CHAPTER 2

RELATIONS AND FUNCTIONS

Ordered Pair

A pair of numbers or elements grouped together in a definite order is known as ordered pair. If a and b are any two numbers, then (a , b) is called ordered pair a, b. Here 'a' is known as first element or x element or x co-ordinate or abscissa and 'b' is known as second element or y element or y co-ordinate or ordinate.

E.g.: $(2,3),(-1,-2),\left(\frac{1}{2},\frac{2}{3}\right),(x,\ y),$ etc. are ordered pairs.

Note: $\{a,b\}=\{b,a\}$ *but* $(a,b)\neq(b,a)$ *unless* $a=b$

Cartesian product of sets

If A and B be any two non-empty sets, then the Cartesian product or cross product of A × B is the set of all ordered pairs of elements from A to B and the Cartesian product or cross product of B × A is the set of all ordered pairs of elements from B to A.

i.e., $A \times B = \{ (x , y) : x \in A, y \in B \}$.

And $B \times A = \{ (x , y) : x \in B, y \in A \}$

Note: If either A or B is a null set, then A × B will also be a null set, i.e., $A \times B = \phi$ and $B \times A = \phi$

Note:

- Two ordered pairs are equal, *iff* the corresponding first elements are equal and the second elements are also equal. i.e., if $(a,b)=(c,d)\Rightarrow a=c\ and\ b=d$
- If there are m elements in A and n elements in B, then there will be 'mn' elements in A × B.
  i.e., if n(A) = m and n(B) = n, then n(A × B) = mn and n(B × A) = nm=mn elements.
- If A and B are non-empty sets and either A or B is an infinite set, then A × B is also infinite.
- If A = B, then A × B becomes A × A and is denoted by A^2.
- $A \times A = \{(a, b) : a, b \in A\}$. Here (a, b) is called an ordered doublet.
- $A \times A \times A = \{(a, b, c) : a, b, c \in A\}$. Here (a, b, c) is called an ordered triplet.
- If set A has m elements and set B has n elements, then number of subsets of $A\times B$ or $A\times B=2^{mn}$.

- The Cartesian product $R \times R = \{(x, y) : x, y \in R\}$ represents the coordinates of all points in the two dimensional space and the Cartesian product $R \times R \times R = \{(x, y, z) : x, y, z \in R\}$ represents the coordinates of all points in the three dimensional space.
- If a set A has n elements, then $n(A \times A) = n^2$ elements.
- If a set A has n elements, then $n(A \times A \times A) = n^3$ elements.

RELATIONS

Relation means an association of two objects according to some property possessed by them.

E.g.:

- Trivandrum is the capital of Kerala,
- Sita is the wife of Rama,
- 12 is greater than 10,
- {a} is the subset of {a,b}, etc..

Relation R from A to B

A relation R in a set A to a set B is the subset of A × B. If (x,y) is a member of a relation R, then we write xRy and read x is the relation R to y.

Domain of R from A to B: The set of all first elements of the ordered pairs in R from A to B is known as domain of R.

Range of R from A to B: The set of all second elements of the ordered pairs in R from A to B is known as range of R.

Co-domain of R: Set B is known as co-domain.

Consider a relation, $R = \{(x, y) : y = x + 1, x \in A \text{ and } y \in B\}$, where A={0,1,2} and B= {1,2,3,4}. Then

$R = \{(0,1), (1,2), (2,4)\}$.

Domain of R ={0,1,2}

Range of R ={1,2,4}

Co-domain of R = set B ={1,2,3,4}

Note: $Range \subseteq Co-domain$

Relation R from B to A

A relation R in a set B to a set A is the subset of B × A. If (x,y) is a member of a relation R, then we write xRy and read x is the relation R to y.

Domain of R from B to A: The set of all first elements of the ordered pairs in R from B to A is known as domain of R.

Range of R from B to A: The set of all second elements of the ordered pairs in R from B to A is known as range of R.

Co-domain of R: Set A is known as co-domain.

Consider a relation, $R=\left\{(x,y): y=x^2+1, x\in B \text{ and } y\in A\right\}$, where A={1,2,3,5,10} and B= {0,1,2,3}. Then

$R=\left\{(0,1),(1,2),(2,5),(3,10)\right\}$.

