6. LINEAR INEQUALITIES

Equation: If two algebraic expressions are separated by '=' sign, it is known as an equation.

E.g.: $2x-3=5,\ x=3x-5,\ x^2-1=0, etc..$

Inequality or inequation: If two real numbers or two algebraic expressions are separated by the symbols $2<3,\ 2x\le 4,\ x+2\ge 3, etc..,$ then it is known as an inequality or inequation.

Solution of an equation: Solution of an inequality is the relation between the variable or variables that satisfy the given inequality.

E.g.: Solve: $x+2\ge 3$

$x+2\ge 3 \Rightarrow x+2-2\ge 3-2 \Rightarrow x\ge 1$

Hence, $x=1\le x<1\ or\ \ x=[1,\infty)$ is the solution.

Graphical solution:

Algebra of inequalities

1. We can add or subtract to both sides of an inequality by a real number, its value is not changed.

E.g.: (i) Solve: $x-2\ge 3$

Adding '2' on both sides, we have

$x-2+2\ge 3+2$

$\therefore\ x\ge 5$

$\therefore$ Solution = $[5,\infty)$ or $5\le x<\infty$.

Graphical Solution:

(ii) Solve: $x+5\le 7$

Subtracting '5' on both sides, we have

$x+5-5\le 7-5$

$\Rightarrow x\le 2$

$\therefore$ Solution = $(-\infty,2]$ or $-\infty<x\le 2$.

Graphical Solution:

2. We can multiply or divide to both sides of an inequality by a positive real number, its value is not changed.

(iii) Solve: $\frac{x}{3} < 1$

Multiplying '3' on both sides, we have

$\frac{x}{3} \times 3 < 1 \times 3 \Rightarrow x < 3$

$\therefore$ Solution = $(-\infty, 3)$ or $-\infty < x < 3$.

Graphical Solution:

(iv) Solve: $2x > 6$

Dividing '2' on both sides, we have

$\frac{2x}{2} > \frac{6}{2} \Rightarrow x > 3$

$\therefore$ Solution = $(3, \infty)$ or $3 < x < \infty$.

Graphical solution:

3. But if we multiply or divide to both sides of an inequality by a negative number, its sign gets changed.

(v) Solve: $-5x \le 10$

Dividing '–5' on both sides, we have

$\frac{-5x}{-5} > \frac{10}{-5} \Rightarrow x < -2$

$\therefore$ Solution = $(-\infty, -2)$ or $-\infty < x < -2$.

(vi) Solve: $2 - \frac{3x}{2} \le 6$

$$2-\frac{3x}{2}\le 6$$

Multiplying throughout by '2', we have

$$2\times 2-2\times\frac{3x}{2}\le 2\times 6$$

$$4-3x\le 12$$

$$-3x\le 12-4$$

$$-3x\le 8$$

Dividing by '–3' on both sides,

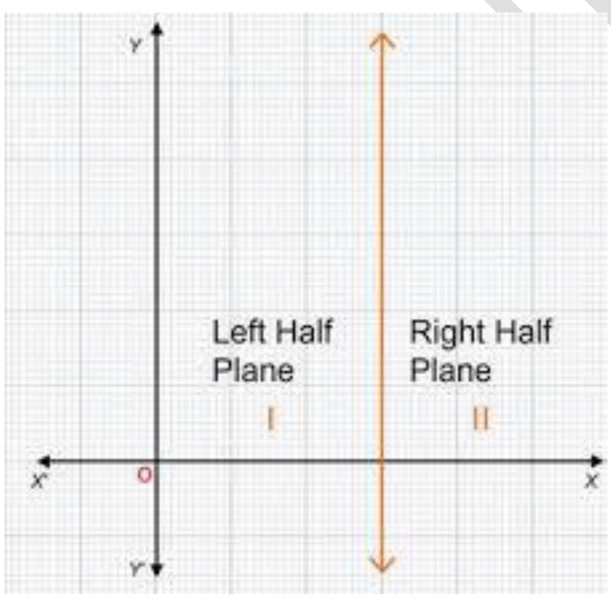
$$\frac{-3x}{-3}\ge\frac{8}{-3}\Rightarrow x\ge-\frac{8}{3}$$

