workers in an industry is Rs. 2000, the average salary of 150 technicians being Rs. 4000, and the non-technicians being Rs. 1,250. The total number of workers is

- a) 450 b) 300
c) 550 d) 500

13. The average age of a husband and a wife who were married four years ago was 20 years then. What will be the average age of the family now if they have a three years old child?

- a) $15\frac{2}{3}$ years b) $16\frac{1}{3}$ years
c) 17 years d) 16 years

14. The average of three consecutive odd numbers is 39. What is the sum of the first two of these numbers?

- a) 78 b) 76 c) 24 d) 11

15. If the average of 9 consecutive numbers is 20, the highest of these numbers is:

- a) 20 b) 21 c) 24 d) 26

16. The sum of two consecutive even numbers is 23 more than the average of these two numbers. What is the second number?

- a) 22 b) 24 c) 26
d) Data inadequate

17. The average of 17 numbers is 10.9. If the average of first nine is 10.5 and that of the last nine is 11.4, the middle number is

- a) 11.8 b) 11.4
c) 10.9 d) 11.7

18. The average monthly expenditure of a family was Rs. 2,200 during first 3 months, Rs. 2,550 during next 4 months and Rs. 3,120 during last 5 months of the year. If the total saving during the year was Rs. 1,260, find average monthly income.

- a) Rs. 3,960 b) Rs. 760.8
c) Rs. 2,805 d) Rs. 3,125

19. 30 pens and 75 pencils were purchased for Rs. 510. If the average price of a pencil was Rs. 2.00, find the average price of a pen.

- a) Rs. 12 b) Rs. 15
c) Rs. 19 d) Rs. 25

20. The average age of the husband and wife who were married 7 years ago was 25 years then. The average age of the family including the husband, wife and the child born during the interval is 22 years, now. How old is the child now?

- a) 2 years b) 3.5 years
c) 1 year d) 4 years

21. Average monthly income of a family of four earning members was Rs. 735. One of the earning members died and therefore the average income came down to Rs. 650. The income of the deceased was:

- a) Rs. 820 b) Rs. 990
c) Rs. 692.50 d) Rs. 1,385

22. A batsman has certain average runs for 20 innings. In the 21st inning, he scored 107 runs thereby increasing his average by 2. What is his average after 21 innings?

- a) 67 b) 65 c) 60 d) 72

ANSWERS TO PRACTICE TEST

1. (b) 2. (a) 3. (d) 4. (d) 5. (b) 6. (b) 7. (a) 8. (c)
9. (d) 10. (c) 11. (a) 12. (c) 13. (c) 14. (b) 15. (c) 16. (b)
17. (a) 18. (c) 19. (a) 20. (a) 21. (b) 22. (a)

Ratio & Proportion

Ratio: The number of times one quantity contains another quantity of the same kind is called ratio of the two quantities. The ratio of a to b is written as

$$a : b = \frac{a}{b} = a \div b$$

In the ratio $a : b$, a and b are called the terms of ratio, ' a ' is the **antecedent** and ' b ' is the **consequent**.

Points to remember:

- i) The order of the terms in a ratio is very important
- ii) The quantities of a ratio must be expressed in the same units.
- iii) The ratio is unaltered if each term is multiplied or divided by the same number.
- iv) When a certain quantity ' q ' is divided in a given ratio $a:b$, the two parts are

$$\frac{aq}{a+b} \text{ and } \frac{bq}{a+b}$$

- v) If $a : b$ and $c : d$ are two ratios, then $ac : bd$ is called the compounded ratio of the given ratios.

Proportion: The equality of the two ratios is called proportion. Suppose the two ratios $a:b$ and $c:d$ are equal, i.e, $a:b = c:d$, then we write,

$$a:b : : c:d$$

Here, a and d are called as extremes and b, c are called means.

Rule:

- i) $ad = bc$ 'or'

Product of extremes = Product of means.

- ii) In $a:b : : c:d$, d is the **fourth proportional** to a, b and c .

- iii) If x is the **third proportional** to a and b then $a : b : : b : x$

- iv) Mean proportional between a and b is

$$\sqrt{(ab)}.$$

Other properties

If $\frac{a}{b} = \frac{c}{d}$ 'or' $a : b : : c:d$.

$$i) \frac{a+b}{b} = \frac{c+d}{d}$$

$$ii) \frac{a-b}{b} = \frac{c-d}{d}$$

$$iii) \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

$$iv) \frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d} = \frac{ka+kc}{kb+kd}$$

Solved examples:

- 1. If $a : b = 4:5$ and $b:c = 6:7$, find the ratios $a:c$ and $a:b:c$

$$\text{Given, } \frac{a}{b} = \frac{4}{5}; \frac{b}{c} = \frac{6}{7}$$

$$\therefore \frac{a}{b} \times \frac{b}{c} = \frac{4}{5} \times \frac{6}{7} \text{ (ie) } \frac{a}{c} = \frac{24}{35}$$

$$\therefore a : c = 24 : 35$$

Here ' b ' term is common to both the equations and so their corresponding values should be made equal.

$$\text{(ie) } a:b = 24 : 30$$

$$b:c = 30 : 35$$

$$\therefore a:b:c = 24 : 30 : 35$$

2. Divide Rs. 54 in the ratio 4 : 5

$$\text{Sum of ratios} = 4+5 = 9$$

$$\therefore \text{First part} = 54 \times \frac{4}{9} = \text{Rs. } 24$$

$$\text{Second part} = 54 \times \frac{5}{9} = \text{Rs. } 30$$

3. In a ratio, which is equal to 7 : 8 , if the antecedent is 35, what is the consequent?

Let the consequent be x

$$\Rightarrow 7x = 8 \times 35; x = \frac{8 \times 35}{7} = 40$$

4. The sides of a triangle are in the ratio of $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$. If the perimeter is 104 cms, find the length of the smallest side.

$$\text{Given ratio is } \frac{1}{2} : \frac{1}{3} : \frac{1}{4} = 6:4:3$$

(Multiplying with the L.C.M. of 2,3, & 4)

$$\text{Sum of ratio} = 6+4+3= 13$$

$$\therefore \text{Smallest side} = \frac{3}{13} \times 104 = 24 \text{ cms.}$$

5. The incomes of A and B are in the ratio 2:3 and their expenditure are in the ratio 1:2, If each saves Rs. 2,400, find A's income.

Let the income of A and B be 2x and 3x

Since, Income - Savings = Expenditure,

$$(2x - 2400) : (3x - 2400) = 1:2$$

$$\Rightarrow 2(2x - 2400) = 3x - 2400$$

$$x = 2400$$

$$\therefore \text{As income } 2x = 2 \times 2400$$

$$= \text{Rs. } 4800.$$

6. In 40 litres mixture of milk and water, the ratio of milk and water is 3:1. How much water should be added in the mixture so that the ratio of milk to water becomes 2:1?

In 40 litres of mixture, quantity of milk

$$= \frac{3}{4} \times 40 = 30 \text{ litres}$$

Quantity of water = 40-30=10 litres

Suppose x litres of water be added in 40 litres of mixture.

$$\therefore \frac{30}{10+x} = \frac{2}{1} \Rightarrow 2(10+x) = 30 \Rightarrow x = 5 \text{ litres}$$

7. Two numbers are such that the ratio between them is 3:5 but if each is increased by 10, the ratio between them becomes 5:7. Find the numbers.

Let the numbers be 3x and 5x

$$\text{Then } \frac{3x+10}{5x+10} = \frac{5}{7}$$

$$\Rightarrow 7(3x+10) = 5(5x+10) \Rightarrow x = 5$$

\therefore The numbers are 15 and 25

8. A bag contains rupees, fifty paise, and twenty five paise coins in the proportion 5:6:8. If the total amount is Rs. 210. Find the number of coins of each kind.

Ans: Let there be 5 rupee coins, 6 fifty paise coins, and 8 twenty five paise coins the value of 6 fifty paise coins

$$= \text{Rs. } 3$$

The value of 8 twenty five paise coins

$$= \text{Rs. } 2$$

The number of rupee coins

$$= \frac{5 \times 210}{10} = 105$$

The number of 50 paise coins

$$= \frac{6 \times 210}{10} = 126$$

The number of 25 paise coins

$$= \frac{8 \times 210}{10} = 168$$

PRACTICE TEST

- If $A:B = 3:2$ $B:C = 4:3$ then $A:B:C = ?$
 - 6:4:3
 - 3:2:3
 - 3:4:3
 - 3:2:1
- Ratio between two numbers is 3:2 and their difference is 225, then the smaller number is:
 - 90
 - 675
 - 135
 - 450
- If $2x + 3y = 4z$, then $x : y : z$ is
 - 4:3:2
 - 6:3:4
 - 3:4:2
 - 6:4:3
- The mean proportion between 9 and 36 is
 - 22.5
 - 18
 - 6
 - 36
- The fourth proportion to 3,6,15 is
 - 15
 - 30
 - 5
 - 18
- Two numbers are in the ratio 7:9. If 12 is subtracted from each of them, the ratio becomes 3:5. The product of the numbers is:
 - 432
 - 567
 - 1575
 - 1263
- What must be added to each term of the ratio 7:13 so that the ratio becomes 2:3?
 - 1
 - 2
 - 3
 - 5
- A total amount of Rs. 1800 is to be divided among A,B and C in such a way that half of A's part, one third of B's part and one-fourth of C's part is equal. The A's part is
 - Rs. 400
 - Rs. 600
 - Rs. 800
 - Rs. 900
- A sum of Rs. 53 is divided among A,B,C in such a way that A gets Rs. 7 more than B and B gets Rs. 8 more than C. Then the ratio of their shares is
 - 10:18:25
 - 18:25:10
 - 25:18:10
 - 15:18:20
- The ratio of number of boys and girls in a school of 720 students is 7:5. How many more girls should be admitted to make the ratio 1:1?
 - 90
 - 120
 - 220
 - 240
- The ratio of the number of boys and girls at a party was 1:2 but when 2 boys and 2 girls left, the ratio became 1:3. then the number of persons initially in the party was
 - 24
 - 36
 - 12
 - 15
- A sum of Rs. 3400 has been divided among A,B and C in such a way that A gets $\frac{2}{3}$ of what B gets and B gets $\frac{1}{4}$ of what C gets. Then, B's share is
 - Rs. 600
 - Rs. 340
 - Rs. 400
 - Rs. 500
- Two numbers are in the ratio 3:5, If 8 is subtracted from each, then they are in the ratio 1:3. Then, the second number is
 - 15
 - 20
 - 4
 - 12
- The proportion of copper and zinc in brass is 13:7. How much zinc will be there in 100 kg of brass?
 - 20 kg
 - 35 kg
 - 45 kg
 - 50kg
- The ratio of the father's age to son's age is 4:1. The product of their ages is 196. The ratio of their ages after 5 years will be:
 - 3:1
 - 10:3
 - 11:4
 - 14:5
- The ages of Manoj and Amit are in the ratio 2:3. After 12 years, their ages will be in the ratio 11:15. The age of Amit is:
 - 32 years
 - 40 years
 - 48 years
 - 56 years
- Rs. 780 is divided among 2 men, 6 women and 8 boys so that the share of a man, a woman and a boy are in the ratio 3:2:1. Then, how much does a boy get?
 - Rs. 130
 - Rs. 60
 - Rs. 240
 - Rs. 40