Domain of R ={0,1,2,3}

Range of R ={1,2,5,10}

Co-domain of R = set A ={1,2,3,5,10}

Representation of a relation:

A relation can be expressed in:

a) Roster Method,
b) Set-builder Method,
c) Arrow diagram and
d) Graphical method.

E.g.: Let A={1,2,3,4}; B={2,3,4}

R is a relation from A to B such that $y=x+2, x\in A \text{ and } y\in B$.

Roster Method

$R=\{(1,3),(2,4)\}$

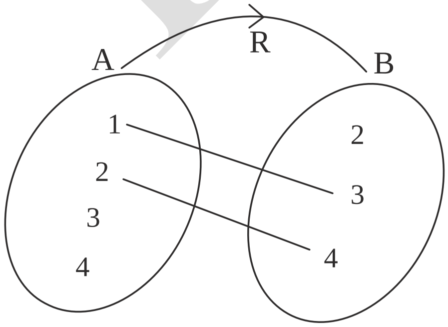
Domain ={1,2}

Range = {3,4}

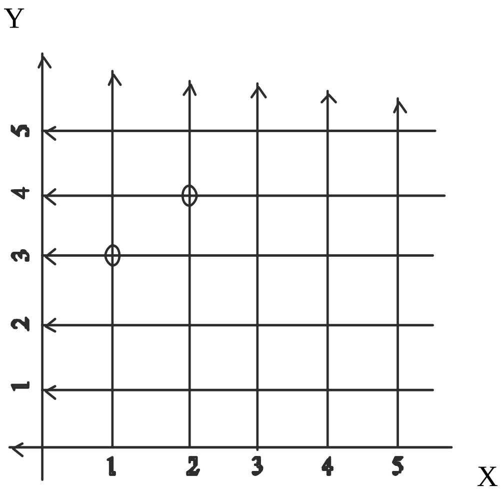
Set-builder Method

$R=\{(x,y): y=x+2, x\in A \text{ and } y\in B\}$

Arrow diagram:



Graphical Method



Note: If a set A has m elements and B has n elements, then

No. of relations from A to B $= 2^{mn}$

No. of relations from B to A $= 2^{nm} = 2^{mn}$

FUNCTIONS

Let A and B be any two non-empty sets. A relation from A to B is said to be a function if and only if,

i) if every x element has y element,

ii) the x element cannot be repeated.

or

i) If every x in A has image in B,

ii) And no element in A has not more than one image in B

E.g.: Let $A = \{0,1,2,3,4\}$; $B = \{1,23,5,7,9\}$

Let $R = \{(x, y) : y = 2x+1, x \in A, y \in B\}$

Note: If a set A has 'm' elements and set B has 'n' elements, then,

i. No. of functions from A to B $= n(B)^{n(A)} = n^m$

ii. No. of functions from B to A $= n(A)^{n(B)} = m^n$

E.g.: If set A has 2 elements and set B has 3 elements, then number of functions from:

i. A to B $= 3^2 = 9$

ii. B to A $= 2^3 = 8$

Domain, Range and co-domain of a function:

If $f : A \to B$ is a function from A to B, then

i) Domain of *f* = set A

ii) Range of *f* = set of all images of elements of A is known as range.

iii) Codomain of f = set B

Similarly, If $f : B \to A$ is a function from B to A, then

iv) Domain of *f* = set B

v) Range of *f* = set of all images of elements of B is known as range.

vi) Codomain of f = set A

Note: Thus $range \subseteq co-domain$.

Equal functions: If two functions *f* and *g* are said to be equal, then,

i. domain of *f* = domain of *g*

ii. codomain of *f* = codomain of *g*

Note: The terms map or mapping are also used to denote function.

If *f* is a function from A to B, we denote f: A $\to$ B or $A \xrightarrow{f} B$. If *f* is a function from A to B and (a, b) $\in$ f, then f(a) = b, where 'b' is called the image of 'a' under *f* and 'a' is called the pre-image of 'b' under *f*.

