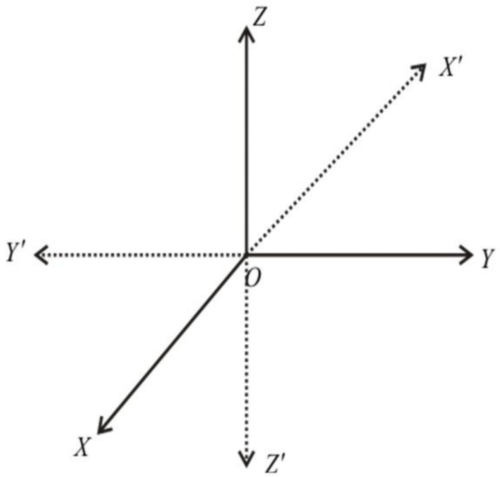
CHAPTER 12

INTRODUCTION TO 3D

1. Three mutually perpendicular planes in space divide the plane into 8 regions and each region is called octant and the lines are known as co ordinate axes.



2. The three axes are XOX', YOY' and ZOZ' are called x-axis, y-axis and z-axis.
3. The coordinates of the point $P(x, y, z)$ are the distances from the origin to the feet of the perpendiculars from the point on the coordinate axes OX, OY and OZ.
4. The distances measured along or parallel to OX, OY and OZ will be positive and that along or parallel to OX', OY' and OZ' will be negative.

$P(x, y, z)$	$XY\ plane,\quad z=0$ $YZ\ plane,\quad x=0$ $ZX\ plane,\quad y=0$

$P(x, y, z)$ lies on the x-axis	$y=0,\ z=0$
$P(x, y, z)$ lies on the y-axis	$z=0,\ x=0$
$P(x, y, z)$ lies on the z-axis	$x=0,\ y=0$

A point on the x-axis	coordinate is $A(x,0,0)$
A point on the y-axis	coordinate is $A(0,y,0)$
A point on the z-axis	coordinate is $A(0,0,z)$

The co-ordinate plane divide the space into 8 regions. Each region is known as an octant.

Octant → / Coordinate ↓	I XOYZ	II $X'OYZ$	III $X'OY'Z$	IV $XOY'Z$	V $XOYZ'$	VI $X'OYZ'$	VII $X'OY'Z'$	VIII $XOY'Z'$
x	+	–	–	+	+	–	–	+
y	+	+	–	–	+	+	–	–
z	+	+	+	+	–	–	–	–

5. **Distance between two points:**

 a) Cartesian equation: Distance between two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is

 $$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \text{ (or)}$$

 $$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

6. Distance of a point $A(x, y, z)$ from the origin is OA = $\sqrt{x^2 + y^2 + z^2}$
7. Distance of a point $A(x, y, z)$ from x axis is $\sqrt{y^2 + z^2}$
8. Distance of a point $A(x, y, z)$ from y axis is $\sqrt{x^2 + z^2}$
9. Distance of a point $A(x, y, z)$ from z axis is $\sqrt{x^2 + y^2}$
10. When three vertices are given, we can prove the following:

 i) Equilateral Δ^{le} - AB = BC = CA

 ii) Isosceles Δ^{le} - Any two sides are equal

 iii) Right angled Δ^{le} - (largest side)^2 = sum of the squares of other two sides.

 iv) Right angled isosceles Δ^{le} – any two sides are equal and (largest side)^2 = sum of the squares of other two sides.

 v) Regular tetrahedron having points O, A, B & C Show that

 $OA = OB = OC = AB = BC = CA$

Note: If three points A, B, C are collinear,

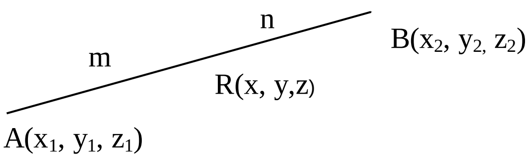
 a. Find AB, BC and CA

 b. Sum of any two sides is equal to the 3rd side.

11. **Section Formula:**

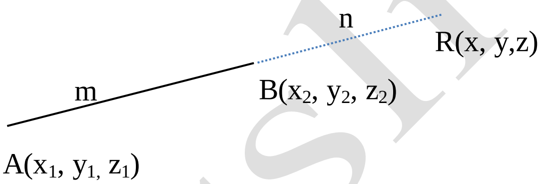
a) Cartesian Equation: Co-ordinates of a point $R(x,y,z)$ dividing the join of two points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ in the ratio $m:n$ internally is

$$R=\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n},\frac{mz_2+nz_1}{m+n}\right)$$



b) Cartesian Equation: Co-ordinates of a point $R(x,y,z)$ dividing the join of two points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ in the ratio $m:n$ externally is

$$R=\left(\frac{mx_2-nx_1}{m-n},\frac{my_2-ny_1}{m-n},\frac{mz_2-nz_1}{m-n}\right)$$



12. Midpoint formula: If $R(x,y,z)$ is the midpoint of two points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$, then coordinates of $R=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2},\frac{z_1+z_2}{2}\right)$

13. The ratio in which the line segment joining the points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ is divided by the:

a) XY plane, then $\frac{mz_2+nz_1}{m+n}=0\Rightarrow mz_2+nz_1=0\Rightarrow mz_2=-nz_1\Rightarrow\frac{m}{n}=-\frac{z_1}{z_2}$

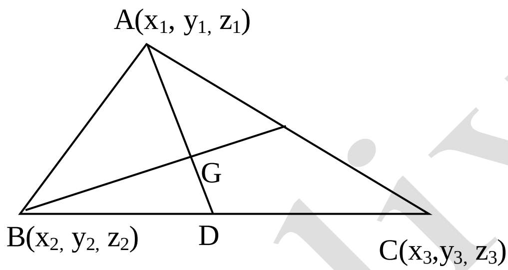
b) YZ plane, then $\frac{mx_2+nx_1}{m+n}=0\Rightarrow mx_2+nx_1=0\Rightarrow mx_2=-nx_1\Rightarrow\frac{m}{n}=-\frac{x_1}{x_2}$

c) XZ plane, then $\dfrac{my_2 + ny_1}{m+n} = 0 \Rightarrow my_2 + ny_1 = 0 \Rightarrow my_2 = -ny_1 \Rightarrow \dfrac{m}{n} = -\dfrac{y_1}{y_2}$

Using midpoint formula, we can prove the following.

Rectangle	: mid pt. of the diagonal AC = mid pt. of the diagonal BD
Square	: – do –
Parallelogram	: – do –
Rhombus	: – do –
Parallelogram but not a rectangle	: Opposite sides are equal but diagonals are not equal

14. Centroid of a Δ[le] having vertices $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ is



$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3} \right)$$

Note: G divides each median in the ratio 2:3.