18. The ratio between the annual incomes of A and B is 5:4 and between their expenditures is 4:3. If at the end of the year, A and B respectively save Rs. 400 and Rs. 500, then the income of A is:
 a) Rs. 4,000 b) Rs. 3,200
 c) Rs. 3,700 d) Rs. 4,800
19. A bag contains one rupee, 50 paise and 25 paise coins in the ratio 5:7:9. If the total amount in the bag is Rs. 430, find the number of coins of 25 paise.
 a) 200 b) 280
 c) 360 d) 300
20. A mixture contains milk and water in the ratio 3:2. If 4 litres of water is added to the mixture, milk and water in the mixture becomes equal. The quantity of milk in the mixture in litre is.
 a) 18 b) 4
 c) 6 d) 12
21. Two equal glasses are $\frac{1}{2}$ and $\frac{2}{3}$ full of milk respectively. The two are completely filled up with water. The contents of the two glasses are then mixed in another vessel. The ratio of milk and water in the vessel is
 a) 5:7 b) 7:5
 c) 1:1 d) 2:3
22. An amount is to be distributed among A,B and C in the ratio 3:7:5 respectively. If the difference in the shares of A and B is Rs. 7,600/- what will be the share of C?
 a) Rs. 5,700 b) Rs. 19,000
 c) Rs. 9,500 d) Rs. 10,000
23. Two varieties of oil are mixed in the ratio 4:3 to produce first quality and if they are mixed in the ratio 2:3 second quality is obtained. How many kg. of the first quality be mixed with 10kg of the second quality so that a third quality having the two varieties in the ratio 5 : 4 may be produced?
 a) 48 kg b) 42 kg
 c) 88 kg d) 98 kg
24. The ratio of the number of gents to ladies in a party was 2:3. When 20 more gents joined the group, the ratio was reversed. The number of ladies in the party was
 a) 16 b) 24 c) 30 d) 36
25. The HCF of three numbers is 12. If they are in the ratio of 1:2:3, the numbers are
 a) 12,24,36 b) 10,20,30
 c) 5,10,15 d) 4,8,12
26. If the ratio of the areas of two squares is 1:4, the ratio of their perimeters is
 a) 1:2 b) 1:4
 c) 1:6 d) 1:8
27. Two numbers are such that their difference, their sum and their product are in the ratio of 1:7:24. The product of the numbers is
 a) 6 b) 12 c) 24 d) 48
28. The incomes of A, B and C are in the ratio 7:9:12 and their spending are in the ratio 8:9:15. If A saves $\frac{1}{4}$ th of his income, then the savings of A,B and C are in the ratio of
 a) 56:99:69 b) 99:56:69
 c) 69:56:99 d) 99:69:56
29. Rs. 180 contained in a box is made up of one rupee, 50 paise, and 25 paise coins in the proportion of 2:3:4. What is the number of 50 paise coins?
 a) 150 b) 180
 c) 240 d) 120
30. 81 is divided into three parts, such that half of the first part, one-third of the second part and one-fourth of the third part are equal. The third part is more than the first by
 a) 9 b) 18
 c) 27 d) 36

ANSWERS TO PRACTICE TEST

1. (a) 2. (d) 3. (d) 4. (b) 5. (b) 6. (b) 7. (d) 8. (a)
 9. (c) 10. (b) 11. (c) 12. (a) 13. (b) 14. (b) 15. (c) 16.(c)
 17. (b) 18. (a) 19. (c) 20. (d) 21.(b) 22. (c) 23. (d) 24.(b)
 25. (a) 26. (a) 27. (d) 28. (a) 29.(d) 30. (b)

QUANTITATIVE APTITUDE

Simplification

VBODMAS

'VBODMAS' (Vinculum - Bracket - Of - Division - Multiplication - Addition - Subtraction) rule should be applied for solving problems involving one or more mathematical operations like multiplication, division, addition, subtraction etc. Such problems are solved in the order of vinculum, bracket, of, division, multiplication, addition and subtraction. Remember 'Of' in VBODMAS means multiplication.

Solved Examples

1. $45 - 4 \times 6 - 5 + 14 \div 7 = ?$
 $45 - 4 \times 6 - 5 + 14 \div 7 = 45 - 24 - 5 + 2 = 18$
2. $21 \div 3 (10 - 3) - 20 + 1 = ?$
 $= 21 \div 3 \times 7 - 20 + 1$
 $= 7 \times 7 - 20 + 1$
 $= 49 - 20 + 1 = 30$
3. 3 of $\frac{4}{5} \div \frac{4}{5} + \frac{1}{6} = ?$
 $3 \text{ of } \frac{4}{5} \div \frac{4}{5} + \frac{1}{6} = \frac{12}{5} \div \frac{4}{5} + \frac{1}{6}$

$$= \frac{12}{5} \times \frac{5}{4} + \frac{1}{6} = 3 + \frac{1}{6} = 3\frac{1}{6}$$

$$3 \div \left[(8 - 5) \div \left\{ (4 - 2) \div \left(2 + \frac{8}{13} \right) \right\} \right] = ?$$

$$4. \quad = 3 \div \left[3 \div \left\{ 2 \div \frac{34}{13} \right\} \right]$$

$$= 3 \div \left[3 \div \left\{ 2 \times \frac{13}{34} \right\} \right] =$$

$$3 \div \left[3 \div \frac{13}{7} \right] = 3 \div \left[\frac{3 \times 7}{13} \right]$$

$$= \frac{3 \times 13}{3 \times 7} = \frac{13}{7}$$

$$5. \quad \frac{(4 + 4 + 4) \div 4}{6 + 6 + 6 \div 6} = ?$$

$$\frac{(4 + 4 + 4) \div 4}{6 + 6 + 6 \div 6} = \frac{12 \div 4}{6 + 6 + 1} = \frac{3}{13}$$

SIMPLIFICATION USING IDENTITIES

- | | |
|---|---|
| <ol style="list-style-type: none"> 1. $a \times (b + c) = a \times b + a \times c$ 2. $(a + b)^2 = a^2 + 2ab + b^2$ 3. $(a - b)^2 = a^2 - 2ab + b^2$ 4. $(a + b)^2 = (a - b)^2 + 4ab$ 5. $(a - b)^2 = (a + b)^2 - 4ab$ 6. $(a - b)(a + b) = a^2 - b^2$ 7. $(a + b)^3 = a^3 + 3ab(a + b) + b^3$ 8. $(a - b)^3 = a^3 - 3ab(a - b) - b^3$ 9. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ 10. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ | <ol style="list-style-type: none"> 11. $\frac{a^3 - b^3}{a^2 + ab + b^2} = a - b$ 12. $\frac{a^3 + b^3}{a^2 - ab + b^2} = a + b$ 13. $\frac{a^2 - ab + b^2}{a^3 + b^3} = \frac{1}{a + b}$ 14. $\frac{a^2 + ab + b^2}{a^3 - b^3} = \frac{1}{a - b}$ 15. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$ 16. $(a + b)^2 - (a - b)^2 = 4ab$ |
|---|---|

FRACTIONS

CLASSIFICATION OF FRACTIONS

1. **Proper Fraction** : A Proper fraction is one whose numerator is less than its denominator.

$$\text{eg. } \frac{1}{3}, \frac{4}{9}$$

2. **Improper Fraction** : An improper fraction is one whose numerator is equal to or greater than its denominator

$$\text{eg. } \frac{6}{5}, \frac{4}{4}$$

3. **Mixed Fraction**: A mixed fraction is a quantity consisting of two parts, one a whole number and other a proper fraction.

$$\text{eg. } 4\frac{1}{8}, 9\frac{3}{4}$$

A mixed fraction can always be expressed as an improper fraction.

$$\text{eg. } 5\frac{2}{3} = 5 + \frac{2}{3} = \frac{(5 \times 3) + 2}{3} = \frac{17}{3}$$

Similarly an improper fraction can always be expressed as a mixed fraction. For that divide the numerator by the denominator and write the quotient as the whole number part of the mixed fraction, the remainder as the numerator and the divisor as the denominator.

$$\text{eg. } \frac{19}{5} = 3\frac{4}{5}; \frac{26}{7} = 3\frac{5}{7}$$

Basic Property of Fractions

1. The value of a fraction is not altered by multiplying the numerator and denominator by the same number.

$$\text{ie. } \frac{a}{b} = \frac{axc}{bxc} = \frac{ac}{bc}$$

2. The value of a fraction is not altered by

dividing the numerator and the denominator by the same number.

$$\text{ie. } \frac{a}{b} = \frac{a \div c}{b \div c}$$

Reduction of a fraction to its lowest terms

To change a fraction to its lowest terms, divide its numerator and denominator by the H.C.F. of the numbers.

$$\text{eg. Reduce } \frac{12}{36} \text{ to its lowest terms.}$$

$$\frac{12}{36} = \frac{12 \div 12}{36 \div 12} = \frac{1}{3}$$

(Since H.C.F. of 12 and 36 is 12)

Reducing fractions to their common denominators

To reduce fractions to their common denominators, change the denominators into their L.C.M.

$$\text{eg. } \frac{3}{4}, \frac{4}{5}$$

L.C.M. of 4 and 5 = 20

To convert the denominator of $\frac{3}{4}$ into 20,

multiply it by 5. To convert the denominator of $\frac{4}{5}$ into 20, multiply it by 4.

$$\text{(ie) } \frac{3 \times 5}{4 \times 5}; \frac{4 \times 4}{5 \times 4} \text{ (ie) } \frac{15}{20}; \frac{16}{20}$$

Comparing Fractions

Let $\frac{a}{c}$ and $\frac{b}{c}$, be two fractions with same denominator c.

Then $\frac{a}{c} > \frac{b}{c}$ if $a > b$ eg. $\frac{4}{5} > \frac{3}{5}$

$\frac{a}{c} < \frac{b}{c}$ if $a < b$ eg. $\frac{1}{3} < \frac{2}{3}$

$\frac{a}{c} = \frac{b}{c}$ if $a = b$ eg. $\frac{1}{2} = \frac{1}{2}$

Addition and Subtraction of Fractions

Method : Convert the fractions with the same denominator by taking L.C.M. and then add or subtract.