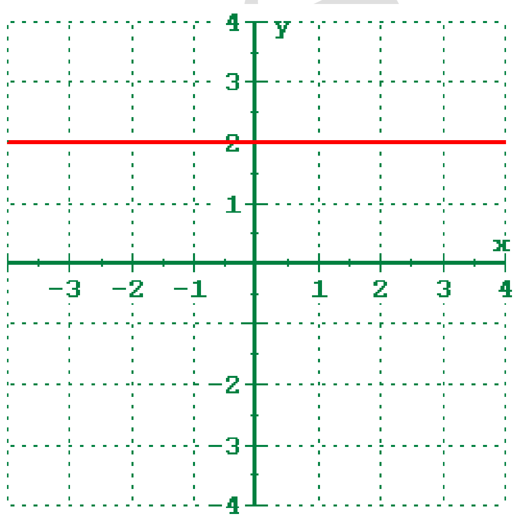
Types of functions:

Real function: A function $f : R \to R$ is said to be a real function, if its domain is a real constant.

Constant function: A function $f : R \to R$ is said to be a constant function if $f(x) = c$, where 'c' is a constant.

Domain: R, Range : c (a constant)

Graph:

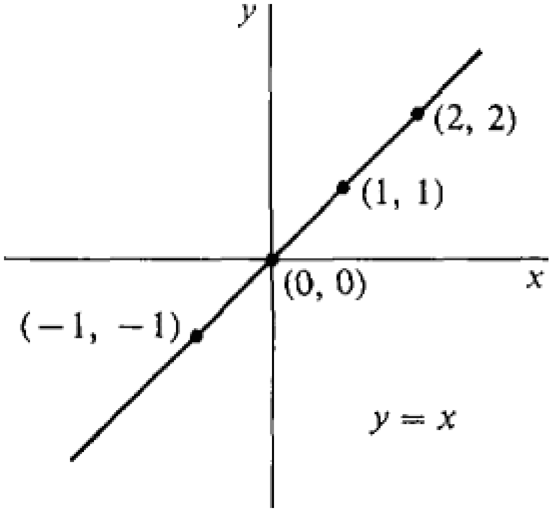


Identity function: A function $f : R \to R$ is said to be an identity function if $f(x) = x$.

Domain: R

Range : R

Graph:

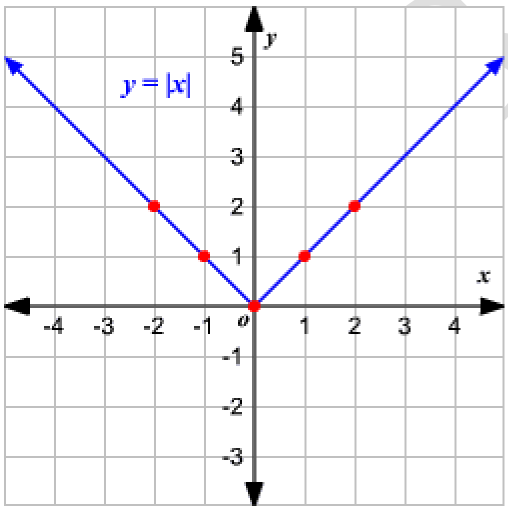


Modulus function: A function $f : R \to R$ is said to be a modulus function, if $f(x) = |x| = \begin{cases} x, & when\, x \geq 0 \\ -x, & when\, x < 0 \end{cases}$.

Domain: R

Range : R^+ (Positive real numbers)

Graph:



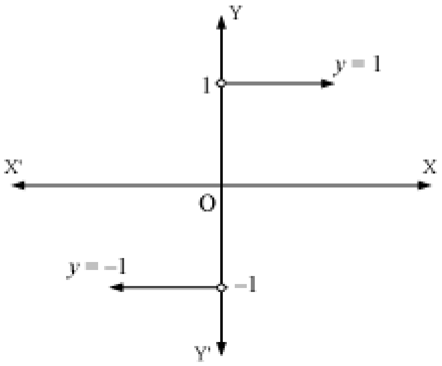
Signum Function: A function $f : R \to R$ is said to be a signum function,if $f(x) = \begin{cases} -1, & if\ x < 0 \\ 0, & if\ x = 0 \\ 1, & if\ x > 0 \end{cases}$

or $f(x)=\frac{|x|}{x}, x \neq 0$ *and* 0 *for* $x=0$ is known as signum function.