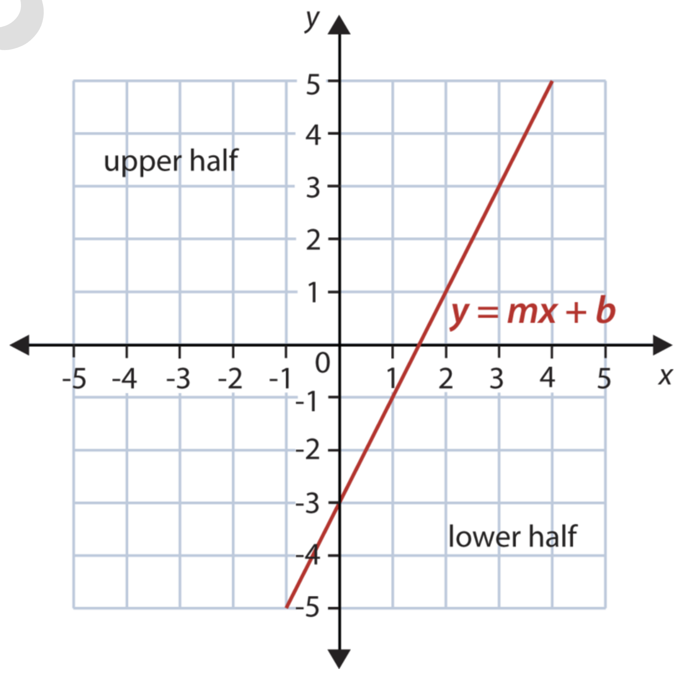
$\therefore$ Solution = $\left[-\frac{8}{3},\infty\right)$ or $-\frac{8}{3}\le x<\infty$.

Graphical solution of inequalities:

A line, vertical or non-vertical divides a plane into two parts, known as half planes.

A vertical line will divide a plane to two parts – left half plane (I half Plane) and right half plane (II half plane). A non-vertical line will divide a plane into two half planes – lower half plane (I half plane) and upper half plane (II half plane).





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Solution region: The region containing all the inequalities is known as solution region.

Method to find the solution region:

1. Take any point (a,b), which is not on the line, and check whether it satisfies the inequality or not.
2. If it satisfies, then the inequality represents the half plane and shades the region containing the point.
3. Otherwise, the inequality represents the half plane and it does not contain the points and shades the other half plane. For easy, we take origin (0,0) as the point.
4. If the inequality is of the form $ax+by\le c$ or $ax+by\ge c$, then the line is also included in the solution region, so we draw a dark line or bold line the in the solution region.
5. If the inequality is of the form $ax+by<c$ or $ax+by>c$, then the line is not included in the solution region, so we draw a broken line or dotted line the in the solution region.

Solve the following inequalities using graphically:

1. $x+y\le 4;\ x\ge 0, y\ge 0$

   Draw the graph of the functions:

   i. $x+y=4$

x	0	4
y	4	0

   ii. $x=0\ \ (y\ axis)$

   iii. $y=0\ (x\ axis)$

   Solution region:

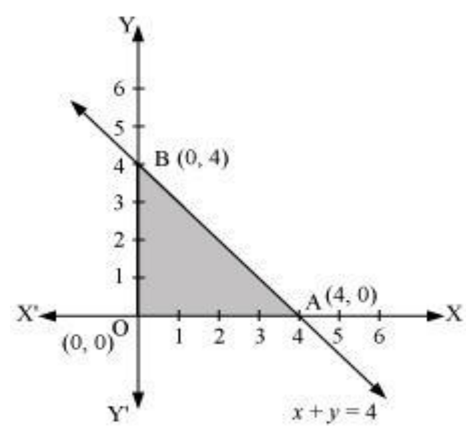
   $x+y\le 4$

   $put\ \ x=0, y=0$

   $0+0=0\le 4, which\ is\ true.$

   Hence, shade the half plane, which contains the point (0,0).
   Since the inequality is of the form $x+y\le 4$, the line is also included in the solution region.

   $x\ge 0\ , y\ge 0$ are always in the first quadrant.



2. Solve: $x+2y\le 10;\ 3x+y\le 15,\ x\ge 0, y\ge 0$

   Draw the graph of the functions:

   i. $x+2y=10$

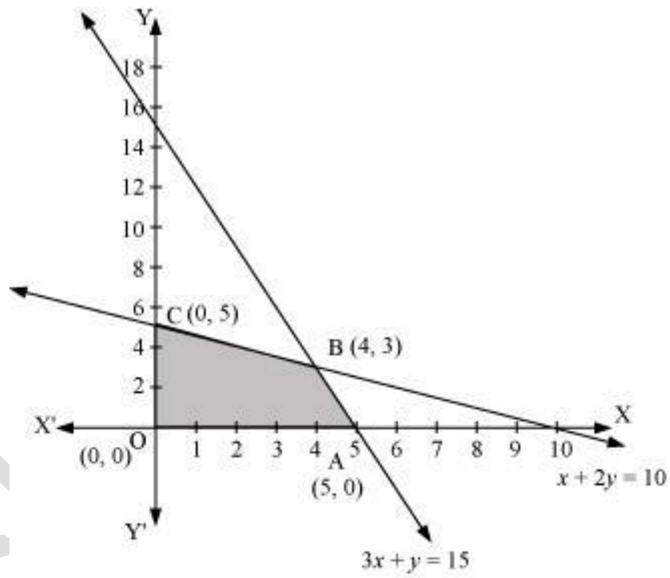
x	0	10
y	5	0

ii. $3x + y = 15$

x	0	5
y	15	0

iii. $x = 0$ (y axis)

iv. $y = 0$ (x axis)