Examples

1. $\frac{3}{7} + \frac{2}{7} = \frac{3+2}{7} = \frac{5}{7}$

2. $\frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$

3. $\frac{2}{7} + \frac{4}{9} = ?$

L.C.M. of 7, 9 = 63

$$\frac{2}{7} + \frac{4}{9} = \frac{18}{63} + \frac{28}{63} = \frac{46}{63}$$

4. $\frac{2}{3} + \frac{3}{4} + \frac{4}{5} = ?$

L.C.M. of 3, 4 and 5 is 60

$$\begin{aligned} \therefore \frac{2}{3} + \frac{3}{4} + \frac{4}{5} &= \frac{40}{60} + \frac{45}{60} + \frac{48}{60} \\ &= \frac{40 + 45 + 48}{60} = \frac{133}{60} = 2 \frac{13}{60} \end{aligned}$$

5. $6\frac{3}{4} - 3\frac{4}{5} = ?$

$$\begin{aligned} 6\frac{3}{4} - 3\frac{4}{5} &= (6-3) + \frac{3}{4} - \frac{4}{5} \\ &= 3 + \frac{15-16}{20} = 3 - \frac{1}{20} \end{aligned}$$

$$= 2 + 1 - \frac{1}{20} = 2 \frac{19}{20}$$

6. $13\frac{1}{3} - 12\frac{3}{4} - 11\frac{5}{6} + 10\frac{11}{12} = ?$

L.C.M. of 3, 4, 6 and 12 is 12

$$\begin{aligned} &= (13+10-12-11) + \frac{1}{3} + \frac{11}{12} - \frac{3}{4} - \frac{5}{6} \\ &= 0 + \frac{4+11-9-10}{12} = \frac{-4}{12} = \frac{-1}{3} \end{aligned}$$

Multiplication of fractions

1. To multiply a fraction by a whole number, multiply the numerator by the whole number.

eg. $2 \times \frac{3}{5} = \frac{(2 \times 3)}{5} = \frac{6}{5}$

2. To multiply a fraction by another fraction multiply corresponding numerators and denominators and then simplify.

eg. $\frac{4}{5} \times \frac{3}{12} = \frac{4 \times 3}{5 \times 12} = \frac{1}{5}$

Division of Fractions

1. To divide a fraction by a whole number, multiply the denominator of the fraction by the whole number.

eg. $\frac{2}{3} \div 7 = \frac{2}{3 \times 7} = \frac{2}{21}$

2. To divide a fraction by a fraction, find the reciprocal of the divisor and then multiply.

eg. $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{10}{12} = \frac{5}{6}$

Note: Cancellation can be performed only to multiplication and division of fractions; it can not be performed in addition or subtraction of fractions.

Point to remember:

1. To multiply a whole number and a mixed

fraction together, perform separate multiplication and then add the results.

$$\begin{aligned} \text{eg. } 18 \times 5 \frac{2}{3} &= (18 \times 5) + 18 \times \frac{2}{3} \\ &= 90 + 12 = 102 \end{aligned}$$

2. To divide a mixed fraction by a whole number divide the whole number part of the mixed fraction by the divisor (let the quotient be a). Reduce the remainder to a single fraction and divide this single fraction by the divisor. (Let the quotient be b). Now the required result is $a+b$.

$$\text{eg. } 21 \frac{2}{3} \div 4$$

$$4 \overline{) 21 \frac{2}{3}} (5$$

20

$$1 \frac{2}{3} = \frac{5}{3}$$

$$\text{Now } \frac{5}{3} \div 4 = \frac{5}{3} \times \frac{1}{4} = \frac{5}{12}$$

$$\therefore 21 \frac{2}{3} \div 4 = 5 + \frac{5}{12} = 5 \frac{5}{12}$$

More Solved Examples

1. There are 40 students in a class. One day only $\frac{7}{10}$ th of total students were present.

Find the number of absentees on that day.

Number of absentees

= Fraction of absentees x Total number

$$= \left(1 - \frac{7}{10}\right) \times 40 = 12 \text{ students}$$

2. A man spends $\frac{2}{5}$ of his salary on food,

$\frac{3}{10}$ of his salary on house rent and $\frac{1}{8}$ of the salary on clothes. He still has Rs. 1,400 left with him. Find his total salary.

Totally he spends $\left(\frac{2}{5} + \frac{3}{10} + \frac{1}{8}\right)$ of his total salary.

\therefore He saves $\left[1 - \left(\frac{2}{5} + \frac{3}{10} + \frac{1}{8}\right)\right]$ part of his salary.

$$\therefore \left(1 - \frac{33}{40}\right) \times \text{total salary} = 1400$$

$$\text{(ie) } \frac{7}{40} \times \text{total salary} = 1400$$

$$\text{total salary} = 1400 \times \frac{40}{7} = \text{Rs. } 8000$$

3. In an examination, a student was asked to find $\frac{3}{14}$ of a certain number. By mistake,

he found $\frac{3}{4}$ of it. His answer was 150 more than the correct answer. Find the given number.

Let the given number be x then

$$\frac{3x}{4} - \frac{3x}{14} = 150$$

$$\Rightarrow x \left(\frac{3}{4} - \frac{3}{14}\right) = 150 \Rightarrow \frac{15}{28} x = 150$$

$$\Rightarrow x = \frac{150 \times 28}{15} = 280$$

4. By how much is $\frac{4}{5}$ of 70 less than $\frac{5}{7}$ of 112?

$$\frac{5}{7} \times 112 - \frac{4}{5} \times 70 = 5 \times 16 - 4 \times 14 = 24$$

5. $\frac{5}{12}$ part of what amount will be equal to

$3\frac{3}{4}$ part of Rs. 100.

Let the amount be Rs.y

$$\frac{5}{12} \text{ of } y = 3\frac{3}{4} \text{ of } 100$$

$$\Rightarrow \frac{5}{12} y = \frac{15}{4} \times 100$$

$$\Rightarrow y = \frac{15 \times 100}{4} \times \frac{12}{5}$$

$$\Rightarrow y = 900$$

Decimal Fractions

Fractions that have powers of 10 in the denominators are called decimal fractions.

(ie) Fractions whose denominators are 10, 10^2 , 10^3 , 10^4 are called decimal fractions.

eg. 0.5, 0.063, 8.98 etc.

Here

$$0.5 = \frac{5}{10}; 0.063 = \frac{63}{1000}; 8.98 = \frac{898}{100}$$

Annexing zeros to the extreme right of decimal fraction does not change its value. $0.47 = 0.470 = 0.4700$ etc.

Addition

For adding a decimal number with another decimal number or with another whole number write the given number in such a way that the number of decimal places are equal for all the numbers.

$$\text{eg. } 2 + 0.63 + 0.712$$

Here maximum number of decimal place = 3

\therefore Convert all the numbers to 3 decimal places.

$$\therefore 2 + 0.63 + 0.712 =$$

$$2.000 + 0.630 + 0.712 = 3.342$$

Subtraction

In subtraction also, the given numbers are to be written in such a way that the number of decimal places become equal for all numbers.

$$\text{eg. } 5 - 0.473$$

$$\text{Maximum number of decimal place} \\ = 3 \text{ (in } 0.473)$$

$$\text{ie. } 5 - 0.473 = 5.000 - 0.473 = 4.527$$

Multiplication

1. Multiplication of a Decimal Fraction by a power of 10:

Shift the decimal point to the right by as many places of decimal as the power of 10.

$$\text{eg. } 4.5291 \times 100 = 452.91$$

2. Multiplication of two or more decimal fractions :

$$0.002 \times 0.08 \times 0.5 = ?$$

Step 1: Multiply the given numbers as if they are without any decimal point.

$$\text{ie. } 2 \times 8 \times 5 = 80$$

Step 2 : Add the total number of decimal places in the given numbers

$$\text{ie } 3 + 2 + 1 = 6$$

Step 3 : Write the result of step 1 and convert it to a number whose number of decimal places is same as the number obtained in step 2 by shifting the decimal point to the left.

$$\therefore 0.002 \times 0.08 \times 0.5 = 0.000080 = 0.00008$$

Division

1. While dividing a decimal fraction by powers of 10, the result is obtained by shifting the decimal point to the left by as many places of decimal as is the power of 10.

$$\text{eg. } 3.45 \div 10 = 0.345$$

$$961.1 \div 100 = 9.611$$

2. While dividing a decimal fraction by a natural number, divide the given fraction without the decimal point by the given natural number. In the answer thus got, place the decimal point

to the left as many places of decimal as are there in the dividend.

eg. $\frac{125}{25} = ?$

First step is $\frac{125}{25} = 5$

$\frac{1.25}{25} = 0.05$

3. While dividing a decimal fraction by a decimal fraction, shift the decimal point to the right of the dividend and the divisor both by equal number of digits such that the divisor is converted into a whole number.

eg. $\frac{3.15}{3.5} = \frac{315}{35} = 0.9$

$\frac{28.6}{0.143} = \frac{28600}{143} = 200$

Expressing a decimal into a vulgar fraction

Put 1 in the denominator under the decimal point and annex with it as many zeros as is the number of digits after the decimal point. Remove the decimal point and reduce the fraction to its lowest terms.

Thus $\frac{0.125}{1000} = \frac{125}{1000} = \frac{1}{8}$

If numerator and denominator of a fraction contain the same number of decimal places, then we may remove the decimal sign.

eg. $\frac{8.86}{9.25} = \frac{886}{925}$

◆ To multiply a decimal by any multiple of ten, move the decimal point as many places to the right as is the number of zeros in the multiplier.

◆ To divide a decimal by any multiple of ten move the decimal point as many places to the left as is the number of zeros in the divisor.

◆ When a divisor as well as dividend is a decimal, we multiply both the dividend and the divisor by suitable multiple of 10 to make the divisor a whole number and then proceed division.

Solved Examples:

1. Evaluate

$237 \times 237 + 363 \times 363 + 2 \times 237 \times 363$

Given expression is of the form

$a^2 + b^2 + 2ab = (a+b)^2$

$= (237 + 363)^2 = (600)^2 = 360000$

2. $\sqrt{221^2 - 220^2} = ?$

Given expression is

$\sqrt{a^2 - b^2} = \sqrt{(a+b)(a-b)}$

$= \sqrt{(221 + 220)(221 - 220)}$

$= \sqrt{441 \times 1} = 21$

3. $\frac{0.45 \times 0.45 \times 0.45 - 0.21 \times 0.21 \times 0.21}{0.45 \times 0.45 + 0.45 \times 0.21 + 0.21 \times 0.21}$

Given expression is of the form

$\frac{a^3 - b^3}{a^2 + ab + b^2} = a - b = 0.45 - 0.21 = 0.24$

4. $\frac{4.7 \times 6.5 + 5.3 \times 6.5}{1.3 \times 7.9 - 1.3 \times 6.9} = ?$

Given expression is $\frac{ax + bx}{cy - dy} = \frac{(a + b)x}{(c - d)y}$

$\frac{(4.7 + 5.3)6.5}{(7.9 - 6.9)1.3} = \frac{10 \times 6.5}{1 \times 1.3} = 50$

5. $\frac{0.75 \times 0.75 - 0.74 \times 0.74}{1.49} = ?$

Given expression is $\frac{a^2 - b^2}{a + b} = a - b$

$$= 0.75 - 0.74 = 0.01$$

$$6. \frac{6.4 \times 6.4 + 2 \times 6.4 \times 3.6 + 3.6 \times 3.6}{(6.4)^2 - (3.6)^2} = ?$$

The given expression is $\frac{a^2 + 2ab + b^2}{a^2 - b^2}$

$$= \frac{(a+b)^2}{(a+b)(a-b)} = \frac{a+b}{a-b} = \frac{6.4+3.6}{6.4-3.6}$$

$$= \frac{10}{2.8} = \frac{100}{28} = \frac{25}{7} = 3\frac{4}{7}$$

$$7. 0.7 \times 0.7 \times 0.7 - 0.3 \times 0.3 \times 0.3 - 3 \times 0.7 \times 0.3 \times 0.4 = ?$$

The given expression is

$$\begin{aligned} & a^3 - b^3 - 3ab(a-b) \\ &= (a-b)^3 = (0.7-0.3)^3 = (0.4)^3 \\ &= 0.064 \end{aligned}$$