Domain: R

Range : $\{-1,0,1\}$, if $x<0, x=0\, and\, x>0$

Graph

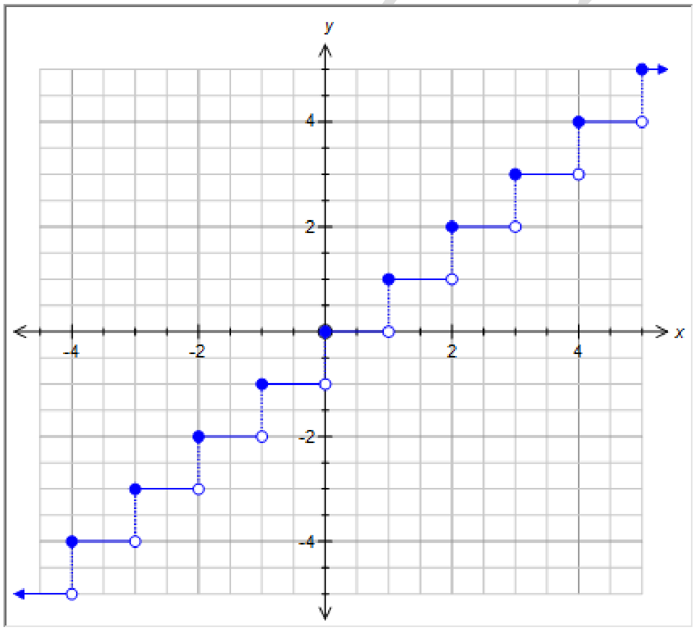


$f(x)=\frac{|x|}{x}$, $x \neq 0$ and 0 for $x = 0$

Greatest Integer Function: A function $f: R \to R$ is said to be a greatest integer function, if $f(x)=[x], x \in R$.

Domain : R
Range : Integer.

Graph



Note: The above graph is also known as step graph.

Note:

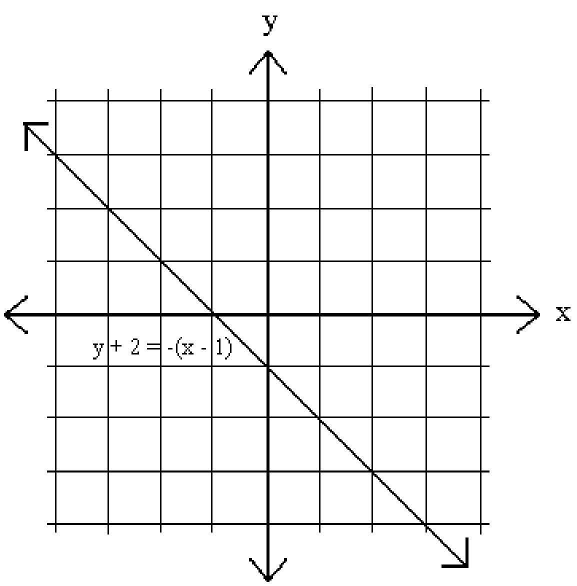
[1]	$0 \le x < 1 = 0$
[2]	$1 \le x < 2 = 1$
[0]	$-1 \le x < 0 = -1$
[1.3]	$1 \le x < 1.3 = 1$
[2.999]	$2 \le x < 2.999 = 2$
[-2.3]	$-3 \le x < -2.3 = -3$

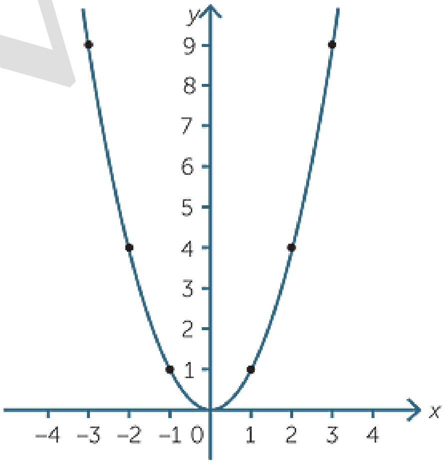
Polynomial Functions: A function $f: R \to R$ is said to be a greatest integer function, if $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_3x^3 + a_2x^2 + a_1x + a_0$.

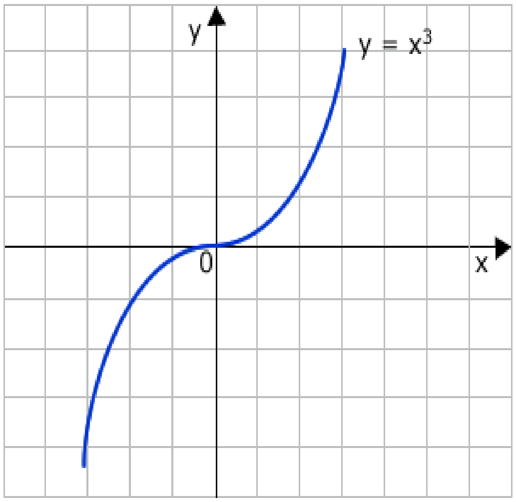
Domain : R
Range : R

E.g.: $f(x) = x^3 - 2x + 5$; $g(x) = 2x^2 + 3x - 1$, etc..