Solution region:

i) $x + 2y \le 10$

$0 + 2(0) \le 10 \Rightarrow 0 \le 10$, which is true. Hence shade the half plane containing the origin.

ii) $3x + y \le 15$

$3(0) + 0 \le 15 \Rightarrow 0 \le 15$, which is true. Hence shade the half plane containing the origin.

iii) $x \ge 0, y \ge 0$ are non-zero positive inequalities, whose graphs are always in the first quadrant. Hence, the shaded region is the solution region.

3. $x + 2y \le 120$, $x + y \ge 60$, $x - 2y \ge 0$, $x \ge 0$, and $y \ge 0$

Draw the graph of the functions:

i. $x + 2y = 120$

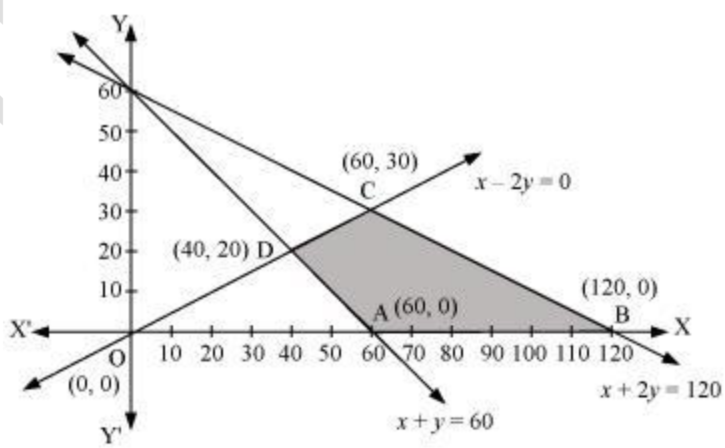
x	0	120
y	60	0

ii. $x + y = 60$

x	0	60
y	60	0

iii. $x - 2y = 0$

x	0	60
y	0	30



iv. $x = 0$ (y axis)

v. $y = 0$ (x axis)

Solution region:

i) $x + 2y \le 120$

$0+2(0)\le 120 \Rightarrow 0\le 120$, which is true. Hence shade the half plane containing the origin.

ii) $x+y\ge 60$

$0+0\ge 60 \Rightarrow 0\ge 60$, which is false. Hence shade the half plane , which does not containing the origin.

iii) $x-2y\ge 0$

Put $x=30, y=10$

$30-2(10)\ge 0 \Rightarrow 10\ge 0$, which is true. Hence shade the half plane, which contain the point (30,10).

iv) $x\ge 0, y\ge 0$ are non-zero positive inequalities, whose graphs are always in the first quadrant.

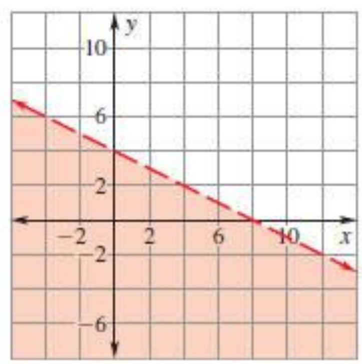
Hence, the shaded region is the solution region.

4. Solve $5x+7y<35$ graphically.

$5x+7y=35$

x	0	7
y	5	0

Solution region:



$5x+7y<35$

$x=0, y=0$

$5(0)+7(0)<35 \Rightarrow 0<35$, which is true.

Hence shade the half plane, which contain the origin.

Also the inequality is of the form $ax+by<c$, the line is not included, so draw a dotted line.

Solve the inequalities:

1. Solve: $2\le 3x-4\le 5$

Adding '4'

$2+4\le 3x-4+4\le 5+4$

$6\le 3x\le 9$

Dividing by '3'