8. Simplify

$$7\frac{1}{2} - \left[2\frac{1}{4} \div \left\{ 1\frac{1}{4} - \frac{1}{2} \left(1\frac{1}{2} - \frac{1}{3} - \frac{1}{6} \right) \right\} \right]$$

$$7\frac{1}{2} - \left[2\frac{1}{4} \div \left\{ 1\frac{1}{4} - \frac{1}{2} \left(1\frac{1}{2} - \frac{1}{3} - \frac{1}{6} \right) \right\} \right]$$

$$= 7\frac{1}{2} - \left[2\frac{1}{4} \div \left\{ 1\frac{1}{4} - \frac{1}{2} \times 1 \right\} \right]$$

$$= 7\frac{1}{2} - \left[2\frac{1}{4} \div \frac{3}{4} \right]$$

$$= 7\frac{1}{2} - \left(\frac{9}{4} \times \frac{4}{3} \right) = 7\frac{1}{2} - 3 = 4\frac{1}{2}$$

$$9. \text{ Find the value of } 4 - \frac{5}{1 + \frac{1}{3 + \frac{1}{2 + \frac{1}{4}}}}$$

$$4 - \frac{5}{1 + \frac{1}{3 + \frac{1}{2 + \frac{1}{4}}}} = 4 - \frac{5}{1 + \frac{1}{3 + \frac{4}{9}}}$$

$$= 4 - \frac{5}{1 + \frac{9}{31}} = 4 - 5 \times \frac{31}{40}$$

$$= 4 - \frac{31}{8} = \frac{32-31}{8} = \frac{1}{8}$$

10. Find the value of

$$\left(1 + \frac{1}{2}\right) \left(1 - \frac{1}{2}\right) \left(1 + \frac{1}{3}\right) \left(1 - \frac{1}{3}\right) \left(1 + \frac{1}{4}\right) \left(1 - \frac{1}{4}\right)$$

$$\left(1 + \frac{1}{2}\right) \left(1 - \frac{1}{2}\right) \left(1 + \frac{1}{3}\right) \left(1 - \frac{1}{3}\right) \left(1 + \frac{1}{4}\right) \left(1 - \frac{1}{4}\right)$$

$$= \left(1 - \frac{1}{4}\right) \left(1 - \frac{1}{9}\right) \left(1 - \frac{1}{16}\right)$$

$$= \frac{3}{4} \times \frac{8}{9} \times \frac{15}{16} = \frac{5}{8}$$

11. Find the value of

$$2 + \sqrt{2} + \frac{1}{2 + \sqrt{2}} + \frac{1}{\sqrt{2} - 2}$$

$$2 + \sqrt{2} + \frac{1}{2 + \sqrt{2}} + \frac{1}{\sqrt{2} - 2} =$$

$$2 + \sqrt{2} + \frac{\sqrt{2} - 2 + 2 + \sqrt{2}}{(2 + \sqrt{2})(\sqrt{2} - 2)}$$

$$= 2 + \sqrt{2} + \left(\frac{2\sqrt{2}}{2 - 4} \right)$$

$$= 2 + \sqrt{2} - \sqrt{2} = 2$$

12. If $\frac{x}{y} = \frac{3}{4}$ then find the value of

$$\frac{6}{7} + \frac{y-x}{y+x}$$

$$\frac{6}{7} + \frac{y-x}{y+x} = \frac{6}{7} + \frac{1-\frac{x}{y}}{1+\frac{x}{y}}$$

$$= \frac{6}{7} + \frac{1-\frac{3}{4}}{1+\frac{3}{4}}$$

$$= \frac{6}{7} + \frac{\frac{1}{4}}{\frac{7}{4}} = \frac{6}{7} + \frac{1}{7} = 1$$

PRACTICE TEST

- $20 - [9 - \{7 + (2 \times 3)\} + 5] = ?$
a) 20 b) 15 c) 17 d) 19
- $6 + [2 + \{4x(8-3) - (2 \times 6) - 1\} + 2] = ?$
a) 17 b) 13 c) 19 d) 15
- $\frac{1}{5}$ of $35 + 4(9-3) = ?$
a) 31 b) 15 c) 24 d) 42
- $\frac{1}{2} + \frac{1}{3} \times \frac{1}{4} - \frac{1}{12}$
a) $\frac{1}{3}$ b) $\frac{4}{5}$ c) $\frac{1}{2}$ d) $\frac{1}{6}$
- $\frac{(7+7+7) \div 7}{3+3+3 \div 3} = ?$
a) $\frac{3}{11}$ b) $\frac{3}{13}$ c) $\frac{5}{7}$ d) $\frac{3}{7}$

6. 2 of $\frac{3}{4} \div \frac{3}{4} + \frac{1}{4} = ?$

- a) $\frac{4}{9}$ b) $\frac{3}{2}$ c) 2 d) $2\frac{1}{4}$

7. $\frac{23.5 \times 23.5 - 3.5 \times 3.5}{9.6 \times 9.6 - 2 \times 9.6 \times 8.6 + 8.6 \times 8.6} = ?$

- a) 540 b) 27 c) 54 d) 670

8. $32.5 \times 32.5 - 2 \times 32.5 \times 2.5 + 2.5 \times 2.5 = ?$

- a) 900 b) 30 c) 500 d) 1225

9. $\frac{8.9 \times 8.9 \times 8.9 - 1.4 \times 1.4 \times 1.4}{8.9 \times 8.9 + 8.9 \times 1.4 + 1.4 \times 1.4} = ?$

- a) 75 b) 10.3 c) 14.5 d) 7.5

10. $\frac{24.4 \times 24.4 - 2 \times 24.4 \times 5.6 + 5.6 \times 5.6}{24.4 \times 24.4 - 5.6 \times 5.6} = ?$

- a) $\frac{188}{0.3}$ b) $\frac{188}{30}$ c) $\frac{47}{75}$ d) $\frac{16}{25}$

11. $0.7 \times 0.7 \times 0.7 + 0.3 \times 0.3 \times 0.3 + 3 \times 0.7 \times 0.3 = ?$

- a) 4 b) 1 c) 10 d) 16

12. $\frac{(0.356)^2 \times 2 - 2 \times 0.356 \times 0.106 + (0.106)^2}{(0.632)^2 + 2 \times 0.632 \times 0.368 + (0.368)^2} = ?$

- a) 0.625 b) 0.0625
c) 0.0345 d) 0.345

13. $\frac{0.637 \times 0.637 - 2 \times 0.637 \times 0.395 + 0.395 \times 0.395}{0.242} = ?$

- a) 1.132 b) 0.242
c) 1.422 d) 1

14. $\frac{4.75 \times 4.75 \times 4.75 + 125 \times 125 \times 125}{4.75 \times 4.75 + 125 \times 125 - 4.75 \times 125} = ?$

- a) 5.25 b) 3.5 c) 0 d) 6

15. $\frac{775 \times 775 + 225 \times 225 - 775 \times 225}{775 \times 775 \times 775 + 225 \times 225 \times 225} = ?$

- a) 1000 b) 0.01
c) 0.001 d) 0.0001

$$16. \frac{(0.337 + 0.126)^2 - (0.337 - 0.126)^2}{0.337 \times 0.126} = ?$$

- a) 0.211 b) 0.463
c) 4 d) 2.11

$$17. \frac{(695 + 345)^2 + (695 - 345)^2}{(695)^2 + (345)^2} = ?$$

- a) 2 b) 345 c) 695 d) 4

$$18. \frac{(4.621 - 2.954)^2 + (4.621 + 2.954)^2}{4.621 \times 4.621 + 2.954 \times 2.954} = ?$$

- a) 4 b) 2 c) 0 d) 1

$$19. \frac{64852 \times 64852 - 24852 \times 24852}{64852 + 24852} = ?$$

- a) 20000 b) 80000
c) 30000 d) 40000

$$20. 126.5 \times 126.5 - 2 \times 126.5 \times 6.5 + 6.5 \times 6.5 = ?$$

- a) 12000 b) 14400
c) 17689 d) 1440

$$21. \frac{0.52 \times 0.52 + 0.4 \times 0.4 - 2 \times 0.52 \times 0.4}{0.52 - 0.4} = ?$$

- a) 1.2 b) 0.92
c) 0.48 d) 0.12

$$22. \frac{(4.8)^3 - 0.027}{(4.8)^2 + 1.44 + 0.09} = ?$$

- a) 4.5 b) 0.45 c) 5.1 d) 2.20

$$23. \frac{1}{1 + \frac{1}{1 + \frac{1}{2}}} = ?$$

- a) 3 b) $\frac{5}{3}$ c) 1 d) $\frac{3}{5}$

$$24. 5 - \left[\frac{3}{4} + \left\{ 2 \frac{1}{2} - \left(0.5 + \frac{1}{6} - \frac{1}{7} \right) \right\} \right] = ?$$

- a) $1 \frac{19}{84}$ b) $2 \frac{61}{84}$ c) $2 \frac{23}{84}$ d) $2 \frac{47}{84}$

$$25. \frac{3 \frac{1}{4} - \frac{4}{5} \text{ of } \frac{5}{6}}{4 \frac{1}{3} \div \frac{1}{5} - \left(\frac{3}{10} + 21 \frac{1}{5} \right)} = ?$$

a) $\frac{1}{6}$ b) $2 \frac{7}{12}$

c) $15 \frac{1}{2}$ d) $21 \frac{1}{2}$

$$26. \frac{1}{2} + \frac{1}{2} \left\{ \frac{3}{4} - \frac{1}{2} \left(\frac{7}{8} - \frac{3}{4} \right) \right\} = ?$$

a) $\frac{27}{16}$ b) $\frac{27}{32}$

c) $\frac{27}{64}$ d) $\frac{107}{112}$

$$27. \frac{\frac{1}{5} \div \frac{1}{5} \text{ of } \frac{1}{5}}{\frac{1}{5} \text{ of } \frac{1}{5} \div \frac{1}{5}} = ?$$

- a) 1 b) 5 c) $\frac{1}{5}$ d) 25

$$28. \frac{(79 - 24) \times (11 - 6)}{5 \times 9 - (13 + 12)} = ?$$

a) $8 \frac{7}{20}$ b) $\frac{55}{4}$

c) $\frac{5}{4}$ d) $\frac{-1}{20}$

29. If $\frac{a}{b} = \frac{7}{8}$, then $\left(\frac{14}{23} - \frac{2b-a}{2b+a} \right)$ is equal to

- a) $\frac{5}{14}$ b) $\frac{5}{9}$ c) $\frac{5}{23}$ d) $\frac{5}{92}$

30. $\left(2-\frac{1}{3}\right)\left(2-\frac{3}{5}\right)\left(2-\frac{5}{7}\right)\left(2-\frac{7}{9}\right)\dots\left(2-\frac{997}{999}\right)=?$
- a) $\frac{5}{999}$ b) $\frac{7}{1000}$
c) $\frac{1000}{7}$ d) $\frac{1001}{3}$
31. If $\frac{a}{b} = \frac{4}{3}$, then the value of $\frac{6a+4b}{6a-5b}$ is
a) -1 b) 3 c) 4 d) 5
32. If $\frac{1}{x}\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) = 1 + \frac{1}{2} + \frac{1}{3}$, then the value of x is
a) $\frac{1}{4}$ b) $\frac{13}{22}$ c) 2 d) 4
33. $\frac{1}{2 + \frac{1}{2 + \frac{1}{2 - \frac{1}{2}}}} = ?$
a) $\frac{8}{19}$ b) $\frac{19}{8}$ c) $\frac{7}{8}$ d) $\frac{8}{9}$
34. $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{9}}} = ?$
- a) $1\frac{5}{9}$ b) $1\frac{10}{19}$ c) $\frac{10}{19}$ d) $\frac{19}{10}$
35. If (a-b) is 6 more than (c+d) and (a+b) is 3 less than (c-d), then (a-c) is
a) 0.5 b) 1.0 c) 1.5 d) 2.0
36. The expression $(7.98x^2 + 7.98x + 0.02)$ will be a perfect square for x equal to
a) 4.0 b) 0.4
c) 0.04 d) 0.004
37. The sum of the smallest six digit number and the greatest five digit number is
a) 199999 b) 201110
c) 211110 d) 1099999
38. The sum of two numbers is 22 and their difference is 14. Find the product of the numbers.
a) 70 b) 75 c) 72 d) 82
39. The sum of squares of two numbers is 80 and the square of their difference is 36. The product of the two numbers is
a) 22 b) 44 c) 58 d) 116
40. The product of two numbers is 120. The sum of their squares is 289. The sum of the two numbers is
a) 20 b) 23 c) 169 d) 150

