CHAPTER 4

PRINCIPLES OF MATHEMATICAL INDUCTION

IMPROVEMENT 2018

1. Consider the statement:

$$P(n):1+3+3^2+\ldots+3^{n-1}=\frac{3^n-1}{2}.$$

   a) Show that P(1) is true. (1)

   b) Prove by principle of Mathematical induction that P(n) is true for all $n\in N$. (3)

   [same as March 2018]

MARCH 2018

2. a) If $3^{2n+2}-8n-9$ is divisible by k for all $n\in N$ is true, then which one of the following is a value of k? (1)

   i) 8 ii) 6 iii) 3 iv) 12

   b) Prove by using the Principle of Mathematical Induction $P(n):1+3+3^2+\ldots+3^{n-1}=\dfrac{3^n-1}{2}$ is true for all $n\in N$.

IMPROVEMENT 2017

3. Consider the statement: $P(n)$:"7^n-3^n is divisible by 4".

   a) Verify the statement for $n=1$ (1)

   b) Prove the statement by using the principle of mathematical induction. (3)

MARCH 2017

4. Consider the statement "$10^{2n-1}+1$ is divisible by 11". Verify that $P(1)$ is true and then prove that the statement by using mathematical induction. (4)

IMPROVEMENT 2016

5. Consider the statement: "$P(n): x^n-y^n$ is divisible by $x-y$".

   a) Show that is $P(1)$ true. (1)

   b) Using the principle of mathematical inductions verify that $P(n)$ is true for all natural numbers. (3)

MARCH 2016

6. Consider the following statement:

$$P(n): a+ar+ar^2+\ldots+ar^{n-1}=\frac{a(r^n-1)}{r-1}$$

   a) Prove that $P(1)$ is true. (1)

   b) Hence by using the principle of Mathematical induction, prove that $P(n)$ is true for all natural numbers n. (3)

IMPROVEMENT 2015

7. Consider the statement: $P(n)=7^n-3^n$ is divisible by 4.

   a) Show that $P(1)$ is true. (1)

   b) Verify, by the method of Mathematical induction that $P(n)$ is true for all $n\in N$. (3)

MARCH 2015

8. A statement $p(n)$ for a natural number n is given by $p(n)=\dfrac{1}{2}+\dfrac{1}{4}+\dfrac{1}{8}+\ldots..+\dfrac{1}{2^n}=1-\dfrac{1}{2^n}$

   a) Verify that $p(1)$ is true. (1)

   b) By assuming that $P(k)$ is true for a natural number k, show that $P(k+1)$ is true. (3)

IMPROVEMENT 2014

9. Using the principal of mathematical induction, prove that $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+........+\frac{1}{2^n}=1-\frac{1}{2^n}$. (4)

MARCH 2014

10. Consider the statement "$3^{2n+2}-8n-9$ is divisible by 8".
    a) Verify the statement is true for $n=1$ (1)
    b) Prove the statement using the principle of mathematical induction for all natural numbers. (3)

IMPROVEMENT 2013

11. Consider the statement

$$P(n):1^3+2^3+3^3+....+n^3=\left[\frac{n(n+1)}{2}\right]^2$$

    a) Verify that $P(n)$ is true. (1)
    b) By mathematical induction show that $P(n)$ is true for all $n\in N$ (3)

MARCH 2013

12. Consider the statement

$$P(n):1+3+3^2+.......+3^{n-1}=\frac{3^n-1}{2}$$

    a) Check whether P(1) is true. (1)
    b) If $P(k)$ is true, prove that $P(k+1)$ is also true. (2)
    c) Is $P(n)$ true for all natural numbers n? Justify your answer. (1)

IMPROVEMENT 2012

13. Prove that

$$1.2+2.3+3.4.....+n(n+1)=\frac{n(n+1)(n+2)}{3}$$

by using the principle of mathematical induction for all $n\in N$ (4)

2012 MARCH

14. Consider the statement, "$n(n+1)(2n+1)$ is divisible by 6".
    a) Verify the statement for $n=2$. (1)
    b) By assuming that $P(k)$ is true for a natural number k, verify that $P(k+1)$ is true. (3)

IMPROVEMENT 2011

No question from this chapter.

MARCH 2011

15. Consider the statement $P(n)$:"9^n-1 is a multiple of 8", where 'n' is a natural number.
    a) Is P(1) true? (1)
    b) Assuming $P(k)$ is true, show that $P(k+1)$ is true. (3)

IMPROVEMENT 2010

16. a) Which among the following is the least number that will divide $7^{2n}-4^{2n}$ for every positive integer n?
    [4,7,11,33] (1)

    b) Prove by mathematical induction, $(\cos\theta+i\sin\theta)^n=(\cos n\theta+i\sin n\theta)$, where $i=\sqrt{-1}$ (3)

MARCH 2010

17. Consider the statement "7^n-3^n is divisible by 4"
    a) Verify the result for n = 2. (1)
    b) Prove the statement using mathematical induction. (3)

IMPROVEMENT 2009
[same as March 2010]

18. Let P(n) be the statement :

"$7^n - 3^n$ is divisible by 4".

a) Verify whether the statement is true for n=2. (1)

b) Prove the result by using mathematical induction. (3)

MARCH 2009

19. a) For every positive integer n, $7^n - 3^n$ should be divisible by (2, 3, 4, 8). (1)

c) Prove by principle of mathematical induction that: $2+2^2+2^3+2^4+...+2^n = 2\left(2^n-1\right)$ (3)

MARCH 2008

20. Consider the statement

$P(n): 1+3+5+....+(2n-1) = n^2$

a) Verify $P(1)$ is true. (1)

b) Prove $P(n)$ by induction. (2)