Sample Questions

Question 1:

Find the radian measures of $25°$

Solution:

$$25° = \frac{\pi}{180} \times 25 \text{ radian}$$
$$= \underline{\underline{\frac{5\pi}{36} \text{ radian}}}$$

Question 2:

Find the degree measures of $\frac{7\pi}{6}$

Solution:

$$\frac{11\pi}{6} \text{ radian} = \frac{11\pi}{6} \times \frac{180}{\pi} \text{ degree}$$
$$= \frac{1980}{6}$$
$$= \underline{\underline{330 \text{ degree}}}$$

Question 3:

Convert 8 radians into degree measure.

Solution:

$$8 \text{ radians} = \frac{180}{\pi} \times 8 \text{ degree}$$
$$= \frac{1440}{22} \times 7 \text{degree}$$
$$= \frac{10080}{22} \text{degree}$$
$$= \frac{5040 \times \cancel{2}}{11 \times \cancel{2}} \text{degree}$$
$$= \frac{5040}{11} \text{degree}$$

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$$= 458\frac{2}{11}\text{degree}$$
$$= 458° + \frac{2\times 60}{11}\text{minute}$$
$$= 458° + 10' + \frac{10}{11}\text{minute}$$
$$= 458° + 10' + \frac{10\times 60}{11}\text{second}$$
$$= 458° + 10' + 55''$$
$$= \underline{\underline{458°10'55''}}$$

Question 4:

Find the radius of the circle in which a central angle of $60°$ intercepts an arc of length 42.5 cm

Solution:

We have $r = \frac{1}{\theta}$

$$= \frac{42.5\times 3}{\pi}$$
$$= \frac{42.5\times 3}{3.14}$$
$$= \underline{\underline{40.61\text{ cm}}}$$

Question 5:

If the arcs of the same lengths in two circles subtend angles $65°$ and $70°$ at the centre, find the ratio of their radii.

Solution:

We have, $l = r_1\theta_1 = r_2\theta_2$

$$\theta_1 = \frac{\pi}{180}\times 65 = \frac{13\pi}{36}$$
$$\theta_2 = \frac{\pi}{180}\times 70 = \frac{7\pi}{18}$$
$$\therefore \quad r_1\theta_1 = r_2\theta_2$$
$$\frac{13\pi}{36}r_1 = \frac{7\pi}{18}r_2$$

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$$\frac{r_1}{r_2} = \frac{7\pi}{18} \times \frac{36}{13\pi}$$

$$\frac{r_1}{r_2} = \frac{14}{13}$$

$$r_1 : r_2 = \underline{\underline{14:13}}$$

Question 6:

If $\cot x = -\frac{5}{12}$, x lies in second quadrant, find the values of other 5 trigonometric functions.

Solution:

Since $\cot x = -\frac{5}{12}$

We have $\tan x = -\frac{12}{5}$

Now $\sec^2 x = 1 + \tan^2 x$

$$\sec^2 x = 1 + \frac{144}{25} = \frac{169}{25}$$

Hence $\sec x = \pm\frac{13}{5}$

Since x lies in second quadrant, secx will be negative.

$\therefore$ $\sec x = -\frac{13}{5}$

$\cos x = -\frac{5}{13}$

Also $\sin x = \tan x \cos x$

$$= -\frac{12}{5} \times -\frac{5}{13}$$

$$= -\frac{12}{13}$$

and $\operatorname{cosec} x = -\frac{13}{12}$

Question 7: (Imp 2009, March 2011)

Find the value of $\sin\frac{31\pi}{3}$

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Solution:

$$\sin\frac{31\pi}{3} = \sin\left(\frac{30\pi+\pi}{3}\right)$$
$$= \sin\left(\frac{30\pi+\pi}{3}\right)$$
$$= \sin\left(10\pi+\frac{\pi}{3}\right)$$
$$= \sin\left(\frac{\pi}{3}\right)$$
$$= \underline{\underline{\frac{\sqrt{3}}{2}}}$$

Question 8:

Find the value of cosec $(-1410°)$.

Solution:

We know that values of cos x repeats after an interval of 2π or $360°$

Therefore,

$$\text{cosec}\,(-1410°) = \text{cosec}\,(-1410° + 4\times 360°)$$
$$= \text{cosec}\,(-1410° + 1440°)$$
$$= \text{cosec}\,30°$$
$$= \underline{\underline{2}}$$

Question 9:

Find the value of $\sin 15°$.

Solution:

We have

$$\sin 75° = \sin(45° + 30°)$$
$$\sin(x+y) = \sin x\cos y + \cos x \sin y$$
$$= \sin 45°\cos 30° + \cos 45° \sin 30°$$
$$= \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\times\frac{1}{2}$$
$$= \frac{\sqrt{3}}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Question 10:

Find the value of $\tan\frac{18\pi}{12}$.