ANSWERS TO PRACTICE TEST

1. (d) 2. (a) 3. (a) 4. (c) 5. (d) 6. (d) 7. (a) 8. (a)
9. (d) 10. (c) 11. (b) 12. (b) 13. (b) 14. (d) 15. (c) 16. (c)
17. (a) 18. (b) 19. (d) 20. (b) 21. (d) 22. (a) 23. (d) 24. (c)
25. (c) 26. (b) 27. (d) 28. (b) 29. (c) 30. (d) 31. (c) 32. (b)
33. (a) 34. (b) 35. (c) 36. (c) 37. (a) 38. (c) 39. (a) 40. (b)

Time and Work

Points to remember

1. If A can finish a piece of work in 'n' days, then A's 1 day's work is $\frac{1}{n}$.
2. If the number of men engaged to do a piece of work is changed in the ratio $a:b$, the time required for the work will be changed in the ratio $b:a$.
3. If A is X times as good a workman as B, then A will take $\frac{1}{X}$ of the time that B takes to do a certain work.
4. If M_1 persons can do 'W₁' works in D_1 days for T_1 hours and M_2 persons can do 'W₂' works in D_2 days for T_2 hours then $M_1 D_1 T_1 W_2 = M_2 D_2 T_2 W_1$.
5. If A can finish a work in 'x' days and B can finish the same work in 'y' days, then time taken by both to finish the work is $\frac{xy}{x+y}$ days
6. If A and B together can do a piece of work in xdays and A alone can do it in y days then B alone can do it in $\frac{xy}{y-x}$ days
7. If A, B and C can do a work in x,y, and z days respectively, then all of them working together can finish the work in $\frac{xyz}{xy + yz + xz}$ days
8. If two taps A and B take a and b hours reselectively to fill a tank, then the two taps together fill $\frac{1}{a} + \frac{1}{b}$ part of the tank in an hour

and the entire tank is filled in $\frac{1}{\left(\frac{1}{2} + \frac{1}{b}\right)} = \frac{ab}{(a+b)}$ hours.

Solved Examples:

1. 8 boys can arrange all the books of school library in 12 days. In how many days can 6 boys arrange them?

Ans: $M_1 D_1 = M_2 D_2$

$$\therefore D_2 = \frac{8 \times 12}{6} = 16 \text{ days}$$

2. A can do a piece of work in 12 days and B alone can do it in 15 days. How much time will both take to finish the work?

Ans: A's 1 day's work = $\frac{1}{12}$

B's 1 day's work = $\frac{1}{15}$

$$(A+B)'s \text{ 1 day's work} = \frac{1}{12} + \frac{1}{15} = \frac{3}{20}$$

\therefore Both together can finish the work in

$$\frac{20}{3} \text{ or } 6\frac{2}{3} \text{ days}$$

Using formula :

Time taken to finish the work

$$= \frac{xy}{x+y} = \frac{12 \times 15}{12 + 15}$$

$$= \frac{12 \times 15}{27} = \frac{20}{3} \text{ or } 6\frac{2}{3} \text{ days}$$

3. A and B together can do a piece of work in 12 days. B alone can finish it in 30 days. In

how many days can A alone finish the work?

Ans: (A+B)'s 1 day's work = $\frac{1}{12}$

B's 1 day's work = $\frac{1}{30}$

A's 1 day's work = $\frac{1}{12} - \frac{1}{30} = \frac{1}{20}$

∴ A alone can finish the work in 20 days

Using formula :

Time taken by A to finish the work = $\frac{xy}{y-x}$

= $\frac{12 \times 30}{30 - 12} = \frac{12 \times 30}{18} = 20$ days

4. 16 men can do a piece of work in 10 days. How many men are needed to complete the work in 40 days?

Ans: Using formula.

$M_1 D_1 = M_2 D_2$

$M_1 = 16, D_1 = 10, D_2 = 40$

$16 \times 10 = M_2 \times 40$

$M_2 = \frac{16 \times 10}{40} = 4$ men

5. A and B can do a piece of work in 18 days, B and C in 24 days, A and C in 36 days. In what time can they do it all working together?

Ans: [(A+B)+(B+C)+(A+C)]'s 1 day's work

= $\frac{1}{18} + \frac{1}{24} + \frac{1}{36} = \frac{1}{8}$

or 2 (A+B+C)'s 1 day's work = $\frac{1}{8}$

or (A+B+C)'s 1 day's work = $\frac{1}{16}$

So they all can finish the work in 16 days

6. 4 men and 6 women finish a job in 8 days, while 3 men and 7 women finish in 10 days. In

how many days will 10 women finish it?

Ans: Considering one day's work,

$4M + 6W = \frac{1}{8}$ (1)

$3M + 7W = \frac{1}{10}$ (2)

(1) x 3 - (2) x 4 gives

$18W - 28W = \frac{1}{10}$ or $10W = \frac{1}{40}$

∴ 10 Women can do the work in 40 days

7. A certain number of men complete a piece of work in 60 days. If there were 8 men more, the work could be finished in 10 days less. How many men were there originally?

Ans: Let the original number of men be x

$M_1 = x, D_1 = 60, M_2 = x+8, D_2 = 50,$

$M_1 D_1 = M_2 D_2$

$x \times 60 = (x+8) \times 50$

$60x - 50x = 400$

$10x = 400 \Rightarrow x = 40$

8. A cistern can be filled separately by two pipes in 12 and 16 minutes respectively. If both pipes are opened together, when will the cistern be filled?

Ans: Work done by 1st pipe in 1 minute = $\frac{1}{12}$

Work done by 2nd pipe in 1 minute = $\frac{1}{16}$

Work done by both in 1 minute = $\frac{1}{12} + \frac{1}{16}$

= $\frac{4+3}{48} = \frac{7}{48}$

∴ Both the pipes together will fill the cistern in $\frac{48}{7}$ minutes ie, $6\frac{6}{7}$ minutes.

Using formula :

Time taken to fill the cistern by both the pipes

$$= \frac{ab}{a+b} = \frac{12 \times 16}{12+16} = 6 \frac{6}{7} \text{ minutes}$$

9. Two inlet pipes of filling rate 10 minutes per cistern and 6 minutes per cistern and one outlet pipe of emptying rate 15 minutes per cistern are all fitted to a cistern and are opened together. Find when the cistern will be full?

Part of the cistern filled by working the three pipes in one minute.

$$= \frac{1}{10} + \frac{1}{6} - \frac{1}{15} = \frac{1}{5}$$

\therefore Time needed to fill the full cistern = 5 minutes

10. A cistern can be filled separately by two pipes A and B in 36 minutes and 45 minutes respectively. A tap C at the bottom can empty the full cistern in 30 minutes. If the tap C is opened 7 minutes after the pipes A and B are opened, find when the cistern becomes full.

Ans: Part of the tank filled by A and B in 7 minutes

$$= 7 \times \left(\frac{1}{36} + \frac{1}{45} \right) = \frac{7}{20}$$

$\therefore 1 - \frac{7}{20} = \frac{13}{20}$ of the tank should be

filled.

Part filled by A, B and C in 1 minutes

$$= \frac{1}{36} + \frac{1}{45} - \frac{1}{30} = \frac{1}{60}$$

\therefore Time needed to fill $\frac{13}{20}$ part of the tank

$$= \frac{13}{20} \times 60 = 39 \text{ minutes}$$

\therefore Total time taken to fill the tank

$$= 39 + 7 = 46 \text{ minutes}$$

PRACTICE TEST

1. Ramesh alone does a piece of work in 4 days and Suresh does it in 12 days. In how many days will the two do it together?

- a) 3 days b) $1\frac{1}{2}$ days
c) 4 days d) 8 days

2. Pranesh and Sumesh can finish a work in 16 days while Pranesh can do the same work in 24 days. In how many days can Sumesh alone finish the same work?

- a) 40 days b) 25 days
c) 48 days d) 20 days

3. Vinod can do a work in 15 days, Vijay in 25 days and Vinay in 30 days. How long will they take to do the work if they work together?

- a) 12 days b) $7\frac{1}{7}$ days
c) 70 days d) 20 days

4. If A, B and C together can finish a piece of work in 4 days, A alone in 12 days and B in 18 days, then C alone can do it in

- a) 21 days b) 15 days
c) 12 days d) 9 days

5. 3 men or 6 women can do a piece of work in 20 days. In how many days will 12 men and 8 women do the same work?

- a) $\frac{7}{2}$ b) $\frac{15}{4}$
c) 5 d) 4

6. Some persons can do a piece of work in 12 days. Two times the number of those persons will do half of that work in

- a) 3 days b) 4 days
c) 6 days d) 12 days

7. 3 men can do a work in 6 days. After 2 days 3 more men joined them. How many days will they take to complete the remaining work?