Graphs of polynomial functions:







Rational Function: A function $f:R\to R$ is said to be a greatest integer function, if $f(x)=\frac{ax+b}{cx+d}, x\neq -\frac{d}{c}$.

E.g.: $f(x)=\frac{2x+1}{x-2}, x\neq 2$; $g(x)=\frac{x-5}{x+1}, x\neq -1$, etc..

1. Find the domain of the rational function $f(x)=\frac{2x-3}{1-x}$:

   $f(x)$ is defined, if $1-x=0\Rightarrow x=1$.

2. Find the domain of the rational function $f(x)=\frac{x^2-3x+5}{x^2-5x+6}$:

   $f(x)$ is defined, if $x^2-5x+6=0\Rightarrow (x-3)(x-2)=0 \Rightarrow x=3 \; or \; x=2$

   $\therefore \; domain = R-\{2,3\}$

3. Find the domain and range of the function: $f(x)=\sqrt{4-x^2}$

Let $f(x)=\sqrt{4-x^2}$

i.e, $y=\sqrt{4-x^2}$ …………….. (1)

In order to find the domain, let $4-x^2\geq 0$

$4\geq x^2 \Rightarrow x^2\leq 4 \Rightarrow x\leq \pm 2$
$\Rightarrow x\geq -2 \; and \; x\leq 2$
$\therefore$ domain of f is $[-2,2]$ or $-2\leq x\leq 2$
From (1), $y\geq 0$ ....(2)

To find the range:

Let $y=\sqrt{4-x^2}$
$y^2=4-x^2\Rightarrow x^2=4-y^2$
$x=\sqrt{4-y^2}$
In order to define x, let $4-y^2\geq 0$
$4\geq y^2 \Rightarrow y^2\leq 4\Rightarrow y\leq \pm 2$
$\Rightarrow y\geq -2 \; and \; y\leq 2$ ....(3)
From (2) and (3), we have
Range of f is $[0,2]$ or $0\leq x\leq 2$

Algebra of functions:

Let $f(x)$ and $g(x)$ be any two functions of x, then

1. $f+g=f(x)+g(x)$
2. $f-g=f(x)-g(x)$
3. $f.g=f(x)\times g(x)$
4. $\frac{f}{g}=\frac{f(x)}{g(x)},\ provided\ g(x)\neq 0$

E.g.: If $f(x)=x^2$ and $g(x)=2x+1$, then

$$f+g=f(x)+g(x)=x^2+2x+1=(x+1)^2$$

$$f-g=f(x)-g(x)=x^2-(2x+1)=x^2-2x-1$$

$$f.g=f(x)\times g(x)=x^2(2x+1)=2x^3+x^2$$

$$\frac{f}{g}=\frac{f(x)}{g(x)}=\frac{x^2}{2x+1},\ x\neq -\frac{1}{2}$$

Objective Questions (Try yourself)

1. If n(A) = 6 and n(B) = 5, then the number of relations on $A\times B$ is

   a) 2^{49} b) 2^{35} c) 2^{25} d) 2^{70} e) $2^{35\times 35}$

2. Suppose the number of element in set A is p, number of elements B is q and the number of elements in $A\times B$ is 7 then p^2+q^2?

   a) 42 b) 49 c) 50 d) 51 e) 55

3. n(A) =18 , n(B) = 15 and $n(A\cap B)$=5 then $n[(A\times B)\cap(B\times A)]$ is

   a) 28 b) 38 c) 35 d) 10 e) 25

4. Let A be the set of first 10 natural numbers and let R be a relation on A defined by $(x,y)\in R\Leftrightarrow x+2y=10\ then\ R^{-1}$.

   a) $\{(2,4),(4,3),(6,2),(8,1)\}$ b) $\{(2,4),(4,3),(2,6),(1,8)\}$

   c) $\{(4,2),(3,4),(2,6),(1,8)\}$ d) $\{(4,8),(4,1),(2,6)\}$ e) None of these.