$\frac{6}{3}\le\frac{3x}{3}\le\frac{9}{3}$

$2\le x\le 3$

or $[2,3]$

2. Solve: $-3\le 4-\frac{7x}{2}\le 18$

Subtracting '4'

$$-3-4 \le 4-\frac{7x}{2}-4 \le 18-4$$

$$-7 \le -\frac{7x}{2} \le 14$$

Multiplying by '2'

$$2\times -7 \le 2\times -\frac{7x}{2} \le 2\times 14$$

$$-14 \le -7x \le 28$$

Dividing by '–7'

$$-\frac{14}{-7} \ge \frac{-7x}{-7} \ge \frac{28}{-7}$$

$2 \ge x \ge -4$ or $[-4,2]$

3. Solve: $5x+1>-24,\ 5x-1<24$

$$5x+1>-24 \quad \text{and} \quad 5x-1<24$$

$$5x>-24-1 \quad \text{and} \quad 5x<24+1$$

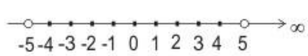
$$5x>-25 \quad \text{and} \quad 5x<25$$

$$x>-\frac{25}{5} \quad \text{and} \quad x<\frac{25}{5}$$

$$x>-5 \quad \text{and} \quad x<5$$

$\therefore$ solution is (–5,5)

Graphical solution:



4. Solve: $3x-7>2(x-6), 6-x>11-2x$

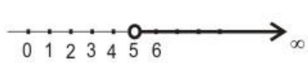
$$3x-7>2x-12 \quad \text{and} \quad 6-x>11-2x$$

$$3x-2x>-12+7 \quad \text{and} \quad -x+2x>11-6$$

$$x>-5 \ \text{and} \ \ x>5$$

$\Rightarrow x>5$ is the solution.

Graphical solution:



Step problems:

1. A solution is to be kept between 68^0 F and 77^0 F. What is the range in temperatures in degree Celsius if the Celsius/Fahrenheit conversion formula is given by $F = \frac{9}{5}C + 32$.

Given $68^0 < F < 77^0$

$$68^0 < \frac{9}{5}C + 32 < 77^0$$

$$68 - 32 < \frac{9}{5}C < 77 - 32$$

$$36 < \frac{9}{5}C < 45$$

$$36 \times \frac{5}{9} < \frac{5}{9} \times \frac{9}{5}C < \frac{5}{9} \times 45$$

$$20 < C < 25$$

$\therefore$ the range of temperature is between 20^0C and 25^0C.

2. A solution of 8% boric acid solution is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 litres of the 8% solution, how many litres of the 2% solution will have to be added?

Let x litres of 2% boric acid are to be added to 640 litres of the 8% boric acid solution.

$\therefore$ The total mixture = (640+x) litres

Given that

$$2\%\ of\ x + 8\%\ of\ 640 > 4\%\ of\ (640 + x)$$

$$\frac{2}{100} \times x + \frac{8}{100} \times 640 > \frac{4}{100} \times (640 + x)$$

$$2x + 8 \times 640 > 4 \times 640 + 4x$$

$$2x + 5120 > 2560 + 4x$$

$$2x - 4x > 2560 - 5120$$

$$-2x > -2560$$

$$x < 1280 \ ....(1)$$

Again,

$$2\%\ of\ x + 8\%\ of\ 640 < 6\%\ of\ (640 + x)$$

$$\frac{2}{100} \times x + \frac{8}{100} \times 640 < \frac{6}{100} \times (640 + x)$$

$$2x + 8 \times 640 < 6 \times 640 + 6x$$

$$2x + 5120 < 3840 + 6x$$

$$2x - 6x < 3840 - 5120$$

$$-4x < -1280$$

$$x > 340 \ ....(2)$$

From (1) and (2), we have,

$x > 340 \ \ and \ \ x < 1280$

$\Rightarrow 340 < x < 1280$

3. How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

Let x litres of water will have to be added to 1125 litres of the 45% solution of acid.

$\therefore$ Total mixture = (x+1125) litres

Given that

$25\% \ of \ (1125+x) < 45\% \ of \ 1125 < 30\% \ of \ (1125+x)$

$25(1125+x) < 45\times 1125 < 30(1125+x)$

$28125+25x < 50625 < 33750+30x$

$28125+25x < 50625 \ \ and \ 33750+30x > 50625$

$25x < 50625-28125 \ \ and \ \ 30x > 50625-33750$

$25x < 22500 \ \ and \ \ 30x > 16875$

$x < \frac{22500}{25} \ \ and \ \ x > \frac{16875}{30}$

$\Rightarrow x > 562.5 \ \ and \ \ x < 900$

$\Rightarrow 562.5 < x < 900$

4. IQ of a person is given by the formula: $IQ = \frac{MA}{CA}\times 100$, where MA is mental age and CA is chronological age. If $80 \le IQ \le 140$ for a group of 12 years old children, find the range of their mental age.

Given that $80 \le IQ \le 140$

$80 \le \frac{MA}{CA}\times 100 \le 140$

$80 \le \frac{MA}{12}\times 100 \le 140$

$80\times 12 \le \frac{MA}{12}\times 100\times 12 \le 140\times 12$

$960 \le MA\times 100 \le 1680$

$9.6 \le MA \le 16.8$