Solution:

$$\tan\left(\frac{17\pi}{12}\right) = \tan\left(\frac{12\pi+5\pi}{12}\right)$$

$$= \tan\left(\pi+\frac{5\pi}{12}\right)$$

$$= \tan\left(\frac{5\pi}{12}\right)$$

$$\tan\left(\frac{5\pi}{12}\right) = \tan\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1-\tan x \tan y}$$

$$\tan\left(\frac{\pi}{4}+\frac{\pi}{6}\right) = \frac{\tan\frac{\pi}{4}+\tan\frac{\pi}{6}}{1-\tan\frac{\pi}{4}\tan\frac{\pi}{6}}$$

$$\tan\left(\frac{\pi}{4}+\frac{\pi}{6}\right) = \frac{1+\frac{1}{\sqrt{3}}}{1-1\times\frac{1}{\sqrt{3}}}$$

$$= \frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

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Question 11:

(March 2013)

Prove that $\dfrac{\tan\left(\dfrac{\pi}{4}+x\right)}{\tan\left(\dfrac{\pi}{4}-x\right)}=\left(\dfrac{1+\tan x}{1-\tan x}\right)^2$

Solution:

$$\text{L.H.S} = \frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}$$

$$= \frac{\left(\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan x\tan\frac{\pi}{4}}\right)}{\left(\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan x\tan\frac{\pi}{4}}\right)}$$

$$= \frac{\left(\dfrac{1+\tan x}{1-\tan x}\right)}{\left(\dfrac{1-\tan x}{1+\tan x}\right)}$$

$$= \left(\frac{1+\tan x}{1-\tan x}\right)\times\left(\frac{1+\tan x}{1-\tan x}\right)$$

$$= \left(\frac{1+\tan x}{1-\tan x}\right)^2$$

$$= \text{R.H.S}$$

Question 12:

Prove that $\dfrac{\tan x+\sec x-1}{\tan x-\sec x+1}=\tan x+\sec x$

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Solution:

L.H.S

$$= \frac{\tan x + \sec x - 1}{\tan x - \sec x + 1}$$

$$= \frac{(\tan x + \sec x - 1)}{(\tan x - \sec x + 1)} \times \frac{(\tan x + \sec x)}{(\tan x + \sec x)}$$

$$= \frac{(\tan x + \sec x - 1)(\tan x + \sec x)}{(\tan x - \sec x)(\tan x + \sec x) + (\tan x + \sec x)}$$

$$= \frac{(\tan x + \sec x - 1)(\tan x + \sec x)}{(\tan^2 x - \sec^2 x) + (\tan x + \sec x)}$$

$$= \frac{(\tan x + \sec x - 1)(\tan x + \sec x)}{-1 + \tan x + \sec x}$$

$$= \frac{(\tan x + \sec x - 1)(\tan x + \sec x)}{\tan x + \sec x - 1}$$

$$= \frac{\cancel{(\tan x + \sec x - 1)}(\tan x + \sec x)}{\cancel{\tan x + \sec x - 1}}$$

$$= (\tan x + \sec x)$$

$$= \text{R.H.S}$$

Question 13:

Prove that

$$2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} = 0$$

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Solution:

L.H.S

$$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$$

$$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$$

$$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\left(\frac{\frac{3\pi}{13} + \frac{5\pi}{13}}{2}\right)\cos\left(\frac{\frac{3\pi}{13} + \frac{5\pi}{13}}{2}\right)$$

$$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\left(\frac{4\pi}{13}\right).\cos\left(\frac{-\pi}{13}\right)$$

$$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\left(\frac{4\pi}{13}\right).\cos\left(\frac{\pi}{13}\right)$$

$$(\because \cos(-\theta) = \cos\theta)$$

$$= 2\cos\frac{\pi}{13}\left[\cos\frac{9\pi}{13} + 2\cos\left(\frac{4\pi}{13}\right)\right]$$

$$= 2\cos\frac{\pi}{13}\left[2\cos\left(\frac{\frac{9\pi}{13}+\frac{9\pi}{13}}{2}\right)\cos\left(\frac{\frac{9\pi}{13}-\frac{9\pi}{13}}{2}\right)\right]$$

$$= 2\cos\frac{\pi}{13}\left[2\cos\left(\frac{13\pi}{26}\right)\cos\left(\frac{13\pi}{26}\right)\right]$$

$$= 4\cos\frac{\pi}{13}.\cos\left(\frac{\pi}{2}\right).\cos\left(\frac{13\pi}{26}\right)$$

$$= 0 \qquad \left(\because \cos\frac{\pi}{2} = 0\right) = \text{R.H.S}$$

Question 14:

Prove that

$$\tan 3x - \tan 2x - \tan x = \tan x \tan 2x \tan 3x$$

Solution:

$$\tan 3x = \tan(x + 2x)$$

$$\tan 3x = \frac{\tan x + \tan 2x}{1 - \tan x \tan 2x}$$

$$\tan 3x\,(1 - \tan x \tan 2x) = \tan x + \tan 2x$$

$$\tan 3x - \tan x \tan 2x \tan 3x = \tan x + \tan 2x$$

$$\therefore \quad \tan 3x - \tan 2x - \tan x = \tan x \tan 2x \tan 3x$$

Question 15:

Prove that $\cos^4 A - \sin^4 A = \cos 2A$

Solution:

L.H.S.

$$= \left(\cos^2 A + \sin^2 A\right)\left(\cos^2 A - \sin^2 A\right)$$

$= 1\,.\cos 2A$

$= \cos 2A = \text{R.H.S.}$

Question 16:

Prove that

$$\cos\left(\frac{B-C}{2}\right) = \frac{b+c}{a}\sin\frac{A}{2}$$

Solution:

R.H.S

$$= \frac{b+c}{a}\sin\frac{A}{2}$$

$$= \frac{2R\sin B + 2R\sin C}{2R\sin A}\ \sin\frac{A}{2}$$

$$\left(\begin{array}{l} \qquad\qquad \text{Any triangle ABC} \\ \because \quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2R \\ \text{R is the radius of the circum circle} \end{array}\right)$$

$$= \frac{\sin B + \sin C}{\sin A}\ \sin\frac{A}{2}$$

$$= \frac{2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)}{2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)}\ \sin\frac{A}{2}$$

$$= \frac{\cancel{2}\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)}{\cancel{2}\cancel{\sin\left(\frac{A}{2}\right)}\cos\left(\frac{A}{2}\right)}\ \cancel{\sin\frac{A}{2}}$$

$$= \frac{\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)}{\cos\left(\frac{A}{2}\right)}$$

$$= \frac{\sin\left(90^\circ - \frac{A}{2}\right)\cos\left(\frac{B-C}{2}\right)}{\cos\left(\frac{A}{2}\right)}$$

$$= \frac{\cos\left(\frac{A}{2}\right)\cos\left(\frac{B-C}{2}\right)}{\cos\left(\frac{A}{2}\right)}$$

$$= \cos\left(\frac{B-C}{2}\right) = \text{R.H.S}$$

Question 17:

(March2015)

Prove that

$$\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}\cot\frac{C}{2}$$

Solution:

$$\text{R.H.S} = \frac{a-b}{a+b}\cot\frac{C}{2}$$

$$= \frac{2R\sin A - 2R\sin B}{2R\sin A + 2R\sin B}\cot\frac{C}{2}$$

$$\left(\begin{array}{c} \text{Any triangle ABC} \\ \because \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R \\ \text{R is the radius of the circum circle} \end{array}\right)$$

$$= \frac{\sin A - \sin B}{\sin A + \sin B}\cot\frac{C}{2}$$

$$= \frac{2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}\cot\frac{C}{2}$$

$$= \cot\left(\frac{A+B}{2}\right)\tan\left(\frac{A-B}{2}\right)\cot\left(\frac{C}{2}\right)$$

$$= \cot\left(90^\circ - \frac{C}{2}\right)\tan\left(\frac{A-B}{2}\right)\cot\left(\frac{C}{2}\right)$$

$$= \tan\left(\frac{C}{2}\right)\tan\left(\frac{A-B}{2}\right)\cot\left(\frac{C}{2}\right)$$

$$= \tan\left(\frac{A-B}{2}\right) = \text{R.H.S}$$

Question 18: (March 2014)

Find the principal solutions of the equation

$\sin x = \frac{\sqrt{3}}{2}$

Solution:

We know that,

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ and

$\sin\frac{\pi}{3} = \sin\left(\pi - \frac{\pi}{3}\right) = \sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

Therefore, principal solutions are $\frac{\pi}{3}, \frac{2\pi}{3}$

Question 19: (Imp 2009, March 2015)

Find the solution of $\sin x = -\frac{\sqrt{3}}{2}$

Solution:

We have

$\sin x = -\frac{\sqrt{3}}{2} = -\sin\frac{\pi}{3} = \sin\left(\pi + \frac{\pi}{3}\right) = \sin\frac{4\pi}{3}$

$\sin x = \sin\frac{4\pi}{3}$

which gives

$$x = n\pi + (-1)^n \frac{4\pi}{3}, \text{where } n \in Z$$

Question 20: (Imp 2012, 2014, 2015, Example 23)

Solve $\sin 2x - \sin 4x + \sin 6x = 0.$

Solution:

$\sin 6x + \sin 2x - \sin 4x = 0$

or $2\sin 4x \cos 2x - \sin 4x = 0$

i.e. $\sin 4x (2\cos 2x - 1) = 0$

$\therefore$ $\sin 4x = 0$ or $\cos 2x = \frac{1}{2}$

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$$\sin 4x = 0 \text{ or } \cos 2x = \cos\frac{\pi}{3}$$

Hence $4x = n\pi$ or $2x = 2n\pi \pm \frac{\pi}{3}$, where $n \in Z$

$$x = n\frac{\pi}{4} \text{ or } x = n\pi \pm \frac{\pi}{6}, \text{where } n \in Z$$

Question 21:

Solve : $2\cos^2\theta + 3\sin\theta = 0$

Solution:

$$2\cos^2\theta + 3\sin\theta = 0$$

$$2\,(1 - \sin^2\theta) + 3\sin\theta = 0$$

$$2\sin^2\theta - 3\sin\theta - 2 = 0$$

$$(2\sin\theta + 1)(\sin\theta - 2) = 0$$

$$\sin\theta = \frac{-1}{2} \quad (\because \sin\theta = 2 \text{ is not possible})$$

$$\sin\theta = -\sin\frac{\pi}{6}$$

$$\sin\theta = \sin\left(-\frac{\pi}{6}\right)$$

$$\theta = -\frac{\pi}{6}$$

$$\theta = n\pi + (-1)^n\left(-\frac{\pi}{6}\right), \text{where } n \in Z$$

Question 22:

Solve : $2\sin^2 x + \sin^2 2x = 2$

Solution:

$$2\sin^2 x + \sin^2 2x = 2$$

$$\therefore \quad \sin^2 2x = 2 - 2\sin^2 x$$

$$= 2(1 - \sin^2 x)$$

$$\sin^2 2x = 2\cos^2 x$$

$$4\sin^2 x \cos^2 x - 2\cos^2 x = 0$$
$$2(1-\cos^2 x)\cos^2 x - \cos^2 x = 0$$
$$2\cos^4 x - \cos^2 x = 0$$
$$\cos^2 x(2\cos^2 x - 1) = 0$$
$$\cos^2 x = 0$$
$$\cos^2 x = \cos^2\frac{\pi}{2}$$
$$x = n\pi \pm \left(\frac{\pi}{2}\right), \text{where } n \in Z$$

EXERCISE

1) $\frac{2\pi}{3}$ radians = ...degree **(Imp 2014)**

Hint or Answer: 120

2) A tree breaks due to a storm and the broken part bends so that the top of the tree touches the ground making an angle of 30^0 with it. The distance between the foot of the tree to the point where the top touches the ground is 8m. Find the height of the tree.

(March 2015)

Hint or Answer: $8\sqrt{3}$ m

3) sin225 = ...

(March 2015)

Hint or Answer: $-\frac{1}{\sqrt{2}}$

4) $\cos(2\pi - x) = ...$

(Imp 2014)

Hint or Answer: cosx

5) The value of $\sin(\pi - x) = \ldots\ldots\ldots\ldots$

(March 2014)

Hint or Answer: sinx

6) If $\tan x = \frac{1}{2}$ and x is in the third quadrant, find sinx and cosx

(March 2012)

Hint or Answer: $-\frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}$

7) Evaluate $\tan\frac{13\pi}{6}$

(March 2012)

Hint or Answer: $\frac{1}{\sqrt{3}}$

8) Show that

$\tan 15° = 2 - \sqrt{3}$

$\tan 15° + \cot 15° = 4$

(Imp 2013)

9) Expand $\cos(x + y)$ and hence prove

$\cos 2x = 1 - 2\sin 2x$

(Imp 2010)

10) Prove that

$$\cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) = \sqrt{2}\cos x$$

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(March 2015)

11) Prove that

$$\tan\left(\frac{B-C}{2}\right)=\frac{b-c}{b+c}\cot\frac{A}{2}$$

(Imp 2015)

12) Prove that

$$\frac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x}=\cot 3x$$

(March 2014)

13) Prove that

$$\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}=\tan 4x$$

(March 2010, 2013)

14) Prove that

$$2\sin\frac{\pi}{6}\sec\frac{\pi}{3}-4\sin\frac{5\pi}{6}\cot\frac{\pi}{4}=1$$

(March 2013)

15) Prove that

$$\frac{\cos 3x+\cos 7x-\cos 2x}{\sin 7x-\sin 3x-\sin 2x}=\cot 2x$$

(March 2012)

16) Show that

$$(\cos x+\cos y)^2+(\sin x+\sin y)^2=4\cos^2\left(\frac{x-y}{2}\right)$$

(March 2011)

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