5. If $R=\{(x,x^3): x\ is\ a\ prime\ number<10\}$, then Range(R) =

   a) $\{125,27,8,341\}$ b) $\{27,353,125,7\}$

   c) $\{18,127,125,343\}$ d) $\{343,125,8,27\}$

6. If a set A has 3 elements and set B has 2 elements, then number of relations from B to A is

a) 32 b) 16 c) 64 d) 32 e) None

7. If $R=\{(1,1),(2,3),(3,5),(4,7)\}$ is a function and this is described by the formula that $g(x)=\alpha x+\beta$, then the value of α *and* β is

a) $\alpha=2;\beta=1$ b) $\alpha=2;\beta=-1$ c) $\alpha=3;\beta=1$ d) $\alpha=2;\beta=-1$ e) $\alpha=-2;\beta=-1$

8. If a set A has 3 elements and B has 2 elements, then the number of functions from B to A is ……..

a) 6 b) 9 c) 8 d) 4 e) None of these

9. If $f(x)=|x|+[x]$ then $f\left(-\frac{3}{2}\right)+f\left(\frac{3}{2}\right)$ is

a) 1 b) 2 c) $\frac{1}{2}$ d) $\frac{3}{2}$ e) $\frac{5}{2}$

10. The domain of the function $f(x)=\frac{1}{\sqrt{4-x^2}}$ is

a) $-2\le x\le 2$ b) $-2<x<2$ c) $-4\le x\le 4$ d) $-4<x<4$ e) $-\infty\le x\le\infty$

11. If $f(x)=\log\left(\frac{1-x}{1+x}\right)$, then the value of $f(a)+f(b)$ is

a) $\log\left(\frac{a-b}{1+ab}\right)$ b) $\log\left(\frac{a+b}{1-ab}\right)$ c) $\log\left(\frac{a+b}{1+ab}\right)$ d) $\log\left(\frac{a-b}{1-ab}\right)$ e) None of these

12. The domain of the function $|x|+|x-1|+|x-2|$ is

a) $R-\{1\}$ b) $R-\{2\}$ c) $R-\{1,2\}$ d) R e) None of these

13. The range of the function $f(x)=\sin x$ is

a) $\{-1\le y\le 1\}$ b) $\{-1\le x\le 1\}$ c) $-1<y<1$ d) $-1<x<1$ e) None of these

14. Let $f\left(x+\frac{1}{x}\right)=x^2+\frac{1}{x^2},(x\ne 0)$, then $f(x)$ equals

a) x^2 b) x^2-1 c) x^2+1 d) x^2-2 e) None of these

15. The domain of the function $f(x)=\sqrt{x-\sqrt{1-x^2}}$ is

a) $\left[-1,-\frac{1}{\sqrt{2}}\right]\cup\left[\frac{1}{\sqrt{2}},1\right]$ b) $[-1,1]$ c) $\left[-\infty,-\frac{1}{2}\right]\cup\left[\frac{1}{\sqrt{2}},\infty\right]$

d) $\left[\frac{1}{\sqrt{2}},1\right]$ e) None of these

16. The domain of $\sqrt{x-1}+\sqrt{8-x}$

a) [1,8) b) (–8,8) c) [1,8] d) (1,8) e) None of these

17. The range of the function $y=\frac{x+2}{x^2-8x+4}$ is

a) $\left(-\infty,-\frac{1}{4}\right)$ b) $R-\left\{-\frac{1}{4},-\frac{1}{20}\right\}$ c) $\left[-\frac{1}{20},\infty\right)$ e) None of these

18. If $f: R \to R$ be defined by $f(x) = 5x - 2$, then $f^{-1}(x)$ is

a) $\dfrac{x+2}{5}$ b) $\dfrac{x-2}{5}$ c) $\dfrac{x}{5}-2$ d) $\dfrac{x}{5}+2$ e) None of these

19. The graph of the function $y = ax + b$, where a and b are constants is a

a) straight line b) parabola c) circle d) hyperbola e) None of these

20. Let $f\left(x+\dfrac{1}{x}\right) = x^2 + \dfrac{1}{x^2}, x \neq -1$, then $f(x) =$

a) x^2 b) $x^2 - 1$ c) $x^2 - 2$ d) $x^2 + 2$

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