Sample Questions

Question 1:

Find the distance between the points $P(1,-3,4)$ and $Q(-4,1,2)$

Solution:

$$PQ = \sqrt{(-4-1)^2 + (1+3)^2 + (2-4)^2}$$
$$= \sqrt{(-5)^2 + (4)^2 + (-2)^2}$$
$$= \sqrt{25+16+4}$$
$$= \sqrt{45} = 3\sqrt{5}$$

Question 2:

Show that the points $P(-2,3,5),(1,2,3),(7,0,-1)$ are collinear.

Solution:

$$PQ = \sqrt{(1+2)^2 + (2-3)^2 + (3-5)^2}$$
$$= \sqrt{9+1+4} = \sqrt{14}$$
$$QR = \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2}$$
$$= \sqrt{36+4+16} = \sqrt{56} = 2\sqrt{14}$$
$$PR = \sqrt{(7+2)^2 + (0-3)^2 + (-1-5)^2}$$
$$= \sqrt{81+9+36} = \sqrt{126} = 3\sqrt{14}$$

Thus, PQ + QR = PR. Hence, P, Q and R are collinear.

Question 3:

Are the points $A(0, 7, -10)$, $B(1, 6, -6)$ and $C(4,9,-6)$, the vertices of a right angled triangle?

Solution:

$$AB^2 = (1-0)^2 + (6-7)^2 + (-6+10)^2 = 1+1+16 = 18$$
$$BC^2 = (4-1)^2 + (9-6)^2 + (-6+6)^2 = 9+9+0 = 18$$
$$AC^2 = (4-0)^2 + (9-7)^2 + (-6+10)^2 = 16+4+16 = 36$$
$$AB^2 + BC^2 = 18+18 = 36 = AC^2$$

$\therefore$ ΔABC is a right angled triangle.

Question 4 :

Find the equation of the set of points which are equidistance from the points $(1,\ 2,\ 3)$ and $(3,\ 2,-1)$.

Solution :

Let $A(1,\ 2,\ 3)$ and $B(3,\ 2,-1)$

We have $PA^2 = PB^2$

$$(x-1)^2+(y-2)^2+(z-3)^2=(x-3)^2+(y-2)^2+(z+1)^2$$

$$x^2-2x+1+y^2-2y+4+z^2-6z+9$$
$$=x^2-6x+9+y^2-2y+4+z^2+2z+1$$
$$-2x+1-2y+4-6z+9=-6x+9-2y+4+2z+1$$
$$4x-8z+14=14$$
$$4x-8z=0$$
$$x-2z=0$$

Question 5 :

Find the coordinates of the point which divides the line segment joining the points $(1,\ 2,\ 3)$ and $(4,\ 2,\ 2)$ in the ratio 1:5 (i) internally, and (ii) externally.

Solution :

The points are $(1,\ 2,\ 3)$ and $(4,\ 2,\ 2)$.

Ratio 1 : 5 internally,

$$(x,y,z)=\left(\frac{1\times 4+5}{6},\frac{1\times 2+5\times 2}{6},\frac{1\times 2+5\times 3}{6}\right)$$
$$=\left(\frac{9}{6},\frac{12}{6},\frac{17}{6}\right)$$
$$=\left(\frac{3}{2},2,\frac{17}{6}\right)$$

Ratio 1 : 5 externally,

$$(x,y,z)=\left(\frac{1\times 4-5}{-4},\frac{1\times 2-5\times 2}{-4},\frac{1\times 2-5\times 3}{-4}\right)$$
$$=\left(\frac{1}{4},\frac{8}{4},\frac{13}{4}\right)$$

$$= \left(\frac{1}{4}, 2, \frac{13}{4}\right)$$

Question 6 :

Find the ratio in which the YZ plane which divides the line segment formed by joining the points (-4,5,1) and (3,-2,1).

Solution :

Let A(0, y, z) be the point in the YZ plane which divides (-4, 5, 1) and (3,-2, 1) in the ratio k:1.

$$\therefore \quad x = \frac{mx_2 + nx_1}{m+n}$$

$$0 = \frac{3k+1(-4)}{k+1}$$

$$4 = 3k$$

$$\frac{4}{3} = k$$

$$\therefore \ k : 1 = 4 : 3$$