- a) 5 days b) 4 days
c) 3 days d) 2 days

8. A is twice as good a workman as B and they took 7 days together to do the work. B alone can do it in:

- a) 12 days b) 18 days
c) 21 days d) 16 days

9. A can do a piece of work in 25 days and B can do the same work in 30 days. They work together for 5 days and then A leaves. B will finish the remaining work in
- a) 21 days b) 11 days
c) 20 days d) 19 days
10. An army of 2000 men had enough food to last for 30 days. After 10 days 500 more men joined them. How long did the food last then?
- a) 20 days b) 15 days
c) 12 days d) 16 days
11. Amar can do a piece of work in 15 days. When he had worked for 3 days, Sameer joined him and the remaining work was finished in 8 days. In how many days can Sameer alone finish the whole work?
- a) 30 days b) 27 days
c) 20 days d) 24 days
12. A, B and C can do a piece of work in 18 days, 27 days and 36 days respectively. They start working together. After working for 4 days, A goes away and B leaves 7 days before the work is finished. Only C remains at work from beginning to end. In how many days was the whole work done?
- a) 17 days b) 18 days
c) 16 days d) 15 days
13. A and B can do a piece of work in 6 days. B and C in 4 days and A and C in 5 days. How long will they take to complete the work if they work together?
- a) $3\frac{9}{37}$ days b) 15 days
c) $1\frac{23}{37}$ days d) $6\frac{9}{37}$ days
14. A man, a woman or a boy can do a piece of work in 3,4 and 12 days respectively. How many boys must assist 1 man and 1 women to do the work in 1 day?
- a) 5 boys b) 6 boys
c) 2 boys d) 20 boys
15. Two pipes can fill a tank in 9 hours and 12 hours respectively. In how much time will they fill the tank when opened together?
- a) $3\frac{1}{2}$ hours b) $5\frac{1}{7}$ hours
c) 5 hours d) $3\frac{2}{3}$ hours
16. A tap can fill a tank in 8 hours and another can empty it in 16 hours. If both the taps are opened simultaneously, the time (in hours) to fill the tank is:
- a) 8 b) 10 c) 16 d) 24
17. A cistern can be filled by a pipe in 15 hours. But due to a leak in the bottom the cistern is just full in 20 hours. When the cistern is full, the leak can empty it in:
- a) 60 hours b) 40 hours
c) 45 hours d) 30 hours
18. A cistern can be filled by pipes A and B in 20 hours and 30 hours respectively. When full, the tank can be emptied by pipe C in 60 hours. If all the taps be turned on at the same time the cistern will be full in
- a) 10 hours b) 15 hours
c) 16 hours d) 30 hours
19. Two pipes A and B can separately fill a tank in 12 minutes and 15 minutes respectively. Both the pipes are opened together but 4 minutes after the start, pipe A is turned off. How much time will it take to fill the tank?
- a) 11 min b) 12 min
c) 6 min d) 8 min
20. Two pipes A and B can fill a cistern in 24 minutes and 32 minutes respectively. If both the pipes are opened together, then after how many minutes B should be closed so that the tank is full in 18 minutes?
- a) 6 b) 8 c) 10 d) 12

ANSWERS TO PRACTICE TEST

1. (a) 2. (c) 3. (b) 4. (d) 5. (b) 6. (a) 7. (d) 8.(c)
9. (d) 10. (d) 11. (a) 12. (c) 13. (a) 14. (a) 15. (b) 16.(c)
17. (a) 18. (b) 19. (c) 20. (b)

Simple & Compound Interest

Interest is the money paid by the borrower to the lender for the use of money lent. Interest is of two kinds, simple and compound. Money borrowed or deposited is called the principal. The sum of principal and interest is called the amount.

i) Simple Interest:

If the interest on a certain sum borrowed for a certain period is reckoned uniformly, it is called Simple Interest.

The simple interest (I) for a principal (P) for (N) years at (R) rate percent per annum is

$$I = \frac{PNR}{100}$$

$$P = \frac{Ix100}{NxR}; R = \frac{Ix100}{PxN}; N = \frac{Ix100}{PxR}$$

ii) Compound Interest:

Money is said to be lent at Compound Interest if the interest is not paid as soon as it falls due, but is added to the principal after a fixed period, so that the amount at the end of the period, becomes the principal for the next period.

a) When interest is compounded annually:

$$\text{Amount} = P \left(1 + \frac{R}{100}\right)^N$$

b) When interest is compounded half yearly:

$$\text{Amount} = P \left(1 + \frac{R}{200}\right)^{2N}$$

c) When interest is compounded quarterly

$$\text{Amount} = P \left(1 + \frac{R}{400}\right)^{4N}$$

$$d) \text{ C.I.} = P \left(1 + \frac{R}{100}\right)^N - P \quad (\text{or})$$

$$= P \left[\left(1 + \frac{R}{100}\right)^N - 1 \right]$$

Solved Examples:

1. Calculate the amount on Rs. 4480 at 8% per annum for 3 years.

$$\text{Ans: S.I.} = \frac{PxNxR}{100}$$

$$= \text{Rs.} \frac{4480 \times 3 \times 8}{100} = \text{Rs.} 1075.20$$

$$\begin{aligned} \text{Amount} &= \text{Rs.} (4480 + 1075.20) \\ &= \text{Rs.} 5555.20 \end{aligned}$$

2. S.I. on Rs. 1500 at 7% per annum for a certain time is Rs. 210. Find the time

$$\text{Ans: Time, } N = \frac{210 \times 100}{1500 \times 7} = 2 \text{ years}$$

3. A certain sum of money at simple interest amounts to Rs. 1260 in 2 years and to Rs. 1350 in 5 years. The rate percent per annum is -----

$$\text{Ans: S.I. for 3 years} = \text{Rs.} (1350 - 1260) = \text{Rs.} 90$$

$$\therefore \text{ S.I. for 2 years} = \text{Rs.} \frac{90}{3} \times 2 = \text{Rs.} 60$$

$$\text{Principal} = \text{Rs.} (1260 - 60) = \text{Rs.} 1200$$

$$\text{Rate, } R = \frac{100 \times 60}{1200 \times 2} \% = 2.5\%$$

4. A man invested $\frac{1}{3}$ of his capital at 7%, $\frac{1}{4}$ at 8% and the remainder at 10%. If his annual income is Rs. 561, the capital is -----

Let the capital be Rs. x Then,

$$\frac{x}{3} \times \frac{7}{100} \times 1 + \frac{x}{4} \times \frac{8}{100} \times 1 + \frac{5x}{12} \times \frac{10}{100} \times 1 = 561$$

$$\Rightarrow \frac{7x}{300} + \frac{8x}{400} + \frac{50x}{1200} = 561$$

$$\Rightarrow \frac{102x}{1200} = 561$$

$$\Rightarrow x = \frac{561 \times 1200}{102} = \text{Rs. } 6,600$$

5. Find the sum of money which increases $\frac{1}{10}$ of itself every year and amounts to Rs. 450 in 5 years at S.I.

Ans: Let P = Rs. 100

$$\text{S.I.} = \text{Rs. } 100 \times \frac{1}{10} = \text{Rs. } 10$$

$$\text{S.I. for 5 years} = \text{Rs. } 50$$

$$\begin{aligned} \text{Amount after 5 years} &= 100 + 50 \\ &= \text{Rs. } 150 \end{aligned}$$

$$\text{If the amount is Rs. } 150, P = \text{Rs. } 100$$

$$\therefore \text{If the amount is Rs. } 450,$$

$$P = \frac{100 \times 450}{150} = \text{Rs. } 300$$

6. A sum was put at simple interest at a certain rate for 2 years. Had it been put at 1% higher rate, it would have fetched Rs. 24 more. Find the sum.

Ans: Let the sum be Rs. x and rate be R% and (R+1)%

$$\text{Then, } \frac{xx(R+1) \times 2}{100} - \frac{xxR \times 2}{100} = 24$$

$$\Rightarrow \frac{2xR}{100} + \frac{2x}{100} - \frac{2xR}{100} = 24$$

$$x = \frac{24 \times 100}{2} = \text{Rs. } 1,200$$

7. Find compound interest on Rs. 5,000 at 10% per annum for 3 years

$$\text{Ans: Amount} = P \left(1 + \frac{R}{100}\right)^N$$

$$= 5000 \left(1 + \frac{10}{100}\right)^3 = \text{Rs. } 6,655$$

\therefore Compound Interest

$$= \text{Rs. } (6,655 - 5,000) = \text{Rs. } 1,655$$

8. If the compound interest on a certain sum for 3 years at 20% per annum is Rs. 728, find the simple interest.

$$\text{Ans: Given that } P \left(1 + \frac{20}{100}\right)^3 - P = \text{Rs. } 728$$

$$\text{or } 1.728 P - P = \text{Rs. } 728$$

$$\therefore P = \text{Rs. } 1,000$$

$$\text{Now, S.I.} = \text{Rs. } \frac{1000 \times 3 \times 20}{100} = \text{Rs. } 600$$

9. The difference between the compound interest and the simple interest on a certain sum at 10% per annum for two years is Rs. 60. Find the sum.

Ans: Let the sum be Rs. x

$$\text{So, S.I.} = \text{Rs. } \frac{xx \times 10 \times 2}{100} = \text{Rs. } \frac{x}{5}$$

$$\text{C.I.} = \text{Rs. } x \left[1 + \frac{10}{100}\right]^2 - x = \text{Rs. } \frac{21x}{100}$$

$$\therefore \text{C.I.} - \text{S.I.} = \frac{21x}{100} - \frac{x}{5} = \frac{x}{100} = \text{Rs.60}$$

$$\therefore x = \text{Rs. } 6,000$$

PRACTICE TEST

1. At what rate percent per annum will a sum of Rs. 3,600 become Rs. 4,500 in 10 years at simple interest?

- a) 5% b) 2.5%
c) 10% d) 6.75%

2. A sum of Rs. 1600 lent at simple interest at 12.5% per annum will become double in

- a) 6 years b) $7\frac{1}{2}$ years
c) 8 years d) $9\frac{1}{4}$ years

3. The difference in simple interest at 13% and 12% p.a. of a sum in one year is Rs. 110. Then the sum is

- a) Rs. 13,000 b) Rs. 15,000
c) Rs. 10,000 d) Rs. 11,000

4. The difference in the interests received from two different banks on Rs. 1000 for 2 years is Rs. 20. Thus, the difference in their rates is

- a) 2% b) 1%
c) 1.5% d) 0.5%

5. Find out the capital required to earn a monthly interest of Rs. 600 at 6% simple interest.

- a) Rs. 1 lakhs b) Rs. 1.2 lakhs
c) Rs. 1.1 lakhs d) Rs. 1.3 lakhs

6. A man invested $\frac{1}{3}$ rd of the sum at 7%, $\frac{1}{4}$ th at 8% and the remaining at 10% for one year. If the annual interest is Rs. 408, then the investment is

- a) Rs. 8,400 b) Rs. 4,800
c) Rs. 5,000 d) Rs. 7,200

7. The difference in simple interest on a certain sum of money for 3 years and 5 years at 18% per annum is Rs. 2,160. Then the sum is

- a) Rs. 6,500 b) Rs. 4,500
c) Rs. 6,000 d) Rs. 7,500

8. At what rate percent per annum simple interest will a sum of money triple itself in 25 years?

- a) 8 b) $8\frac{1}{3}$
c) $9\frac{1}{11}$ d) 10

9. What sum of money lent out at compound interest will amount to Rs. 968 in 2 years at 10% per annum, interest being charged annually?

- a) Rs. 900 b) Rs. 825
c) Rs. 780 d) Rs. 800

10. The difference between compound interest and simple interest on certain sum of money in 2 years at 4% per annum is Rs. 50. Find the sum

- a) Rs. 30,550 b) Rs. 31,250
c) Rs. 25,670 d) Rs. 35,400

11. A sum of money lent at compound interest amounts to Rs. 1210 in two years and to Rs. 1464.10 in 4 years. Find the rate of interest.

- a) 12% b) 10%
c) 8% d) 15%

12. A man borrows Rs. 4,000 at 8% per annum on compound interest. At the end of every year he pays Rs. 1,500 as part payment of loan and interest. How much does he still owe to the bank after 3 such annual payments?

- a) Rs. 1,799 b) Rs. 169.25
c) Rs. 2,000 d) Rs. 234

ANSWERS TO PRACTICE TEST

1. (b) 2. (c) 3. (d) 4. (b) 5. (b) 6. (b) 7. (c) 8.(a)
9. (d) 10. (b) 11. (b) 12. (b)

Problems on Age

Solved Examples

1. A father was 4 times as old as his son 8 years ago. Eight years hence, father will be twice as old as his son. Find their present ages.

Ans: Let son's age 8 years ago be x years.

Thus, father's age at that time = $4x$ years

After 8 years, son's age

$$= (x-8) + 8 = (x-16) \text{ years}$$

After 8 years, father's age

$$= (4x-8)+8 = (4x+16) \text{ years}$$

$$\therefore 2(x-16) = 4x + 16 \text{ or } x=8$$

\therefore The present age of the son = $x-8 = 16$ years

\therefore The present age of the father

$$= 4x-8 = 32+8= 40 \text{ years}$$

2. A is twice as old as B was two years ago. If the difference in their ages be 2 years, find A's age.

Ans: Let B's age 2 years ago be x years

$$\therefore \text{A's present age} = 2x \text{ years}$$

$$\text{Also } 2x - (x-2) = 2 \text{ or } x=4$$

$$\therefore \text{A's age} = 2x = 8 \text{ years}$$

3. The age of a father 10 years ago was thrice the age of his son. Ten years hence, the father's age will be twice that of his son. The ratio of their present ages is:

Ans: Let the present ages of father and son be x and y years respectively.

$$\text{Then } (x-10) = 3(y-10) \text{ or}$$

$$3y-x = 20 \text{ ----- (1)}$$

$$\text{and } (x+10) = 2(y+10) \text{ or}$$

$$x-2y = 10 \text{ ----- (2)}$$

$$(1) + (2) \Rightarrow y = 30$$

Substituting $y = 30$ in equation (1) we get $x = 70$

$$\text{Ratio of their ages} = 70 : 30 \text{ or } 7:3$$

4. Ratio of Ashok's age to Pradeep's age is equal to 4:3. Ashok will be 26 years old after 6 years. How old is Pradeep now?

Ans: Ashok's present age = $(26-6)$

$$= 20 \text{ years}$$

$$\text{Pradeep's present age} = 20 \times \frac{3}{4}$$

$$= 15 \text{ years}$$

5. The ratio of the ages of father and son at present is 6:1. After 5 years the ratio will become 7:2. The present age of the son is:

Ans: Let their present ages be $6x$ and x years respectively.

$$\text{Then } \frac{6x+5}{x+5} = \frac{7}{2}$$

$$= 2(6x+5) = 7(x+5) \Rightarrow x=5$$

$$\text{Present age of the son} = 5 \text{ years.}$$

6. Three years ago the average age of A and B was 18 years. With C joining them now, the average becomes 22 years. How old is C now?

Ans: (A+B)'s total present age

$$= (2 \times 18 + 3 + 3) = 42 \text{ years}$$

$$(A+B+C)'s \text{ total present age}$$

$$= 22 \times 3 = 66 \text{ years}$$

$$\text{C's age} = 66 - 42 = 24 \text{ years}$$

PRACTICE TEST

1. A father is twice as old as his son. 20 years ago, the age of the father was 12 times the age of the son. The present age of the son is
 - a) 44 years
 - b) 22 years
 - c) 40 years
 - d) 20 years
2. The respective ages of a father and his son are 41 and 16 years. In how many years will the father be twice as old as his son?
 - a) 19 years
 - b) 9 years
 - c) 15 years
 - d) 10 years
3. The ratio of ages of Mohan and Sohan is 4:3. The sum of their ages is 42 years. The age of Mohan is
 - a) 24 years
 - b) 18 years
 - c) 32 years
 - d) 30 years.
4. The ratio of ages of Rani and Vinita is 3:5. The difference in their ages is 12 years. Then the age of Vinita is
 - a) 20 years
 - b) 15 years
 - c) 18 years
 - d) 30 years
5. Two years ago, Vinod was four times as old as Indhu. 8 years hence, Vinod's age will exceed Indhu's age by 12 years. The ratio of the present ages of Vinod and Indhu
 - a) 5:1
 - b) 4:1
 - c) 3:1
 - d) 2:1
6. The ages of A and B are in the ratio 3:5. After 9 years the ratio of their ages will be 3:4. The present age of B is
 - a) 9 years
 - b) 15 years
 - c) 20 years
 - d) 16 years
7. A's mother was four times as old as A ten years ago. After 10 years she will be twice as old as A. Then, A's present age is
 - a) 30 years
 - b) 25 years
 - c) 20 years
 - d) 15 years
8. A father's age is three times the sum of the ages of his two children, but 20 years hence his age will be equal to sum of their ages. Then the father's age is -
 - a) 30 years
 - b) 40 years
 - c) 35 years
 - d) 45 years
9. The ratio of the father's age to the son's age is 4:1. The product of their ages is 196. The ratio of their ages after 5 years will be:
 - a) 3:1
 - b) 10:3
 - c) 11:4
 - d) 14:5
10. In 10 years, A will be twice as old as B was 10 years ago. If A is now 9 years older than B, find the present age of B.
 - a) 39
 - b) 27
 - c) 45
 - d) 26
11. A is as much younger than B as he is older than C. If the sum of B's and C's ages is 40 years, find the age of A.
 - a) 40 years
 - b) 10 years
 - c) 25 years
 - d) 20 years
12. The ages of Ram and Mohan differs by 16 years. Six years ago, Mohan's age was thrice as that of Ram's. Then Ram's present age is
 - a) 15 years
 - b) 20 years
 - c) 14 years
 - d) 30 years
13. A father is 4 times as old as his son; in 20 years he will be only twice as old as his son. Then the respective ages of father and son are
 - a) 40, 10 years
 - b) 80, 20 years
 - c) 60, 15 years
 - d) 48, 12 years
14. The difference between the ages of two persons is 8 years. 15 years ago, the elder one was twice as old as the younger one. Then the present age of the elder person is
 - a) 23 years
 - b) 31 years
 - c) 34 years
 - d) 40 years

ANSWERS TO PRACTICE TEST

1. (b) 2. (b) 3. (a) 4. (d) 5. (c) 6. (b) 7. (c) 8.(a)
9. (c) 10. (a) 11. (d) 12. (c) 13. (a) 14. (b)

L.C.M & H.C.F

Factors and Multiples: If a number 'm' divides another number 'n' exactly, then we say that 'm' is a factor of 'n' and that 'n' is a multiple of 'm'.

eg. 3 is a factor of 12 and therefore 12 is a multiple of 3.

Least Common Multiple (L.C.M.)

L.C.M. is the least non-zero number in common multiples of two or more numbers.

Multiple of 6 = 6, 12, 18, 24, 30,

Multiple of 8 = 8, 16, 24, 32, 40,

Common Multiple of 6 and 8 = 24, 48

.....

Least Common Multiple = 24

Factorisation Method:

Find the L.C.M. of 12, 27 and 40

Factors of 12

$$= 2 \times 2 \times 3 = 2^2 \times 3$$

Factors of 27

$$= 3 \times 3 \times 3 = 3^3$$

Factors of 40

$$= 2 \times 2 \times 2 \times 5 = 2^3 \times 5$$

| | | | | | |
|---|----|---|----|---|----|
| 2 | 12 | 3 | 27 | 2 | 40 |
| | 6 | | 9 | | 20 |
| | 3 | | 3 | | 10 |
| | 1 | | 1 | | 5 |
| | | | | | 1 |

$$\therefore \text{L.C.M.} = 2^3 \times 3^3 \times 5 = 1080$$

SHORTCUT METHOD

(Division Method)

Find the L.C.M. of 12, 27, 40

| | |
|---|------------|
| 2 | 12, 27, 40 |
| 2 | 6, 27, 20 |
| 3 | 3, 27, 10 |
| | 1, 9, 10 |

$$\therefore \text{L.C.M.} = 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 1080$$

HIGHEST COMMON FACTOR (H.C.F)

The highest common factor of two or more numbers is the greatest number which divides each of them exactly.

eg. Find the H.C.F. of 24 and 56

Factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24

Factors of 56 = 1, 2, 4, 7, 8, 14, 28, 56

Common factors of 24 and 56 are 1, 2, 4, 8

\therefore H.C.F. of 24 and 56 = 8

Factorisation Method: H.C.F. can be found by resolving the given numbers into prime factors and then taking the product of least powers of all common factors, that occur in these numbers.

Eg. Find H.C.F. of 48, 108, 140

| | | | | | | |
|--|---|----|---|-----|---|-----|
| Factors of 48 | 2 | 48 | 2 | 108 | 2 | 140 |
| = 2x2x2x2x3 | | 24 | | 54 | | 70 |
| = 2 ⁴ x3 | | 12 | | 27 | | 35 |
| Factors of 108 | | 6 | | 9 | | 7 |
| = 2x2x3x3x3 = 2 ² x3 ³ | | 3 | | 3 | | |

Factors of 140

$$= 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$$

H.C.F. = 2² = 4

Division Method

Find the H.C.F. of 48, 108, 140

| | |
|---|--------------|
| 2 | 48, 108, 140 |
| 2 | 24, 54, 70 |
| | 12, 27, 35 |

H.C.F.=2x2= 4

QUICKER & SHORT CUT METHOD

Find the H.C.F. of 777 and 1147

$$\begin{array}{r} 777) 1147 \quad (1 \\ \underline{777} \\ 370) 777 \quad (2 \\ \underline{740} \\ 37) 370 \quad (10 \\ \underline{370} \\ 0 \end{array}$$

H.C.F. of 777 and 1147 is 37

* The product of two given numbers is equal to the product of their H.C.F. and L.C.M.

L.C.M. of two numbers

$$= \frac{\text{Product of numbers}}{\text{H.C.F. of numbers}}$$

L.C.M. of given fractions

$$= \frac{\text{L.C.M. of numerators}}{\text{H.C.F. of denominators}}$$

H.C.F. of given fractions

$$= \frac{\text{H.C.F. of numerators}}{\text{L.C.M. of denominators}}$$

- ◆ The L.C.M. of a given set of numbers would be either the highest or higher than the highest of the given numbers.
- ◆ The H.C.F. of a given set of numbers would be either the lowest or lower than the lowest.

Solved Examples

1. Find the L.C.M. of 125, 64, 8 and 3.

Ans : Given numbers are 5^3 , 2^6 , 2^3 and 3

$$\therefore \text{L.C.M. } 5^3 \times 2^6 \times 3 = 24,000$$

2. Find the L.C.M. of $\frac{1}{3}, \frac{5}{6}, \frac{5}{9}, \frac{10}{27}$?

Ans: L.C.M. of fractions

$$= \frac{\text{L.C.M. of numerators}}{\text{H.C.F. of denominators}}$$

L.C.M. of 1, 5 and 10 is 10

H.C.F. of 3, 6, 9 and 27 is 3

$$\text{L.C.M. of given fractions} = \frac{10}{3}$$

3. Find the H.C.F. of $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{9}{10}$

Ans: H.C.F. of fractions

$$= \frac{\text{H.C.F. of numerators}}{\text{L.C.M. of denominators}}$$

H.C.F. of 1, 3, 5, 7 and 9 is 1

L.C.M. of 2, 4, 6, 8 and 10 is 120

$$\text{H.C.F. of given fractions} = \frac{1}{120}$$

4. The L.C.M. of two number is 2310. Their H.C.F. is 30. If one number is 210, the other is:

Ans: The other number

$$= \frac{\text{L.C.M.} \times \text{H.C.F.}}{\text{given number}} = \frac{2310 \times 30}{210} = 330$$

5. The H.C.F. and L.C.M. of two numbers are 44 and 264 respectively. If the first number is divided by 2, the quotient is 44, The other number is

Ans: First number = $2 \times 44 = 88$

$$\text{Second number} = \frac{44 \times 264}{88} = 132$$

6. The least square number which is divisible by 6, 8 and 15 is:

Ans: The least number divisible by 6, 8 and 15 is their L.C.M. which is 120

$$\text{Now } 120 = 2 \times 2 \times 2 \times 3 \times 5$$

To make it a perfect square, it must be multiplied by $2 \times 3 \times 5$

$$\therefore \text{Required Number} = 120 \times 2 \times 3 \times 5 = 3600$$

7. The least number of square tiles required to pave the ceiling of a room 15m 17cm long and 9m. 2cm broad is:

Ans: Size of largest square tile

= H.C.F. of 1517 cm and 902 cm

= 41 cm.

∴ Least number of tiles required

$$\begin{aligned} &= \frac{\text{Area of the room}}{\text{Area of one tile}} \\ &= \frac{1517 \times 902}{41 \times 41} = 814 \end{aligned}$$

8. Find the least number which when divided separately by 15, 20, 36 and 48 leaves 3 as remainder in each case.

Ans : Required number

= L.C.M. of (15, 20, 36 and 48) + 3

= 720 + 3 = 723

9. Find the greatest number that will divide 197 and 269 and leaves 5 as remainder in each case.

Required number = H.C.F. of [(197-5) and (269-5)]

= H.C.F. of (192 and 264) = 8

12. Five bells begin to toll together and toll respectively at intervals of 6, 7, 8, 9 and 12 seconds. How many times they will toll together in one hour, excluding the one at the start?

Ans: L.C.M. of 6, 7, 8, 9 and 12

= $2 \times 2 \times 3 \times 7 \times 2 \times 3 = 504$

ie, The bells will toll together after each 504 seconds. In one hour, they will toll together

$$= \frac{60 \times 60}{504} = 7 \text{ times}$$

PRACTICE TEST

1. Find the L.C.M of 12, 15, 18 and 27.

a) 1,080 b) 540

c) 270 d) 760

2. Find the H.C.F. of 72, 48 and 30.

a) 30 b) 12 c) 6 d) 3

3. Find the L.C.M. of $2^2 \times 3^3 \times 5^3$ and $2^3 \times 3^2 \times 5$.

a) 27,000 b) 180

c) 36 d) 13,500

4. Find the L.C.M. of $\frac{2}{5}$, $\frac{3}{10}$ and $\frac{4}{15}$

a) $\frac{1}{30}$ b) $2\frac{2}{5}$

c) $\frac{24}{750}$ d) $\frac{2}{5}$

5. Find the H.C.F. of $\frac{4}{5}$, $\frac{3}{10}$ and $\frac{7}{15}$

a) $\frac{1}{5}$ b) $\frac{84}{5}$

c) $\frac{84}{30}$ d) $\frac{1}{30}$

6. If the L.C.M of x and y is z, their H.C.F. is.

a) $\frac{xy}{z}$ b) xyz

c) $\frac{x+y}{z}$ d) $\frac{z}{xy}$

7. H.C.F of two numbers is 24 and their L.C.M is 1080. If one of the numbers is 120, find the other.

a) 216 b) 532 c) 108 d) 820

8. L.C.M. of 2.5, 0.5 and 0.175 = ?

a) 2.5 b) 0.5

c) 0.175 d) 17.5

9. H.C.F. of two numbers is 24 and their L.C.M is 1344. If the difference between the numbers is 80, their sum is:

a) 368 b) 356

c) 332 c) 304

10. Find the greatest number which can divide 1354, 1866 and 2762 leaving the same remainder 10 in each case.

a) 64 b) 124 c) 156 d) 260

11. Find the least perfect square which is di-

visible by 3, 4, 5, 6 and 8.

- a) 2500 b) 1200
c) 3600 d) 900

12. The least number which when divided by 15, 27, 35 and 42 leaves in each case a remainder 7 is:

- a) 1897 b) 1987
c) 1883 d) 2007

13. Two containers contain 60 and 165 litres of milk respectively. Find the maximum capacity of a container which can measure the milk in each container an exact number of times (in litres)

- a) 15 b) 3 c) 5 d) 10

14. Two baskets contain 195 and 250 bananas respectively, which are distributed in equal number among children. Find the largest number of bananas that can be given, so that 3 bananas are left over from the first basket and 2 from the second.

- a) 4 b) 18 c) 8 d) 6

Qn: (15- 18) :- Write in ascending order

15. $\frac{1}{2}, \frac{2}{5}, \frac{3}{4}, \frac{3}{2}$

a) $\frac{2}{5}, \frac{1}{2}, \frac{3}{4}, \frac{3}{2}$ b) $\frac{3}{4}, \frac{1}{2}, \frac{2}{5}, \frac{3}{2}$

c) $\frac{3}{2}, \frac{3}{4}, \frac{1}{2}, \frac{2}{5}$ d) $\frac{3}{2}, \frac{1}{2}, \frac{2}{5}, \frac{3}{4}$

16. $\frac{5}{3}, \frac{11}{9}, \frac{5}{6}, \frac{7}{12}$

a) $\frac{11}{9}, \frac{7}{12}, \frac{5}{3}, \frac{5}{6}$ b) $\frac{7}{12}, \frac{5}{6}, \frac{11}{9}, \frac{5}{3}$

c) $\frac{5}{6}, \frac{7}{12}, \frac{11}{9}, \frac{5}{3}$ d) $\frac{5}{3}, \frac{11}{9}, \frac{5}{6}, \frac{7}{12}$

17. $\frac{5}{6}, \frac{7}{8}, \frac{3}{4}, \frac{1}{3}$

a) $\frac{7}{8}, \frac{1}{3}, \frac{3}{4}, \frac{5}{6}$ b) $\frac{5}{6}, \frac{7}{8}, \frac{3}{4}, \frac{1}{3}$

c) $\frac{3}{4}, \frac{7}{8}, \frac{1}{3}, \frac{3}{4}$ d) $\frac{1}{3}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}$

Qn: 18- 20 Write in descending order

18. $\frac{1}{3}, \frac{2}{5}, \frac{3}{4}, \frac{1}{6}$

a) $\frac{1}{3}, \frac{2}{5}, \frac{3}{4}, \frac{1}{6}$ b) $\frac{1}{6}, \frac{2}{5}, \frac{3}{4}, \frac{1}{3}$

c) $\frac{2}{5}, \frac{3}{4}, \frac{1}{3}, \frac{1}{6}$ d) $\frac{3}{4}, \frac{2}{5}, \frac{1}{3}, \frac{1}{6}$

19. $\frac{5}{6}, \frac{7}{8}, \frac{11}{12}, \frac{3}{10}$

a) $\frac{5}{6}, \frac{7}{8}, \frac{11}{12}, \frac{3}{10}$ b) $\frac{7}{8}, \frac{5}{6}, \frac{11}{12}, \frac{3}{10}$

c) $\frac{11}{12}, \frac{7}{8}, \frac{5}{6}, \frac{3}{10}$ d) $\frac{7}{8}, \frac{5}{6}, \frac{11}{12}, \frac{3}{10}$

20. $\frac{5}{3}, \frac{11}{9}, \frac{5}{6}, \frac{7}{12}$

a) $\frac{5}{3}, \frac{11}{9}, \frac{5}{6}, \frac{7}{12}$ b) $\frac{11}{9}, \frac{5}{3}, \frac{7}{12}, \frac{5}{6}$

c) $\frac{5}{3}, \frac{11}{9}, \frac{5}{6}, \frac{7}{12}$ d) $\frac{11}{9}, \frac{5}{6}, \frac{5}{3}, \frac{7}{12}$

Qn 21-23 Find the greatest of the given fractions

21. $\frac{2}{3}, \frac{4}{15}, \frac{3}{5}, \frac{3}{4}$

a) $\frac{4}{15}$ b) $\frac{3}{4}$

c) $\frac{3}{5}$ d) $\frac{2}{3}$

22. $\frac{5}{8}, \frac{6}{11}, \frac{13}{22}, \frac{9}{13}$

- a) $\frac{5}{8}$ b) $\frac{6}{11}$
 c) $\frac{13}{22}$ d) $\frac{9}{13}$
23. $\frac{3}{4}, \frac{5}{7}, \frac{2}{3}, \frac{8}{11}$
 a) $\frac{3}{4}$ b) $\frac{5}{7}$
 c) $\frac{2}{3}$ d) $\frac{8}{11}$

Qn: (24 - 26) Find the smallest of the given fraction.

24. $\frac{2}{3}, \frac{5}{7}, \frac{9}{13}, \frac{9}{14}, \frac{7}{4}$
 a) $\frac{9}{14}$ b) $\frac{2}{3}$
 c) $\frac{7}{4}$ d) $\frac{5}{7}$
25. $\frac{11}{14}, \frac{14}{17}, \frac{17}{20}, \frac{23}{26}, \frac{29}{32}$
 a) $\frac{29}{32}$ b) $\frac{11}{14}$
 c) $\frac{17}{20}$ d) $\frac{14}{17}$
26. $\frac{5}{6}, \frac{3}{4}, \frac{5}{8}, \frac{6}{7}$
 a) $\frac{3}{4}$ b) $\frac{6}{7}$
 c) $\frac{5}{8}$ d) $\frac{5}{6}$

27. A heap of stones can be made in groups of 21 but when made up into groups of 16, 20, 25 and 45 there are 3 stones left in each case, The number of stones in the heap is
 a) 3600 b) 3603
 c) 7200 d) 7203
28. Three measuring rods are 64cm, 80cm

- and 96 cm in length. The least length of cloth (in metres) that can be measured exact number of times using any of the three rods is
 a) 0.96m b) 9.6m
 c) 96m d) 960m
29. The largest number, which exactly divides the product of any three consecutive integers is
 a) 2 b) 3 c) 6 d) 12
30. The L.C.M. of two numbers is 63 and their H.C.F. is 9. If one of the numbers is 27, the other number will be
 a) 9 b) 21 c) 17 d) 189
31. The HCF of two numbers is 32 and their product is 10240. Find their L.C.M?
 a) 640 b) 320 c) 324 d) 230
32. A gardener had a number of shrubs to plant in rows. At first he tried to plant 8, then 12 and then 16 in a row but he had always 3 shrubs left with him. On trying 7 he had none left. Find the total number of shrubs.
 a) 147 b) 150 c) 137 d) 154
33. Six bells commencing tolling together and toll at intervals of 2,4,6,8,10 and 12 seconds respectively. In 30 minutes, how many times do they toll together.
 a) 17 b) 15 c) 16 d) 20
34. In a seminar the number of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively. Find the minimum number of rooms required, where in each room the same number of participants are to be seated; and all of them being in the same subject.
 a) 20 b) 22 c) 25 d) 21

ANSWERS TO PRACTICE TEST - 6

1. (b) 2. (c) 3. (a) 4. (b) 5. (d)
 6. (a) 7. (a) 8. (d) 9. (a) 10. (a)
 11. (c) 12. (a) 13. (a) 14. (c) 15. (a)
 16. (b) 17. (d) 18. (d) 19. (c) 20. (a)
 21. (b) 22.(d) 23. (a) 24. (a) 25. (b)
 26. (c) 27. (d) 28.(b) 29. (c) 30. (b)
 31. (b) 32. (a) 33. (b) 34. (d)